Numerical Modeling of Crown Forest Fires Spread Using Averaged Setting

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Abstract: - The forest fires spread in the pine forests has been numerically simulated using a three-dimensional mathematical model. The model was integrated with respect to the vertical coordinate because horizontal sizes of forest are much greater than the heights of trees. In this paper, the assignment and theoretical investigations of the problems of crown forest fires spread pass the firebreaks were carried out. In this context, a study - mathematical modelling - of the conditions of forest fire spreading that would make it possible to obtain a detailed picture of the change in the temperature and component concentration fields with time, and determine as well as the limiting condition of fire propagation in forest with these fire breaks.

Key-Words: - Forest Fire; Mathematical Model; Crown Fire; Fire Spread; Control Volume; Numerical Method

1. Introduction

The forest fires are very complicated phenomena. At present, fire services can forecast the danger rating of, or the specific weather elements relating to, forest fire. There is need to understand and predict forest fire initiation, behaviour and spread. This paper's purposes are the improvement of knowledge on the fundamental physical mechanisms that control forest fire spread. A great deal of work has been done on the theoretical problem of how forest fire spread. Crown fires are initiated by convective and radiative heat transfer from surface fires. However, convection is the main heat transfer mechanism. Crown fires a more difficult to control than surface. The first accepted method for prediction of crown fires was given by Rothermal [1] and Van Wagner [3]. The semi-empirical models [1-2] allow to obtain a quite good data of the forest fire rate of spread as a function of fuel bulk and moisture, wind velocity and the terrain slope. But these models use data for particular cases and do not give results for general fire conditions. Also crown fires initiation and hazard have been studied and modeled in detail (eg: Alexander [3], Van Wagner [3], Xanthopoulos, [4], Van Wagner, [5], Cruz [6], Albini [7], Scott, J. H. and Reinhardt, E. D. [8].The discussion of the problem of modeling forest fires is provided by a group of co-workers at Tomsk University (Grishin [9], Grishin and Perminov [9], Perminov [11]). A mathematical model of forest fires was obtained by Grishin [9] based on an analysis of known and original experimental data [10,13], and using concepts and methods from reactive media mechanics. The physical two-phase models used in [14] may be considered as a development and extension of the formulation proposed by Grishin and Perminov [10-12]. However, the investigation of crown fires initiation has been limited mainly to cases studied of forest fires propagation without take into account the mutual interaction of crown forest fires with different obstacles(roads, glades and etc.). This paper's purpose is to demonstrate the influence of these firebreaks on crown forest fires spread.

2. Physical and mathematical model of crown forest fire

It is assumed that the forest during a forest fire can be modelled as 1) a multi-phase, multi-storeyed, spatially heterogeneous medium; 2) in the fire zone the forest is a porous-dispersed, two-temperature, single-velocity, reactive medium; 3) the forest canopy is supposed to be non - deformed medium (trunks, large branches, small twigs and needles), which affects only the magnitude of the force of resistance in the equation of conservation of momentum in the gas phase, i.e., the medium is assumed to be quasi-solid (almost non-deformable during wind gusts); 4) let there be a so-called "ventilated" forest massif, in which the volume of fractions of condensed forest fuel phases, consisting of dry organic matter, water in liquid state, solid

pyrolysis products, and ash, can be neglected compared to the volume fraction of gas phase (components of air and gaseous pyrolysis products); 5) the flow has a developed turbulent nature and molecular transfer is neglected; 6) gaseous phase density doesn't depend on the pressure because of the low velocities of the flow in comparison with the velocity of the sound. Let the point x_1, x_2 , x_3= 0 is situated at the centre of the surface forest fire source at the height of the roughness level, axis $0x_1$ directed parallel to the Earth's surface to the right in the direction of the unperturbed wind speed, axis $0x_2$ directed perpendicular to $0x_1$ and axis $0x_3$ directed upward (Fig. 1). Problem formulated above reduces to the solution of systems of equations (1)-(7):

