Effects of Radiation on Free Convection Flow past an Upward Facing Horizontal Plate in a Nanofluid in the Presence of Internal Heat Generation

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Abstract: - This article aims to present a numerical investigation for the effects of radiation on heat and mass transfer characteristics over an upward facing horizontal flat plate embedded in a porous medium filled with a nanofluid when internal heat generation is present. The Buongiornio nanofluid model is implemented wherein Brownian motion and thermophoresis effects are present. The transformation of the governing boundary layer equations into ordinary differential equations has been carried out by employing similarity transformations. The solutions of the transformed governing equations have been obtained by using MATLAB BVP solver bvp4c. The results are presented graphically and discussed for various parameters such as the buoyancy ratio Nr, Brownian motion Nb, thermophoresis Nt, radiation R and Lewis number Le in the presence of internal heat generation. Thermal radiation leads to an appreciable increase in temperature and nanoparticle volume fraction profiles. The Brownian motion and the thermophoresis of nanoparticles increases the effective thermal conductivity of the nanofluid. The slip velocity, the local heat transfer rate and mass transfer rate are significantly increased when the Lewis number and the radiation increase in the presence of internal heat generation.

Key-Words: - Free Convection, Porous Medium, Radiation, Nanofluid, Internal heat generation.

1 Introduction

Nanofluids are suspensions of nanoparticles in common fluids that show remarkable enhancement of their properties at modest nanoparticle concentrations. Most commonly used nanoparticles are aluminum, copper, iron and titanium or their oxides. Consideration of nanofluids in a variety of processes results in noteworthy applications in engineering and sciences since materials with sizes of nanometers possesses unique physical and chemical properties. Many research works on convective heat transfer in nanofluids in porous medium have been paid a considerable attention in literature because of its widespread applications in industries.

Numerous investigations in literature reveals the fact that nanofluids have been found to have enhanced thermo-physical properties such as thermal conductivity, thermal diffusivity, viscosity and convective heat transfer coefficients compared to those of base fluids like oil or water. Moreover, the enhanced thermal behavior of nanofluids will be useful for so many innovations based on the heat transfer improvement such as power generation, chemical sectors, thermal therapy for cancer treatment, heating or cooling of buildings, ventilation and air-conditioning.

Choi [1] is the first who introduced the term nanofluids to refer to the fluid with suspended nanoparticles. Choi *et al.* [2] confirmed that the addition of a small amount of nanoparticles to conventional heat transfer liquids increases the thermal conductivity of the fluid up to approximately two times. Buongiorno [3] concluded that only Brownian diffusion and thermophoresis are essential slip mechanisms in nanofluids. Kuznetsov and Nield [4] found that the reduced Nusselt number is a decreasing function of each of the nanofluids numbers Nr, Nb and Nt. Nield and Kuznetsov [5] employed the Darcy model for the momentum equation and set the boundary conditions so that the temperature and the nanoparticle fraction are constant along the wall. Kakac and Pramuanjaroenkij [6] proved that nanofluids notably improve the heat transfer capability of conventional heat transfer fluids. A review of heat transfer characteristics of nanofluids was carried out by Wang and Mujumdar [7]. Gorla and Chamkha [8] confirmed that nanofluids reduce drag force, heat and mass transfer rate. Khan and Pop [9] applied similarity solution for the steady free convection boundary layer flow past a horizontal flat plate embedded in a porous medium filled with nanofluids. The boundary layer flow past a horizontal flat plate embedded in a porous medium saturated with a water-based nanofluid was numerically investigated by Aziz *et al.* [10]. They found that the numerical data indicates no appreciable effect of the bio convection parameters on temperature and nanoparticle concentration distributions in the flow field. Hady *et al.* [11] examined the radiation effect on viscous flow of a nanofluid and heat transfer over a nonlinearly stretching sheet.

