Flat Plates under Compression, an Extension of Column Buckling Theory: Analytical and Numerical Comparison

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Abstract: - Flat plates under compression primarily fail by buckling, an instability where the panel suddenly deflects laterally while material stresses may still be below yield. This buckling phenomenon is widely utilised in designing aircraft structures, ship hulls, and bridge girders, where thin-walled structures are subjected to high compressive loading conditions. In this work, the main objective is to present an analytical and numerical method that connects column theory to the plate buckling response. Additionally, different parametric studies are conducted to compare results and advocate the use of the finite element method as an alternative to other approaches.

Key-Words: - Flat Plate, Buckling, Finite Element Method, Euler Load, Slenderness, Column.

Received: October 4, 2025. Revised: April 14, 2026. Accepted: May 11, 2026. Published: July 7, 2026.

1 Introduction

For thin plates, elastic or elasto-plastic buckling governs capacity, strongly influenced by slenderness ratio, aspect ratio, boundary conditions, and loading type (uniaxial, biaxial, shear). Increasing slenderness reduces critical buckling stress, with the reduction more pronounced at high slenderness, where behaviour becomes purely elastic, [1]. Nevertheless, biaxial compression is generally the most unfavourable loading condition compared to uniaxial cases, [1].

The theory of flat plates under compression generalises Euler column buckling by replacing the beam element with a two-dimensional plate whose stability is governed by classical plate theory, [2]. In this approach, the flexural rigidity of the plate, aspect ratio, and support conditions enter the buckling equation through a nondimensional coefficient, effectively extending column formulas to plates.

While columns buckle in a single, global mode, plates can buckle locally in complex, multi-wave patterns, and they often possess significant post-buckling strength, unlike most columns. The effective width concept is used to design these components beyond their initial buckling load.

Advanced finite element and semi‑analytical methods are now standard tools to assess elastic, elasto‑plastic, and post-buckling responses of flat plates under compression, [1]. Numerical methods, especially finite elements, can provide detailed mode shapes, sensitivity to imperfections, enabling efficient yet safe design of thin-walled structures such as aircraft skins, bridge decks, and ship hull panels.

Recent work [3] proposes a design formula that uses the shape of the elastic plate buckling equation and a correction factor to directly predict ultimate compressive strength, simplifying many earlier empirical curves and explicitly tying ultimate strength back to elastic buckling, analogous to column curves. Other different works are developed to be used as alternative numerical and experimental methodologies that help safe design, [4], [5], [6].

The main objective of this work is to use the analytical plate theory, a numerical method, to connect "column-like" and "plate‑like" responses, and to compare the theory with the numerical model, as proposed to use the finite element method using shell elements.

2 Analytical Study of Flat Plates

The following analytical methodology is applied to the columns in compression [6] and will be extended to a flat plate.

According to the literature, short columns fail with a critical buckling stress that can be larger than the material yield. In this zone, the material no longer performs by elasticity [6], [7], [8], and the buckling of this region is called inelastic, [7], [8].

The long columns will tend to fail with the application of lower loads by elastic buckling, reaching the critical buckling stress [7], [8], and it is called elastic buckling [8], [9] or Euler critical load N_{cr}.

This critical load N_{cr} gives the value of the slenderness of a member in compression [6], [9] according to Equation (1).

$$N_{cr} = \frac{\pi^2}{L_e^2} E\, I \tag{1}$$

where:
E is the modulus of elasticity, and I is the minimum moment of inertia. For columns with pinned-fixed ends, the effective length L_e is equal to 70% of the column length L [6], [7]. For flat plates, the same boundary conditions will be applied.

The non-dimensional slenderness $\bar{\lambda}$ is obtained according to Equation (2), a function of the cross-section A, the yield stress f_y and the critical load N_{cr}.

$$\bar{\lambda} = \sqrt{\frac{A\, f_y}{N_{cr}}} \tag{2}$$

This equation represents a fundamental parameter that indicates the flat plate prone to buckling, defining the ratio of its actual slenderness to the theoretical limit where elastic buckling equals yield strength.