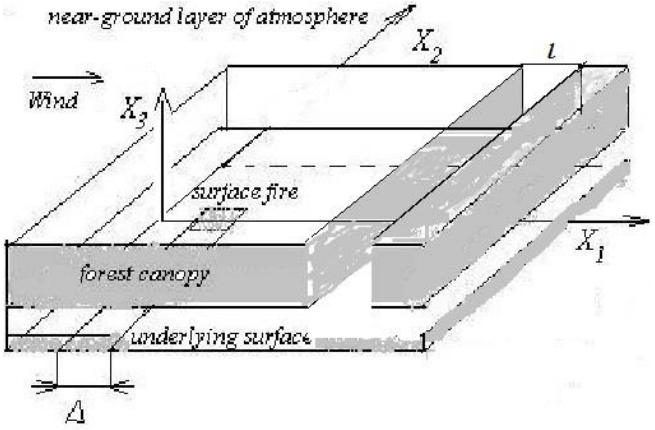

Fig. 1.

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x_j}(\rho v_j)=Q,\ j=1,2,\ i=1,2; \tag{1}$$

$$\rho\frac{dv_i}{dt}=-\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_j}(-\rho\overline{v_i'v_j'})- -\rho s c_d v_i\,|\vec{v}\,|-\rho g_i-Qv_i; \tag{2}$$

$$\rho c_p\frac{dT}{dt}=\frac{\partial}{\partial x_j}(-\rho c_p v_j'\overline{T'})+q_5R_5- -\alpha_v(T-T_s)+k_g(cU_R-4\sigma T^4); \tag{3}$$

$$\rho\frac{dc_\alpha}{dt}=\frac{\partial}{\partial x_j}(-\rho\overline{v_j'c_\alpha'})+R_{5\alpha}-Qc_\alpha,\alpha=1,5; \tag{4}$$

$$\frac{\partial}{\partial x_j}\left(\frac{c}{3k}\frac{\partial U_R}{\partial x_j}\right)-kcU_R+4k_S\sigma T_S^4+4k_g\sigma T^4=0, \tag{5}$$

$$k=k_g+k_S;$$

$$\sum_{i=1}^{4}\rho_i c_{pi}\varphi_i\frac{\partial T_S}{\partial t}= =q_3R_3-q_2R_2-k_S(cU_R-4\sigma T_S^4)+\alpha_v(T-T_S); \tag{6}$$

$$\rho_1\frac{\partial\varphi_1}{\partial t}=-R_1,\rho_2\frac{\partial\varphi_2}{\partial t}=-R_2,\ \rho_3\frac{\partial\varphi_3}{\partial t}=\alpha_C R_1-\frac{M_C}{M_1}R_3,\rho_4\frac{\partial\varphi_4}{\partial t}=0; \tag{7}$$

$$\sum_{\alpha=1}^{5}c_\alpha=1,\ p_e=\rho RT\sum_{\alpha=1}^{5}\frac{c_\alpha}{M_\alpha},\vec{v}=(v_1,v_2,v_3),\vec{g}=(0,0,g)$$

The system of equations (1)–(7) must be solved taking into account the initial and boundary conditions

$$x_1=-x_{1e}: v_1=V_e,\ v_2=0,\frac{\partial v_3}{\partial x_1}=0,T=T_e,\ c_\alpha=c_{\alpha e}, -\frac{c}{3k}\frac{\partial U_R}{\partial x_1}+cU_R/2=0; \tag{8}$$

$$x_1=x_{1e}:\frac{\partial v_1}{\partial x_1}=0,\ \frac{\partial v_2}{\partial x_1}=0,\frac{\partial v_3}{\partial x_1}=0,\ \frac{\partial c_\alpha}{\partial x_1}=0, \frac{\partial T}{\partial x_1}=0,\frac{c}{3k}\frac{\partial U_R}{\partial x_1}+\frac{c}{2}U_R=0; \tag{9}$$

$$x_2=x_{20}:\frac{\partial v_1}{\partial x_2}=0,\ \frac{\partial v_2}{\partial x_2}=0,\ \frac{\partial v_3}{\partial x_2}=0,\frac{\partial c_\alpha}{\partial x_2}=0, \frac{\partial T}{\partial x_2}=0,-\frac{c}{3k}\frac{\partial U_R}{\partial x_2}+\frac{c}{2}U_R=0; \tag{10}$$

