On the Solution to Pre-Buckling of a Thin Rectangular Plate Subjected to a Thick Strip In-Plane Loading

JACOB NAGLER
NIRC
Haifa, Givat Downes
ISRAEL

Abstract: - The current paper deals with the problem of the simply supported thin rectangular plate subjected to the intermediate strip in-plane loading. Based on the strain energy method (Fourier ansatz), the critical (minimum value) of buckling stress occurrence was determined in a general form dependent only on the strip thickness, strip location, plate width and stress magnitude. Compatible with the classical columns Euler method it was found that the plate stability is decreased with the increasing of the plate width due to larger induced stresses. Also, strip location relative to the support region was found to influence the buckling (same analogy to the Euler buckling theory; consider the strip as a both sides pressed rod). Additionally, the strip width parameter increase is likely to cause larger buckling stress. Moreover, expressions that includes both axial and transverse loads for different extended cases configurations were also derived and examined based on the strain energy method alongside explanation for possible applications (thin aluminum plate welding). In a general view, it was found that the cases of combined axial and perpendicular loading action are less stabilized than cases where only one kind of loading configuration is participated. Finally, the buckling stress was found to agree qualitatively with the cited literature.

Key-Words: - static, buckling; in-plane loading; buckling stress; rectangular plate; strip.

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1 Introduction

Rectangular plate subjected to an in-plane stress in the geometry shape of rectangular strip has a lot of importance in the ship industry (side plates [1] and stiffened ribs plates [2]), and structural engineering (stiffened plates [3], perforated thin plates [4]), manufacturing (cold rolling process [5-6]) and composite plates (ply and film plates [7-9] FGM plates [10-11] and other types of plate involving the viscoelastic form [12-13]). The prevailing common classical solution of compressed stress due to rib influence can be found in classical books of solid & structural mechanics [14-17].

The loading configurations at the plate boundaries can be divided into static and dynamic loads [19]. In addition, post- or pre- buckling state [19] is another way of classification and final method to distinguish is the applying loads type and way of operation [13, 20-24].

The calculations methods to cope with the problem diverse from Rayleigh – Ritz methods [10], numerical methods [22], exact analytic solutions [25] and semi-analytic solution [26-27].

Specifically, the case of intermediate load in the pre-buckling state in the in-plane was investigated by [25-29] which based on the direct thin plate theory displacement equation. However, the current study extent the studies of Xiang et al. [25], Yao et al., [26], Mijušković et al. [27] and Wang et al. [28-29] by developing a general analytic solution to the pre –buckling state of the simply supported rectangular thin plate subjected to a thick strip that is generally located in the plate plane based on the strain energy theory. The obtained expression will be examined and compared qualitatively to the relevant literature references [2, 14, 16, 18, 25].

Before beginning with the analytic model, it will be mentioned that there are several main plate theoretical model in the literature for different regimes, like: Kirchhoff-Love [31-33] (simple displacement relations, all kinds of thinner plate thickness), Ulfyand-Mindlin [34-35] (plate thickness to planar dimensions ratio is of the order of one tenth), Mindlin–Reissner [36-38] (the bending stress is assumed to behave linearly while the shear stress is supposed to be quadratic through the plate thickness), Reissner–Stein [39] (improved Saint-Venant theory for cantilever plates) and von Karman [40-44] (general and most veteran model from 1910 describing the large displacements of thin flat plate based on nonlinear partial differential equation). Moreover, von Karman model for thin flat elastic plate subjected to stress is an extension of Kirchhoff-Love model and based on differential kinematic equations depicting stress-strain relationships [40-44]. The solution for the obtained non-linear differential equation (large deflection) that is based on Airy's function is not easy and immediate to solve [44]. There are methods that convert the non-linear set of equilibrium [40, 43 - 44] to energy [40] or guessing appropriate displacement function [41] method which is similar to what we propose here. However, here we concern only partial stress loading region (strip) that act on a supported thin flat elastic plate.

Ulfyand-Mindlin [34 - 35] and Mindlin–Reissner [36, 38] plates have their own complicated assumptions (as noted above) with appropriated differential equations relationships between stresses and strains. Yet, Brunelle and Robertson [34] (deals with different boundary conditions and full stress loading on the opposite edges) and Bui *et al*. [37] (only generalized scheme without solving or specifying a particular case) have formulated it by using energy form which is similar to the current study.

Another dominant model-case which is similar to the current study energy formulation has been produced by Reissner–Stein [39] for cantilever elastic thing plate under torsional and transverse moments bending which is different from our model (boundary conditions).

As though, particularly, the current model is based on total potential energy function minimization formulation that appear (for example) Reissner–Stein [39] and also known to be called Fourier Ansatz model which is derived from the linear theory (i.e. governing energy is functional quadratic). Concentrating on thin, flat elastic plate under partial loading in order to have simple closed analytic solution for future evaluation of applications based on (or involved with) pre-buckling stress (heating, punching, etc.).

