Compelling Heat Transfer Augmentation of Laminar Natural Convection from Vertical Plates to Binary Gas Mixtures Formed with Light Helium and Selected Heavier Gases

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Abstract: - The present study addresses a remarkable behavior of certain binary gas mixtures in connection to laminar natural convection along a heated vertical plate at constant temperature. The binary gas mixtures are formed with light helium (He) as the primary gas and selected heavier secondary gases. The heavier secondary gases are nitrogen (N_2), oxygen (O_2), xenon (Xe), carbon dioxide (CO_2), methane (CH_4), tetrafluoromethane (CF_4) and sulfur hexafluoride (SF_6). The central objective in the study is to investigate the attributes of the set of seven He–based binary gas mixtures for heat transfer enhancement with respect to the heat transfer with helium (He) and air. From heat convection theory, four thermo–physical properties: viscosity η, density ρ, thermal conductivity λ, and isobaric heat capacity C_p affect the thermo–buoyant convection of fluids. In this study it became necessary to construct a particular correlation equation consigned to the set of seven He–based binary gas mixtures, which operate in the Prandtl number closed sub–interval [0.1, 1]. The heat transfer rate Q_{mix} with the set of seven He–based binary gas mixtures from the vertical plate involves four thermo–physical properties: density ρ_{mix}, viscosity η_{mix}, thermal conductivity λ_{mix}, and isobaric heat capacity $C_{p,\,mix}$ that vary with the molar gas composition w. A case study is performed to elucidate the unique characteristics of the modified convective heat transport that the set of seven He–based binary gas mixtures brings forward as compared to the convective heat transport of He and air. It was found that the He+SF_6 binary gas mixture renders an absolute maximum heat transfer enhancement rate that is 39 times higher than the heat transfer rate provided by He and 78 times higher than air.

Key–Words: Natural convection; vertical plate; laminar boundary layer flow; light helium is the primary gas; selected heavier gases are the secondary gases; binary gas mixtures; heat transfer enhancement.

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1 Introduction

The art and science of heat transfer enhancement has evolved into an important component of thermal science over the last decades. State–of–the–art review articles on heat transfer enhancement authored by Bergles [1] and Manglik [2] have appeared regularly in specialized journals and handbooks on heat transfer. According to Bergles [1], the heat transfer enhancement schemes can be classified either as passive, which require no direct external power, or active, which do require external power.

The effectiveness of the two schemes is strongly dependent on the heat transfer mode, which may range from single–phase natural convection to two–phase boiling and two–phase condensation passing through single–phase forced convection. A great amount of effort has been devoted to search for heat transfer enhancement in forced convection flows during the past decades, but unfortunately the effort associated with heat transfer enhancement in natural convection flows has remained practically unnoticed in the specialized literature.

With regards to passive schemes in solid media, Bhavnani and Bergles, [3], measured the average heat transfer coefficients for vertical plates owning an incrusted bundle of transverse ribs exposed to air. The authors determined that the heat transfer enhancement ratio relative to a similar plain vertical plate of equal projected area was possible using transverse ribs of certain size. Thereby, the maximum heat transfer enhancement ratio reached 23%.

With regards to passive schemes in fluid media, a distinction must be made between liquids and gases.

First in the case of liquids, Kitagawa et al. [4] carried out an experimental study to investigate the effects of sub–millimeter air bubble injection on the heat transfer characteristics of laminar natural convection of water along a heated vertical plate. The simultaneous measurements of velocity and temperature demonstrated that the heat transfer enhancement ratio is directly affected by the flow modification due to the rising air bubbles near the heated vertical plate. The ratios of the convection heat transfer coefficient with injection relative to the convection heat transfer coefficient without injection, intensifies with increments in the air bubble flow rate. This phenomenon translated into modest heat transfer enhancement ratios that ranged from 1.35 to 1.85. Natural convective boundary layer flows of a nano fluid past a vertical plate was analyzed by Kuznetsov and Nield, [5], using a standard similarity methodology of the conservation equations followed by numerical computations. The outcome of the analysis included velocity, temperature and solid volume fraction of the nano fluid in their respective boundary layers. An enlargement in the heat transfer performance of the nano fluid with respect to the case of the base fluid was found for most cases treated.

Second, in the case of gases, Petri and Bergman, [6] conducted a dual numerical and experimental investigation on natural convection heat transfer next to a vertical plate using a binary gas mixture in laminar regime. The authors found that a helium–rich binary gas mixture seeded with a small amount of xenon yielded higher heat transfer rates relative to those situations connected to pure helium. In sum, the authors concluded that the convective heat transfer rates are increased by a factor of 8% induced by the seeding of light helium with heavy xenon. Arpaci, [7], investigated theoretically the coupling of natural convection and thermal radiation from a vertical plate to a stagnant gray gas The study employed the integral method based on the two limiting approximations, i.e., thin and thick gases.

The objective of the present work is to investigate the behavior of several binary gas mixtures composed by light helium as the primary gas and heavier secondary gases pertinent to thermal–driven boundary layers adjacent to a heated vertical plate at constant temperature T_W. The central idea is to explore the interaction between the density ρ_{mix}, viscosity η_{mix}, thermal conductivity λ_{mix} and isobaric heat capacity $C_{p,mix}$ of the binary gas mixtures composed with light He and heavier gases. The heavier gases are: nitrogen (N_2), oxygen (O_2) xenon (Xe), carbon dioxide (CO_2), methane (CH_4), sulfurhexafluoride (SF_6), tetrafluoro–
methane and carbon tetrafluoride (CF_4).

2 Heat Transport by the Natural Convection Mechanism

Heat convection theory (Jaluria [8] stipulates the following relation

Natural convection mechanism =
heat conduction mechanism +
fluid motion due to thermo–
buoyant forces

Within the heat conduction mechanism, the heat transfer enhancement is simple because the immobile air can be replaced with an immobile gas having higher thermal conductivity λ, like for instance light helium (He). Contrarily, the heat transfer enhancement promoted by the natural heat convection mechanism is more complicated because the acting thermo–buoyancy forces need to be stimulated as a whole. Focusing on this aspect, a viable option strongly insinuates the exploration of better gases. For instance, binary gas mixtures formed with light helium (He) as the primary gas and heavier nitrogen (N_2), oxygen (O_2) xenon (Xe), carbon dioxide (CO_2), methane (CH_4), sulfur hexafluoride (SF_6), and tetrafluoromethane or carbon tetrafluoride (CF_4) as the secondary gases.

The heat transfer rate Q from a heated vertical plate to a surrounding cold fluid is given by Newton's "equation of cooling" (Arpaci [9]):

$$Q = \bar{h} A_S (T_w - T_\infty) \qquad (1)$$

where $\bar{h}$ is the average convective coefficient, A_s is the surface area and $(T_w - T_\infty)$ is the plate–to–fluid temperature difference.

The estimation of the average convective coefficient $\bar{h}$ for a laminar boundary layer in Eq. (2) depends on the temperature gradient of the fluid in contact with the vertical plate, namely –T' (0) (Jaluria [8]). For the thermal boundary layer, the similarity variables θ and η are defined by the ratios

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \eta = \frac{x}{H} \qquad (2)$$

where H is the height of the vertical plate.

3 Correlation Equations for the Prandtl Number

For laminar boundary layer up–flows near a heated vertical plate, Ostrach [10] calculated numerically the velocity and temperature fields of a multitude of fluids with Prandtl numbers ranging from Pr = 0.01 to 1,000. Subsequently, the author developed the so–called Prandtl number function $-\theta'(0) = g\ (Pr)$. This finding led to the comprehensive correlation equation for the average Nusselt number $\overline{Nu}_H$:

$$\overline{Nu}_H = Ra_H^{1/4} \times g(Pr) \qquad (3)$$

which complies with the Boussinesq approximation (Jaluria, [8]). Herein, the thermo–physical properties of the fluid are evaluated at the film temperature $T_f = \frac{T_w + T_\infty}{2}$. The coefficient of volumetric thermal expansion for ideal gases is $\beta = \frac{1}{T_\infty}$, where T_∞ is in absolute degrees. In contrast, the coefficient of volumetric thermal expansion must be obtained experimentally for non ideal gases and liquids (Jaluria, [8]).

