Heat Energy Transfers Inside the Double Circular Flow Heating System

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Abstract: - The paper discusses the transfer of heat energy within the system containing double heating . Presented is an original mathematical model that describes the considered transfer of heat energy. The paper describes mathematical functions applicable for the model. Defined are typical scenarios that may arise within the system and given are conditions of their occurrence. Presented is a problem of measurement of heat energy using available calorimeters and proposed a solution for its elimination.

Key-Words: - mathematical model, transfer of heat energy, regulation, heating system, calorimeter, fluid flow, heat transfer rate

1 Introduction

Today, heating systems are more complex than they were in the past. Heating systems in the past normally had only one heating circuit consisting of the device for heat energy[1] production (input device of system) through which heat enters the system and the device on which it is consumed (output device of system). When the heating system consists of more than one heating circuits, the question is raised whether the mathematical model used for the system with one heating circuit may be applied to each of heating circuits. This paper describes mathematical model of the heating system with two heating circuits, describes the transfer of heat energy within the system, and shows that the answer to the above question is negative.

2 Description of Considered System

Studied heating system (see Figure 1.) consists of two heating circuits (N1-N3-N4-N6-N1 and N2-N3-N4-N5-N2) connected on the common part (N2-N3-N4-N5). Heating devices U1 and U2, through which heat leaves the heating system, are positioned on individual parts of each of the two heating circuits. Heating device U3, through which heat enters the heating system, is positioned on the common part of the two heating circuits. The heat flow rate[2] at devices U1, U2 and U3 are $\dot{Q}_1$, $\dot{Q}_2$ and $\dot{Q}_3$, respectively.

The following figure shows the observed heating system:

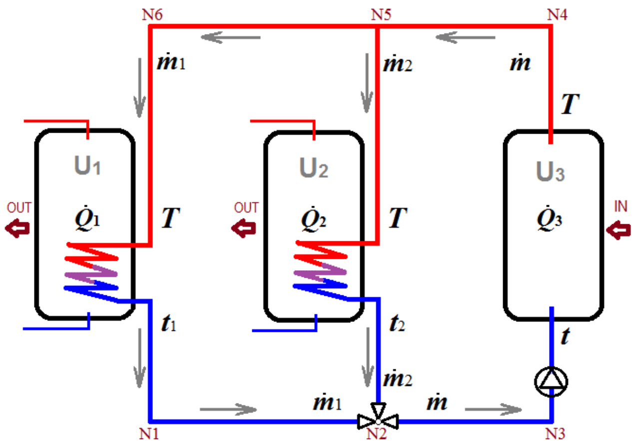

Fig.1 Double circular flow heating system

Through the system circulates fluid. Its thermal losses may be neglected. The total mass of the fluid circulating through the system we consider a constant. The temperatures of the fluid leaving the devices U1, U2 and U3 are t_1, t_2 and T, respectively. The temperatures of the fluid entering the devices U1, U2 and U3 are T, T and t, respectively. A three-way valve is positioned at the junction of independent parts of the circles and the

[1] If this energy is a positive we are talking about the energy of heating, if it is negative we are talking about the energy of cooling.

[2] The heat transfer rate. We will say heat flow.

common part in the direction of fluid flow to the device U3 (the knob N2). Our focus is the mass fluid flow rate[3] $\dot{m}$ of the fluid which passes through the three-way valve to the device U3. The three-way valve regulates the ratio of the mass fluid flow rates $\dot{m}_1$ and $\dot{m}_2$, coming from the direction of the devices U1 and U2 in the total mass fluid flow $\dot{m}$ entering the U3. Obviously $\dot{m} = \dot{m}_1 + \dot{m}_2$.

For example, the heating device U3 may be a solar collector or a gas heater, while heating devices U1 and U2 may be a heat storages with a heat exchanger.

3 Mathematical Expressions of the System

It is assumed that $t_1 < t_2$. Assuming that there is no loss of energy, equation $\dot{Q}_1 + \dot{Q}_2 = \dot{Q}_3$ is valid. The share of fluid flow $\dot{m}_1$ in the fluid flow $\dot{m}$ is denoted by α, $(0 \le \alpha \le 1)$. The parameter α is regulated by the three-way valve and it is its characteristics. Now we have that $\dot{m}_1 = \alpha\,\dot{m}$, and $\dot{m}_2 = (1-\alpha)\dot{m}$. As $t_1 < t_2$, at three-way valve, heat energy is moving from the fluid flow $\dot{m}_2$ to the fluid flow $\dot{m}_1$. With $\dot{q}_1$ is denoted heat flow of the heat energy received by the fluid flow $\dot{m}_1$ at time of mixing with fluid flow $\dot{m}_2$. With $-\dot{q}_2$ is denoted heat flow of the heat energy transfers from the warmer fluid flow $\dot{m}_2$ to the cooler fluid flow $\dot{m}_1$ at their interference. Due to energy conservation law we have $\dot{q}_1 = -\dot{q}_2$. Suppose that the fluid heat capacity c is constant for observed temperatures of the fluids.