2 Problem Formulation

In a general form, consider a simply supported thin flat elastic plate (Δ) subjected to thick strip loading with thickness as appear in Fig. 1. Using Cartesian coordinates system as shown in Fig. 1, alongside the general equation of strain energy density for plane stress ($\sigma_z = 0$) [2, 14, 18]:

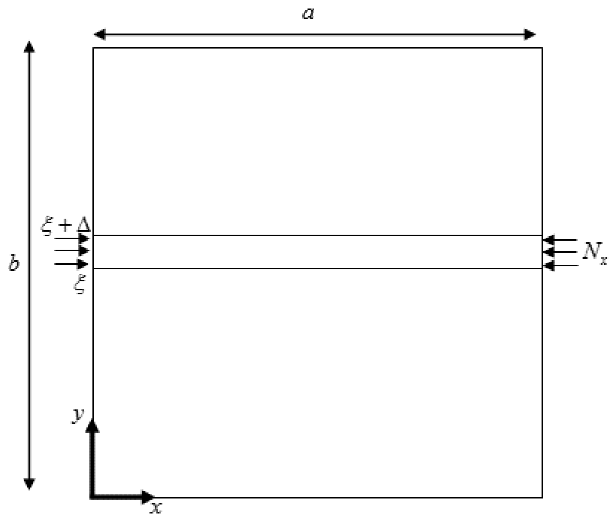

Fig. 1: Simply supported plate subjected to a thick strip loading

$$U = \frac{D}{2}\int_0^a\int_0^b\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\nu)\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2 w}{\partial x\partial y}\right)^2 dydx = \frac{D}{2}\int_0^a\int_0^b\left[(\Delta w)^2 - 2(1-\nu)\left(w_{,xx}\cdot w_{,yy} - w_{,xy}\right)^2\right]dxdy \tag{1}$$

where $D = \dfrac{Et^3}{12(1-\nu^2)}$, E is the Young modulus and ν is the Poisson ratio. Additionally, the work expression (V) that is defined only for the strip region ($\xi \le y \le \xi + \Delta$) and controlled by the concentrated normalized (outer) pressures/stresses N_x in the strip distributed uniformly over the displacement w, will be:

$$V = \frac{1}{2}\{f\}w^2 = -\frac{N_x t}{2}\int_0^a\int_\xi^{\xi+\Delta}\left(\frac{\partial w}{\partial x}\right)^2 dydx\,. \tag{2}$$

Now, we will assume that the deflection parameter can be represented using the following function:

$$w(x,y) = \sum_{m=1}\sum_{n=1} A_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \tag{3}$$

which fulfils the following simply supported (S-S) boundary conditions (B.C.):

One simple support on the one end side:

$w_1(x,0) = w_{1,yy}(x,0) = 0$

One simple support on the one end side:

$w_1(x,0) = w_{1,yy}(x,0) = 0$

One simple support on both end sides:

$w(x,0) = w_{yy}(x,0) = 0,$

$w(x,b) = w_{yy}(x,b) = 0$

First continuous condition: $\begin{cases} w_1(x,\xi) = w_2(x,\xi) \\ w_{1,yy}(x,\xi) = w_{2,yy}(x,\xi) \end{cases}$

Second continuous condition:

$$\begin{cases} w_2(x,\xi+\Delta) = w_3(x,\xi+\Delta) \\ w_{2,yy}(x,\xi+\Delta) = w_{3,yy}(x,\xi+\Delta) \end{cases}$$

while the constants A_n are dependent on the boundary is a scalar function, however, the conditions. Note that w index numbers 1,2,3 are aided to separate three plate subdomains (1 – right side, 2 – strip, 3 – left side).

Hence the stored energy and the virtual work reads, are after substituting (3) into expressions (1) and (2), as:

$$U = \frac{D}{2}\int_0^a\int_0^b\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\nu)\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2 w}{\partial x\partial y}\right)^2 dydx =$$

$$\sum_m\sum_n A_{mn}^2\frac{\pi^4 Dab}{8}\left(\frac{m^4}{a^4}+\frac{n^4}{b^4}+2\nu\left(\frac{mn}{ab}\right)^2 - \frac{9}{8}\pi^4 A_{mn}^2(1-\nu)\left(\frac{mn}{ab}\right)^4\right) \quad (4)$$

In order to simplify, we will assume $\nu = 1$ and consider only the bending influence, such as Eq. (4) will become:

$$U = \frac{D}{2}\int_0^a\int_0^b\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 dydx = \sum_{nm} A_{mn}^2\frac{Dab\pi^4}{8}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 \quad (5)$$

Note that m and n indicate the numbers of half-waves in each direction.

$$V = \frac{1}{2}\{f\}w_{,x}^2 = -\frac{N_x t}{2}\int_0^a\int_\xi^{\xi+\Delta}\left(\frac{\partial w}{\partial x}\right)^2 dydx =$$

$$-\frac{N_x t}{2}\left(\frac{m\pi}{a}\right)^2\int_0^a\sum_m\cos^2\left(\frac{m\pi x}{a}\right)dx\cdot\int_\xi^{\xi+\Delta}\left\{\sum_n A_{mn}\sin\left(\frac{n\pi y}{b}\right)\right\}^2 dy =$$

$$-\frac{N_x t m^2\pi^2}{8a}\cdot\sum_{nm}A_{mn}^2\left\{\Delta - \frac{b}{2n\pi}\left[\begin{array}{l}\sin\left(\frac{2n\pi(\xi+\Delta)}{b}\right)\\ -\sin\left(\frac{2n\pi\xi}{b}\right)\end{array}\right]\right\} \quad . (6)$$

since $\int_0^a\sum_m\cos^2\left(\frac{m\pi x}{a}\right)dx = a/2$

Hence, the overall potential energy of the system will supply:

$$\Pi = U + V =$$

$$\int_0^a\int_0^b\left[\begin{array}{l}\frac{D}{2}\left(\nabla^2 w\right)^2 - 2(1-\nu)\left(w_{,xx}\cdot w_{,yy}-w_{,xy}\right)^2\\ -\frac{1}{2}\{f\}w_{,x}^2\end{array}\right]dxdy =$$

$$\sum_{nm}A_{mn}^2\frac{Dab\pi^4}{8}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2$$

$$-\frac{N_x t\pi^2}{8a}\cdot\sum_{nm}m^2A_{mn}^2\left\{\Delta - \frac{b}{2n\pi}\left[\begin{array}{l}\sin\left(\frac{2n\pi(\xi+\Delta)}{b}\right)\\ -\sin\left(\frac{2n\pi\xi}{b}\right)\end{array}\right]\right\}\geq 0 \quad . (7)$$

Therefore the limited general minimum expression value for N_x will be:

$$N_x \leq \frac{\sum_{nm}A_{mn}^2\frac{\pi^2 Da^2 b}{t}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}{\sum_{nm}m^2A_{mn}^2\left\{\Delta - \frac{b}{2n\pi}\left[\begin{array}{l}\sin\left(\frac{2n\pi(\xi+\Delta)}{b}\right)\\ -\sin\left(\frac{2n\pi\xi}{b}\right)\end{array}\right]\right\}} \quad (8)$$

The critical value of N_x will be (for one A_{mn} term):

$$N_x\big|_{cr} \leq \frac{\pi^2 D a^2 b}{m^2 t} \cdot \frac{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}{\Delta - \frac{b}{2n\pi}\left[\sin\left(\frac{2n\pi(\xi+\Delta)}{b}\right) - \sin\left(\frac{2n\pi\xi}{b}\right)\right]}. \tag{9}$$

Alternative method, finding the extreme values of N_x by differentiating the inequality Π along constant $A_{mn} \neq 0$ as $\frac{\partial \Pi}{\partial A_{mn}} = 0$ gives the same value. Now, expression (9) will be approximated for the first minimal critical buckling load with the parameters pair (m,n) to yield:

$$N_x\big|_{cr} = \frac{\pi^2 D a^2 b}{m^2 t} \cdot \frac{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}{\Delta + \frac{b}{n\pi}}, \tag{10}$$

such as the sinus expression in the denominator will be bounded as $\left|\sin\left(\frac{2n\pi(\xi+\Delta)}{b}\right) - \sin\left(\frac{2n\pi\xi}{b}\right)\right| \leq 2$ since $|\sin(x)| < 1$. Note that the solution $N_x\big|_{cr} = \frac{\pi^2 D a^2 b}{m^2 t} \cdot \frac{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}{\Delta - \frac{b}{n\pi}}$ is not physical (due to the negative denominator, $b > \Delta$). Expressions (9)-(10) seem to make sense in the context of Δ behavior since the critical load N_x required to buckle the plate is diminished with the strip width increase. Next step, for $n = 1$ (10) has the minimum value and becomes:

$$N_x\big|_{cr} = \frac{\pi^2 D a^2 b}{m^2 t} \cdot \frac{\left(\frac{m^2}{a^2}+\frac{1}{b^2}\right)^2}{\Delta + \frac{b}{\pi}} = \frac{\pi^2 D b}{a^2 t} \cdot \frac{\left[m + \frac{1}{m}\left(\frac{a}{b}\right)^2\right]^2}{\Delta + \frac{b}{\pi}}. \tag{11}$$

Now, using some algebra can lead us to the reduced relation (11), presented as Euler's formula in the form of boundary support:

$$N_x\big|_{cr} = \frac{\pi^2 D}{bt} \cdot \frac{k}{\Delta \mp \frac{b}{\pi}}; \quad k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2. \tag{12}$$

Note that for the case $\Delta = b, \xi = 0$ (inserted into Eq. (9)) we obtain the classical solution [2, 14, 20]:

$$N_x\big|_{cr} = \frac{\pi^2 D}{b^2 t} \cdot k; \quad k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2. \tag{13}$$

Note that the classical solution (13) is not dependent on the plate width length *b* whereas expressions (9) and (10) are dependent on *b* due to the additional strip width Δ.

Finally, the maximum buckling stress for the problem posed by [25, 28-29] including two extensions will be presented. Following the previous development, three prominent extent problems shown in Figs. 2 are solved in Table 1 based on the superposition principal. Note that the subdomain 2 definition in case (c) relates to the whole plate without the loads N_1.