As a point of reference, Bejan and Lage [11] have demonstrated that the Grashof number Gr_H (and not the Rayleigh number Ra_H) marks the transition from laminar–to–turbulent natural convection flows adjacent to vertical plates. The critical Grashof number recommended stays around $Gr_{H,cr} = 10^9$.

Focusing on the Prandtl number spectrum 0 < Pr < ∞, there are two extreme limits: one limit deals with Pr → 0 for metallic liquids and the other limit deals with Pr → ∞ for oils and thick liquids. Within this vscenario, LeFevre, [12], discovered that the average Nusselt number $\overline{Nu}_H$ adheres to the following two asymptotes

$$\overline{Nu}_H = 0.8\,(Ra_H\,Pr)^{1/4} \qquad \text{as } Pr \to 0 \qquad (4a)$$

$$\overline{Nu}_H = 0.671\,Ra_H^{1/4} \qquad \text{as } Pr \to \infty \qquad (4b)$$

The specialized heat convection literature contains comprehensive correlation equations for the average Nusselt number $\overline{Nu}_H = f\,(Ra_H, Pr)$ covering the entire Pr spectrum 0 < Pr < ∞. For instance, in the laminar regime restricted to $Gr_H < 10^9$, Churchill and Chu, [13], developed the correlation equation

$$\overline{\mathrm{Nu}}_H = 0.68 + \frac{0.67\,\mathrm{Ra}_H^{1/4}}{f(\mathrm{Pr})} \qquad (4c)$$

with the companion Prandtl number function

$$f(Pr) = \left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9} \qquad (4d)$$

According to Martynenko and Polijakov [14], the pair of Eqs. (4c) and (4d) can also be used for liquid metals (Pr < 0.1) when Ra_H is substituted by $Gr_H\, Pr^2$.

It is convenient to scrutinize the individual dependence of fluids on the Prandtl number as it relates to the Prandtl number function g (Pr) in the double–valued function of Eq. (3) from Reference, [11]. More specifically, four sub–groups can be categorized with regards to Prandtl number fluids:

(1) metallic liquids with Pr << 1;
(2) air, pure gases and vapors with Pr ≈ 1;
(3) water and light liquids with Pr around 10
(4) oils and thick liquids with Pr >> 1.

First and foremost, a specific correlation equation for the binary gas mixtures owing $\mathrm{Pr} \in (0.1, 1)$ has to be constructed from the original data in the broad Prandtl number spectrum spanning from 0.001 (liquid metals) to 1000 (oils and heavy liquids reported by Ostrach [10]. In this sense, the Pr data taken from [10] was re–tabulated later by Suryanarayana, [15]. The Pr data is analyzed with regression analysis using the software TableCurve [16]. The outcome of the regression analysis delivers the following four–part correlation equations:

$$g(\mathrm{Pr}) = \begin{cases} 0.671\,\mathrm{Pr}^{1/4}, & \mathrm{Pr} \to 0 \\ 0.549\,\mathrm{Pr}^{0.171}, & \mathrm{Pr} \in [0.01, 1] \\ 0.549\,\mathrm{Pr}^{0.044}, & \mathrm{Pr} \in [1, 100] \\ 0.8\,\mathrm{Pr}^{0}, & \mathrm{Pr} \to \infty \end{cases} \qquad (5a,b,c,d)$$

All correlation coefficients R–square are high ~ 0.999, while the largest error of 3.58% occurs around Pr = 0.1.

In the definitive Pr classification exposed in Eqs. (5), it can be seen in more detail that Eq. (5a) handles liquid metals, Eq. (5b) handles air, regular gases, and vapors, Eq. (5c) handles water and light liquids and Eq. (5d) handles oils and thick liquids.

If a new sub–group for binary gas mixtures denoted Pr_{mix} is added to the entire Pr spectrum ranging from 0.001 (liquid metals) to 1000 (oils and heavy liquids, then a fifth sub–group is squeezed inside the Pr sub–interval [0.1,1] in Eq. (5b), but closer to Pr = 1. Fundamentally, the Prandtl number of binary gas mixtures Pr_{mix} occupies the narrow sub–interval [0.1, 1] as seen in Figure 2 taken from Campo and Papari [17]. It is observable in the figure that the absolute minimum Pr_{mix} for the binary gas mixtures are: 0.1 for the He+Xe binary gas mixture, 0.2 for the He+CF_4 binary gas mixture and 0.3 for the He+SF_6 binary gas mixture.

Liu and Ahlers, [18], performed an in– depth investigation to provide an explanation for the nature of the "anomalously" low values of Prandtl number linked to binary gas mixtures. Using a combined methodology based on statistical–mechanical theory, experimental measurements of other researchers as well as their own findings, the authors have demonstrated the possible existence of lower Prandtl numbers for a hydrogen–xenon binary gas mixture decreasing down to 0.16.

3.1 Specific Correlation Equation for the Average Convective Coefficient of Binary Gas Mixtures

Heat convection theory (Jaluria [8]) dictates that laminar boundary layer flows promoted by thermo–buoyant forces depend upon four thermo–physical properties: dynamic viscosity η, thermal conductivity λ, density ρ, and isobaric heat capacity C_p. In the particular case of binary gas mixtures, the four thermo–physical properties are adequately re–expressed by η_{mix}, λ_{mix}, ρ_{mix} and $C_{p,mix}$.

Eq. (3) particularized to the proper Pr sub–interval [0.1, 1] stated in Eq. (5b), along with the corresponding Prandtl number function g (Pr) = $0.549 Pr^{0.171}$ delivers the correlation equation

$$\overline{Nu}_H = 0.549\, G_R^{1/4} Pr^{0.42} \qquad (6)$$

Isolating the average convective coefficient $\bar{h}$ in Eq. (6), the equation representative of the average convective coefficient of a binary gas mixture $\bar{h}_{mix}$ turns out to be

$$\bar{h}_{mix} / B = \left(\frac{\lambda_{mix}^{0.58}\, \rho_{mix}^{0.50}\, C_{p,\,mix}^{0.42}}{\eta_{mix}^{0.08}} \right) \qquad (7)$$

where the subscript 'mix' signifies binary gas mixture.

The overall thermo–geometric parameter B in Eq. (7) absorbs four quantities: the acceleration of gravity g, the height of the vertical plate H, the plate temperature T_w and the free–stream temperature of the binary gas mixture T_∞. That is

$$B = 0.549 \left[\frac{g \beta (T_w - T_\infty)}{H} \right]^{0.25} \qquad (7a)$$

with units $s^{-1/2}$. Alternatively, using the coefficient of volumetric thermal expansion $\beta = \frac{1}{T_\infty}$ for ideal gases, B could be rewritten compactly as

$$B = 0.549 \left[\frac{g}{H} \left(\frac{T_w}{T_\infty} - 1 \right) \right]^{0.25} \qquad (7b)$$

As mentioned in the Introduction, the main objective of the study is to explore the possible intensification of the heat transfer rate Q in laminar natural convection gas flows past a vertical plate. If the surface area of the vertical plate A_s in Eq. (2) remains unchanged and the plate–to–fluid temperature difference $(T_w - T_\infty)$ is constant, the only possible way for augmenting Q is by enlarging the magnitude of $\bar{h}$ Thereby, the analysis of binary gas mixtures formed with light He and selected heavier secondary gases will be pursued in this work.

Let us pause here momentarily to describe the ideal values of the thermo–physical properties in the binary gas mixture should possess in harmony with the structure of Eq. (7). Thereby, it is evident that in order to intensify the heat transfer rate Q_{mix} for laminar natural convection with binary gas mixtures along a vertical plate, the trio of thermo–physical properties λ_{mix}, ρ_{mix}, and $C_{p,mix}$ in the numerator of Eq. (7) must be large and/or the single thermo–physical property η_{mix} in the denominator of Eq. (7) must be small.