Chamkha and Aly [12] concluded that the local Sherwood number increased as either the thermophoresis parameter, Lewis number, whereas it decreased as either the buoyancy ratio, Brownian motion parameter, or magnetic field parameter increased. Shateyi and Prakash [13] observed that the local temperature rises as the Brownian motion, thermophoresis and radiation effects intensify. Uddin *et al.* [14] confirmed that the increasing hydrodynamic (momentum) slip significantly retards the boundary layer flow, whereas it markedly increases temperature and nanoparticle concentration values. Noghrehabadi *et al.* [15] summarized that the concentration gradient of nanoparticles (because of thermophoresis and Brownian motion forces) affects the local viscosity and thermal conductivity of nanofluids and consequently heat transfer of nanofluids. Makinde and Aziz [16] found that the effect of Lewis number on the temperature distribution is minimal.

An extensive collection of reviews on the convection through porous media is available in Ingham and Pop [17], Nield and Bejan [18] and Vafai [19]. Cheng and Minkowycz [20] obtained similarity solutions for free convection flow about an isothermal plate embedded in a porous medium by assuming the wall temperature as a power function of distance from the leading edge. Chen and Chen [21] presented similarity solutions for the natural convection of non-Newtonian fluids about a horizontal surface in a porous medium. Moorthy and Govindarajulu [22] analyzed the free convection flow of non-Newtonian fluids along a horizontal plate in a porous medium. The basic scales of natural convection heat and mass transfer in fluids and fluid-saturated porous media were discussed by Bejan [23]. A comparative survey of the results that may be derived at different Rayleigh numbers from these correlations reported by Corcione [24].

The addition of internal heat generation in a problem reveals that it affects the temperature distribution strongly. Since internal heat generation is associated in the fields of disposal of nuclear waste, storage of radioactive materials, nuclear reactor safety analyses, fire and combustion studies and in many industrial processes, considering internal heat generation plays a major role in the engineering applications. Heat generation can be assumed to be constant, space dependent or temperature dependent. Crepeau and Clarksean [25] considered a space dependent heat generation in their investigation on flow and heat transfer from vertical plate. They showed that the exponentially decaying heat generation model can be used in mixtures where a radioactive material is surrounded by inert alloys. Ali [26] explicated the effect of lateral mass flux on the natural convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with internal heat generation. Similarity solutions of free convection boundary layers over vertical and horizontal surfaces in porous media with internal heat generation were discussed by Postelnicu and Pop [27]. They concluded that the heat transfer is high when the internal heat generation is present.

Radiation increases the fluid velocity, temperature and the heat transfer rate. Radiative heat transfer has been given a significant importance in literature due to its extensive applications in physics and engineering including in space technology and other high-temperature processes. Thermal radiation effects have a great influence to control heat transfer rate in the processes of manufacturing whenever the quality of the final product depends on heat control factors. Radiative heat transfer from a vertical wall to conductive traditional fluids involves in some interesting applications such as the cooling of nuclear reactors, liquid metal fluids and power generation systems. In high-temperature chemical operations, for instance, combustion and fire science, it is essential to simulate thermal radiation heat transfer effects. Steady free convection flow through a porous medium bounded by a vertical infinite porous plate in the presence of radiation was examined by Raptis [28]. Chen *et al.* [29] found that both the wall shear stress and the surface heat transfer rate increase with increasing radiation interaction. Magyari and Pantokratoras [30] emphasized that the effect of thermal radiation in the linearized Rosseland approximation is quite trivial, both physically and computationally. Hossain and Takhar [31] proved that both the momentum and the thermal boundary layer thicknesses increase when the radiation-conduction interaction and the surface temperature increase. Hossain *et al.* [32] studied the effect of radiation on the natural convection flow of an optically dense incompressible fluid along a uniformly heated vertical plate with a uniform suction.