The studied flat plates are in steel material S235 [10], with a modulus of elasticity of 210 GPa and a yield stress of 235 MPa.

Table 1 represents the dimensions and the geometric characteristics of the studied plates.

All geometric parameters allow the calculation of the mass geometry (A and I) needed to be used in previously equations: L is the plate length, W is the width, and t is the plate thickness.

Table 1. Characteristics of the steel flat plates

L, W, mm	t, mm	A, mm^2	I, mm^4
L=250 W=125	1.5	2.38	4.65
	1.0	0.70	3.55
	0.8	0.36	2.51
L=500 W=250	1.5	1.19	4.23
	1.0	0.35	4.97
	0.8	0.18	5.28
L=1000 W=500	1.5	0.59	4.23
	1.0	0.18	2.13
	0.8	0.09	0.26

Source: created by the author

3 Numerical Study of Flat Plates

A numerical program was used to simulate the critical load in flat plates. Ansys® program based on the finite element method was chosen. For this type of simulation, a SHELL281 shell element with 8 nodes and 6 degrees of freedom per node was selected. This element is suitable for analysing thin or moderately thick shell elements, for linear, high rotation, and/or large nonlinear deformation applications [6]. A mesh size with a length of 10 mm was imposed in all numerical models, after a convergence criterion.

The numerical procedure with the Ansys ® program employs a linear eigenvalue buckling analysis to extract the elastic buckling mode shapes of each flat plate. These eigenmodes are then normalized and scaled to a prescribed imperfection amplitude, providing representative local buckle shapes that are introduced into the plate geometry for subsequent nonlinear or parametric analyses for the future work of this research. Pre-stress effects reinforce the computation of the stress stiffness matrix. This is completed by first completing a structural analysis on an ideally loaded structure and then evaluating the stress field to continue a modal analysis that results in the buckling mode. The eigenvalue solver applies a unit load on the column to establish the needed buckling load, [6].

The procedure for the eigenbuckling analysis is solved according to Equation (3) as a homogeneous system, [11]. It solves an eigenvalue problem:

$$(K + \lambda\, K_\sigma)\, \phi = 0 \tag{3}$$

where:
K is the elastic stiffness matrix of the structure, which represents the usual linear relationship between nodal displacements and internal forces for the undeformed/base configuration, assuming linear elastic material behavior;

K_σ is the geometric (or initial‐ stress) stiffness matrix, with the main purpose to capture how existing stresses in the structure (due to a reference load pattern) change the stiffness and stability of the system. This matrix depends on the stress state in the base configuration and represents the destabilising effect of compressive stresses;

λ is the eigenvalue (buckling load factor), a scalar multiplier to be used with the reference load, which must be scaled to reach an elastic critical buckling state in the associated mode;

ϕ is the eigenvector (buckling mode shape) of nodal displacements/rotations that describe the shape the structure tends to adopt at buckling in that mode. It is usually normalised, and it gives the shape of the buckle, not its physical amplitude.

For plates, the first few eigenmodes often represent local plate buckling patterns (one, two, or more half-waves across the panel). The first mode also represents the critical mode to be obtained. In practice, it is needed to:

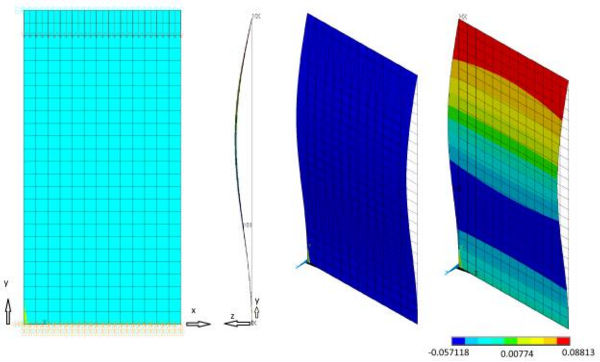
-Run an eigenbuckling analysis of the flat plate;

-Extract the relevant local buckling mode shape;

-Multiply this mode by a chosen amplitude to obtain a scaled imperfection field.