3.2 Thermo–physical Property Called the Thermal Effusivity

From an in–depth perspective, it is observable that $\bar{h}_{mix}$ in Eq. (7) varies directly proportional with the square root of the product λ_{mix} x ρ_{mix} x $C_{p,\,mix}$ and inversely proportional with the 0.10 power of η_{mix}. The numerator may be viewed through the optic of the thermo–physical property called the thermal effusivity ε (Grigull and Sandner, [19]):

$$\varepsilon = (\lambda\, \rho\, C_p)^{1/2}$$

In particular, the thermal effusivity of the He–based binary gas mixtures related to laminar natural convection along a vertical plate may be expressed in an approximate manner as

$$\varepsilon_{mix} = (\lambda_{mix}\, \rho_{mix}\, C_{p,mix})^{1/2}$$

Accordingly, Eq. (7) could be rewritten compactly in terms of the thermal effusivity ε_{mix} as follows

$$\bar{h}_{mix}/B \approx \frac{\varepsilon_{mix}}{\eta_{mix}^{0.08}} \qquad (7c)$$

with minimal contribution from η_{mix}.

4 Thermo–physical Properties of Binary Gas Mixtures

To form the binary gas mixtures with light He as the primary gas, the seven heavier secondary gases are: N_2, O_2, Xe, CO_2, CH_4, CF_4 and SF_6. The molar masses M of the participating gases are taken from Poling et al., [20], and are listed in Table 1. In addition, the molar mass difference ΔM of the eight binary gas mixtures are listed in Table 2.

Table 1. Molar mass of the pure gases (in ascending order)

Gas	M (g/mol)
He	4.00
CH_4	16.04
N_2	28.01
O_2	32.00
CO_2	44.01
CF_4	88.00
Xe	131.29
SF_6	146.06

Table 2. Molar mass difference ΔM (in ascending order) of the He–based binary gas mixtures

Binary gas mixture	ΔM (g/mol)
He+CH_4	12.04
He+N_2	24.01
He+O_2	28.00
He+CO_2	40.01
He+CF_4	84.00
He+Xe	127.29
He+SF_6	142.06

Notice that the three heavier secondary gases having the largest molar mass factors with respect to the primary light gas He are: CF_4 with a factor of 20, Xe with a factor of 30 and SF_6 with a factor of 40 approximately.

4.1 Molar Gas Composition

The molar gas composition w_i of a binary gas mixture is defined as the mass fraction of pure gas i

$$w_i = x_i\left(\frac{M_i}{M_{mix}}\right), \qquad \text{for } i = 1, 2 \tag{8}$$

where x_i is the mole fraction, M_i is the molar mass of pure gas i. Here, the molar mass of the binary gas mixture (Poling et al. [20]) is defined by

$$M_{mix} = x_1M_1 + x_2M_2 \tag{8a}$$

The molar mass of the chosen pure gases is listed in Table 1.

4.2 Density

The density of a binary gas mixture ρ_{mix} at low pressure is determined with the truncated virial equation of state (Poling et al. [20]):

$$Z = \frac{p}{RT\rho_{mix}} = 1 + 2B_2\,\rho_{mix} \tag{9}$$

where Z is the compressibility factor, p is the pressure, R is the gas constant. Herein, the second virial coefficient B_2 is evaluated with the simple correlation attributed to Tsonopoulos [21]

$$\frac{B_2 p_c}{RT_c} = B^{(0)} + \omega B^{(1)} \tag{10}$$

where the Pitzer acentric factor ω, the critical temperature T_c and the critical pressure p_c for the pure gases are taken from Poling et al. [20]. The numerical values of $B^{(0)}$ and $B^{(1)}$ are computed from the pair of relations:

$$B^{(0)} = 0.1445 - \frac{0.33}{T_r} - \frac{0.1385}{T_r^{\,2}} - \frac{0.0121}{T_r^{\,3}} - \frac{0.000607}{T_r^{\,8}} \tag{11a}$$

and

$$B^{(1)} = 0.0637 + \frac{0.331}{T_r^2} - \frac{0.423}{T_r^3} - \frac{0.008}{T_r^8} \tag{11b}$$

where the temperature ratio $T_r = T/T_c$.

4.3 Isobaric Heat Capacity

The isobaric heat capacity of a binary gas mixture $C_{p,\,mix}$ at low density obeys the mixing rule (Poling et al. [20]):

$$C_{p,\,mix} = \sum_i x_i C^0_{p,i} \tag{12}$$

where x_i denotes for the mole fraction of pure gas i. The isobaric molar heat capacity of the pure gas i identified by $C^0_{p,i}$ satisfies the equality

$$C^0{}_p - C^0{}_v = R \qquad (13)$$

Herein, C_v^0 being the molar heat capacity at constant volume of pure gas i, is quantified from

$$\frac{C_v^0}{R} = S + \sum_{j=1}^{k}\left(\frac{\Theta_{vj}}{T}\right)^2 \frac{\exp\left(\frac{\Theta_{vj}}{T}\right)}{\left[\exp\left(\frac{\Theta_{vj}}{T}\right)\right]^2} \qquad (14)$$

In this equation, the symbol Θ_{vj} stands for the characteristic vibrational temperature corresponding to the vibrational degree of freedom j. In addition, the symbol S equates to 5/2 for linear molecules and to 3 for nonlinear molecules as noted by McQuarrie [22].

4.4 Viscosity

Based on the Kinetic Theory of Gases (Hirschfelder et al. [23] and Chapman and Cowling [24]), the viscosity of a binary gas mixture η_{mix} is calculated with the matrix formula:

$$\eta_{mix} = -\frac{\begin{vmatrix} H_{AA} & H_{AB} & x_A \\ H_{BA} & H_{BB} & x_B \\ x_A & x_B & 0 \end{vmatrix}}{\begin{vmatrix} H_{AA} & H_{AB} \\ H_{AB} & H_{BB} \end{vmatrix}} \qquad (15)$$

First, the element H_{AA} along the main diagonal in the two matrices is:

$$H_{AA} = \frac{x_A^2}{\eta_A} + \frac{2x_A x_B}{\eta_{AB}} \frac{m_A m_B}{(m_A + m_B)^2}\left(\frac{5}{3A^*_{AB}} + \frac{m_B}{m_A}\right) \qquad (15a)$$

Second, the element H_{BB} along the main diagonal of the two matrices is deduced from the expression for H_{AA} by interchanging the subscripts A and B. Third, the elements located off the main diagonal in the two matrices are:

$$H_{AB}(A \neq B) = -\frac{2x_A x_B}{\eta_{AB}} \frac{m_A m_B}{(m_A + m_B)^2}\left(\frac{5}{3A^*_{AB}} - 1\right) \qquad (15b)$$

Further, the interaction viscosity η_{AB} appearing in the two preceding equations (15a) and (15b) is given by the formula

$$\eta_{AB} = \frac{5}{16}\left[\left(\frac{2m_A m_B}{m_A + m_B}\right)\frac{kT}{\pi}\right]^{1/2} \frac{1}{\sigma_{AB}^2\, \Omega_{AB}^{*(2,2)}(T^*_{AB})}. \qquad (15c)$$

in which the subscript A identifies the heavier gas and the subscript B identifies the light gas, the pair m_A and m_B designate the masses of A and B and the pair x_A and x_B are the mole fractions of A and B. In addition, $A_{AB}{}^*$ represents for the ratio collision integral at T^*_{AB}, which is defined by Bzowski et al. [25].

4.5 Thermal Conductivity

For the thermal conductivity of a binary gas mixture λ_{mix}, Schreiber at al. [26] developed the matrix formula:

$$\lambda_{mix} = -\frac{\begin{vmatrix} L_{AA} & L_{AB} & x_A \\ L_{AB} & L_{BB} & x_B \\ x_A & x_B & 0 \end{vmatrix}}{\begin{vmatrix} L_{AA} & L_{AB} \\ L_{AB} & L_{BB} \end{vmatrix}} \qquad (16)$$

To save journal space, the elements L_{AA}, H_{AB} and L_{BB} in the upper and lower matrices in Eq. (16) are not included. Instead, L_{AA}, H_{AB} and L_{BB} are available in Reference [26]. Besides, the pair x_A and x_B are the mole fractions of A and B.