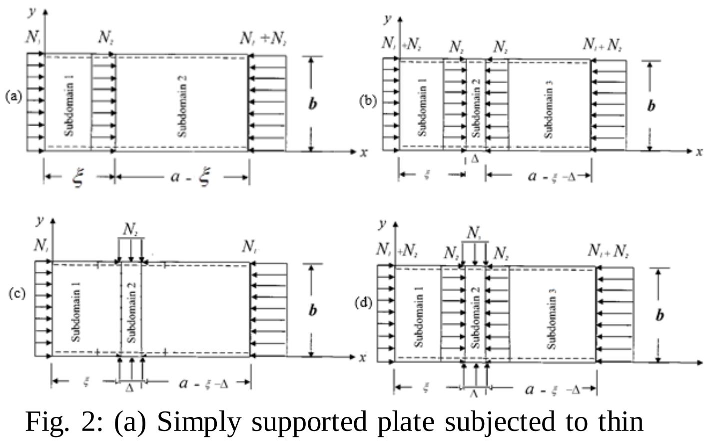

Fig. 2: (a) Simply supported plate subjected to thin intermediate and ended uniaxial loads (taken from [25, 28-29]) (b) Rectangular plate subjected to thick intermediate and ended uniaxial loads (c) Rectangular plate subjected to perpendicular thick load and ended uniaxial loads (d) Rectangular plate subjected to intermediate perpendicular combined with uniaxial thick loads and ended uniaxial loads

Table 1. Comparison between different load configurations vs. maximum buckling stress

Problem Definition	Maximum superposition buckling stress
Rectangular plate subjected to thin intermediate and ended uniaxial loads [27, 30-31] as appear in Fig. 2(a)	$N_{1,\max}=\frac{\pi^2 D\xi^2}{m^2 t}\cdot\left(\frac{m^2}{\xi^2}+\frac{n^2}{b^2}\right)^2$ $N_{2,\max}=\frac{\pi^2 D(a-\xi-\Delta)^2}{m^2 t}\cdot\left(\frac{m^2}{(a-\xi-\Delta)^2}+\frac{n^2}{b^2}\right)^2-\frac{\pi^2 D\xi^2}{m^2 t}\cdot\left(\frac{m^2}{\xi^2}+\frac{n^2}{b^2}\right)^2$
Rectangular plate subjected to thick intermediate and ended uniaxial loads as appear in Fig. 2(b)	$N_{1,\max}=\frac{\pi^2 D\Delta^2}{m^2 t}\cdot\left(\frac{m^2}{\Delta^2}+\frac{n^2}{b^2}\right)^2$ $N_{2,\max}=\frac{\pi^2 D\xi^2}{m^2 t}\cdot\left(\frac{m^2}{\xi^2}+\frac{n^2}{b^2}\right)^2+\frac{\pi^2 D(a-\xi-\Delta)^2}{m^2 t}\cdot\left(\frac{m^2}{(a-\xi-\Delta)^2}+\frac{n^2}{b^2}\right)^2-\frac{\pi^2 D\Delta^2}{m^2 t}\cdot\left(\frac{m^2}{\Delta^2}+\frac{n^2}{b^2}\right)^2$
Rectangular plate subjected to perpendicular intermediate thick load and ended uniaxial loads as appear in Fig. 2(c)	$N_{1,\max}=\frac{\pi^2 D a^2}{m^2 t}\cdot\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2$ $N_{2,\max}=\frac{\pi^2 D a^2 b}{m^2 t}\cdot\frac{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}{\Delta-\frac{b}{2n\pi}\left[\sin\left(\frac{2n\pi(\xi+\Delta)}{b}\right)-\sin\left(\frac{2n\pi\xi}{b}\right)\right]}$
Rectangular plate subjected to intermediate perpendicular combined with uniaxial thick loads and ended uniaxial loads as appear in Fig. 2(d)	$N_{1,\max}=\frac{\pi^2 D\Delta^2}{m^2 t}\cdot\left(\frac{m^2}{\Delta^2}+\frac{n^2}{b^2}\right)^2$ $N_{2,\max}=\frac{\pi^2 D a^2 b}{m^2 t}\cdot\frac{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}{\Delta-\frac{b}{2n\pi}\left[\sin\left(\frac{2n\pi(\xi+\Delta)}{b}\right)-\sin\left(\frac{2n\pi\xi}{b}\right)\right]}$ $N_{3,\max}=\frac{\pi^2 D\xi^2}{m^2 t}\cdot\left(\frac{m^2}{\xi^2}+\frac{n^2}{b^2}\right)^2+\frac{\pi^2 D(a-\xi-\Delta)^2}{m^2 t}\cdot\left(\frac{m^2}{(a-\xi-\Delta)^2}+\frac{n^2}{b^2}\right)^2-\frac{\pi^2 D\Delta^2}{m^2 t}\cdot\left(\frac{m^2}{\Delta^2}+\frac{n^2}{b^2}\right)^2$

3 Results and Discussion

In this section the following parameters will be substituted into Eqs. (9)-(10) and (13) which are represented in Fig 3, as:

$$\begin{gathered}b=1,\ a=0.5b,\Delta=0.3b\\ 0\le a/b\ \le 3, 0\le\Delta/b\ \le 3, 0\le\xi/b\ \le 3\end{gathered}. \tag{14}$$

Respectively, the representative parameters in Figs. 4-7 are as follows:

$$\begin{gathered}a=1,\ a=0.5b,\Delta=0.3b\\ 0\le b/a\ \le 3, 0\le\Delta/a\ \le 3, 0\le\xi/a\ \le 3\end{gathered}. \tag{15}$$

Examining Fig. 3(a) teaches that all solutions are very close quantitatively and behaves qualitatively the same. Also, the approximate solution (10) seems to be an average solution between the exact solution (9) and the classic (13) solution. The buckle loading seems to decrease until they reach certain critical value of *a*/*b* ratio from which they slightly increase but keeps stabilized as soon as the plate is more elongated (*b* parameter is reduced or *a* is increased). The opposite (minimum critical buckling load is decreased) occurs if the plate is flattened (fit with the classical columns Euler method). Note that similar phenomenon occurs in Figs. 4(a)-7(a). Accordingly, it can be assumed approximately that the minimum buckling load is varied linearly according with the geometrical ratio *a*/*b* (Fig. 3(a)) and *b*/*a* (Figs. 4(a)-7(a)), respectively.