The above mentioned studies broadly deal the effects on free convection flow and the heat transfer characteristics in horizontal plate with or without radiative heat transfer in detail. The inclusion of internal heat generation is of great interest in many research works. Most of the studies have been paid a good attention on the free convection of nanofluids in porous media. Obviously, the use of nanofluids in porous media would be very much helpful in heat transfer enhancement. In this paper, the authors target to analyze the effects of radiation and internal heat generation with heat and mass transfer over an upward facing horizontal flat plate embedded in a porous medium filled with a nanofluid as a first attempt of its kind.

2 Mathematical Model

Consider the steady, two dimensional, free convection boundary layer flow past an upward facing horizontal flat plate embedded in a porous medium filled with a nanofluid in the presence of internal heat generation and radiation. The schematic diagram for the present flow model and the coordinate system is illustrated in Fig. 1. The model under investigation involving nanofluid incorporates the effects of Brownian motion and thermophoresis with Rosseland diffusion approximation. The thermo-physical properties of the nanofluid are assumed to be constant except the density variation in the buoyancy force which is estimated based on the Boussinesq approximation. The direction of the flow is along x-axis and that of

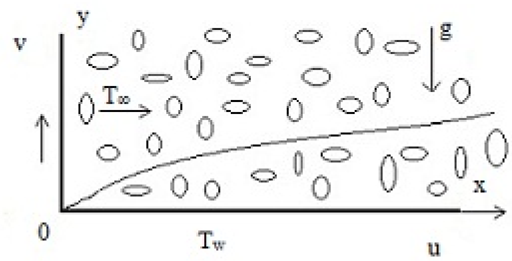


Fig. 1: Physical model and co-ordinate system

y-axis is normal to it. The gravitational acceleration g is in the direction opposite to the x-coordinate. The uniform wall temperature of the plate T_W and uniform nanoparticle volume fraction C_W are assumed to be higher than the ambient temperature T_∞ and ambient nanoparticle volume fraction C_∞, respectively.

The flow is assumed to be slow so that an advective term and a Forchheimer quadratic drag term do not appear in the Darcy Equation. Using the Darcy-Boussinesq approximation and assume that the nanoparticle concentration is dilute, the boundary layer equations are written as follows,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\mu}{k}\frac{\partial u}{\partial y}=-\left[(1-C_\infty)\rho_{f\infty}\beta\frac{\partial T}{\partial x} -(\rho_p-\rho_{f\infty})\frac{\partial C}{\partial x}\right]g \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} +\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{q_w}{(\rho C_p)_f}-\frac{1}{(\rho C_p)_f}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where u and v are the velocity components along x and y axes. C is the local nanoparticle volume fraction, T is the local temperature. D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient. $\alpha_m=\frac{k_m}{(\rho C_p)_f}$ is a thermal diffusivity of the

porous medium, β is volumetric thermal expansion coefficient of the base fluid, $(\rho C_p)_f$ and $(\rho C_p)_p$ are effective heat capacity of the fluid and nanoparticle material respectively. $\tau = \frac{\varepsilon(\rho C_p)_p}{(\rho C_p)_f \upsilon}$ is the ratio of nanoparticle heat capacity and the base fluid heat capacity. The subscript ∞ denotes the values at large values of y where the fluid is quiescent.The – sign on the right hand side of Eq. (2) refers to the case of a heated plate facing upward. The first term in the square bracket on the right hand side of Eq. (2) is the positive (upward) buoyancy term due to the thermal expansion of the base fluid and the second term is the negative (downward) buoyancy term due to the difference in densities of the nanoparticles and the base fluid. The first term in the square bracket in Eq. (3) is the thermal energy transport due to Brownian diffusion, while the second term is the energy diffusion due to thermophoretic effect. In a similar fashion, the terms on the right hand side of Eq. (4) can be interpreted.
The appropriate boundary conditions for the flow model are written as follows:

$$y = 0 : v = 0, T = T_w, C = C_w \tag{5}$$

$$y \to \infty : u \to 0, T \to T_\infty, C \to C_\infty \tag{6}$$

3 Method of Solution

The equation of continuity is satisfied for the choice of a stream function $\psi(x, y)$ such that $u = \psi_y, v = -\psi_x$.