5 Heat Transfer Analysis for Binary Gas Mixtures

When the accurate formulas for the four thermo–physical properties η_{mix} (w) in Eq. (9), λ_{mix} (w) in Eq. (12), ρ_{mix} (w) in Eq. (15) and $C_{p,mix}$ (w) in Eq. (16) at a given pressure p and temperature T are introduced into Eq. (6), the average convective coefficient of a binary gas mixture $\bar{h}_{mix}$ respond to continuous changes in the molar gas composition w of the seven binary gas mixtures $He+N_2$, $He+O_2$, He+Xe, $He+CO_2$, $He+CF_4$, $He+CH_4$, $He+SF_6$ in the proper w–domain [0, 1].

The methodical algebraic calculations for the quantification of $\bar{h}_{mix}$ related to the seven binary gas mixtures are carried out with fixed small steps $\Delta w = 0.01$ utilizing the spreadsheet software Excel [27].

The four thermo–physical properties of The seven binary gas mixtures $He+N_2$, $He+O_2$, He+Xe, $He+CO_2$, $He+CH_4$, $He+CF_4$ and $He+SF_6$ exhibit a variety of curve shapes. Parabolic down curves λ (w) from primary light He (w = 0) to the heavier secondary gases (w = 1) are shown in Figure 3. There are curves up and curves down η (w) from light He (w = 0) to the heavier gases SF_6 (w = 1), CF_4 (w = 1) and O_2 (w = 1) are shown in Figure 4. The other binary gas mixtures $He+N_2$, He+Xe, $He+CO_2$, $He+CH_4$ follow parabolic down curves. Negative sloped straight lines Cp (w) from light He (w = 0) to the heavier gases (w = 1) are displayed in Figure 5. The exponentially increasing curves ρ(w) from light He (w = 0) to the heavier gases (w = 1) are illustrated in Figure 6.

5.1 Maximum Heat Transfer Rates Rendered by Light He–based Binary Gas Mixtures

The objective function is the relative heat transfer rate Q_{mix} (w)/B associated with Eq. (2).

The goal of the sub–section is to search for the absolute maximum of the objective function. Conceptually, a point $w = w_{opt}$ is an absolute maximum of a single–valued function $Q_{mix}(w)/B$ if

$$Q_{mix}(w_{opt})/B \geq Q_{mix}(w)/B \qquad (17)$$

for all w values inside the w–domain [0, 1] (Sioshansi and Conejo [28]). In other words, the location of the optimal molar gas composition w_{opt} corresponds to the w value that renders an absolute maximum $Q_{mix,\,abs\,max}/B$ for all ranges of $Q_{mix}(w)/B$ contained in the w–domain [0, 1]. Besides the absolute maximum, the other are called relative maxima.

Table 3. Values of the thermo–physical properties at T = 300K and p = 1 atm. (The highest and lowest values are highlighted)

Gas	M g/mol	ρ_ kg/m^3	η_ μPa.s	λ mW/(m.K)	C_p^0 J/(kg.K)
He	**4.00**	**0.1624**	19.92	**155.70**	**5199.11**
CH_4	16.04	0.6553	**11.19**	34.89	2230.47
N_2	28.01	1.1379	17.96	25.88	1039.66
O_2	32.00	1.3004	20.78	26.64	918.21
CO_2	44.01	1.7964	15.08	16.79	848.40
CF_4	88.00	3.5410	17.34	15.16	699.36
Xe	131.29	5.3610	23.20	**5.52**	**158.49**
SF_6	**146.06**	**5.8585**	**29.70**	13.20	671.39

6 Presentation and Discussion of Results

The presentation and discussion of results will be conveniently divided in two parts.

6.1 Natural Convection Heat Transfer using Light Helium

First, it was deemed appropriate to contrast the natural convection heat removal from the large vertical plate with light He against standard air standard air before going to the binary gas mixtures formed with primary light He and the secondary heavier gases N_2, O_2, Xe, CO_2, CH_4, CF_4 and SF_6. In this respect, using Eq. (7) at a temperature of 300K, the heat transfer enhancement ratio E_{ht} caused by light He with respect to air is

$$E_{ht} = \frac{\left(\frac{Q}{B}\right)_{He}}{\left(\frac{Q}{B}\right)_{air}} = 2 \qquad (18)$$

The physical explanation for this behavior is that the thermal conductivity of He is eight times larger than the thermal conductivity of air, the viscosities of He and air are about the same and the isobaric heat capacities of He and air are very close.

6.2 Natural Convection Heat Transfer using the He–based Binary Gas Mixtures

Second, the natural convection heat removal by the seven He–based binary gas mixtures is discussed next. At this point, it is instructive to highlight the characteristics of the four thermophysical properties for He. In comparison with the seven secondary gases, it is observable in Table 3 that He possesses the highest thermal conductivity λ_{He} = 155.70 mW/m.K, the highest isobaric heat capacity $C_{p,\ He}$ = 5199.11 J/kg.K, but the lowest density ρ_{He} = 0.1624 kg/m^3 and an intermediate viscosity η_{He} = 19.92 µPa.s. Therefore, in reference to Eq. (7a) the lowest ρ_{He} value needs to be compensated with a high ρ value coming from the heavier secondary gases N_2, O_2, Xe, CO_2, CH_4, CF_4 and SF_6. In addition, the intermediate η_{He} value needs to be compensated with a low η value coming from the heavier secondary gas.

An additional figure has been prepared to illustrate how the target parameter for the seven He based binary gas mixtures, i.e., the relative heat transfer rate Q_{mix}/B varies with the molar gas composition w in the proper w–domain [0, 1]. In the figure format, the abscissa associates the light primary gas He with the left extreme w = 0, whereas the right extreme w = 1 represents each of the seven heavier secondary gases N_2, O_2, Xe, CO_2, CH_4, CF_4 and SF_6. In this regard, Figure 7 displays the family of seven curves for the relative heat transfer rate Q_{mix}/B varying with the molar gas composition w at the film temperature T = 300K. Obviously, the point of reference is the primary light He that owns a relative heat transfer rate Q_{He}/B = 12. Among the binary gas mixtures examined, it is seen that three He+SF_6, He+CF_4 and He+Xe binary gas mixtures exhibit maxima relative heat transfer rates $Q_{mix,max}$/B. Further, the optimal molar gas compositions w_{opt} for these three binary gas mixtures are located near the right extreme w = 1 occupied by the heavier secondary gases SF_6, CF_4 and Xe. First, the He+SF_6 binary gas mixture produces the absolute maximum relative heat transfer rate $Q_{mix,max}$/B = 16.70 that happens at the optimal molar gas composition w_{opt} = 0.960. As compared to $(Q/B)_{He}$, and $(Q/B)_{air}$, the He+SF_6 binary gas mixture generates remarkable heat transfer enhancement ratios

$$E_{ht} = \frac{\left(\frac{Q}{B}\right)_{He+SF_6}}{\left(\frac{Q}{B}\right)_{He}} = 39 \qquad (19a)$$

$$E_{ht} = \frac{\left(\frac{Q}{B}\right)_{He+SF_6}}{\left(\frac{Q}{B}\right)_{air}} = 78 \qquad (19b)$$

Another indicator is that among the seven binary gas mixtures under scrutiny, the He+SF_6 binary gas mixture has the largest molar gas difference ΔM = 146.06.

One relative maximum for the relative convective coefficient $Q_{mix,max}$/B = 14.89 is provided by the He+CF_4 binary gas mixture, which takes place at the optimal molar gas composition w_{opt} = 0.936. Herewith, the heat transfer enhancement ratio for the He+CF_4 binary gas mixture referred to He and air have a significant magnitude

$$E_{ht} = \frac{\left(\frac{\bar{Q}}{B}\right)_{He+CF_4}}{\left(\frac{\bar{Q}}{B}\right)_{He}} = 24 \qquad (20a)$$

$$E_{ht} = \frac{\left(\frac{\bar{Q}}{B}\right)_{He+CF_4}}{\left(\frac{\bar{Q}}{B}\right)_{air}} = 48 \qquad (20b)$$

The He+Xe binary mixture supplies another relative maximum for the relative convective coefficient $Q_{mix,max}/B = 13$ operating at $w_{opt} = 0.785$. Here, the heat transfer enhancement ratio provided by the He+Xe binary gas mixture when paired against Q_{He}/B descends to the modest values

$$E_{ht} = \frac{\left(\frac{Q}{B}\right)_{He+Xe}}{\left(\frac{Q}{B}\right)_{He}} = 8 \qquad (21a)$$

$$E_{ht} = \frac{\left(\frac{Q}{B}\right)_{He+Xe}}{\left(\frac{Q}{B}\right)_{air}} = 16 \qquad (21b)$$

Attention is now turned to the seeded heavier secondary gases in the He–based binary gas mixtures under study. Thereby, the amount of seeded SF_6 at $w_{SF_6} = 0.04$ is slightly smaller than the amount of seeded CF_4 at $w_{CF_4} = 0.064$ and much smaller than the amount of seeded Xe at $w_{Xe} = 0.215$.