Analyzing the strip width (Δ) influence; one might conclude from observing 3(b)-7(b) that the required critical loading to buckle the plate is being reduced due to the fact that the increase of the strip width is accompanied with larger induced stresses that accelerating the buckling process. Although it was not illustrative presented, the parameters *m*, *n* has almost no effect on the buckling load for (relative to the) high strip width length.

Analyzing the strip location (ξ) appearing in Figs. 3(c)-7(c) that expressed in sinus wave of maximum loading in reversed direction at the center of the plate might be explained by the fact that the existing induced stresses are located in further distances from the supports locations. Therefore when the stresses acting in a further distance relative to the supports locations the buckling might be occurred sooner which is also adjust to Euler compressed column theory. Remark that the jump in Figs. 3(c)-7(c) resembles numerically the fact that the location of the loading is the same as its width.

One application that can be used here is when thin aluminium plate is subjected to heating through welding process - by making row of thin welding strips with spaces between them as close as possible to the supports in order to avoid concentrated induced stresses that might cause buckling.

Observing Figs. 4-7 we can see the influence (disturbance) of the transverse perpendicular loading as appear in 6(a) and 7(a). Generally speaking, the cases where both axial and perpendicular loading are acting are less stable than cases where only one kind of loading configuration is participated. Finally, qualitative confirmation with literature [2, 14, 16, 18, 25] has been achieved.

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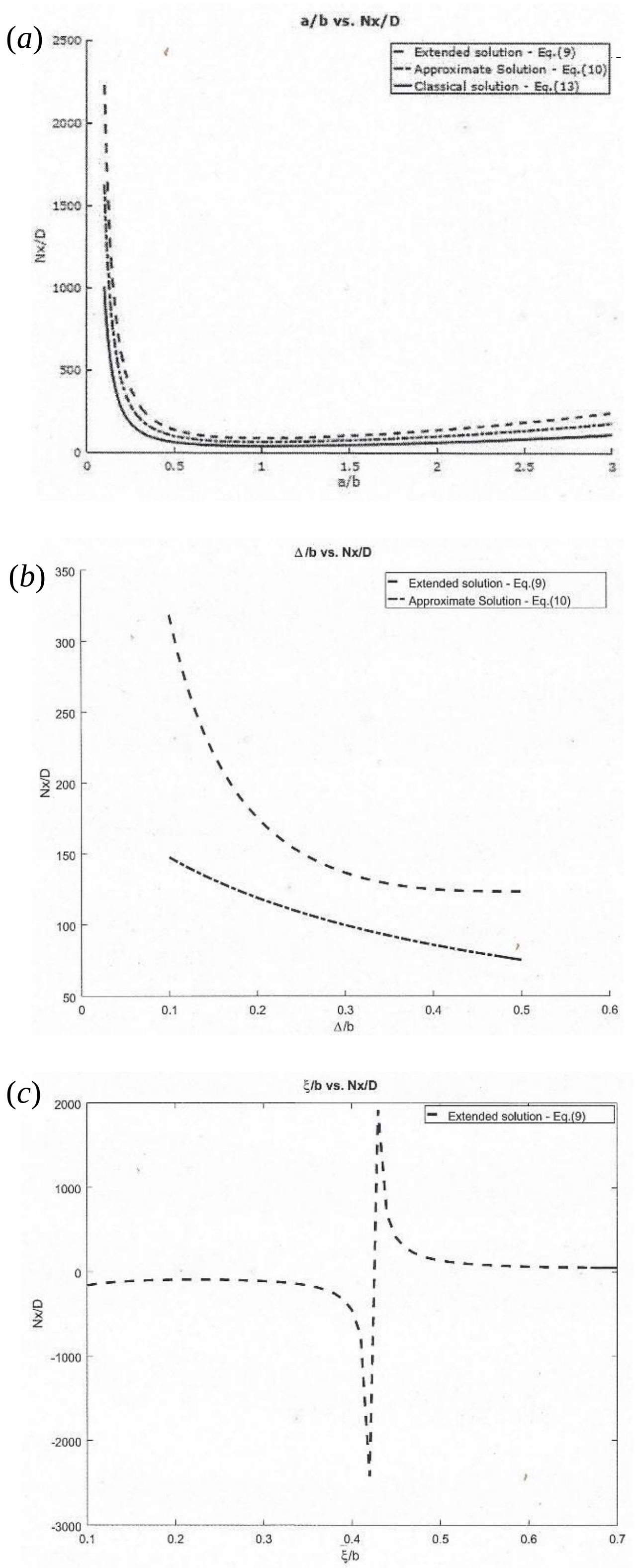

Fig. 3: Stability criteria (critical load) for simply supported plates under intermediate load vs. (a) different geometrical ratio *(a/b)* values (b) different delta (Δ) width length values (c) different ξ distance values.