$$x_2=x_{2e}:\frac{\partial v_1}{\partial x_2}=0,\ \frac{\partial v_2}{\partial x_2}=0,\frac{\partial v_3}{\partial x_2}=0,\frac{\partial c_\alpha}{\partial x_2}=0, \frac{\partial T}{\partial x_2}=0,\frac{c}{3k}\frac{\partial U_R}{\partial x_2}+\frac{c}{2}U_R=0. \tag{11}$$

$$x_3=0: v_1=0,\ v_2=0,\frac{\partial c_\alpha}{\partial x_3}=0,-\frac{c}{3k}\frac{\partial U_R}{\partial x_3}+\frac{c}{2}U_R=0, v_3=v_{30},T=T_g,\ |x_1|\le\Delta,|x_2|\le\Delta v_3=0,\ T=T_e,\ |x_1|>\Delta,|x_2|>\Delta; \tag{12}$$

$$x_3=x_{3e}:\frac{\partial v_1}{\partial x_3}=0,\ \frac{\partial v_2}{\partial x_3}=0,\frac{\partial v_3}{\partial x_3}=0,\frac{\partial c_\alpha}{\partial x_3}=0, \frac{\partial T}{\partial x_3}=0,\frac{c}{3k}\frac{\partial U_R}{\partial x_3}+\frac{c}{2}U_R=0. \tag{13}$$

Here and above $\frac{\mathrm{d}}{\mathrm{d}t}$ is the symbol of the total (substantial) derivative; α_v is the coefficient of phase exchange; ρ - density of gas – dispersed phase, t is time; v_i - the velocity components; T, T_S, - temperatures of gas and solid phases, U_R - density

of radiation energy, k - coefficient of radiation attenuation, P - pressure; c_p – constant pressure specific heat of the gas phase, c_{pi}, ρ_i, φ_i – specific heat, density and volume of fraction of condensed phase (1 – dry organic substance, 2 – moisture, 3 – condensed pyrolysis products, 4 – mineral part of forest fuel), R_i – the mass rates of chemical reactions, q_i – thermal effects of chemical reactions; k_g , k_S - radiation absorption coefficients for gas and condensed phases; T_e - the ambient temperature; c_α - mass concentrations of α - component of gas - dispersed medium, index α=1,2,...,5, where 1 corresponds to the density of oxygen, 2 - to carbon monoxide CO, 3 - to carbon dioxide and inert components of air, 4 - to particles of black, 5 - to particles of smoke; R – universal gas constant; M_α , M_C, and M molecular mass of α - components of the gas phase, carbon and air mixture; g is the gravity acceleration; c_d is an empirical coefficient of the resistance of the vegetation, s is the specific surface of the forest fuel in the given forest stratum. To define source terms which characterize inflow (outflow of mass) in a volume unit of the gas-dispersed phase, the following formulae were used for the rate of formulation of the gas-dispersed mixture Q, outflow of oxygen R_{51}, changing carbon monoxide R_{52}, generation of black R_{54} and smoke particles R_{55}.

$$Q = (1-\alpha_c)R_1 + R_2 + \frac{M_c}{M_1}R_3 + R_{54} + R_{55}$$

$$R_{51} = -R_3 - \frac{M_1}{2M_2}R_5, R_{52} = \nu_g(1-\alpha_c)R_1 - R_5,$$

$$R_{53} = 0, R_{54} = \alpha_4 R_1, R_{55} = \frac{\alpha_5 \nu_3}{\nu_3 + \nu_{3*}}R_3.$$

Here ν_g – mass fraction of gas combustible products of pyrolysis, α_4 and α_5 – empirical constants. Reaction rates of these various contributions (pyrolysis, evaporation, combustion of coke and volatile combustible products of pyrolysis) are approximated by Arrhenius laws whose parameters (pre-exponential constant k_i and activation energy E_i) are evaluated using data for mathematical models [10,12].