Furthermore, it can be observed in Figure 7 that as $Q_{mix,max}/B$ decreases from the upper He+SF_6 binary gas mixture to the intermediate He+CF_4 binary gas mixture ending in the lower He+Xe binary gas mixture, so that the optimal molar composition w_{opt} shifts gradually toward the left extreme on the w abscissa.

The physical explanation for the heat transfer enhancement delivered by the trio of He+SF_6, He+CF_4 and He+Xe binary gas mixtures that own the largest molar mass difference ΔM may be explained as follows. The curves in the pair of Figures 3 and 5 reveal that the thermal conductivity λ_{mix} and the isobaric heat capacity $C_{p,mix}$ of the seven He–based binary gas mixtures decrease with the molar gas composition w. Therefore, this behavior means that the two thermo–physical properties λ_{mix} and $C_{p,mix}$ do not contribute to the heat transfer enhancement. Consequently, the heat transfer enhancement will depend solely on the interplay between the density ρ_{mix} and the viscosity η_{mix}. The numbers listed in the three Tables 4, 5 and 6 reveal that the density ρ_{mix} is the major contributor to the heat transfer enhancement. First, the density ρ_{mix} of the He+SF_6 binary gas mixture at the optimal molar gas composition $w_{opt} = 0.96$ is 15.25 times higher than the density of pure He. Second, the density ρ_{mix} of He+CF_4 binary gas mixture at the optimal molar gas composition, $w_{opt} = 0.936$ is 9.44 times higher than the density of pure He. Third, the density ρ_{mix} of the He+Xe binary gas mixture at the optimal molar gas composition, $w_{opt} = 0.785$ is 4.06 times higher than the density of pure He. In contrast, for the three best binary gas mixtures He+SF_6, He+CF_4 and He+Xe, the corresponding ratios for the viscosity between w_{opt} and w = 0 for He range between 0.95 and 1.61, so that the variability is not that significant.

The remaining four Q_{mix}/B vs. w curves related to the sub–group of He+N_2, He+O_2, He+CO_2 and He+CH_4 binary gas mixtures form a tight cluster, all exhibiting monotonic decreasing trends with increments in the molar gas composition in the w–domain [0, 1]. Obviously, for this particular trio, the primary light He is preferred as the coolant, instead of the four binary gas mixtures He+N_2, He+O_2, He+CO_2 and He+CH_4.

Table 4. He+Xe binary gas mixture

Thermo–physical property	Molar gas composition of He, w = 0	Optimal molar gas composition $w_{opt} = 0.785$	Thermo–physical property ratio
Density, ρ	0.16	0.65	4.06
Viscosity, $\eta \times 10^5$	1.99	2.51	1.26

Table 5. He+CF_4 binary gas mixture

Thermo–physical property	Molar gas composition of He, w = 0	Optimal molar gas composition, $w_{opt} = 0.936$	Thermo–physical property ratio
Density, ρ	0.16	1.51	9.44
Viscosity, $\eta \times 10^5$	1.99	1.89	0.95

Table 6. He+SF_6 binary gas mixture

Thermo–physical property	Molar gas composition of He, w = 0	Optimal molar gas composition, w_{opt} = 0.96	Thermo–physical property ratio
Density, ρ	0.16	2.44	15.25
Viscosity, $\eta \times 10^5$	1.99	3.21	1.61

6.3 Beneficial Characteristics of the Best He–based Binary Gas Mixtures

The three best He–based binary gas mixtures related to laminar natural convection along a heated vertical plate that yield maximum heat transfer shares the following characteristics.

1) As indicated in the last three lines of Table 2, the three largest molar mass difference ΔM are: He+CF_4 with ΔM = 84.00, He+Xe with ΔM = 127.29 and He+SF_6 with ΔM = 142.06.

2) The three lowest Prandtl numbers are: He+Xe with Pr_{mix} = 0.1, He+CF_4 with Pr_{mix} = 0.2 and He+SF_6 with Pr_{mix} = 0.3. These three numbers can be observable in the lower right part of Figure 2.

3) The absolute maximum relative heat transfer rate $Q_{mix,max}/B$ = 16.70 is delivered by the He+SF_6 binary gas at the optimal molar gas composition w_{opt} = 0.960. One relative maximum for the relative convective coefficient $Q_{mix,max}/B$ = 14.89 is provided by the He+CF_4 binary gas mixture at w_{opt} = 0.936. The He+Xe binary mixture supplies another relative maximum for the relative convective coefficient $Q_{mix,max}/B$ = 13 operating at w_{opt} = 0.785. These numbers can be observable in the upper right part of Figure 7.

6.4 Comparison of Numerical Calculations with Experimental Data

The magnitude of the heat transfer enhancement ratio E_{ht} = 8 for the He+Xe binary gas mixture in Eq. (21) coincides with the heat transfer enhancement ratio obtained numerically and experimentally by Petri and Bergman [6], but at a slightly different molar gas composition w.

7 Concluding Remarks

The conclusions that may be drawn from the present study are enumerated next.

The first conclusion is that the He+SF_6 binary gas mixture yields a remarkable absolute heat transfer enhancement ratio E_{ht} = 39% at the optimal molar gas composition w_{opt} = 0.960 compared to the primary light He. The corresponding Pr_{mix} = 0.3.

The second conclusion is that the He+CF_4 binary gas mixture at the optimal molar gas composition w_{opt} = 0.960 delivers a significant relative heat transfer enhancement ratio E_h = 24% at the optimal molar gas composition w_{opt} = 0.936 compared to the primary light He. The corresponding Pr_{mix} = 0.2.

The third conclusion is that the He+Xe binary gas mixture provides a modest relative heat transfer enhancement ratio E_{ht} = 8% at the optimal molar gas composition w_{opt} = 0.785 compared to the primary light He. The corresponding Pr_{mix} = 0.1.

In the global picture, it could be pointed out that usage of exotic binary gas mixtures, like the trio He+CF_4, He+SF_6 and He+Xe formed with light He and heavier gases CF_4, SF_6 and Xe may be envisioned for special engineering tasks that demand high heat transfer rates in a multitude of industries over the world.

Nomenclature

A_S	surface area of vertical plate (m^2)
B	thermogeometric parameter in Eq. (7a) ($s^{-1/2}$)
B_2	second virial coefficient (m^3 $mole^{-1}$)
C_p	mass isobaric heat capacity (J kg^{-1} K^{-1})
C_p^0	molar isobaric heat capacity of ideal gas (J $mole^{-1}$ K^{-1})
C_v^0	molar isobaric heat capacity of ideal gas (J $mole^{-1}$ K^{-1})

E_{ht} heat transfer enhancement ratio
g acceleration of gravity (m s^{-2})
Gr_H Grashof number, $g\beta \frac{\rho^2}{\eta^2}(T_s - T_\infty)H^3$
$\bar{h}$ average convective coefficient ($W\ m^{-2}\ K^{-1}$)
H height of vertical plate (m)
m molecular mass (kg)
M_i molar mass of pure gas i (kg $mole^{-1}$)
$\overline{Nu}_H$ average Nusselt number, $\frac{\bar{h} H}{\lambda}$
p pressure (bar)
p_c critical pressure (bar)
Pr Prandtl number, $\frac{\eta C_p}{\lambda}$
Q heat transfer rate (W)
R gas constant (J $mole^{-1}$ K^{-1})
Ra_H Rayleigh number, Gr_H Pr
T temperature (K)
T_c critical temperature (K)
T_W plate temperature (K)
T_∞ free–stream temperature (K)
w molar gas composition of a binary gas mixture
w_{opt} optimal molar gas composition of a binary gas mixture
x mole fraction
Z compressibility factor

