MHD nanofluid flow containing gyrotactic microorganisms

R.KANDASAMY AND RADIAH MUHAMAD

Research Centre for Computational Fluid Dynamics,

FSTPi, Universiti Tun Hussein Onn Malaysia, Malaysia, 86400, Parit Raja, Batu Pahat, Johor, Malaysia

Tel./Fax: 006 07 453 7416/006 07 453 6051

Email: future990@gmail.com

Abstract: In this paper, we investigate the bioconvection induced by the magnetohydrodynamic flow of water based nanofluid containing nanoparticles and motile microorganisms over a vertical plate. Nanofluid bioconvection is generated by buoyancy forces on the interaction of motile microorganisms and nanoparticles. The microorganisms are imposed into the nanofluid to stabilize the nanoparticles to suspend due to a phenomenon called bioconvection. The bioconvection parameters tend to decelerate the concentration of motile microorganisms depend strongly upon the chemical reaction, magnetic, buoyancy, nanofluid and bioconvection parameters.

Keywords: Bioconvection, MHD boundary layer nanofluid flow, Gyrotactic microorganisms, Brownian motion, thermophoresis particle deposition, chemical reaction.

NOMENCLATURE

B_0 Magnetic flux density, $kg\ s^{-2} A^{-1}$

C_w Concentration of the wall, K

C_∞ Concentration of the fluid far away from the wall, K

C_T Temperature ratio, K

c_p Specific heat due to constant pressure, $J\ kg^{-1} K^{-1}$

Ec Eckert number, $\frac{\alpha^2 Ra_x}{x^2 C_p \Delta T_f}$ (-)

k_1 Chemical reaction rate, $mol\ m^{-1} s^{-1}$

k_f Thermal conductivity of the base fluid, $kg\ m\ s^{-3}\ K^{-1}$

k_s Thermal conductivity of the nanoparticle, $kg\ m\ s^{-3}\ K^{-1}$

Lb Bioconvection Lewis number, $\frac{\alpha_f}{D_n}$ (-)

Le Lewis number $\frac{\alpha_f}{D_B}\left(\frac{m^2 s^{-1}}{m^2 s^{-1}}\right)$(-)

M Magnetic parameter, $\frac{\sigma B_0{}^2 x}{U\ \rho_f}, \left(\frac{\Omega^{-1} m^{-1} B_0^2\ m}{m\ s^{-1}\ kg\ m^{-3}}\right)$(-)

Nb Brownian motion parameter, $\frac{\tau\ D_B \Delta C_w}{\alpha}\left(\frac{K^{-1} m s^{-1} K}{m\ s^{-1}}\right)$(-)

n Density of the microorganism, K

Nt Thermophoresis parameter, $\dfrac{\tau\ D_T \Delta T_w}{\alpha\ T_\infty}\left(\dfrac{K^{-1} m^2\ s^{-1} K}{m^2\ s^{-1}}\right)$ (-) Pr Prandtl number, $\dfrac{\nu_f}{\alpha_f}\left(\dfrac{m^2 s^{-1}}{m^2 s^{-1}}\right)$ (-)

Nr Buoyancy ratio parameter , $\dfrac{(\rho_P - \rho_{f_\infty})\Delta C_w}{\rho_f \beta\ \Delta T_w (1-\phi_\infty)}, \left(\dfrac{kg\ m^{-3} K}{kg\ m^{-3} K^{-1} KK}\right)$ (-) T Temperature, K

n_w Density of microorganism of the wall, K n_∞ Density of the microorganism far away from the wall, K

Pe Bioconvection Peclet number , $\dfrac{bW_c}{D_n}\left(\dfrac{m\ ms^{-1}}{m^2 s^{-1}}\right)$ (-) Q_0 Rate of source/sink, $kg\ m^{-2}$

Rb Bioconvection Rayleigh number, $\dfrac{\gamma\ \Delta n_w\ \Delta\rho}{\rho_f \beta\ (1-\phi_w)\Delta T_w}\left(\dfrac{m^3 K\ kg\ m^{-1}}{kg\ m^{-3} K^{-1} KK}\right)$ (-) u, v Velocity components, $m\ s^{-1}$

x, y Streamwise coordinate and cross-stream coordinate, $\left(x = \left(\dfrac{\xi}{k}\right)^{\frac{2}{1-m}}\right), m$

Greek symbols

α_f Thermal diffusivity, $m^2\ s^{-1}$ β_f Thermal expansion coefficients of the base fluid, K^{-1}

ρ_f Density, $kg\ m^{-3}$ σ Electric conductivity, $\Omega^{-1} m^{-1}$

σ_1 Stefan – Boltzman constant, $kg\ s^{-3}\ K^{-4}$ μ_f Dynamic viscosity, $kg\ m^{-1}\ s^{-1}$

ν_f Dynamic viscosity of the nanofluid, $m^2\ s^{-1}$ η Similarity variable, (-)

γ Chemical reaction parameter, $\dfrac{(\alpha\ \nu\ x)^{\frac{1}{2}} K_1}{D_B\left((1-\phi_\infty)\beta\ g\ \Delta T_f\right)^{\frac{1}{2}}}\left(\dfrac{\left(m\ s^{-1}\left(m\ s^{-1}\right)m\right)^{\frac{1}{2}} s^{-1}}{\left(m\ s^{-1}\right)\left(K^{-1} m\ s^{-2} K\right)^{\frac{1}{2}}}\right)$ (-)

1. Introduction

Nanofluids have been treated for industrial applications as gaining heat transfer fluids for almost two decades, [1-7]. Mechanisms controlling to heat transfer improvement in nanofluids are analyzed in many publications, [8-12]. The object of the current work is to nominate a new kind of a nanofluid that includes nanoparticles and gyrotactic microorganisms. Wang and Fan [13] predicted that nanofluids involve molecular, micro, macro and majuscule. Bioconvection refers to a macroscopic convection motion of nanofluid caused by the density gradient induced by collective swimming of motile microorganisms [14-19]. These self-propelled motile microorganisms enhance the

Corresponding Author: Ramasamy Kandasamy, Email: future990@gmail.com

density of the base fluid by swimming in a particular direction, thus causing bioconvection. Kuznetsov[20-22] analyzed on nanofluids containing gyrotactic microorganisms and reaffirm that the resultant large-scale motion of fluid caused by self-propelled motile microorganisms increases mixing and prevent nanoparticle agglomeration in fluids.

Bioconvection has large amount of applications in biomedical. The uses of including microorganisms to the suspension add increased mass transfer, especially in microvolumes and developed nanofluid stability [20]. Nanofluid and bioconvection is consequently quite alluring for new microfluidic devices and bioconvection has many applications in biomedical systems. Nanofluid bioconvection is investigated to be possible if the density of nanoparticles is low, so that the particles do not produce any significant increase in the viscosity of the base fluid [21]. Chemical reaction in Brownian motion has a dominant application in thermodynamics.