Since $\dot{q}_1 = \dot{m}_1(t-t_1)c = \alpha(t-t_1)\dot{m}c$, and $\dot{q}_2 = \dot{m}_2(t-t_2)c = (1-\alpha)(t-t_2)\dot{m}c$ we obtain that the fluid temperature entering the U3 is

$$t = \alpha t_1 + (1-\alpha)t_2\,. \quad (1)$$

Obviously, $t_1 \le t \le t_2$. Substituting this equality in terms of $\dot{q}_1$ and $\dot{q}_2$ we have

$$\dot{q}_1 = -\dot{q}_2 = \alpha(1-\alpha)(t_2-t_1)\dot{m}c = \\ = [-(t_2-t_1)\alpha^2 + (t_2-t_1)\alpha]\dot{m}c, \quad (2)$$

i.e., flow rate of the heat passes from the fluid flow $\dot{m}_2$ to the fluid flow $\dot{m}_1$ is a square function of the parameter α. The temperature of the fluid at the outlet of the device U3, in relation to the temperature at the entrance, increases by $T-t$, and the heat flow rate of the fluid flow $\dot{m}$ is increased to $\dot{Q}_3 = \dot{m}(T-t)c$. A part of the heat flow of the heat energy generated in the device U3 $\dot{d}_1 = \alpha\dot{m}(T-t)c$ fluid flow $\dot{m}_1$ transfers to the device U1 and the rest of the generated heat flow $\dot{d}_2 = (1-\alpha)\dot{m}(T-t)c$ fluid flow $\dot{m}_2$ transfers to the device U2. The heat flows $\dot{Q}_1$ and $\dot{Q}_2$ that comes out of the system through the devices U1 and U2 are given by:

$$\dot{Q}_1 = \dot{m}_1(T-t_1)c = \\ = \alpha(T-t+t-t_1)\dot{m}c = \\ = \dot{d}_1 + \dot{q}_1\,, \quad (3)$$

$$\dot{Q}_2 = \dot{m}_2(T-t_2)c = \\ = (1-\alpha)(T-t+t-t_2)\dot{m}c = \\ = \dot{d}_2 + \dot{q}_2 = \dot{d}_2 - \dot{q}_1\,. \quad (4)$$

Thus, the aforementioned shares of the heat flow of transmitted energy entered into the system via the device U3 ($\dot{d}_1$ and $\dot{d}_2$) are not equal to amounts of heat flow $\dot{Q}_1$ and $\dot{Q}_2$ that comes out of the system via the devices U1 and U2. The amount $\dot{Q}_1$ is larger and $\dot{Q}_2$ is smaller just for the amount $\dot{q}_1$ which is the amount of the heat flow transferred from the device U2 to the device U1.

While $\dot{d}_1$ and $\dot{d}_2$ are the shares of the heat flow of the incoming heat energy moved from the heating device U3 to the heating devices U1 and U2, $\dot{q}_1$ is the heat flow of heat energy moved from the warmer device U2 to the cooler device U1 and depends on the three-way valve mixing parameter α.

Example function $\dot{d}_1(\alpha)$, $\dot{d}_2(\alpha)$, $\dot{q}_1(\alpha)$ and $\dot{q}_2(\alpha)$, is shown in Figure 2 (case: $\dot{m}c = 0{,}660$; $T-t = 14$; $t_2 - t_1 = 12$,). Each of functions $\dot{q}_1$ and $\dot{q}_2$ its extreme value $\pm 0{,}25(t_2-t_1)\dot{m}c$ achieve for $\alpha = 0{,}5$, and for $\alpha = 0$ and $\alpha = 1$ their value is zero. Functions $\dot{d}_1$ and $\dot{d}_2$ are linear functions of the parameter α and worth ratio:
$\dot{d}_1 : \dot{d}_2 = \dot{m}_1 : \dot{m}_2 = \alpha : (1-\alpha)$.

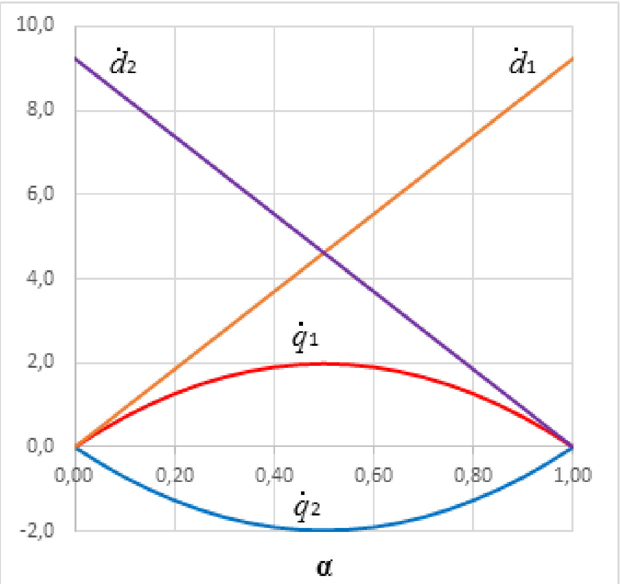

Fig.2 Amounts of transferred heat flow

[3] We will say fluid flow.