Greek letters

β coefficient of volumetric thermal expansion (K^{-1})
ε thermal effusivity ($W\ m^{-2}\ K^{-1}\ s^{1/2}$)
η viscosity (μ Pa s)
λ thermal conductivity ($W\ m^{-1}\ K^{-1}$)
ρ density (kg m^{-3})
ω Pitzer acentric factor

Subscripts

mix binary gas mixture
max maximum
opt optimal

References;

[1] A. E. Bergles, Techniques to Enhance Heat Transfer. In *Handbook of Heat Transfer*, Chapter 11.1–11.76, Rohsenow, W. M., Hartnett, J. P., Cho, Y. I., eds., McGraw–Hill, New York, NY, 1998.
[2] R. M. Manglik, Heat Transfer Enhancement, Chapter 14. In *Heat Transfer Handbook,* eds. Bejan, A. and Kraus, A. D., Wiley and Sons, Hoboken, NJ, 2003.
[3] S. H. Bhavnani and A. E. Bergles, Effect of surface geometry and orientation on laminar natural convection heat transfer from a vertical flat plate with transverse roughness elements, *International Journal of Heat and Mass Transfer*, Vol. 33, Issue 5, 1990, pp. 965–981.
[4] A. Kitagawa, K. Uchida and Y. Hagiwara, Effects of bubble size on heat transfer enhancement by sub–millimeter bubbles for laminar natural convection along a vertical plate, *International Journal of Heat and Fluid Flow*, Vol. 30, Issue 4, 2009, pp. 778–788.
[5] A. V. Kuznetsov and D. A. Nield, Natural convective boundary–layer flow of a nanofluid past a vertical plate: A revised model, *International Journal of Thermal Sciences*, Vol. 77, 2014, pp. 126–129.
[6] J.G. Petri and T. L. Bergman, Augmentation of natural convection heat transfer using binary gas coolants, *International Journal of Heat and Mass Transfer*, Vol. 33, Issue 7, 1990, pp. 1441–1449.
[7] V. Arpaci, Effect of thermal radiation on the laminar free convection from a heated vertical plate, *International Journal of Heat and Mass Transfer*, Vol. 11, 1968, pp. 871–881.
[8] Y. Jaluria, *Natural Convection Heat and Mass Transfer*, Pergamon Press, New York, NY, 1980.

[9] V. Arpaci and P. S. Larsen, *Convection Heat Transfer*, Prentice–Hall, Englewood Cliff, NJ, 1984.

[10] S. Ostrach, An Analysis of Laminar Free Convection Flow and Heat Transfer about a Flat Plate Parallel to the Direction of the Generating Body Force, *NACA Technical Note No. 1111*, 1953.

[11] A. Bejan and J. L. Lage, The Prandtl number effect on the transition in natural convection along a vertical plate, *Journal of Heat Transfer*, Vol. 112, 1990, pp. 787–790.

[12] E. J. LeFevre, Laminar free convection from a vertical plane surface, *Proceedings of the 9th International Congress on Applied Mechanics*, Vol. 4, Paper # 1–168, pp. 168–179, Brussels, Belgium, 1956.

[13] S. W. Churchill and H. H. S. Chu, Correlating equations for laminar and turbulent free convection from a vertical plate, *International Journal of Heat and Mass Transfer*, Vol. 18, 1975, pp. 1323–1229.

[14] O. G. Martynenko and A. F. Polijakov, *Free. Convection. In Thermopedia: A–to–Z Guide to Thermodynamics, Heat Transfer, Mass Transfer and Fluids Engineering*. Available in https://www.thermopedia.com.

[15] N. V. Suryanarayana, *Engineering Heat Transfer*, West Publishing Co., New York, NY, 1995.

[16] https://www.systatsoftware.com

[17] A. Campo and M. M. Papari, Calculation of the minimum Prandtl number for selected binary gas mixtures formed with light helium and selected heavier gases, *Applied Thermal Engineering*, Vol. 31, Issue 16, 2011, pp.31423146.

[18] J. Liu and G. Ahlers, Rayleigh–Benard convection in binary gas mixtures: Thermo–physical properties and the onset of convection, *Physics Reviews E*, Vol. 55, No. 6, Paper # 6950, 1977.

[19] U. Grigull and H. Sandner, *Heat Conduction*, Hemisphere, New York, NY, 1984.

[20] B. E. Poling, J. M. Prausnitz and J. P. O'Connell, *The Properties of Gases and Liquids*, pp. A.5–A.19, McGraw–Hill, New York, NY, 2001.

[21] C. Tsonopoulos, An empirical correlationof second virial coefficients, *AIChE Journal*, Vol. 20, 1974, pp. 45–56.

[22] D. A. McQuarrie, *Statistical Mechanics*, Harper and Row, New York, NY, 2000.

[23] J. O. Hirschfelder, C. F. Curtis and R. B. Bird, *Molecular Theory of Gases and Liquids*, Wiley and Sons, Hoboken, NJ, 1964.

[24] S. Chapman and C. F. Cowling, *The Mathematical Theory of Non–uniform Gases*, Cambridge University Press, Cambridge, London, England, UK, 1970.

[25] J. Bzowski, J. Kestin, E. A. Mason and F. J. Uribe, Equilibrium and transport properties of gas mixtures at low density: eleven poly-atomic gases and five noble gases, *Journal of Physical Chemical Reference Data*, Vol. 19, 1990, pp. 1179–1232.

[26] M. Schreiber, V. Vesovic and W. A.Wakeham, Thermal conductivity of atom–molecule dilute gas mixtures, *High Temperature–High Pressure*, Vol. 29, 1997, pp. 653–658.

[27] https://www.microsoft.com/en-us/microsoft-365/excel

[28] R. Sioshansi and A. J. Conejo, Optimization *in Engineering: Models and Algorithms*, Springer, Berlin, Germany, 2017.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

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Conflicts of Interest

Please declare anything related to this study.

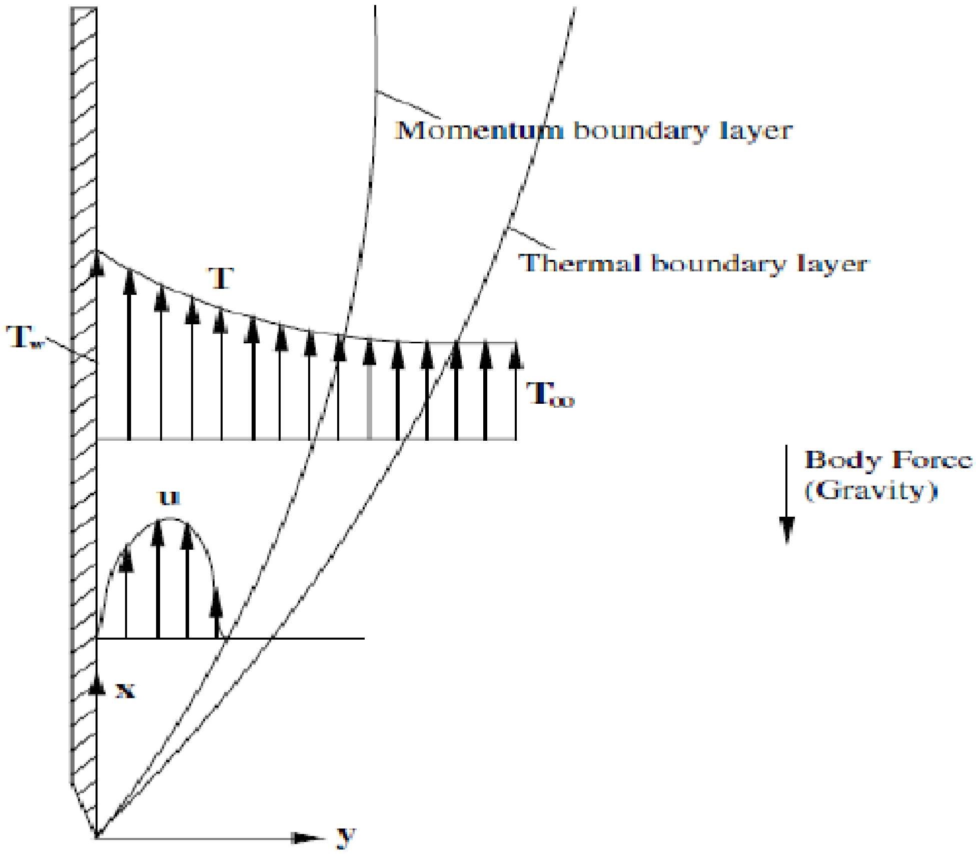

Fig. 1: Laminar natural convection from a heated vertical plate: (a) ---- Momentum boundary layer, (b) —— Thermal boundary layer