The current work is initiated by the past investigation in the literature. This work analyzes the behaviour of water suspension that contains nanoparticles and motile gyrotactic microorganisms in Navier slip. Interaction of microorganisms and nanoparticles with magnetic field presents an interesting fluid dynamics problem. It is constructed that the magnetic field can be utilized as a good controller of the nanofluid flow field incorporating bioconvection, motile microorganisms and Brownian motion. .

2. Mathematical Analysis

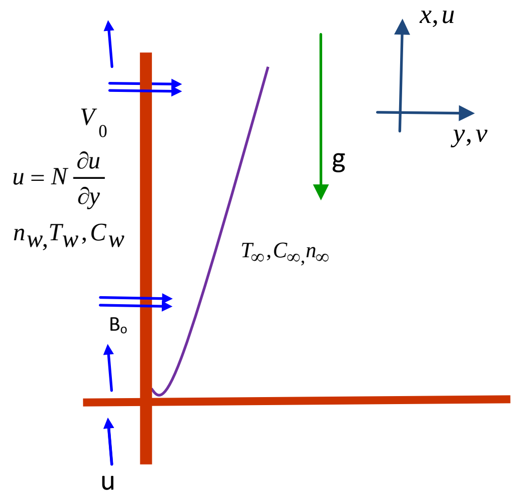

Fig.1. Physical model and coordinate system

Consider steady two-dimensional flow of water-based electrically conducting fluid containing oxytactic microorganisms over a vertical plate with Navier slip kept at T_w, C_w and n_w along plate and

T_∞, C_∞ and n_∞. for away from the plate, Fig. 1. Magnetic strength B_0 is constructed parallel to the y-axis and the induced magnetic field and the electric polarization charges are negligible. The presence of nanoparticles is assumed that there is no effect on microorganisms swim and the bioconvection takes place in suspension of nanoparticles. Based on Oberbeck–Boussinesq approximation, the governing equations are derived as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial p}{\partial x}=\mu\frac{\partial^2 u}{\partial y^2}-\left(\sigma B_0^2\right)u-\rho_f\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+ \left[g\rho_{f_\infty}(1-\phi_\infty)\beta(T-T_\infty)-(\rho_p-\rho_{f_\infty})g(C-C_\infty)-ng\gamma(\rho_{m_\infty}-\rho_{f_\infty})\right] \tag{2}$$

$$\frac{\partial p}{\partial y}=0 \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_f\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{\mu_f\alpha_f}{k_f}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\alpha_f\sigma B_0^2u^2}{k_f} \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-K_1(C-C_\infty) \tag{5}$$

$$u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial y}+\frac{bW_c}{(C_w-C_\infty)}\left[\frac{\partial}{\partial y}\left(n\frac{\partial C}{\partial y}\right)\right]=D_n\frac{\partial^2 n}{\partial y^2} \tag{6}$$

$$y=0: \; u=N\frac{\partial u}{\partial y}, v=0, T=T_w, n=n_w, C=C_w, \quad y\to\infty: \; u\to 0, C\to C_\infty, T\to T_\infty, n\to n_\infty \tag{7}$$

N - the Navier slip coefficient, where $N=0$ corresponds to no slip and full lubrication is described in the limit $N\to\infty$. By Kuznetsov and Nield [23], we get

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\nu_f\frac{\partial^2 u}{\partial y^2}-\left(\frac{\sigma B_0^2}{\rho_f}\right)u+\left(\frac{1-\phi_\infty}{\rho_f}\right)g\rho_{f_\infty}\beta(T-T_\infty) -\frac{(\rho_p-\rho_{f_\infty})}{\rho_f}g(C-C_\infty)-\frac{(\rho_p-\rho_{f_\infty})}{\rho_f}ng\gamma \tag{8}$$

Stream function ψ by $u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}$ and utilizing the dimensionless quantities

$$\eta=\frac{y}{x}Ra_x^{1/4}, \psi=\alpha Ra_x^{1/4}f(\eta), \theta=\frac{T-T_\infty}{T_f-T_\infty}, \chi=\frac{n-n_\infty}{n_w-n_\infty}, \quad \phi=\frac{C-C_\infty}{C_w-C_\infty} \text{ and } Ra_x=\frac{(1-\phi_\infty)}{\alpha\nu}\beta g\Delta T_f x^3$$

The transformed ODEs as

$$f'''+\frac{3}{4\,\mathrm{Pr}}ff''+\theta-Nr\phi-Rb\chi-(M)f'=0 \tag{9}$$

$$\theta'' + \frac{3}{4} f\theta' + Nb\theta'\phi' + Nt\theta'^2 + Ec\,\Pr\left(f''^2 + Mf'^2\right) = 0 \tag{10}$$

$$\phi'' + \frac{3}{4} Lef\phi' + \frac{Nt}{Nb}\theta'' - \gamma\,\phi = 0 \tag{11}$$

$$\chi'' + \frac{3}{4} Lbf\chi' - Pe\left[\phi'\chi' + \phi''(\sigma + \chi)\right] = 0 \tag{12}$$

Subject to the boundary conditions

$$\begin{aligned} &\eta = 0 : f(0) = 0,\quad f'(0) = \delta\ f''(0), \theta(0) = 1, \phi(0) = 1, \chi(0) = 1; \\ &\eta \to \infty : f'(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0, \chi(\infty) \to 0 \end{aligned} \tag{13}$$

$\Pr = \frac{\nu_f}{\alpha_f}$ - Prandtl number, $Lb = \frac{\alpha_f}{D_n}$ - bioconvection Lewis number and $Le = \frac{\alpha_f}{D_B}$ - regular Lewis number, $Pe = \frac{bW_c}{D_n}$ - Peclet number, $Nr = \frac{(\rho_P - \rho_{f_\infty})\Delta C_w}{\rho_f \beta\ \Delta T_w (1-\phi_\infty)}$ - Buoyancy ratio parameter, $\delta = \frac{N_1 Ra_x^{1/4}}{x}$ - Slip parameter, $Rb = \frac{\gamma\ \Delta n_w\ \Delta\rho}{\rho_f \beta\ (1-\phi_w)\Delta T_w}$ - bioconvection Rayleigh number, $Nb = \frac{\tau\ D_B \Delta C_w}{\alpha}$ -Brownian motion parameter, $Nt = \frac{\tau\ D_T \Delta T_w}{\alpha\ T_\infty}$ -thermophoresis parameter, $M = \frac{\sigma B_0^2 x^2}{\mu_f Ra_x^{1/2}}$ - magnetic parameter , $\sigma = \frac{n_\infty}{\Delta n_w}$ -bioconvection constant, $Ec = \frac{\alpha^2 Ra_x}{x^2 C_p \Delta T_f}$ - Eckert number, $\gamma = \frac{(\alpha\ \nu\ x)^{\frac{1}{2}} K_1}{D_B\left((1-\phi_\infty)\beta\ g\ \Delta T_f\right)^{\frac{1}{2}}}$ - chemical reaction parameter. The local Nusselt number $Nu_x = \frac{q_w x}{k\Delta T}$, the local Sherwood number $Sh_x = \frac{q_m\ x}{D_B \Delta C}$, the local density number of the motile microorganisms $Sh_x = \frac{q_m\ x}{D_B \Delta C}$ where q_w, q_m and q_n are the wall heat, mass and motile microorganisms fluxes are defined as