$$R_1 = k_1\rho_1\varphi_1 \exp(-\frac{E_1}{RT_S}), R_2 = k_2\rho_2\varphi_2 T^{-0.5}\exp(-\frac{E_2}{RT_S}),$$

$$R_3 = k_3\rho\varphi_3 S_\sigma c_1 \exp(-\frac{E_3}{RT_S}),$$

$$R_5 = k_5 M_2\left(\frac{c_1 M}{M_1}\right)^{0.5}\left(\frac{c_2 M}{M_2}\right)T^{-2.25}\exp(-\frac{E_5}{RT}).$$

The initial values for volume of fractions of condensed phases are determined using the expressions:

$$\varphi_{1e} = \frac{d(1-\nu_z)}{\rho_1}, \varphi_{2e} = \frac{Wd}{\rho_2}, \varphi_{3e} = \frac{\alpha_c\varphi_{1e}\rho_1}{\rho_3}$$

where d -bulk density for surface layer, ν_z – coefficient of ashes of forest fuel, W – forest fuel moisture content. It is supposed that the optical properties of a medium are independent of radiation wavelength (the assumption that the medium is "grey"), and the so-called diffusion approximation for radiation flux density were used for a mathematical description of radiation transport during forest fires. To close the system (1)–(7), the components of the tensor of turbulent stresses, and the turbulent heat and mass fluxes are determined using the local-equilibrium model of turbulence (Grishin, [10]). The system of equations (1)–(7) contains terms associated with turbulent diffusion, thermal conduction, and convection, and needs to be closed. The components of the tensor of turbulent stresses $\rho\overline{v_i'v_j'}$, as well as the turbulent fluxes of heat and mass $\overline{\rho v_j' c_p T'}$, $\overline{\rho v_j' c_\alpha'}$ are written in terms of the gradients of the average flow properties using the formulas

$$-\rho\overline{v_i v_j} = \mu_t\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{2}{3}K\delta_{ij},$$

$$-\rho\overline{v_j c_p T'} = \lambda_t\frac{\partial T}{\partial x_j}, \ -\rho\overline{v_j c_\alpha'} = \rho D_t\frac{\partial c_\alpha}{\partial x_j},$$

$$\lambda_t = \mu_t c_p / \mathrm{Pr}_t, \rho D_t = \mu_t / Sc_t, \mu_t = c_\mu\rho K^2/\varepsilon,$$

where μ_t, λ_t, D_t are the coefficients of turbulent viscosity, thermal conductivity, and diffusion, respectively; Pr_t, Sc_t are the turbulent Prandtl and Schmidt numbers, which were assumed to be equal to 1. In dimensional form, the coefficient of dynamic turbulent viscosity is determined using local equilibrium model of turbulence [11]. The length of the mixing path is determined using the formula $l = x_3 k_t/(1+2.5x_3\sqrt{c_d s/h})$ taking into account the fact that the coefficient of resistance c_d in the space between the ground cover and the

forest canopy base is equal to zero, while the constants k_t = 0.4 and $h=h_2-h_1$ (h_2, h_1 – height of the tree crowns and the height of the crown base). It should be noted that this system of equations describes processes of transfer within the entire region of the forest massif, which includes the space between the underlying surface and the base of the forest canopy, the forest canopy and the space above it, while the appropriate components of the data base are used to calculate the specific properties of the various forest strata and the near-ground layer of atmosphere. This approach substantially simplifies the technology of solving problems of predicting the state of the medium in the fire zone numerically. The thermodynamic, thermophysical and structural characteristics correspond to the forest fuels in the canopy of a different (for example pine [10,12]) type of forest. Because of the horizontal sizes of forest massif more than height of forest – *h*, system of equations of general mathematical model of forest fire (1)-(7) was integrated between the limits from height of the roughness level - 0 to *h*. Besides, suppose that

$$\int_0^h \varphi dx_3 = \overline{\varphi} h$$

$\overline{\varphi}$ - average value of φ. The problem formulated above is reduced to a solution of the following system of equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = Q - (\dot{m}^- - \dot{m}^+)/h, \; j = 1,2,3; \quad (14)$$

$$\rho \frac{dv_i}{dt} = -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j}(-\rho \overline{v_i' v_j'}) - \rho s c_d v_i |\vec{v}| - \quad (15)$$
$$- \rho g_i - Q v_i + (\tau_i^- - \tau_i^+)/h, \; i = 1,2,3;$$