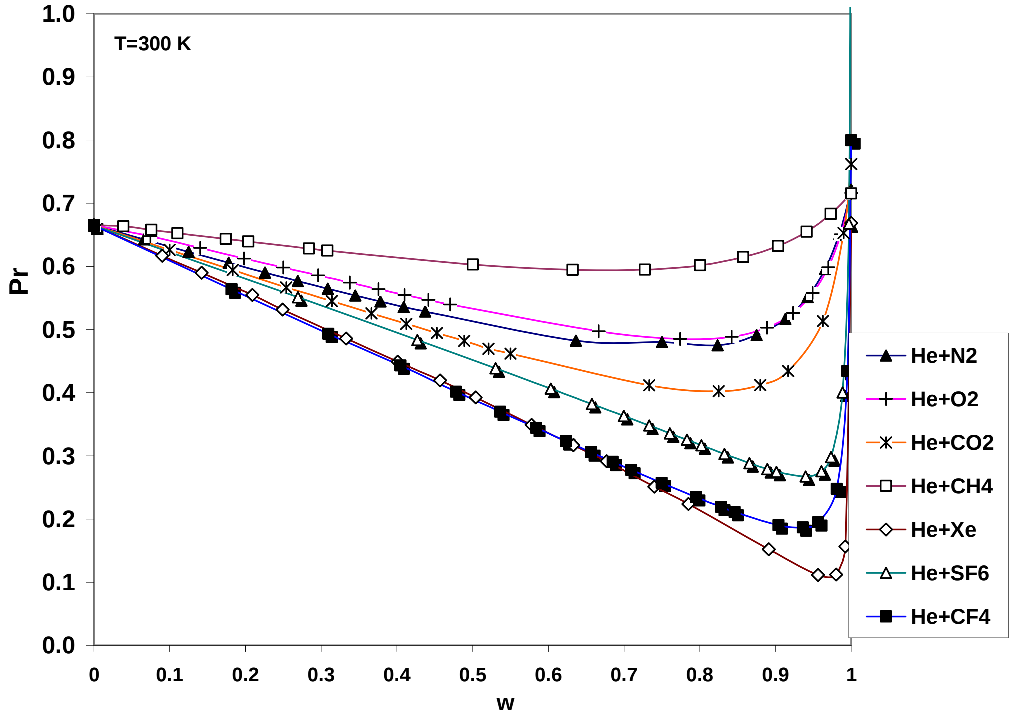

Fig. 2: Variation of Prandtl number with molar gas composition w of the seven He–based binary gas mixtures at T = 300 K, p = 1 atm.

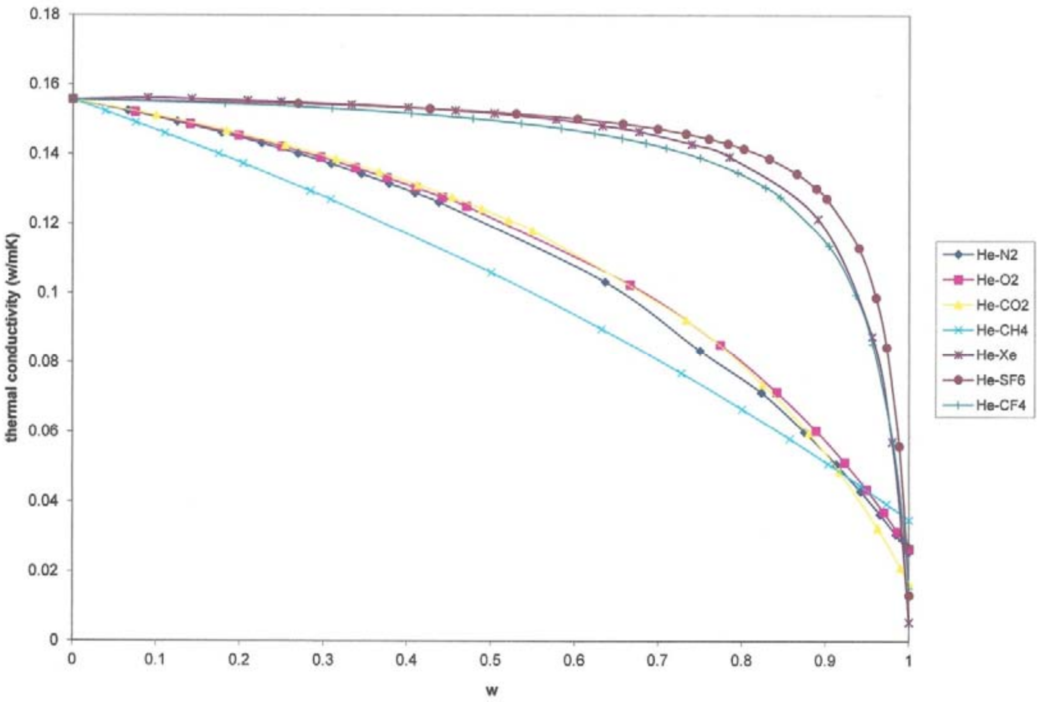

Fig. 3: Variation of the thermal conductivity with molar gas composition w of the seven He–based binary gas mixtures at T = 300 K, p = 1 atm.

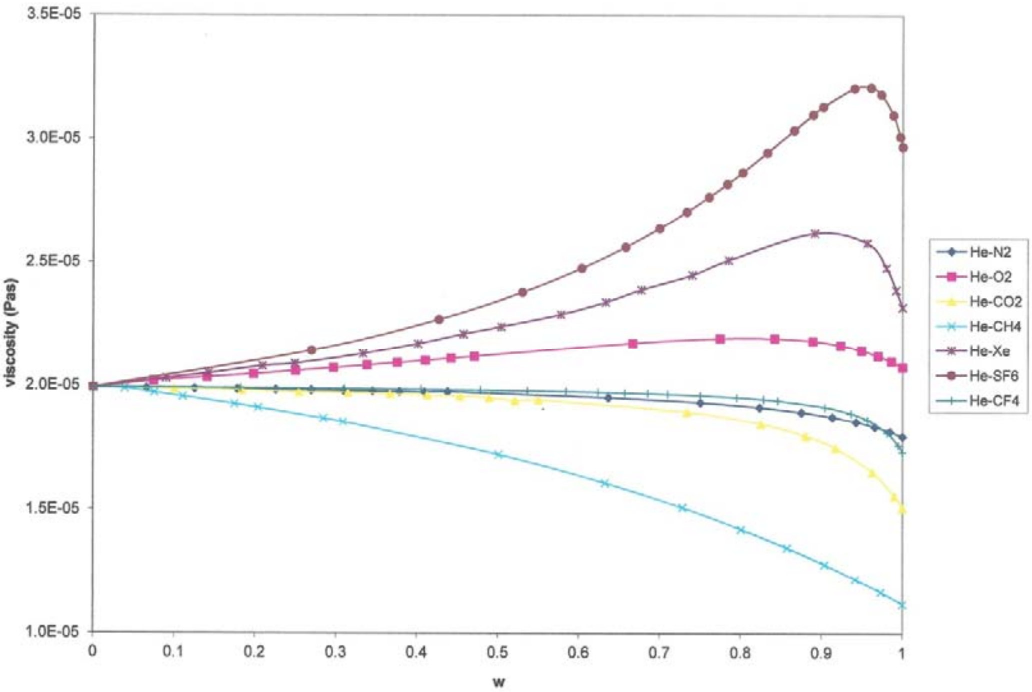

Fig. 4: Variation of the viscosity with molar gas composition w of the seven He–based binary gas mixtures at T = 300 K, p = 1 atm.