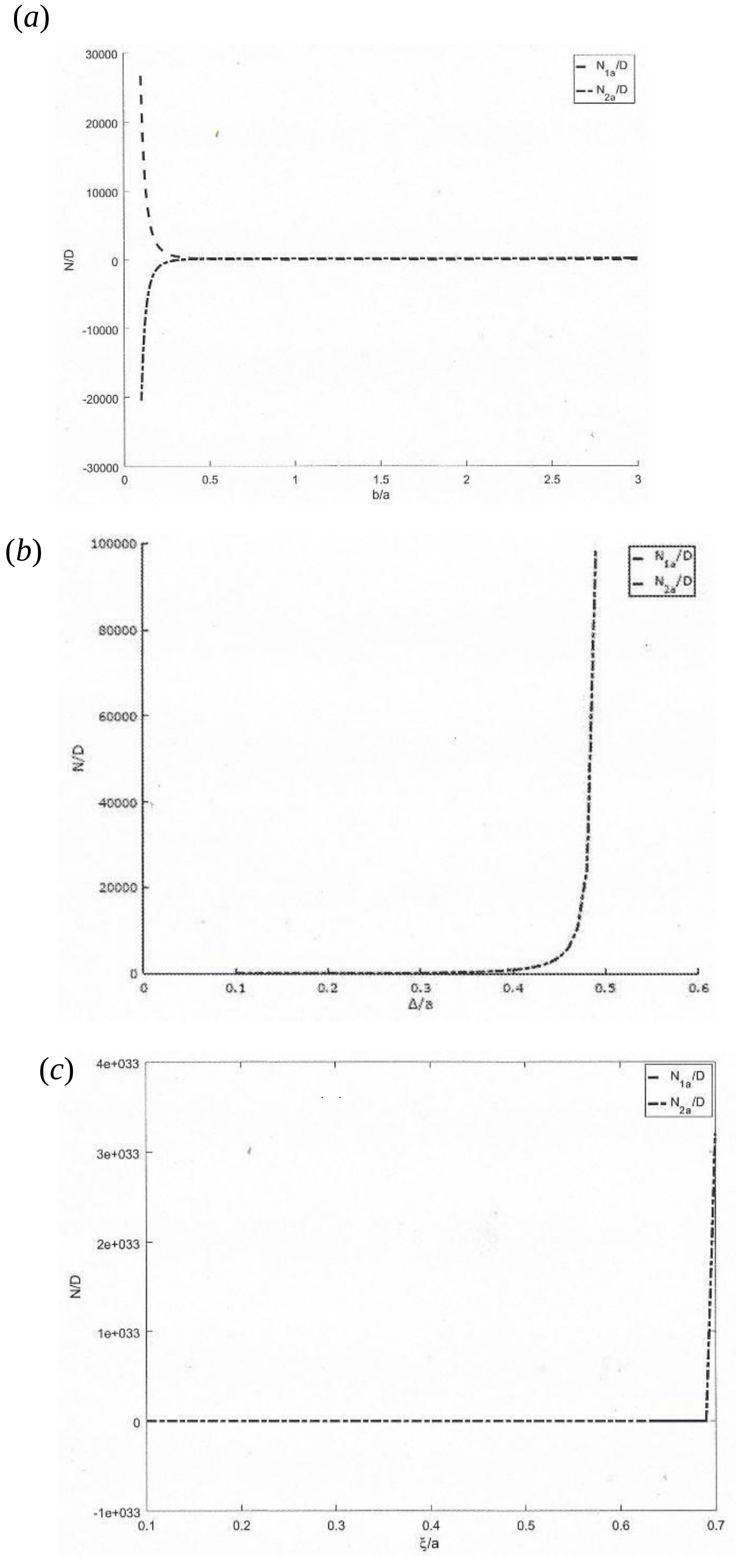

Fig. 4: Stability criteria for simply supported plates under intermediate load N_2 and end load N_1 vs. (a) different geometrical ratio *(a/b)* values (b) different delta (Δ) width length values (c) different ξ distance values.