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x_j}(-\rho c_p \overline{v_j' T'}) + q_5 R_5 - \alpha_v (T - T_s) + \quad (16)$$
$$+ k_g (cU_R - 4\sigma T^4) - (q_T^- - q_T^+)/h;$$

$$\rho \frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j}(-\rho \overline{v_j' c_\alpha'}) + R_{5\alpha} - Q c_\alpha + \quad (17)$$
$$+ (J_\alpha^- - J_\alpha^+)/h, \; \alpha = \overline{1,5};$$

$$\frac{\partial}{\partial x_j}\left(\frac{c}{3k}\frac{\partial U_R}{\partial x_j}\right) - kcU_R + 4k_S \sigma T_S^4 + \quad (18)$$
$$+ 4k_g \sigma T_S^4 + (q_R^- - q_R^+)/h = 0;$$

$$\sum_{i=1}^{4} \rho_i c_{pi} \varphi_i \frac{\partial T_S}{\partial t} = q_3 R_3 - q_2 R_2 + k_S (cU_R - \quad (19)$$
$$- 4\sigma T_S^4) + \alpha_V (T - T_S);$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_1, \rho_2 \frac{\partial \varphi_2}{\partial t} = -R_2, \quad (20)$$
$$\rho_3 \frac{\partial \varphi_3}{\partial t} = \alpha_C R_1 - \frac{M_C}{M_1} R_3, \rho_4 \frac{\partial \varphi_4}{\partial t} = 0;$$
$$\sum_{\alpha=1}^{5} c_\alpha = 1, \; p_e = \rho R T \sum_{\alpha=1}^{5} \frac{c_\alpha}{M_\alpha}, \vec{v} = (v_1, v_2, v_3), \vec{g} = (0,0,g)$$

In system of equations (14)-(20) are introduced the next designations:

$$\dot{m} = \rho v_3, \tau_i = -\rho \overline{v_i' v_3'}, J_\alpha = -\rho \overline{v_3' c_\alpha'}, J_T = -\rho \overline{v_3' T'}$$

Upper indexes "+" and "-" designate values of functions at $x_3=h$ and $x_3=0$ correspondingly. It is assumed that heat and mass exchange of fire front and boundary layer of atmosphere are governed by Newton law and written using the formulas:

$$(q_T^- - q_T^+)/h = -\alpha (T - T_e)/h,$$
$$(J_\alpha^- - J_\alpha^+)/h = -\alpha (c - c_{\alpha e})/h c_p.$$

The system of equations (14)–(20) must be solved taking into account the initial and boundary conditions.

3. Numerical solution and results

The boundary-value problem (14)–(20) is solved numerically. In order to efficiently solve this problem in a reactive flow the method of splitting according to physical processes (Perminov [11] was used. The basic idea of this method is based on the information that the physical timescale of the processes is great than chemical. In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions was calculated. Then the system of ordinary differential equations of chemical kinetics obtained as a result of splitting was then integrated. The time step for integrating each function has to be smaller than the characteristic time of physical process to ensure the convergence of the numerical method. The time step was selected automatically. A discrete analogue was obtained by means of the control volume method using the SIMPLE like algorithm (Patankar [15]). The accuracy of the program was checked by the method of inserted analytical solutions. Analytical expressions for the unknown functions were substituted in (14)–(20) and the closure of the equations was calculated. This was then treated as the source in each equation. Next, with the aid of the algorithm described above, the values of the functions used were inferred with an accuracy of not less than 1%. The effect of the dimensions of the control volumes on the solution was studied by diminishing them.