Let us denote the rise in temperature of the fluid in the U3 $T - t$ with ΔT and denote difference of output temperature at the devices U2 and U1 $t_2 - t_1$ with $\Delta t_{1,2}$. By including ΔT, $\Delta t_{1,2}$ and expression for the temperature $t = \alpha t_1 + (1-\alpha)t_2$ in (3) and (4) we get that $\dot{Q}_1$ and $\dot{Q}_2$ are quadratic function of the parameter α as follows:

$$\dot{Q}_1(\alpha) = \left[-\Delta t_{1,2}\,\alpha^2 + \left(\Delta t_{1,2} + \Delta T\right)\alpha\right]\dot{m}c =$$
$$= -\Delta t_{1,2}\,\alpha\left[\alpha - \left(1 + \frac{\Delta T}{\Delta t_{1,2}}\right)\right]\dot{m}c\,,$$

(5)

$$\dot{Q}_2(\alpha) = \left[\Delta t_{1,2}\,\alpha^2 - \left(\Delta t_{1,2} + \Delta T\right)\alpha + \Delta T\right]\dot{m}c =$$
$$= \Delta t_{1,2}\,(1-\alpha)\left[-\alpha + \frac{\Delta T}{\Delta t_{1,2}}\right]\dot{m}c\,.$$

(6)

It is valid that: $\dot{Q}_1(0) = 0$, $\dot{Q}_1(1) = \Delta T\dot{m}c$, $\dot{Q}_2(0) = \Delta T\dot{m}c$, $\dot{Q}_2(1) = 0$.

Also is true that:

$\dot{Q}_1(\alpha) + \dot{Q}_2(\alpha) = \dot{Q}_1(1) = \dot{Q}_2(0) = \Delta T\dot{m}c = \dot{Q}_3$.

Generally, for $\Delta T \neq -\Delta t_{1,2}$ function $\dot{Q}_1(\alpha)$ has two roots (zeros):

$$\alpha_1 = 0 \text{ and } \alpha_2 = 1 + \frac{\Delta T}{\Delta t_{1,2}}\,. \quad (7)$$

For $\Delta T = -\Delta t_{1,2}$ function $\dot{Q}_1(\alpha)$ has one root $\alpha_1 = \alpha_2 = 0$.

Function $\dot{Q}_2(\alpha)$, for $\Delta T \neq \Delta t_{1,2}$ has two roots:

$$\alpha_1 = \frac{\Delta T}{\Delta t_{1,2}}, \qquad \alpha_2 = 1\,. \quad (8)$$

For $\Delta T = \Delta t_{1,2}$, function $\dot{Q}_2(\alpha)$ has one root $\alpha_1 = \alpha_2 = 1$.

Function $\dot{Q}_1(\alpha)$ has a maximum for

$\alpha_M = \frac{1}{2}\left(1 + \frac{\Delta T}{\Delta t_{1,2}}\right)$ and

$\dot{Q}_1(\alpha_M) = \Delta t_{1,2}\alpha_M^2\dot{m}c$.

Function $\dot{Q}_2(\alpha)$ has a minimum for

$\alpha_m = \frac{1}{2}\left(1 + \frac{\Delta T}{\Delta t_{1,2}}\right)$ and

$\dot{Q}_2(\alpha_m) = \Delta T\dot{m}\,c - \Delta t_{1,2}\alpha_m^2\dot{m}c$.

Because we assume that $\Delta t_{1,2} > 0$ (i.e. $t_1 < t_2$), always is true that $\dot{Q}_1(\alpha_M) > 0$ and $\dot{Q}_2(\alpha_m) < 0$.

We will be interested in only the case when the zeros are within the interval [0,1].

<u>For $\Delta T > 0$</u>: $\dot{Q}_1(\alpha)$ has one root, $\alpha_1 = 0$, on [0,1]; $\dot{Q}_2(\alpha)$ on [0,1] has one root $\alpha_2 = 1$ if $\Delta T \geq \Delta t_{1,2}$, and two earlier mentioned roots (see 8) when $0 < \Delta T < \Delta t_{1,2}$.

$\dot{Q}_1(\alpha)$ has maximum in α_M on [0,1], and $\dot{Q}_2(\alpha)$ has minimum in α_m on [0,1] if $0 < \Delta T \leq \Delta t_{1,2}$.

<u>For $\Delta T < 0$</u>: $\dot{Q}_1(\alpha)$ on [0,1] has two earlier mentioned roots (see 7) when $-\Delta t_{1,2} < \Delta T < 0$, and only one root $\alpha_1 = 0$ when $\Delta T \leq -\Delta t_{1,2}$; $\dot{Q}_2(\alpha)$ has only one root, $\alpha_2 = 1$, on [0,1].

$\dot{Q}_1(\alpha)$ has maximum in α_M, a $\dot{Q}_2(\alpha)$ has minimum in α_m on [0,1] when $-\Delta t_{1,2} \leq \Delta T < 0$.

See Figure 3. (case: $\dot{m}c = 0{,}66$; $t_2 - t_1 = 12$). It clearly shows that, in some cases, $\dot{Q}_2$ can take a negative value. In these cases through the device U2 heat energy enters the system, i.e. system cools the heating device U2. The heat energy that is in this case entered into the system is transferred to the cooler device U1.