$$\frac{Nu_x}{Ra_x^{1/4}} = -\theta'(0), \frac{Sh_x}{Ra_x^{1/4}} = -\phi'(0), \frac{Nn_x}{Ra_x^{1/4}} = -\chi'(0) \tag{14}$$

$Ra_x = \frac{(1-\phi_\infty)}{\alpha\nu}\beta g \Delta T_f x^3$ - local Rayleigh number.

3. Results and Discussion

The system of equations (9) - (12) along with (13) are not solved analytically and numerical solutions are obtained utilizing the software Maple 18 (fourth-fifth order Runge–Kutta–Fehlberg method with shooting technique) and the Maple worksheet is presented in Appendix A. Computational calculation, we have fixed the parameters as $Nr = 0.3, \Pr = 6.2, Nb = Nt = 0.3, Pe = 0.5$ and $Le = 3.0, \gamma = 0.5, Lb = 0.5, \sigma = 0.1, \delta = 0.5$. It is predicted that the value of $-\theta'(0)$ is nearly coincided with Kuznestov and Nield [24] and Khan et al. [25] .

Table 1 : Comparison of $-\theta'(0)$ in the absence of nanofluid and bioconvection parameters

Pr	Khan et al. [25]	Kuznestov and Nield [24]	present result
1.0	0.40135	0.401	0.40097342
10.0	0.46903	0.449	0.46795281
100.0	0.49260	0.458	0.47563913
1000.0	0.49878	0.459	0.49184186

Table 2: $f''(0)$, $-\theta'(0)$, $-\phi'(0)$ and $-\chi'(0)$ for different values of M

Pe	M	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-\chi'(0)$
0.0	0.1	1.2088833084590	-0.483679011349	-0.661359533909	-.19273625848839
	0.5	0.9448813454660	-0.413242802219	-0.671974680691	-0.1696762555882
	1.0	0.7842598780007	-0.363035505367	-0.681866789137	-0.1580467798365
1.0	0.1	0.9984950454487	-0.423127792494	-0.669078412744	-1.324880102671
	0.5	0.8212083398026	-0.367261034648	-0.680267227806	-1.340034931474
	1.0	0.6960990219322	-0.324225137448	-0.689355837473	-1.353550362098

The skin friction decreases, the rate of heat transfer increases, the rate of mass transfer and the rate density of motile microorganisms decreases (see Table 2) with increase of magnetic parameter for bioconvection Peclet number, $Pe = 1.0$ because of the combined effect of decrease of the density of nanofluid and increase of electric conductivity of the nanofluid. Theoretical solution of nanoparticle volume fraction profile for Nr (Fig.2) is significantly correlated with Fig. 4(a) of Khan et al. [25].

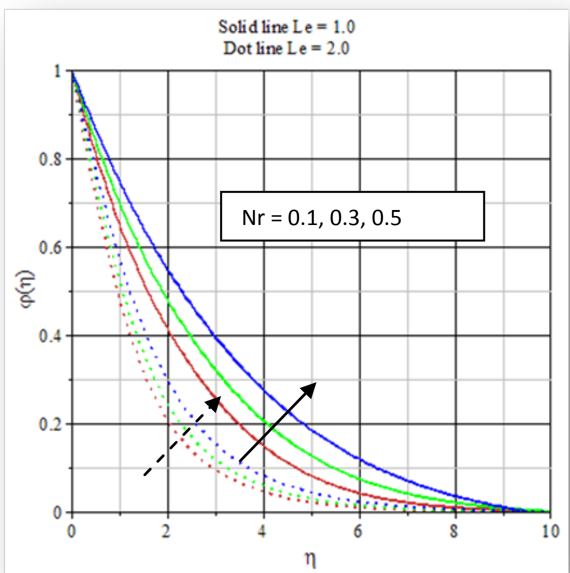

Fig. 2: Comparison of nanoparticle volume fraction profile for Nr with Fig. 4(a) of Khan et al. [25]