Fields of temperature, velocity, component mass fractions, and volume fractions of phases were obtained numerically. The first stage is related to increasing maximum temperature in the place of ignition with the result that a crown fire source appears. At this process stage over the fire source a thermal wind is formed a zone of heated forest fire prolysis products which are mixed with air, float up and penetrate into the crowns of trees. As a result, forest fuels in the tree crowns are heated, moisture evaporates and gaseous and dispersed pyrolysis products are generated. Ignition of gaseous pyrolysis products of the crown occurs at the next stage, and that of gaseous pyrolysis products in the forest canopy occurs at the last stage. As a result of heating of forest fuel elements of crown, moisture evaporates, and pyrolysis occurs accompanied by the release of gaseous products, which then ignite and burn away in the forest canopy. At the moment of ignition, the gas combustible products of pyrolysis burn away, and the concentration of oxygen is rapidly reduced. The temperatures of both phases reach a maximum value at the point of ignition. The ignition processes is of a gas - phase nature. Figures 2 present the distribution for different instants of time for temperature *a)* $\overline{T}$ ($\overline{T} = T / T_e, T_e = 300K$) (1 – 1.5, 2- 2., 3 – 2.6, 4 – 3, 5 – 3.5, 6 – 4.) for gas phase, *b)* oxygen $\overline{c}_1$ (1 – 0.1, 2 – 0.5, 3 – 0.6, 4 – 0.7, 5 – 0.8, 6 – 0.9), *c)* volatile combustible products of pyrolysis $\overline{c}_2$ concentrations (1 – 1., 2- 0.1, 3 – 0.05, 4 – 0.01) ($\overline{c}_\alpha = c_\alpha / c_{1e}, c_{1e} = 0.23$) for different instants of time. The isotherms are moved in the forest canopy and deformed by the action of wind (Figure 2*a*). Similarly, the fields of component concentrations oxygen (Figure 2b) are deformed. It is concluded that the forest fire begins to spread. The results of calculation give an opportunity to evaluate critical condition of the forest fire spread, which allows applying the given model for preventing fires. It overestimates the rate of crown forest fire spread that depends on crown properties: bulk density, moisture content of forest fuel and etc.

The model proposed there give a detailed picture of the change in the velocity, temperature and component concentration fields with time, and determine as well as the influence of different conditions on the crown forest fire initiation.

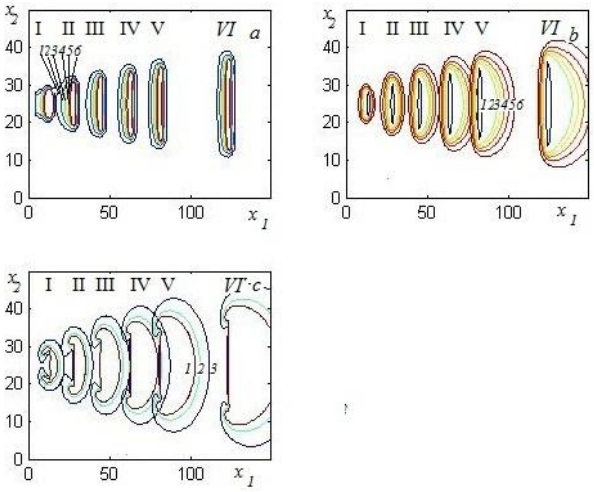

Fig. 2. I – t=3 s, II – 7 s, III - 12 s, IV - 18 s, V - 24 s, VI -38 s; *V*=5m/s.

It is important to study the interaction of forest fire front with firebreak of finite size (glade) (Figure 3. *a)* temperature *T* for gas phase, *b)* oxygen $\overline{c}_1$, *c)* volatile combustible products of pyrolysis $\overline{c}_2$ concentrations).