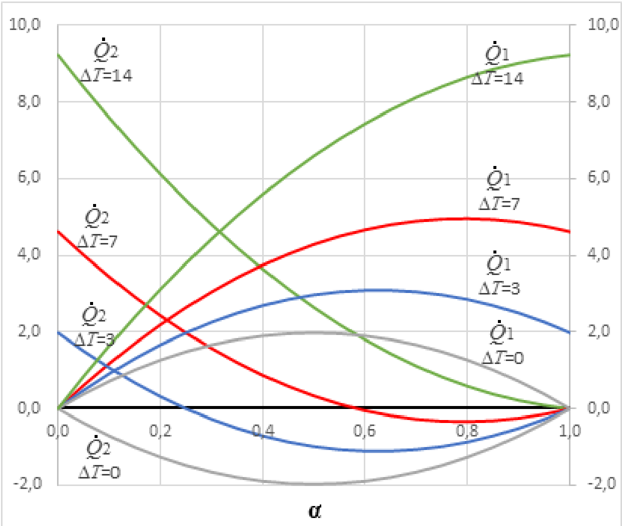

Fig.3 Heat flows emitted at devises U2 and U1

In case $\Delta t_{1,2} = 0$ (i.e., $t_1 = t_2$) we have:

$\dot{q}_1 = -\dot{q}_2 = 0$,

$\dot{Q}_1(\alpha) = \dot{d}_1 = \alpha\Delta T\dot{m}c$,

$\dot{Q}_2(\alpha) = \dot{d}_2 = (1-\alpha)\Delta T\dot{m}c$.

In case $\Delta t_{1,2} < 0$ (i.e. $t_1 > t_2$) terms for $\dot{q}_1$, $\dot{q}_2$, $\dot{d}_1$, $\dot{d}_2$, Q_1 and $\dot{Q}_2$ remain the same.

If we consider functions $\dot{Q}_1$ and $\dot{Q}_2$ in (5) and (6) as a function of three variables α, ΔT and $\Delta t_{1,2}$, it is easy to show that:

$$\dot{Q}_1(\alpha, \Delta T, \Delta t_{1,2}) = \dot{Q}_2(1-\alpha, \Delta T, -\Delta t_{1,2}),$$
$$\dot{Q}_2(\alpha, \Delta T, \Delta t_{1,2}) = \dot{Q}_1(1-\alpha, \Delta T, -\Delta t_{1,2}),$$

(9)

$$\dot{Q}_1(\alpha, \Delta T, \Delta t_{1,2}) = -\dot{Q}_2(1-\alpha, -\Delta T, \Delta t_{1,2}),$$
$$\dot{Q}_2(\alpha, \Delta T, \Delta t_{1,2}) = -\dot{Q}_1(1-\alpha, -\Delta T, \Delta t_{1,2}),$$

(10)

$$\dot{Q}_1(\alpha, \Delta T, \Delta t_{1,2}) = -\dot{Q}_1(\alpha, -\Delta T, -\Delta t_{1,2}),$$
$$\dot{Q}_2(\alpha, \Delta T, \Delta t_{1,2}) = -\dot{Q}_2(\alpha, -\Delta T, -\Delta t_{1,2}).$$

(11)

Notice that $\dot{Q}_1$ and $\dot{Q}_2$ given in (5) and (6) are linear functions in ΔT. Functions $\dot{d}_1$, $\dot{d}_2$ and $\dot{Q}_3$ are also linear functions in ΔT. Figure 4 (the case $\dot{m}c = 0{,}66$; $\Delta t_{1,2} = 12$; $\alpha = 0{,}7$) shows graphs of functions $\dot{d}_1(\Delta T)$, $\dot{d}_2(\Delta T)$, $\dot{Q}_1(\Delta T)$, $\dot{Q}_2(\Delta T)$ and

$\dot{Q}_3(\Delta T)$. The abscissas of points of intersections of lines in the picture do not depend on $\dot{m}c$.

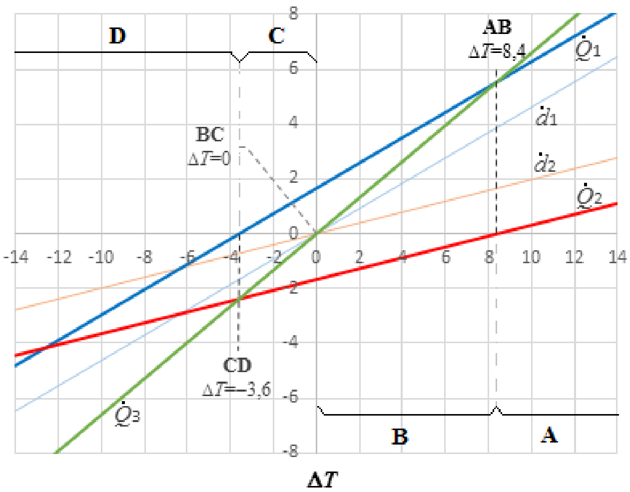

Fig.4 Heat flows emitted at devises U1, U2 ($\dot{Q}_1, \dot{Q}_2$) and generated at device U3 ($\dot{Q}_3$) as a function of ΔT and typical cases

Further we discuss several possible cases and thereby we allow that energy $\dot{Q}_3$ may be zero or negative, i.e. it is possible case of cooling system through U3, that is $T \leq t$ ($\Delta T \leq 0$).