-For a Geometrically and Materially Nonlinear Analysis with Imperfections (GMNIA), superimpose the scaled shape on the perfect geometry and then use it in the GMNIA analysis, [11].

According to the numerical procedure, Figure 1 represents the model of a plate (L=250, W=125, t=1.5) mm with the applied boundary conditions pinned-fixed ends and the respective results.



a) b) c) d)

Fig. 1: Numerical flat plate: a) mesh, b) buckling mode, c) deformed plate and d) vertical displacement in mm

Source: created by the author

In Figure 1, it is possible to identify the mesh type, the first buckling mode, which corresponds to the critical Euler load 2.49 kN, the deformed shape and the maximum vertical displacement. The other numerical results in all flat plates represent the same behaviour, but with different critical load values.

4 Results and Discussion

In Table 2, all analytical and numerical values are represented. A relative error is also calculated in relation to the analytical method.

Table 2. Analytical and numerical critical load in compressed flat plates with pinned-fixed ends

L, W, mm	t, mm	N_{cr_An}, kN	N_{cr_Nu}, kN	Error, %
L=250 W=125	1.5	2.38	2.49	4.65
	1.0	0.70	0.73	3.55
	0.8	0.36	0.37	2.51
L=500 W=250	1.5	1.19	1.24	4.23
	1.0	0.35	0.37	4.97
	0.8	0.18	0.19	5.28
L=1000 W=500	1.5	0.59	0.62	4.23
	1.0	0.18	0.18	2.13
	0.8	0.09	0.09	0.26

Source: created by the author

It is noticeable that the numerical values agree with the analytical calculation. The relative error is lower and decreases with thin flat plates.

Figure 2 shows the obtained curves of the critical load vs the non-dimensional slenderness in all studied flat plates.

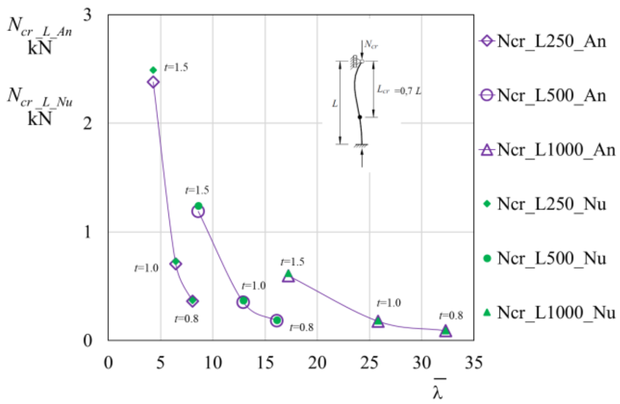


Fig. 2: Results of the critical load vs. non-dimensional slenderness

Source: created by the author

The curves represent a decrease in critical load when the slenderness increases. Furthermore, for different lengths, longer plates have a lower capacity to reach the maximum critical load. The same behavior if the thickness of the plate decreases.

5 Conclusion

Flat plate buckling under compression is a critical structural phenomenon that serves as a fundamental extension of Euler's classical theory of column buckling.

This paper presents a simplified study for flat plates under compression, treated as a two-dimensional generalization of columns, enriched with plate buckling modes. This work combines analytical plate theory with parametric finite element studies to construct simple design rules that effectively extend the classical theory of column buckling to real plate elements.

As future work, the numerical model will incorporate geometric imperfections and material nonlinearity, leading to unified treatments of the plates, with the calibration of design reduction factors in stability codes.

Acknowledgement:
The author acknowledges Fundação para a Ciência e a Tecnologia (FCT) for its financial support to LAETA via the project UID/50022/2025 (DOI: https://doi.org/10.54499/UID/50022/2025).

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- The author contributed with the following: Investigation; Methodology; Formal analysis; and Writing - review and editing.

Sources of Funding for Research Presented in a Scientific Article or the Scientific Article Itself
No funding was received for conducting this study.

Conflict of Interest
No conflicts of interest to declare that are relevant to the content of this article.