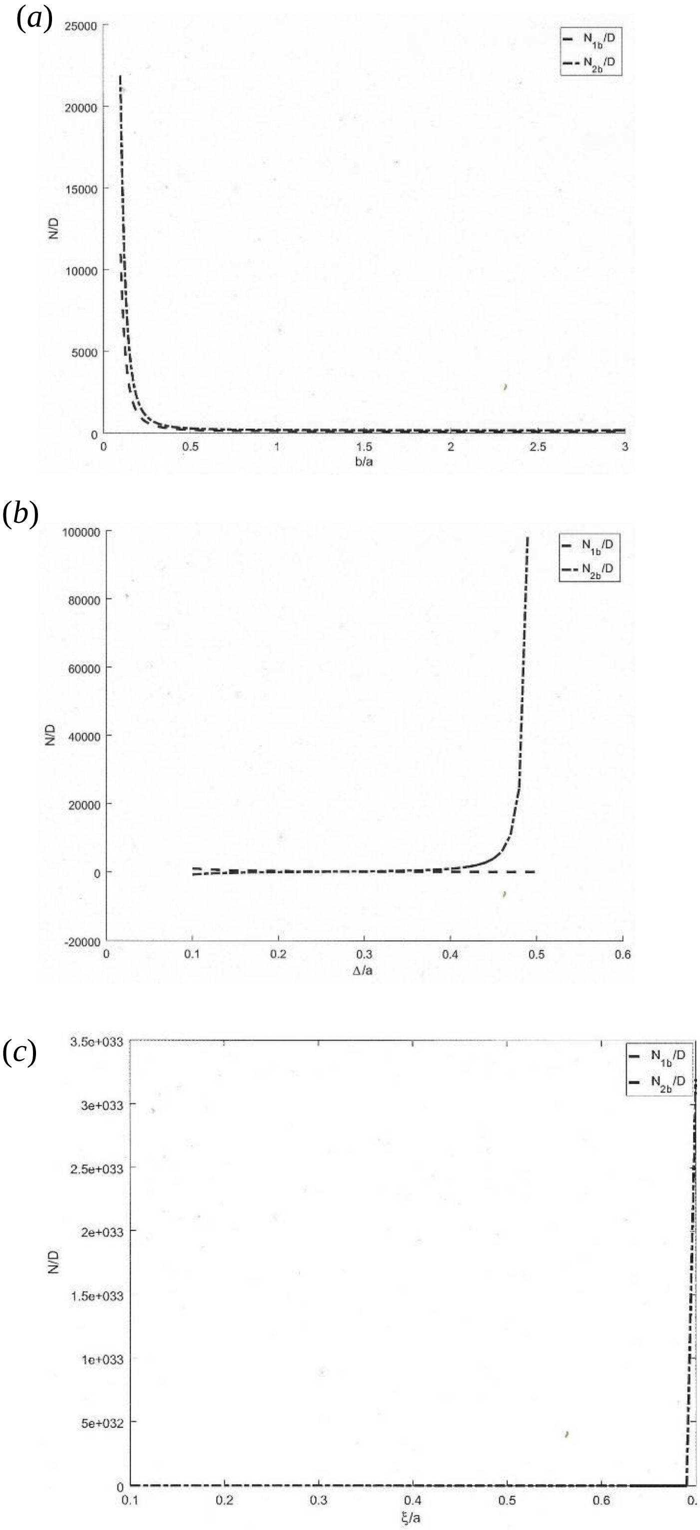

Fig. 5: Stability criteria for simply supported plates under intermediate load N_2 and end load N_1 vs. (a) different geometrical ratio *(a/b)* values (b) different delta (Δ) width length values (c) different ξ distance values.

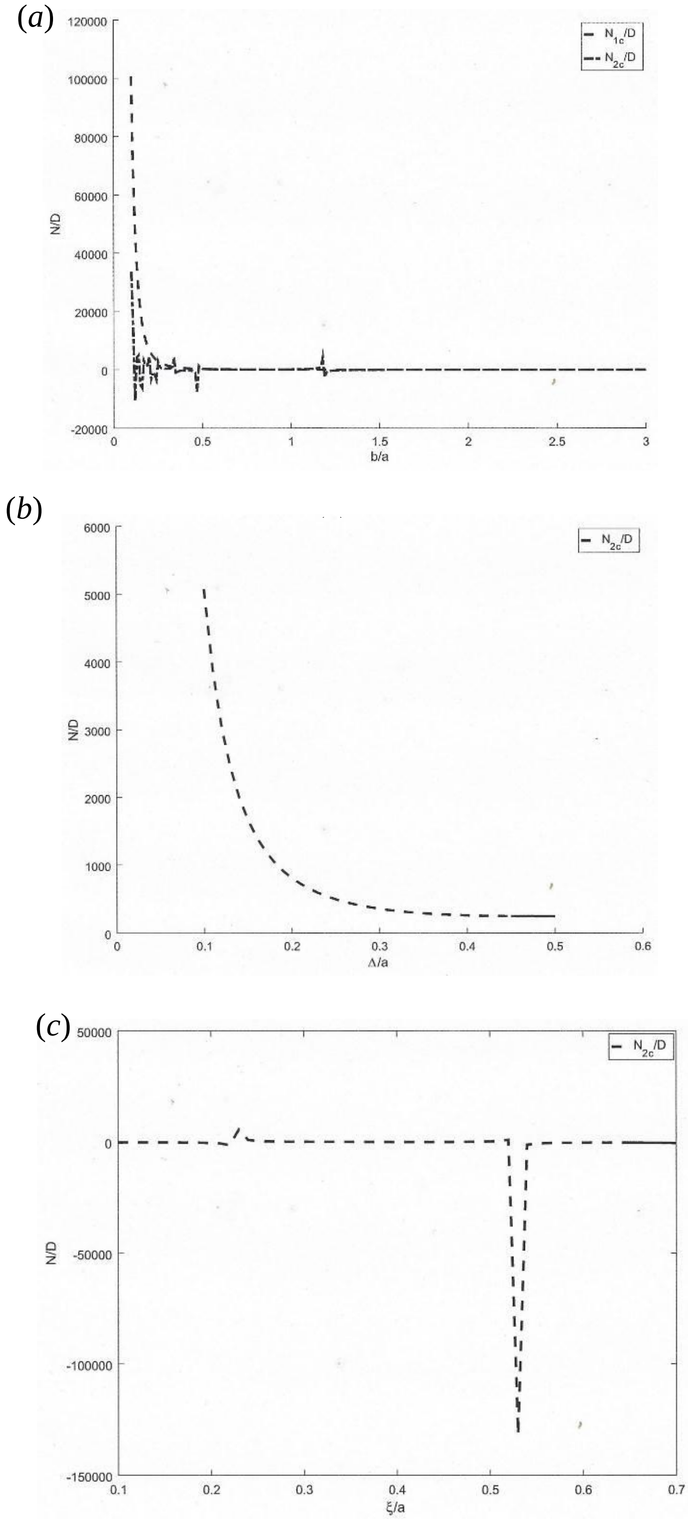

Fig. 6: Stability criteria for simply supported plates under intermediate load N_2 and end load N_1 vs. (a) different geometrical ratio *(a/b)* values (b) different delta (Δ) width length values (c) different ξ distance values.