We now introduce the following similarity transformations:

$$\eta = (Ra_x)^{\frac{1}{2}} \frac{y}{x} \tag{7}$$

$$\psi = \alpha_m (Ra_x)^{\frac{1}{2}} f(\eta) \tag{8}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{9}$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{10}$$

The internal heat generation which decays exponentially is given by

$$q_w = \frac{(\rho C_p)_f \alpha_m (T_w - T_\infty) Ra_x e^{-\eta}}{x^2} \tag{11}$$

By using the Rosseland approximation, the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y} \tag{12}$$

where σ_s is the Stephen Boltzmann constant and k_e is the mean absorption coefficient.

It is obvious that by using the Rosseland approximation, only, optically thick fluids (i.e., intensive absorption) are considered under the present investigation. If the temperature differences within the flow are sufficiently small, then equation (12) can be linearized by expanding T^4 into the Taylor series about T_∞, which after neglecting higher order terms takes the form

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{13}$$

Then the radiation term in equation (3) takes the form

$$\frac{\partial q_r}{\partial y} = \frac{16\sigma_s T_\infty^3}{3k_e} \frac{\partial^2 T}{\partial y^2} \tag{14}$$

After the substitution of these transformations (7) to (14) along with the boundary conditions (5) and (6) into equations (2) to (4), the resulting non-linear ordinary differential equations are written as,

$$f'' - \frac{2}{3}\eta(\theta' - Nr\phi') = 0 \tag{15}$$

$$(1 + R)\theta'' + \frac{1}{3} f\theta' + Nb\theta'\phi' + Nt\theta'^2 + e^{-\eta} = 0 \tag{16}$$

$$\phi'' + \frac{Le}{3} f\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{17}$$

together with the boundary conditions

$$f = 0,\ \theta = 1, \phi = 1 \ at\ \eta = 0 \tag{18}$$

$$f' \to 0,\ \theta \to 0, \phi \to 0\, as\ \eta \to \infty \tag{19}$$

where the prime denotes differentiation with respect to the variable η.

The four parameters in Equations (15)–(17) are Le, Nr, Nb and Nt they denote the Lewis number, the buoyancy-ratio parameter, the Brownian motion parameter and the thermophoresis parameter, respectively, which are defined by

$$Le = \frac{\alpha_m}{\varepsilon D_B}$$

$$Nr = \frac{(\rho_p - \rho_{f\infty})(C_w - C_\infty)}{(1 - C_\infty)\rho_{f\infty}\beta(T_w - T_\infty)}$$

$$Nb = \frac{(\rho C_p)_p D_B (C_w - C_\infty)}{(\rho C_p)_f \upsilon}$$

$$Nt = \frac{(\rho C_p)_p D_T (T_w - T_\infty)}{(\rho C_p)_f T_\infty \upsilon}$$

The generalized local Rayliegh number is defined as

$$Ra_x = \left(\frac{(1 - C_\infty) k g \beta (T_w - T_\infty) x}{\alpha_m \upsilon} \right)^{2/3}$$

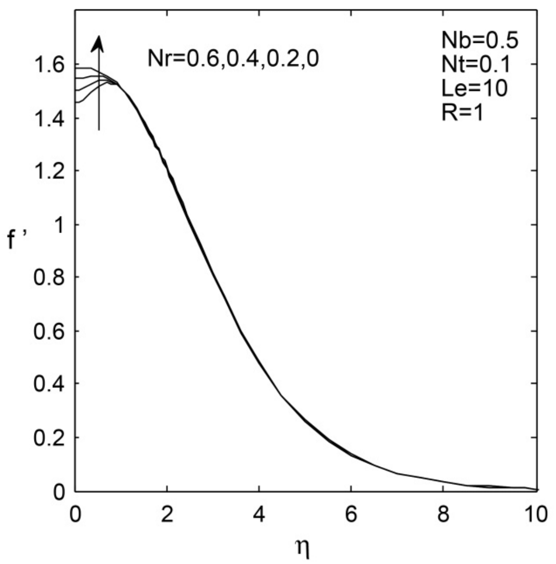
The most important characteristics of the problem are the rates of heat and mass transfer which are described by the local Nusselt number and the local Sherwood number are defined as