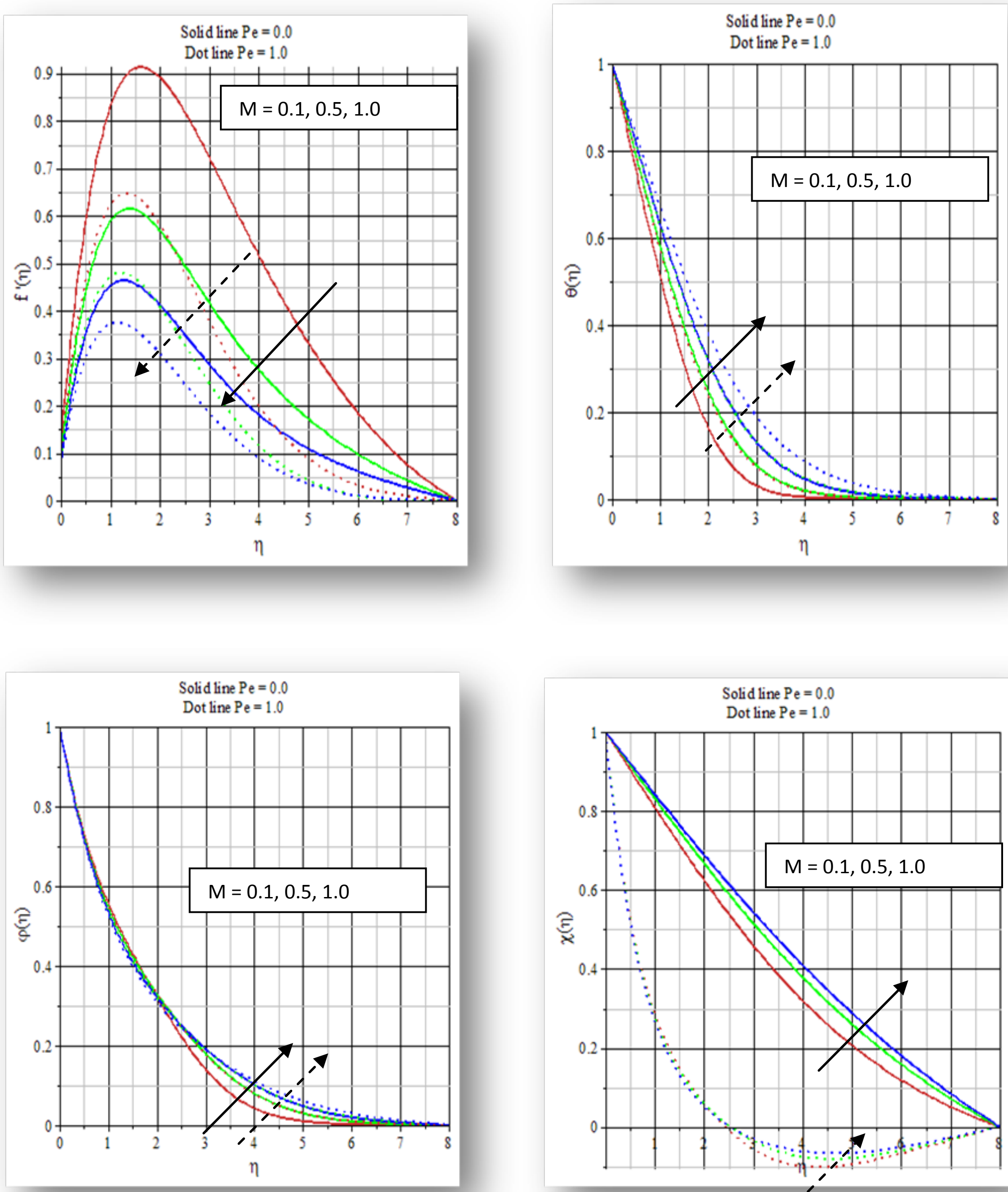

Fig. 3: Magnetic effects on velocity, temperature, nanoparticle volume fraction and density of the motile microorganisms profiles

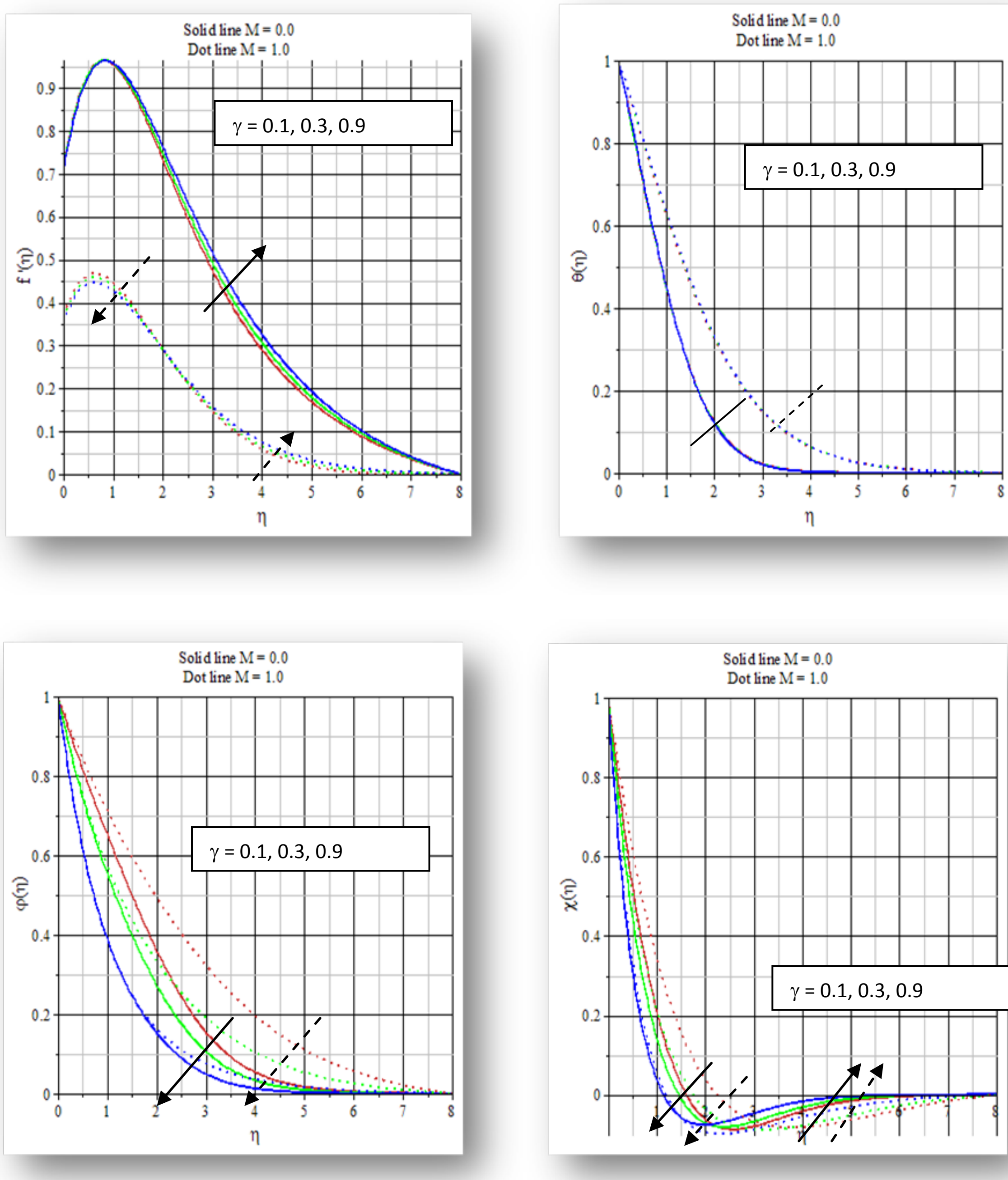

Fig. 4: Chemical reaction on velocity, temperature, nanoparticle volume fraction and density of the motile microorganisms profiles