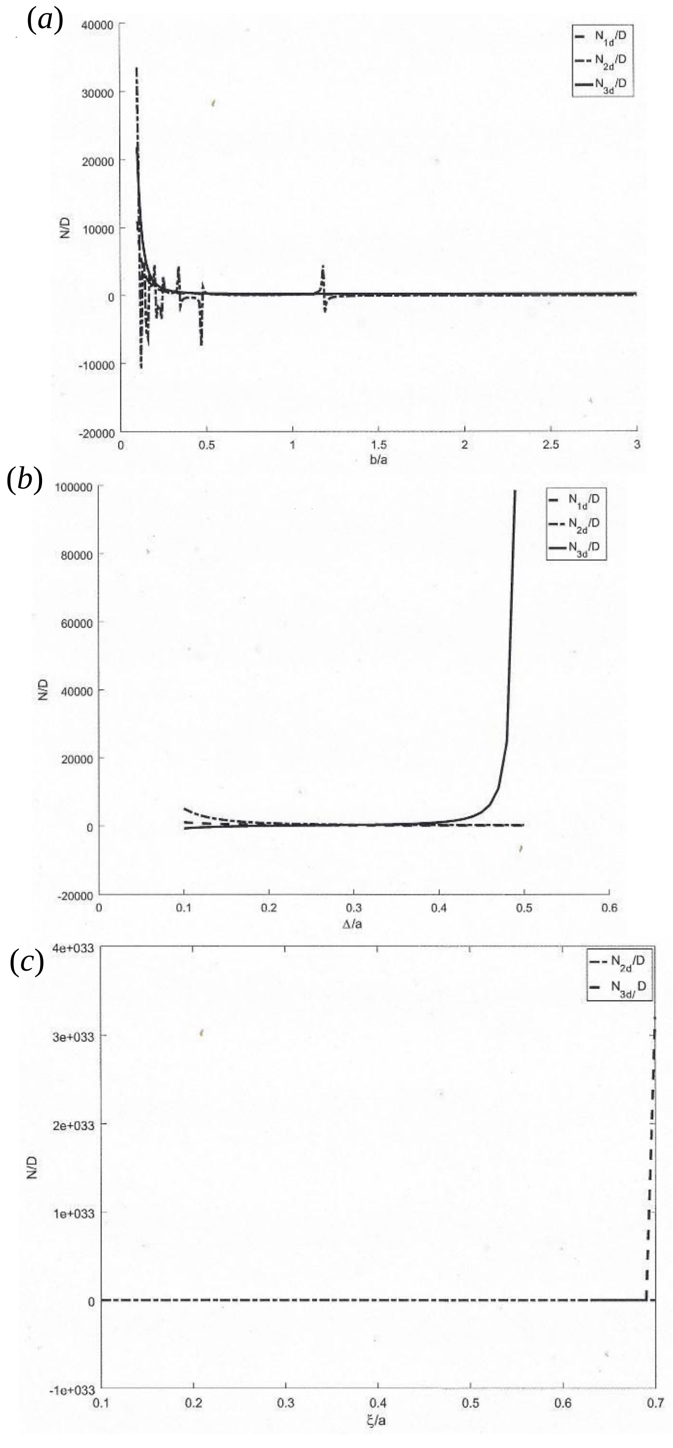

Fig. 7: Stability criteria for simply supported plates under intermediate load *N*2 and end load *N*1 vs. (a) different geometrical ratio (*a*/*b*) values (b) different delta (Δ) width length values (c) different ξ distance values.

4 Conclusion

Current study presents generalized static analytic solution for the critical buckling load of the simply supported rectangular plate subjected to the intermediate strip in-plane loading, based on the strain energy method. The expression was found to be dependent only on the strip thickness, strip location, plate width and stress magnitude. The developed solutions were found to agree quantitatively and qualitatively with the classical solution.

Compatible with the classical columns Euler method it was found that the plate stability is decreased with the increasing of the plate width. Accordingly, it can be assumed approximately that the minimum buckling load is linearly proportional to the geometrical ratio between plate length and plate width (*a*/*b*).

Moreover, the strip width (Δ) parameter analysis has shown that increase is likely to cause buckling stress development due to the fact that the increase of the strip width is accompanied with larger induced stresses that accelerating the buckling process. In addition, strip location (ξ) was found to be of importance due to the fact that the stresses acting in a further distance relative to the supports locations might accelerating the buckling process which is compatible with the Euler compressed column theory.

Additionally, expressions that includes both axial and transverse loads for different extended cases configurations were also derived and examined based on the strain energy method alongside explanation for possible applications (thin aluminum plate welding). In general view, it was found that the cases of combined axial and perpendicular loading action are less stabilized than cases where only one kind of loading configuration is participated.

Finally, comparison to literature references shows good qualitative agreement. Further studies in the field might include the dynamic case (with simulation performance) of plates and shells. Also, combination of out of plane load might contribute for the whole comprehension.

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Data Availability
All data, models, and code generated or used during the study appear in the submitted article.

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