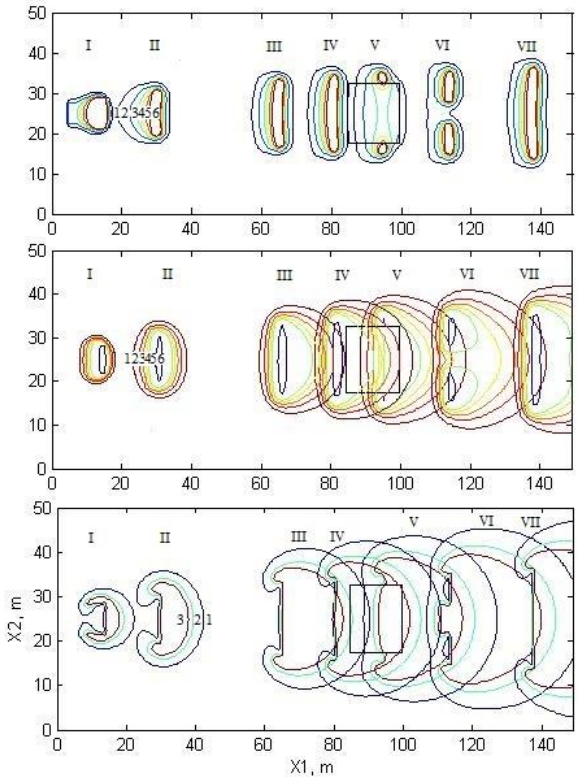

Fig. 3. I – t=3 s, II – 7 s, III - 12 s, IV - 18 s, V - 24 s, VI – 32 s, VII - 38 s; *V*=5m/s.

The distance between forest fire source and glade

equals 84 m. Figures 3(*a, b,c*) show the results of numerical simulation of a forest fire spreading around the glade under the action of wind blowing through it at a speed 5 m/s in the direction of the Ox_1-axis. Initially, the source of the fire has the shape of a rectangular. Then isotherms are deformed under the action of wind and the contour of forest fire is look as crescent. When the fire (isotherms II in Figure 3 *a*) moves around the forest glade it is divided in two parts. But after that two fire fronts were joined in united fire (isotherms VI in Figure.3 *a*). Figures 3 (*b, c*) present the distribution of concentration of oxygen and volatile combustible products of pyrolysis $\bar{c}_2$ for this case. If in our case the distance between the initial forest fire source and glade is increased to 74 m the crown forest fire does not spread around the glade and the forest fire dies before this clearing (Figure 4. *a)* temperature $\bar{T}$ for gas phase, concentrations for *b)* oxygen $\bar{c}_1$, *c)* volatile combustible products of pyrolysis $\bar{c}_2$ concentrations).). The results of calculation give an opportunity to evaluate critical condition of the forest fire spread, which allows applying the given model for preventing fires.

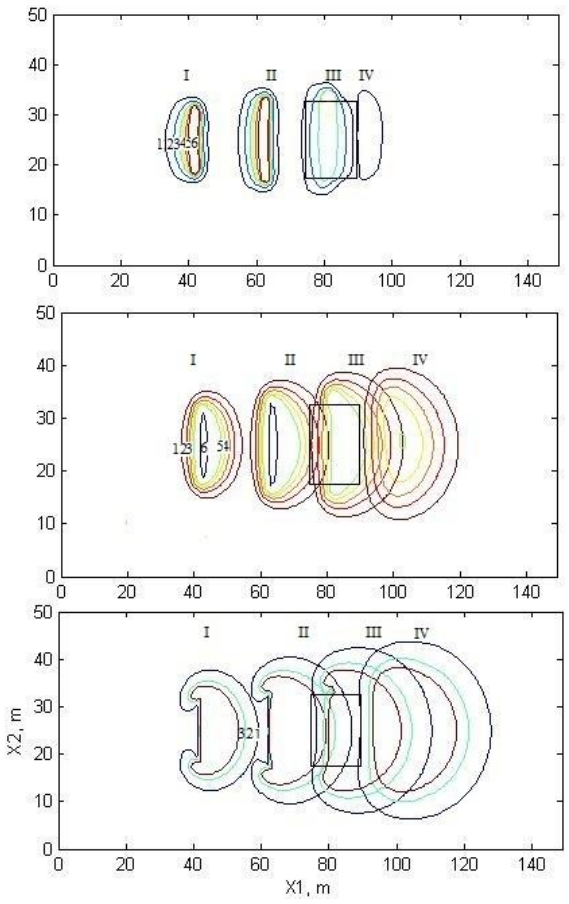

Fig. 4. I – t=11 s, II – 18 s, III - 24 s, IV - 28 s; *V*=5m/s.

4. Conclusions

It overestimates the rate of crown forest fire spread that depends on crown properties: bulk density, moisture content of forest fuel and etc. The model proposed there give a detailed picture of the change in the velocity, temperature and component concentration fields with time, and determine as well as the influence of different conditions on the crown forest fire initiation. The results obtained agree with the laws of physics and experimental data (Grishin [9]; Konev [13]).

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