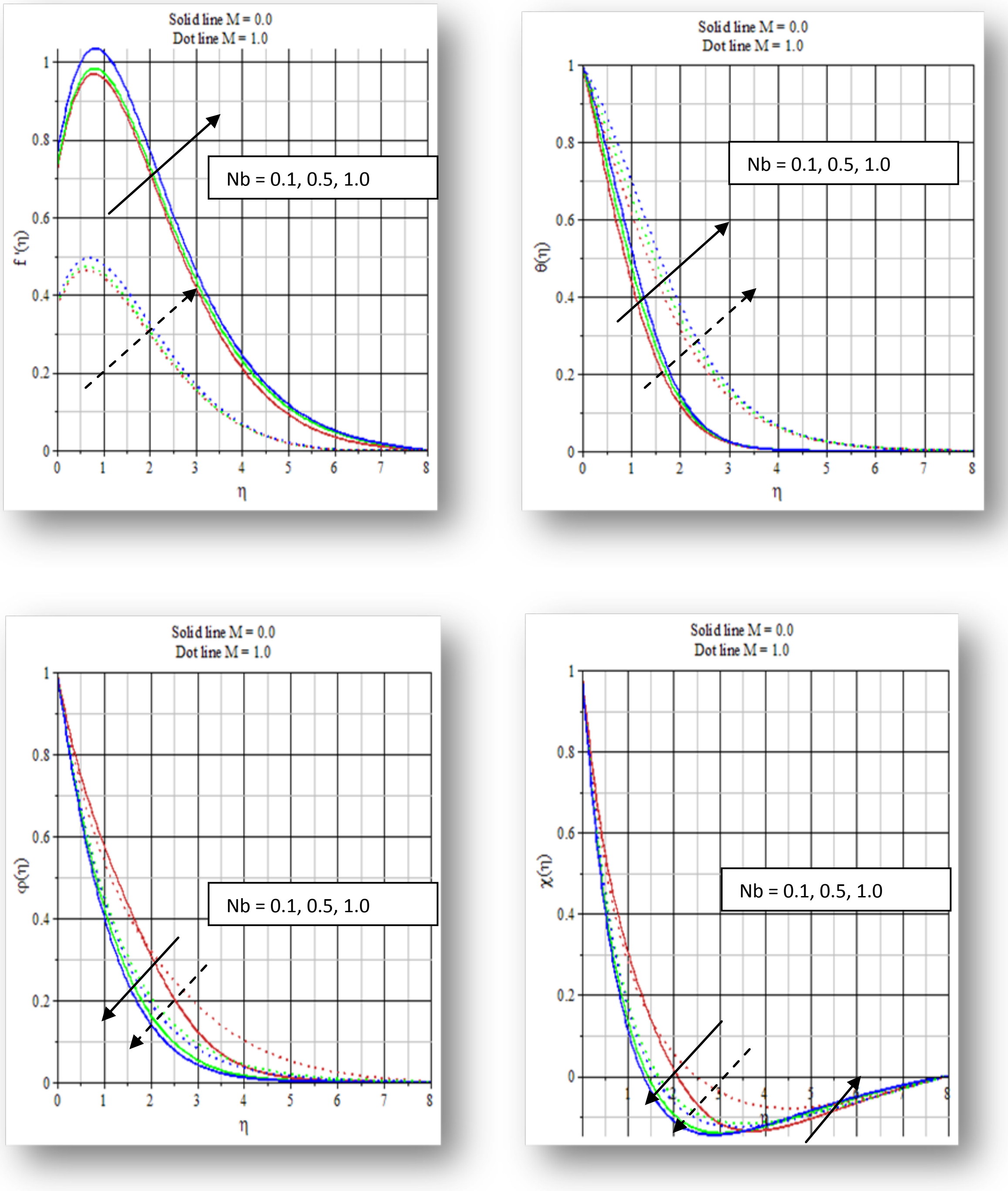

Fig. 5: Brownian motion on velocity, temperature, nanoparticle volume fraction and density of the motile microorganisms profiles

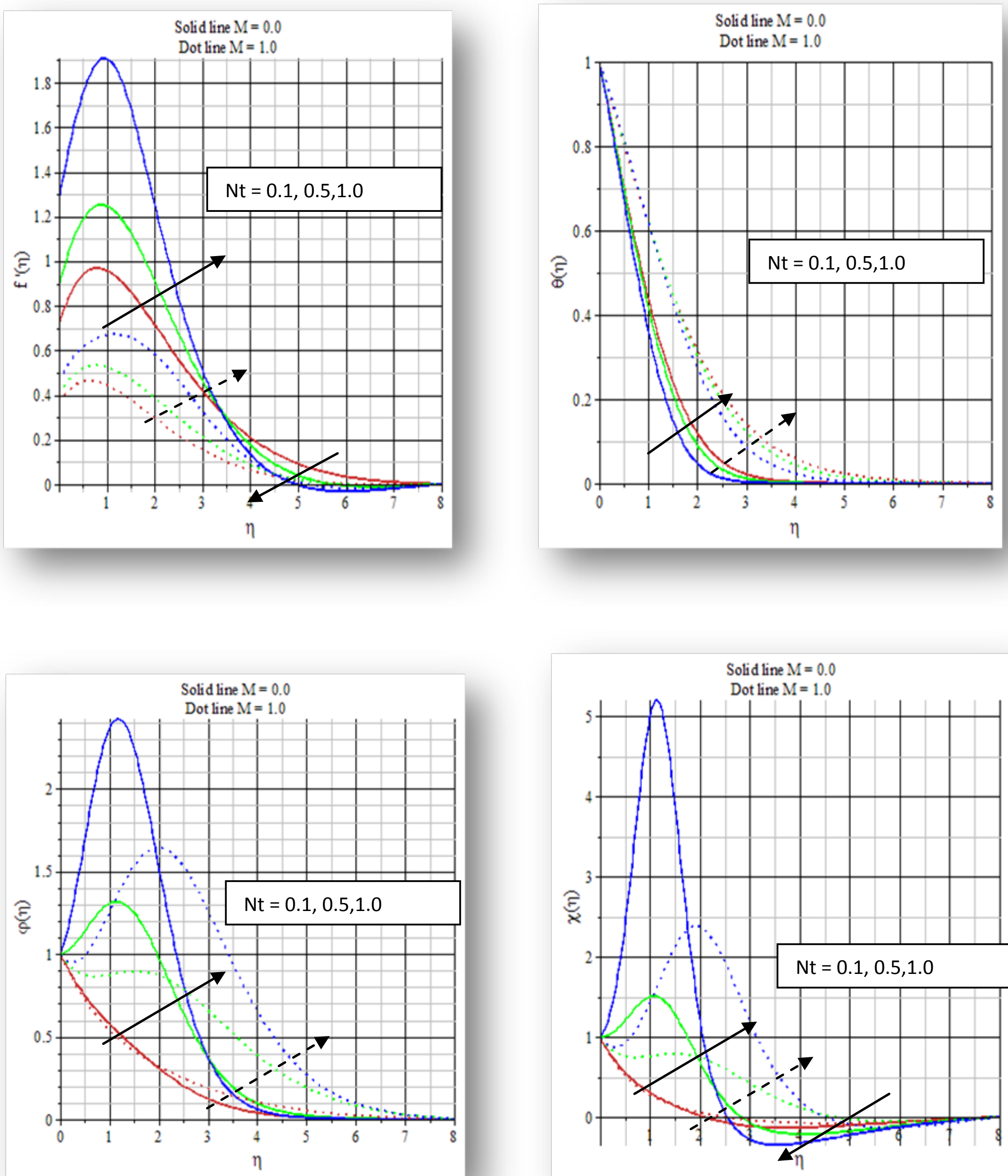

Fig. 6: Thermophoresis particle deposition on velocity, temperature, nanoparticle volume fraction and density of the motile microorganisms profiles