4 Possible Typical Cases of Energy Transfer System

If $\Delta t_{1,2} = 0$, i.e. $t_1 = t_2$ then $\dot{q}_1 = \dot{q}_2 = 0$, $\dot{Q}_1 = \alpha\, \Delta T m\, c$ and $\dot{Q}_2 = (1-\alpha)\Delta T \dot{m} c$.
If $\alpha = 0$ then $\dot{q}_1 = \dot{q}_2 = 0$, $\dot{Q}_2 = \dot{Q}_3 = \Delta T \dot{m} c$ and $\dot{Q}_1 = 0$.
If $\alpha = 1$ then $\dot{q}_1 = \dot{q}_2 = 0$, $\dot{Q}_1 = \dot{Q}_3 = \Delta T \dot{m} c$ and $\dot{Q}_2 = 0$.
If $\Delta T = 0$ it is true that $\dot{Q}_1 = \dot{q}_1$, and $\dot{Q}_2 = \dot{q}_2$, i.e. heat is transferred only within the system from the warmer device U2 to the colder device U1 when $\Delta t_{1,2} > 0$ and from the warmer device U1 to the colder device U2 when $\Delta t_{1,2} < 0$.

Furthermore, we observe the case when $\Delta t_{1,2} > 0$ (i.e. $t_1 < t_2$). We assume that $0 < \alpha < 1$ and that ΔT can take both positive and negative values. Consider the typical cases (scenarios) depending on the values that ΔT can take.

Consider a combination of possible cases when $\dot{Q}_1, \dot{Q}_2$ and ΔT are bigger, smaller and equal to zero. Then all observed cases can be reduced to four disjunctive typical cases determined by a four interval values for ΔT. We denote each of these cases, i.e. each of these intervals, with large letters A, B, C and D. They are shown in Figure 5. with its three contact points. Transitional cases, i.e. the join points of two intervals, are denoted by AB, BC and CD.

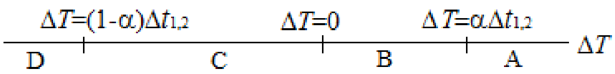

Fig.5 Typical case intervals

From equation (5) it follows that $\dot{Q}_1$ is > 0 when $\Delta T > -(1-\alpha)\Delta t_{1,2}$, and from (6) that $\dot{Q}_2$ is > 0 when $\Delta T > \alpha \Delta t_{1,2}$. Because $-(1-\alpha)\Delta t_{1,2} < 0$ and $\alpha \Delta t_{1,2} > 0$ we have that for $\Delta T > \alpha\Delta t_{1,2}$, $\dot{Q}_1$, $\dot{Q}_2$ and ΔT are > 0. Let us denote this typical case with A. Inequality $\Delta T > \alpha \Delta t_{1,2}$ is equivalent to the simpler one $T > t_2$. If in $\Delta T > \alpha \Delta t_{1,2}$ we replace ΔT with $T - t$, then include equality (1), and all but the T switch to the right side we get the inequality $T > t_2$. We do the same for all other cases and get the result in the Table 1 (see Table 1 in Appendix).

Now by measuring values t_1, t_2, t and T we can easily determine the typical cases in a transfer of heat energy in the system with dual heat circular flow. We see (Table 1) that in a typical case B, despite of warming on input device U3, the output device U2 is cooling. Also at typical case C, despite of cooling the input device U3, the output device U1 is heating.

Notice that the transitional case AB occurs when the proportion of ΔT in $\Delta t_{1,2}$ is α; BC occurs when $\Delta T = 0$, and CD when the proportion of $-\Delta T$ in $\Delta t_{1,2}$ is $1-\alpha$. So, $\dot{Q}_2$ becomes negative when the share of ΔT in $\Delta t_{1,2}$ is less than α.

For example, consider the case when $\Delta t_{1,2} = 12$ and $\alpha = 0{,}70$ (see Figure 4.). Then for $\Delta T > 8{,}40$ we have a typical case A, for $0 < \Delta T < 8{,}40$ case B, for $-3{,}60 < \Delta T < 0$ case C and for $\Delta T < -3{,}60$ typical case D.

We can raise the question whether it is possible, and if so how, to choose parameter α such to obtain a desired or to avoid unwanted case. For example, when $\Delta t_{1,2} = 12$ and $\Delta T = 3$, for $0 < \alpha < 0{,}25$ we have the typical case A, but for $0{,}25 < \alpha < 1$ we have the typical case B. The answer to the raised question is not considered in this paper.

Consider the typical cases depending on the values that can take α ($0 < \alpha < 1$). Then all observed cases can be reduced to three disjunctive typical cases determined by three intervals value for α. They are clearly shown in Figure 6 with the two join points.