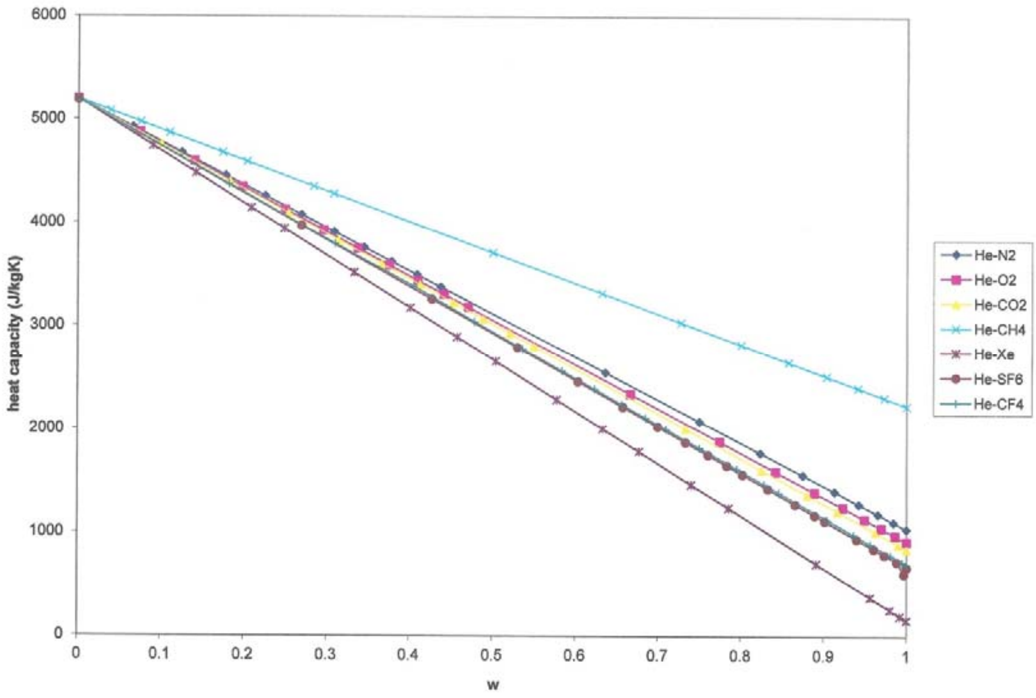

Fig. 5: Variation of the isobaric heat capacity heat capacity with molar gas composition w of the seven He–based binary gas mixtures at T = 300 K, p = 1 atm.

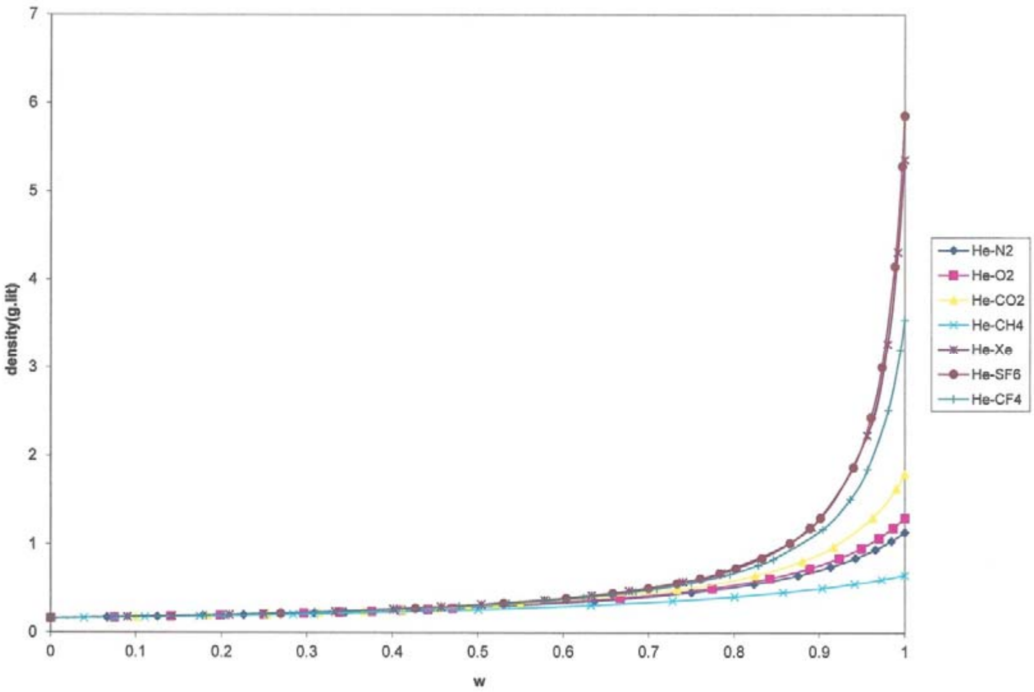

Fig. 6: Variation of the density with molar gas composition w of the seven He–based binary gas mixtures at T = 300 K, p = 1 atm.

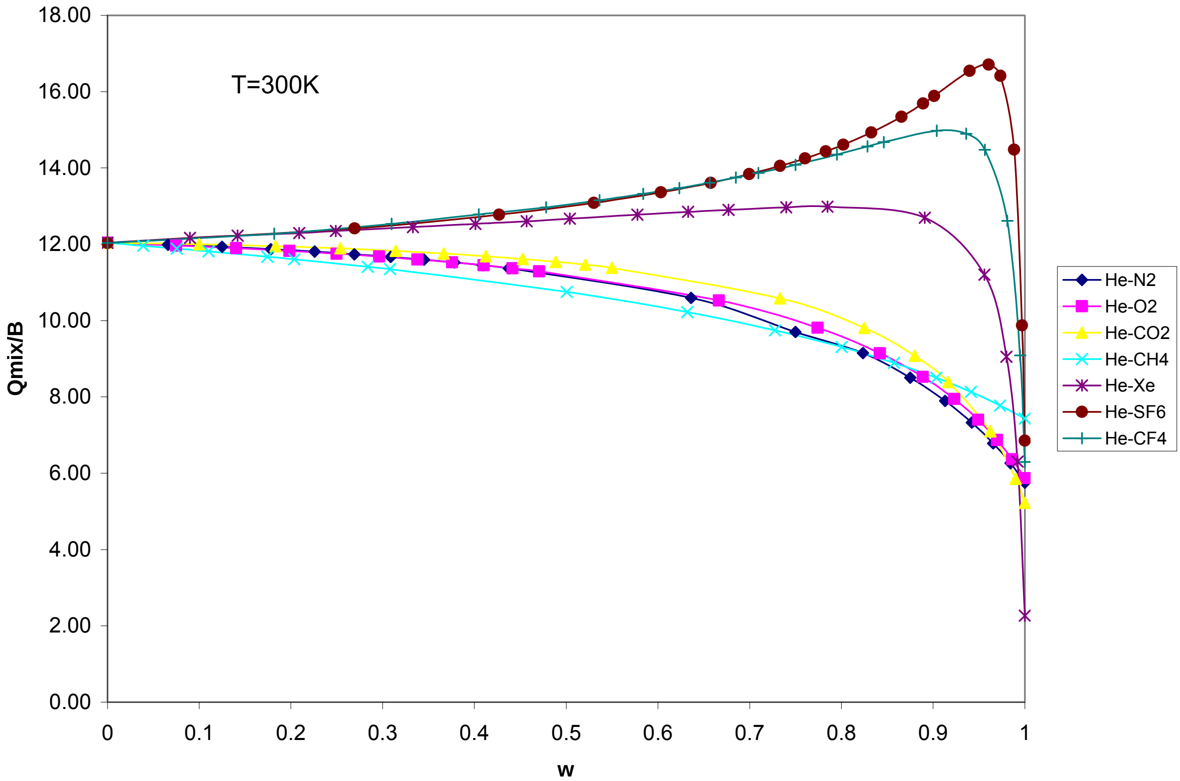

Fig. 7: Variation of the relative heat transfer rate Q_{mix}/B with the molar gas composition w of the seven He–based binary gas mixtures at T = 300 K, p = 1 atm.