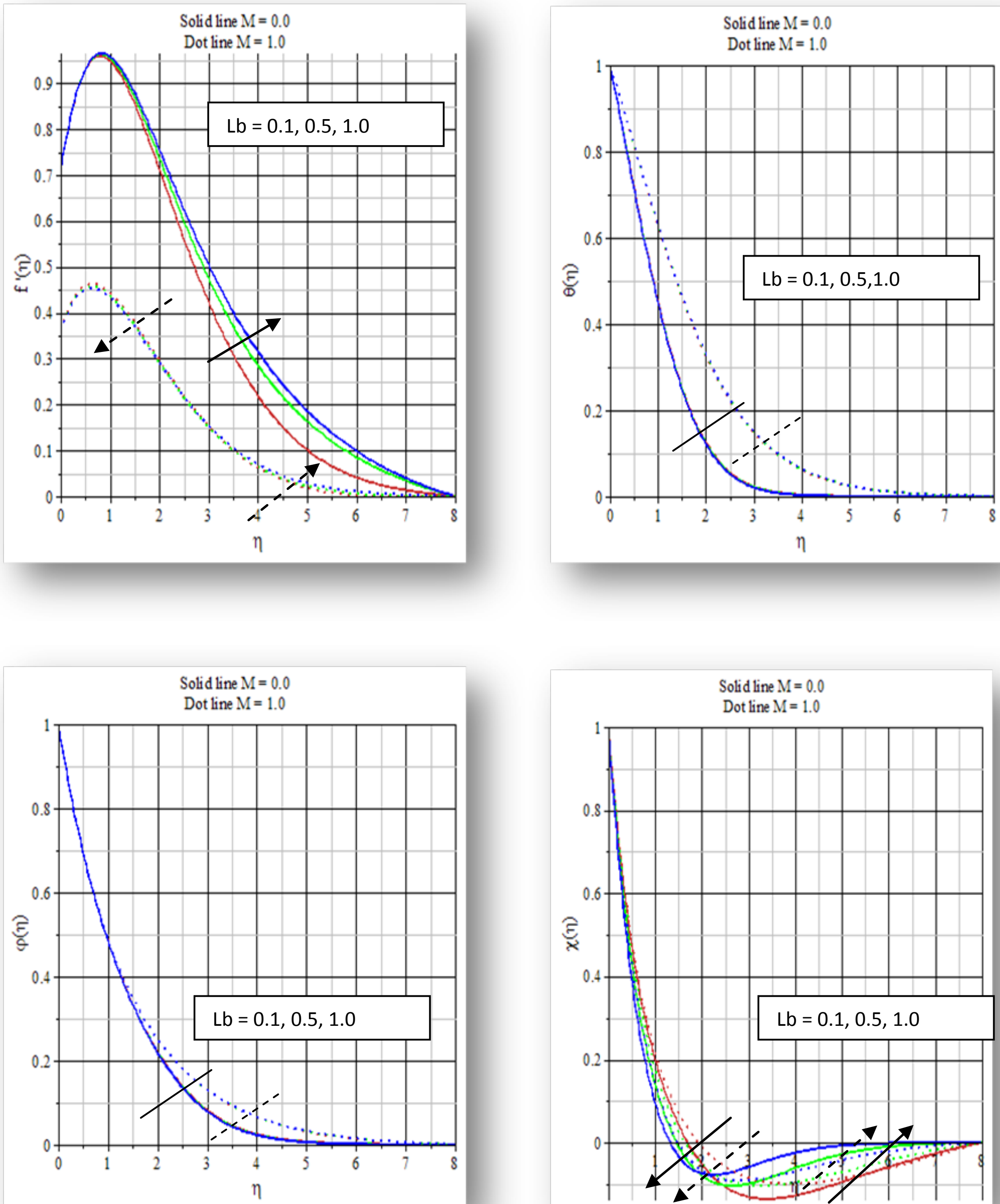

Fig. 7: Bioconvection Lewis number on velocity, temperature, nanoparticle volume fraction and density of the motile microorganisms profiles