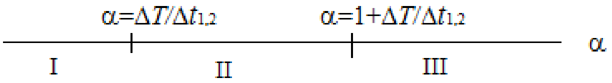

Fig.6 Typical case intervals

From equation (5) it follows that $\dot{Q}_1 > 0$ when $\alpha < 1 + \Delta T/\Delta t_{1,2}$, and from (6) it follows that $\dot{Q}_2 < 0$ when $\alpha > \Delta T/\Delta t_{1,2}$. Thus, on the interval $\left(\frac{\Delta T}{\Delta t_{1,2}}, 1 + \frac{\Delta T}{\Delta t_{1,2}}\right)$ $\dot{Q}_1$ is positive and $\dot{Q}_2$ is negative. Only one of the edges of this interval can be inside (0,1). For $\Delta T > 0$ it is $\frac{\Delta T}{\Delta t_{1,2}}$, for $\Delta T < 0$ it is $1 + \frac{\Delta T}{\Delta t_{1,2}}$. Other cases as well as comparison with cases in the previous table are given in the following Table 2 (see Table 2 in Appendix).

By measuring, we can get the values t_1, t_2, t and T and then calculate ΔT and $\Delta t_{1,2}$. When $\Delta T > \Delta t_{1,2}$ we have the case A regardless of the value of a α $(0 < \alpha < 1)$.
When $0 < \Delta T < \Delta t_{1,2}$, then for $\alpha < \frac{\Delta T}{\Delta t_{1,2}}$ we have the case A, and for $\alpha > \frac{\Delta T}{\Delta t_{1,2}}$ the case B.
When $-\Delta t_{1,2} < \Delta T < 0$, then for $\alpha < 1 + \frac{\Delta T}{\Delta t_{1,2}}$ we have the case D, and for $\alpha > 1 + \frac{\Delta T}{\Delta t_{1,2}}$ the case C. When $\Delta T < -\Delta t_{1,2}$ we have the case D regardless of the value of α $(0 < \alpha < 1)$.

Furthermore, we observe the case when $\Delta t_{1,2} < 0$ (i.e. $t_2 < t_1$). By using equation (9) the results obtained earlier (in the case when $\Delta t_{1,2} > 0$) can be used in this case
By substituting: α with $1 - \alpha$, t_1 with t_2, t_2 with t_1, $\dot{Q}_1$ with $\dot{Q}_2$ and $\dot{Q}_2$ with $\dot{Q}_1$ in Table 1, we get a new table, Table 3 with a new typical cases A', B', C' and D' which applies to the case $\Delta t_{1,2} < 0$ (see Table3 in Appendix).
The same could be done and for Table 2.

5 The Problem with Calorimeters

Let us observe the operation of the system during defined time interval τ, e.g. one month. Heat energy that enters or leaves the system through devices U3, U1 and U2 is measured using calorimeters. Calorimeters with a single counter can measure either energy or heating or cooling, while those with two counters[4] can measure both the energy of heating and cooling energy.

For further consideration we define

$$\dot{Q}_1^+ = \begin{cases} \dot{Q}_1, & \dot{Q}_1 > 0 \\ 0, & \dot{Q}_1 \le 0 \end{cases}, \quad \dot{Q}_1^- = \begin{cases} 0, & \dot{Q}_1 \ge 0 \\ \dot{Q}_1, & \dot{Q}_1 < 0 \end{cases},$$

$$\dot{Q}_2^+ = \begin{cases} \dot{Q}_2, & \dot{Q}_2 > 0 \\ 0, & \dot{Q}_2 \le 0 \end{cases}, \quad \dot{Q}_2^- = \begin{cases} 0, & \dot{Q}_2 \ge 0 \\ \dot{Q}_2, & \dot{Q}_2 < 0 \end{cases},$$

$$\dot{Q}_3^+ = \begin{cases} \dot{Q}_3, & \dot{Q}_3 > 0 \\ 0, & \dot{Q}_3 \le 0 \end{cases}, \quad \dot{Q}_3^- = \begin{cases} 0, & \dot{Q}_3 \ge 0 \\ \dot{Q}_3, & \dot{Q}_3 < 0 \end{cases}.$$

Let us observe the case of heating the system through the heating device U3 ($\Delta T > 0$, i.e. $\dot{Q}_3 > 0$). In this case only typical cases A, B, A' and B' are possible. The total energy of heating that during the observed time interval enters the system through the device U3 is denoted with K_3^+. Then

$$K_3^+ = \int_0^\tau d\dot{Q}_3.$$

Suppose calorimeters on the devices U1 and U2 measure only energy of heating (positive heat energy, energy that through the devices U1 and U2 leaves system). The total energy of heating that during the observed time interval, in which $\Delta T > 0$, leaves the system through the devices U1 and U2 is denoted with K_1^{++} and K_2^{++} respectively. These amounts are measured[5] on the calorimeters. Then we have

$$K_1^{++} = \int_0^\tau d\dot{Q}_1^+ \text{ and}$$

$$K_2^{++} = \int_0^\tau d\dot{Q}_2^+ .$$

If in a given time interval typical case B or B' did not appear then is valid the equation

$$K_1^{++} + K_2^{++} = K_3^+.$$

However, it is generally

$$K_1^{++} + K_2^{++} \ne K_3^+, \text{ moreover}$$

$$K_1^{++} + K_2^{++} \ge K_3^+.$$

Therefore, the sum of the measured energy of heating on devices U1 and U2 is not equal to the energy of heating generated by at heating device U3 and generally is higher than it. This is because, in typical cases of B and B ', heat energy through the warmer devices (U2 or U1) entered the system (negative output of heat energy) and moved to the cooler device[6] (U1 or U2). This transition is not taken into account on the calorimeter attached to the warmer device, which measures only positive energy, but on the calorimeter at the colder device it is taken into account as a positive heat energy. Therefore, if the device U1 is cooler than U2, the amount K_1^{++} is a sum of energy of heating generated