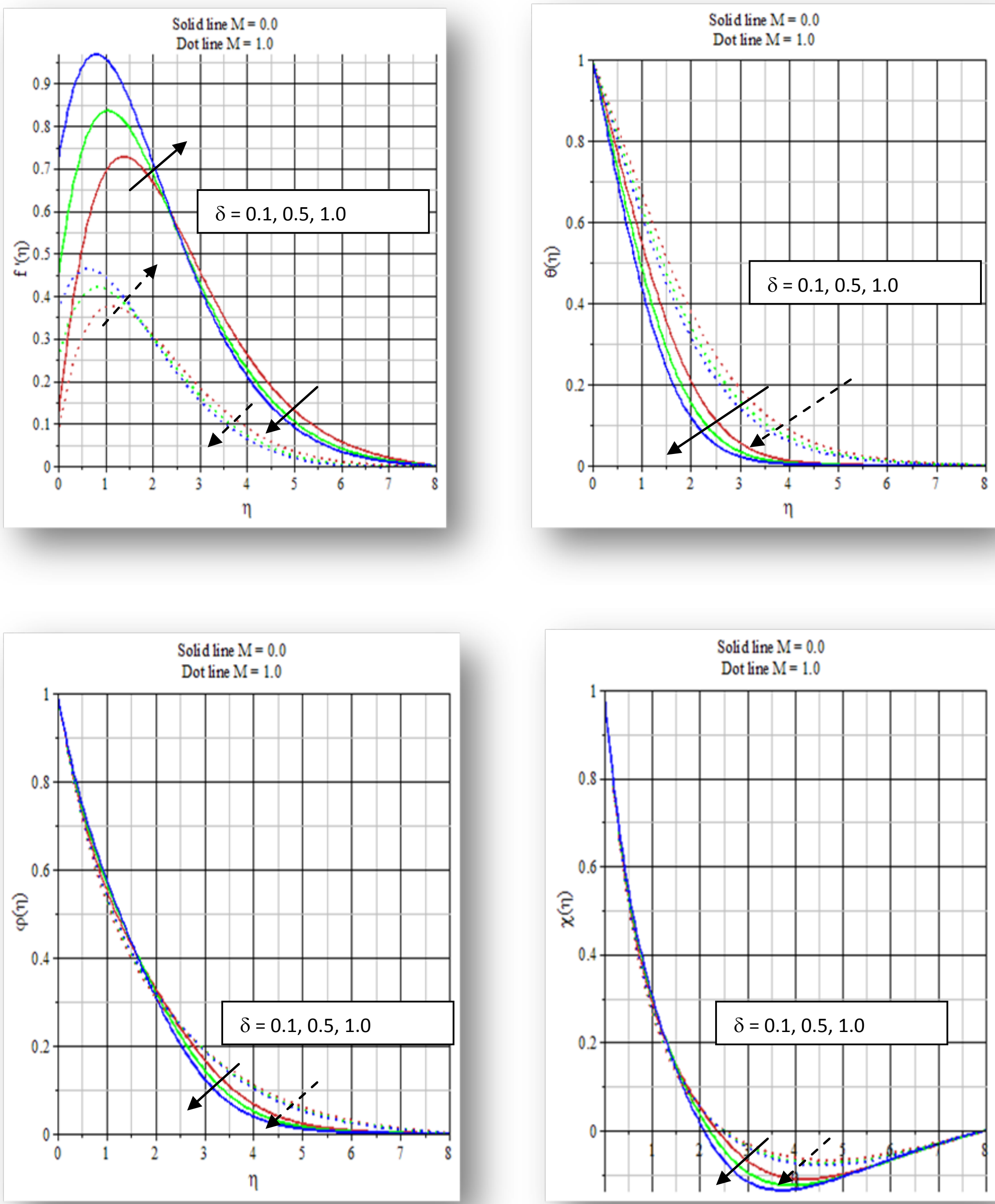

Fig. 8: Slip parameter on velocity, temperature, nanoparticle volume fraction and density of the motile microorganisms profiles

A constant bioconvection Peclet number, velocity, temperature, nanoparticle volume fraction and the density of motile microorganisms of the nanofluid for different values of magnetic strength are presented in Fig.3. Velocity of the nanofluid decelerates and the temperature, the nanoparticle volume fraction and the density of the motile microorganisms increase with increase of the strength of magnetic field because of Lorentz force, which suppresses the velocity of the nanofluid. Hydrodynamic and density of the motile microorganisms boundary layer thickness for $Pe = 1.0$ are lower than that of $Pe = 0.0$ while the thermal and the nanoparticle volume fraction boundary layers thickness for $Pe = 1.0$ are higher than that of $Pe = 0.0$. This is due to an additional resistance induced by the magnetic field plays a dominant role on flow field.

Effects of chemical reaction in magnetic field play an important role on the density of the motile microorganism field, see Fig.4. When $M = 0$, velocity increases, nanoparticle volume fraction decreases and density of the motile microorganism firstly decrease $0 \le \eta \le 2.189$ and then increases $\eta > 2.189$ with increase of chemical reaction. When the magnetic strength $M = 1.0$, the velocity / density of the motile microorganisms of the nanofluid firstly decreases $0 \le \eta \le 1.781 / 0 \le \eta \le 3.417$ and then increases $\eta > 1.781 / \eta > 3.417$ with increase of chemical reaction because of density and the electric conductivity of the nanofluid. Both $M = 0.0$ and $M = 1.0$, it is important to note that there is no significant change in temperature field whereas nanoparticle volume fraction of the nanofluid decreases as increase of chemical reaction. Further, nanoparticle volume fraction boundary layer thickness for $M = 1.0$ is stronger than that of $M = 0.0$.

Fig. 5 presents Brownian motion on velocity, temperature, nanoparticle volume fraction and the density of the motile microorganisms of the nanofluid. When $M = 1.0$ and $M = 0.0$, $\theta(\eta)$ increase, $\varphi(\eta)$ decreases, $\chi(\eta)$ firstly decreases $0 \le \eta \le 3.274$ and then increases $\eta > 3.274$ with raises of Brownian motion for $M = 0.0$ whereas the density of the motile microorganisms of the nanofluid decreases with enhances of Brownian motion for $M = 1.0$. Hydrodynamic boundary layer for $M = 1.0$ is lower than that of $M = 0.0$ whereas the thermal, diffusion and motile microorganisms boundary layers for $M = 1.0$ are higher than that of $M = 0.0$ with accelerate of Brownian motion.

Fig. 6 displays thermophoresis Nt on velocity, temperature, nanoparticle volume fraction and the density of the motile microorganisms. When $M = 1.0$, velocity, temperature, nanoparticle volume fraction and density of the motile microorganisms of the nanofluid increase with increase of Nt whereas $M = 0.0$, $f'(\eta)$ / $\chi(\eta)$ firstly increases $0 \le \eta \le 3.426 /$ $0 \le \eta \le 2.258$ and then decreases $\eta > 3.426 /$ $\eta > 2.258$ and the temperature and nanoparticle volume fraction increases with enhance of Nt. When $M = 1.0$, nanoparticle volume fraction and the density of the motile microorganisms profiles for $Nt = 1.0$ are higher compared to that of $Nt = 0.1, 0.5$ with enhance of thermophoresis because of high strength of thermal diffusivity of nanofluid. When $M = 1.0$, $f'(\eta)$, $\theta(\eta)$ and $\varphi(\eta)$ are uniform with increase of bioconvection Lewis number whereas the density of motile microorganisms firstly decreases $0 \le \eta \le 3.813$ and then increases $\eta > 3.813$ with increase of bioconvection Lewis number (Fig.7) because the density of motile microorganisms depends upon the bioconvection Lewis number. The density of motile microorganisms boundary layer for $M = 1.0$ is thinner than that of $M = 0.0$ because of strength of thermal conductivity and diffusivity of the nanofluid.

Slip factor of the surface in $M = 1.0$, plays a dominant role on the momentum, thermal and motile microorganisms boundary layer,Fig.8. $\theta(\eta)$, $\varphi(\eta)$ and $\chi(\eta)$ decrease with increase of slip parameter whereas the velocity of the nanofluid immediately increases $0 \le \eta \le 1.792$ and then decreases $\eta > 1.792$ with increase of slip parameter due to thermal diffusivity and kinematic viscosity of the nanofluid..

4. Conclusions

From the current investigation, the following conclusions are predicted:

- Density of motile microorganisms plays a dominant role on in the magnetic field and it decreases $0 \le \eta \le 3.417$ and then increases $\eta > 3.417$ with increase of chemical reaction.
- Brownian motion and thermophoresis affect the nanoparticle volume fraction and density of motile microorganisms monotonically.
- Density of motile microorganisms firstly decreases and then increases with increase of bioconvection Lewis number because

of strength of thermal conductivity and diffusivity of the nanofluid.

- The velocity of the nanofluid immediately increases $0 \le \eta \le 1.792$ and then decreases $\eta > 1.792$ with increase of slip parameter due to the kinematic viscosity of the nanofluid.

Finally, the nanoparticles and motile microorganisms of convective nanofluid flow in the vertical region in can be controlled by changing the quantity of the Brownian motion. The problem of nanofluid flow in electric conductivity has powerful implementation in many fields such as the power generation in nuclear reactors, nuclear reactor coolant, the aerodynamic of plastic sheet, the centralized cooling system of high grade machinaries, and so forth.

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Appendix A

> *restart*;

> *with*(*Shoot*) :

> *with*(*plots*) :

> $Pr := 6.2; s := 0; \lambda := 0.1; Pe := 0; \delta := 0.1; \sigma := 1; c := 0.5;$

$Pr := 6.2$, $s := 0$,$\lambda := 0.1$,$Pe := 0$,$\delta := 0.1$,$\sigma := 1$,$c := 0.5$

> $M := 0.1; Z := 0.1; \Xi := 0.5; Nt := 0.1; Nb := 0.1; l := 10; Ec := 0.001; Le := 0.5; Lb := 0.1; Nr := 0.1; Rb := 0.3;$

$M := 0.1$,$Nt := 0.1$,$Nb := 0.1$,$Ec := 0.001$,$Le := 0.5$,$Lb := 0.1$,$Nr := 0.1$,$Rb := 0.3$

> $A := \frac{3}{4.\,Pr};$ $A := 0.1209677420$

> $FNS := \{f(\eta), u(\eta), v(\eta), \theta(\eta), r(\eta), \varphi(\eta), h(\eta), \chi(\eta), g(\eta)\} :$

>

$$ODE := \Big\{\mathrm{diff}(f(\eta),\eta) = u(\eta), \mathrm{diff}(u(\eta),\eta) = v(\eta), \mathrm{diff}(\theta(\eta),\eta) = r(\eta), \mathrm{diff}(\varphi(\eta),\eta) = h(\eta), \mathrm{diff}(\chi(\eta),\eta) = g(\eta), \mathit{diff}(v(\eta),\eta) + A.f(\eta).v(\eta) + \theta(\eta) - Nr.\,\varphi(\eta) + Rb\,.\chi(\eta) - \mathrm{M}.u(\eta) = 0, \mathrm{diff}(r(\eta),\eta) + \frac{3}{4}f(\eta).r(\eta) + Nb.r(\eta).h(\eta) + Nt.r(\eta)^2 + Ec\,.Pr.\big(v(\eta)^2 + M.u(\eta)^2\big) = 0, \mathit{diff}(h(\eta),\eta) + \frac{3}{4}\,.Le.f(\eta).\,h(\eta) - \left(\frac{Nt}{Nb}\right).\left(\frac{3}{4}f(\eta)\,.r(\eta) + Nb.r(\eta).h(\eta) + Nt.r(\eta)^2 + Ec.Pr.\big(v(\eta)^2 + M.u(\eta)^2\big)\right) - c.\varphi(\eta) = 0, \mathrm{diff}(g(\eta),\eta) + \frac{3}{4}.Lb.f(\eta).\,g(\eta) - Pe.\Big(h(\eta).g(\eta) - \Big(\frac{3}{4}\,.Le.f(\eta).\,h(\eta) - \left(\frac{Nt}{Nb}\right).\left(\frac{3}{4}f(\eta).r(\eta) + Nb.r(\eta).h(\eta) + Nt.r(\eta)^2 + Ec.Pr.\big(v(\eta)^2 + M.u(\eta)^2\big)\right) - c.\varphi(\eta)\Big).(\sigma + \chi(\eta))\Big) = 0\Big\};$$

$$ODE := \left\{ \frac{d}{d\eta} g(\eta) + 0.07500000000\,(f(\eta).g(\eta)) = 0, \frac{d}{d\eta} r(\eta) + \frac{3}{4} f(\eta).r(\eta) + 0.1\,(r(\eta).h(\eta)) + 0.1\,r(\eta)^2 + 0.0062\,v(\eta)^2 + 0.00062\,u(\eta)^2 = 0, \frac{d}{d\eta} v(\eta) + 0.1209677420\,(f(\eta).v(\eta)) + \theta(\eta) - 0.1\,\varphi(\eta) + 0.3\,\chi(\eta) - 0.1\,u(\eta) = 0, \frac{d}{d\eta} h(\eta) + 0.3750000000\,(f(\eta).h(\eta)) - 0.7500000000\,(f(\eta).r(\eta)) - 0.1000000000\,(r(\eta).h(\eta)) - 0.1000000000\,r(\eta)^2 - 0.006200000000\,v(\eta)^2 - 0.0006200000000\,u(\eta)^2 - 0.5\,\varphi(\eta) = 0, \frac{d}{d\eta} \chi(\eta) = g(\eta), \frac{d}{d\eta} f(\eta) = u(\eta), \frac{d}{d\eta} \theta(\eta) = r(\eta), \frac{d}{d\eta} u(\eta) = v(\eta), \frac{d}{d\eta} \varphi(\eta) = h(\eta) \right\}$$

> $B := \delta.\alpha;$ **,** $B := 0.1\,\alpha$

> $IC := \{f(0) = s, u(0) = B, \theta(0) = 1, \varphi(0) = 1, \chi(0) = 1, v(0) = \alpha, r(0) = \tau, h(0) = \varsigma, g(0) = \zeta\};$

$$IC := \{\chi(0) = 1, f(0) = 0, g(0) = \zeta, h(0) = \varsigma, r(0) = \tau, \theta(0) = 1, u(0) = 0.1\,\alpha, v(0) = \alpha, \varphi(0) = 1\}$$

> $L := 8;$ $L := 8$

> $BC := \{u(L) = 0, \theta(L) = 0, \varphi(L) = 0, \chi(L) = 0\};$ $BC := \{\chi(8) = 0, \theta(8) = 0, u(8) = 0, \varphi(8) = 0\}$

> $infolevel[shoot] := 1:$

> $S := shoot(ODE, IC, BC, FNS, [\alpha = 1.1754060620356857, \tau = -0.4731175910735134, \varsigma = -0.6621299385504876, \zeta = -0.21461856363447018]):$

shoot: Step # 1

shoot: Parameter values : alpha = 1.1754060620356857 tau = -.4731175910735134 ς = -.6621299385504876 Zeta = -.21461856363447018

shoot: Step # 2

shoot: Parameter values : alpha = HFloat(1.206734092233355) tau = HFloat(-0.48349913706723946) ς = HFloat(-0.6613438511096211) Zeta = HFloat(-0.19357271909683235)