[4] The amount on each counter is positive.

[5] It means, the difference between amounts at calorimeter counter at the end and at the beginning of the interval observed.

[6] The colder device is one with lower fluid temperature at the outlet.

by the device U3 that is passed to U1, and the energy passed from U2 to U1 when heat is entering the heating system through U2.

The problem can be avoided if devices U1 and U2 are supplied with calorimeters which measure both heating energy (output, positive thermal energy) and cooling energy (input, negative thermal energy). Let K_1^{-+} and K_2^{-+} is heat energy which entered into the system through U1 and U2 (cooling energy) during the observation time interval, in which $\Delta T > 0$. These amounts are measured on the calorimeters as energy of cooling. In observed case we have

$K_1^{-+} = -\int_0^\tau d\dot{Q}_1^-$ and

$K_2^{-+} = -\int_0^\tau d\dot{Q}_2^-$.

Then $K_1^{++} - K_2^{-+}$ is thermal energy generated in the U3, which passed to U1 and $K_2^{++} - K_1^{-+}$ is thermal energy generated in U3, which passed to U2. It can be seen that the amount of these thermal energy is easily calculated. So, we have the equality:

$$K_1^{++} - K_2^{-+} + K_2^{++} - K_1^{-+} = K_3^+.$$

Because, in this case, K_3^+ can be calculated using the known values K_1^{++}, K_1^{-+}, K_2^{++} and K_2^{-+}, there is no need for calorimeter at the device U3 which measures the amount of K_3^+. Then only two calorimeters on devices U1 and U2, which measures the heating energy (heat supply to devices) and cooling energy (heat removal from the device) are sufficient**.**

The same problem arises in the case of cooling system (bringing negative heat energy in the system, $\Delta T < 0$, i.e. $\dot{Q}_3 < 0$) through the device U3. Total energy of cooling (negative heat) that during the observed time interval enters the system through U3 denote with K_3^-. Similar to $\Delta T > 0$ and for $\Delta T < 0$, ie, in the case of cooling, we introduce K_1^{--}, K_1^{+-}, K_2^{--} and K_2^{+-}. Similar to the earlier we get that

$K_1^{--} + K_2^{--} \geq K_3^-$ and

$$K_1^{--} - K_2^{+-} + K_2^{--} - K_1^{+-} = K_3^-.$$

In this case too, only two calorimeters are sufficient, one per each of the devices U1 and U2.

Suppose that during the observed time interval it is possible both heating and cooling of the system through the U3, i.e. that ΔT can take both positive and negative values. Let us denote the heating energy which is measured on the calorimeters at devices U1 and U2 with K_1^+ and K_2^+, and cooling energy with K_1^- and K_2^-. In observed case we have

$K_1^+ = \int_0^\tau d\dot{Q}_1^+$, $\quad K_1^- = -\int_0^\tau d\dot{Q}_1^-$,

$K_2^+ = \int_0^\tau d\dot{Q}_2^+$, $\quad K_2^- = -\int_0^\tau d\dot{Q}_2^-$,

$K_3^+ = \int_0^\tau d\dot{Q}_3^+$, $\quad K_3^- = -\int_0^\tau d\dot{Q}_3^-$.

Then we have

$K_1^+ = K_1^{++} + K_1^{+-}$,

$K_2^+ = K_2^{++} + K_2^{+-}$,

$K_1^- = K_1^{--} + K_1^{-+}$,

$K_2^- = K_2^{--} + K_2^{-+}$.

$K_1^+ - K_1^- + K_2^+ - K_2^- = K_3^+ - K_3^-$,

but it is obvious that

$K_1^+ + K_2^+ \geq K_3^+$ and

$K_1^- + K_2^- \geq K_3^-$.

The question is whether it is possible and if so how to get a share of heating energy K_3^+ generated at the U3, which is transmitted to the devices U1 and U2, i.e. $K_1^{++} - K_2^{-+}$ and $K_2^{++} - K_1^{-+}$.

The same question is also for the share of energy of cooling K_3^- generated by the device U3, which is transmitted to the devices U1 and U2, i.e. $K_1^{--} - K_2^{+-}$ and $K_2^{--} - K_1^{+-}$. The problem represents a system of 6 linear equations with 8 unknowns and there is no single solution[7]. To be able to get a share of energy of heating and energy of cooling which move from device U3 to devices U1 and U2, calorimeters on the devices U1 and U2 should be able to measure the energy of heating and cooling energy separately in each case when $\Delta T > 0$ and when is $\Delta T < 0$.

6 Conclusion

When the heating system contains more heating circuit transitions of heat energy between heating circuits appears. In the case of the observed heating system with two heat circular flows the heat transfer rate (heat flow) of heat which transitions from the warmer to the cooler circuit is given explicitly by formula. It is a quadratic function of the three-way valve parameter, and is a linear function of the difference of the output temperature of thermal devices U2 and U1. In the considered case, it is possible to explicitly determine the amount of heat flow of heat energy that comes out of the system through each heating circle through devices U1 and U2. There are typical cases that depend on the parameter of the three-way valve, the difference of thermal devices U2 and U1 output temperature and the difference between output and input temperature of the device U3 which brings energy into the system. They clearly set the conditions under which the input and two output of heat flow are positive or

[7] The rank of the associated matrix of the system is 5.

negative. The total heat flow of a heating or a cooling system, which, in a given time interval, via the device U1 and U2 leaves the system generally does not have to be equal to the heat flow entering the system via the device U3 if typical cases B, D, B' and D' occur.

In order to calculate the share of input thermal energy generated at U3 that is transferred to the output devices U1 and U2, in cases of only cooling or only heating, calorimeters embedded on the devices U1 and U2 must be capable of measuring energy of cooling as well as energy of heating i.e. they must have two counters. Then the input thermal energy on U3 can be calculated and there is no need for installing the calorimeter at U3.

If in a given time interval both heating and cooling of systems through U3 occur it is not possible to uniquely determine the share of input thermal energy leaving the system via output devices U1 and U2 using calorimeters with two counters. It should be used calorimeters with four counters. Two counters are needed for measuring energy of heating and energy of cooling in the case when $\Delta T > 0$ and another two for measuring energy of heating and energy of cooling in the case when $\Delta T < 0$.

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Appendix

Typical cases	Condition		Signs of function		
			$\dot{Q}_1$	$\dot{Q}_2$	$\dot{Q}_3$
A	$0 < \alpha(t_2 - t_1) < \Delta T$	$t_1 < t < t_2 < T$	$+$	$+$	$+$
AB	$0 < \Delta T = \alpha(t_2 - t_1)$	$t_1 < t < t_2 = T$	$+$	0	$+$
B	$0 < \Delta T < \alpha(t_2 - t_1)$	$t_1 < t < T < t_2$	$+$	$-$	$+$
BC	$\Delta T = 0$	$t_1 < t = T < t_2$	$+$	$-$	0
C	$-(1-\alpha)(t_2 - t_1) < \Delta T < 0$	$t_1 < T < t < t_2$	$+$	$-$	$-$
CD	$\Delta T = -(1-\alpha)(t_2 - t_1) < 0$	$t_1 = T < t < t_2$	0	$-$	$-$
D	$\Delta T < -(1-\alpha)(t_2 - t_1) < 0$	$T < t_1 < t < t_2$	$-$	$-$	$-$

Table 1

Typical cases	Condition	Cases from Table 1	Signs of function			
			ΔT	$\dot{Q}_1$	$\dot{Q}_2$	$\dot{Q}_3$
I	$0 < \alpha < \frac{\Delta T}{t_2 - t_1}$	A	+	+	+	+
	$0 < \alpha = \frac{\Delta T}{t_2 - t_1}$	AB	+	+	0	+
II	$0 < \frac{\Delta T}{t_2 - t_1} < \alpha < 1 < 1 + \frac{\Delta T}{t_2 - t_1}$	B	+	+	−	+
II	$0 < \alpha < 1$	BC	0	+	−	0
II	$\frac{\Delta T}{t_2 - t_1} < 0 < \alpha < 1 + \frac{\Delta T}{t_2 - t_1} < 1$	C	−	+	−	−
	$0 < \alpha = 1 + \frac{\Delta T}{t_2 - t_1} < 1$	CD	−	0	−	−
III	$1 + \frac{\Delta T}{t_2 - t_1} < \alpha < 1$	D	−	−	−	−

Table 2

Typical cases	Condition		Signs of function		
			$\dot{Q}_1$	$\dot{Q}_2$	$\dot{Q}_3$
A'	$0 < (1 - \alpha)(t_1 - t_2) < \Delta T$	$t_2 < t < t_1 < T$	+	+	+
A'B'	$0 < \Delta T = (1 - \alpha)(t_1 - t_2)$	$t_2 < t < t_1 = T$	0	+	+
B'	$0 < \Delta T < (1 - \alpha)(t_1 - t_2)$	$t_2 < t < T < t_1$	−	+	+
B'C'	$\Delta T = 0$	$t_2 < t = T < t_1$	−	+	0
C'	$-\alpha(t_1 - t_2) < \Delta T < 0$	$t_2 < T < t < t_1$	−	+	−
C'D'	$\Delta T = -\alpha(t_1 - t_2) < 0$	$t_2 = T < t < t_1$	−	0	−
D'	$\Delta T < -\alpha(t_1 - t_2) < 0$	$T < t_2 < t < t_1$	−	−	−

Table 3