$$Nu_x = -(Ra_x)^{\frac{1}{2}} \theta'(0) \quad (20)$$

$$Sh_x = -(Ra_x)^{\frac{1}{2}} \phi'(0) \quad (21)$$

4 Results and Discussion

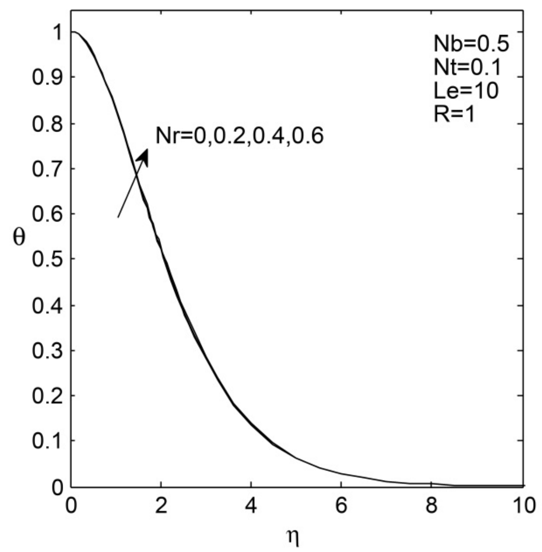
The governing equations for the present problem were transformed into a set of coupled nonlinear differential equations by applying similarity transformation. The equations (15)-(17) subject to the boundary conditions (18)-(19) are reduced to a system of first-order differential equations and solved using a MATLAB boundary value problem solver called bvp4c. This package solves boundary value problems for ordinary differential equations of the form $y' = f(x, y, p)$, $a \le x \le b$, by employing a collocation method subject to general nonlinear, two-point boundary conditions $f(y(a), y(b), p) = 0$. Here p is a vector of unknown parameters. Boundary value problems (BVPs) arise in most various forms. Just about any boundary value problems can be formulated for solution with bvp4c.



(a)

The first step is to write the *ODEs* as a system of first order ordinary differential equations. This software uses the higher order finite difference code (Shampine *et al*. [33]).

The effects on velocity, heat and mass transfer characteristics are investigated by varying the flow controlling parameters Brownian motion (Nb), thermophoresis (Nt), buoyancy-ratio (Nr), Lewis number (Le) and Radiation parameter (R) in the presence of internal heat generation.



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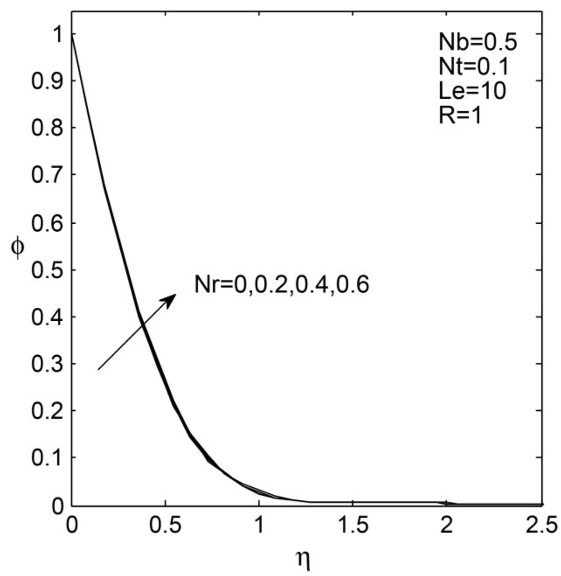


(c)

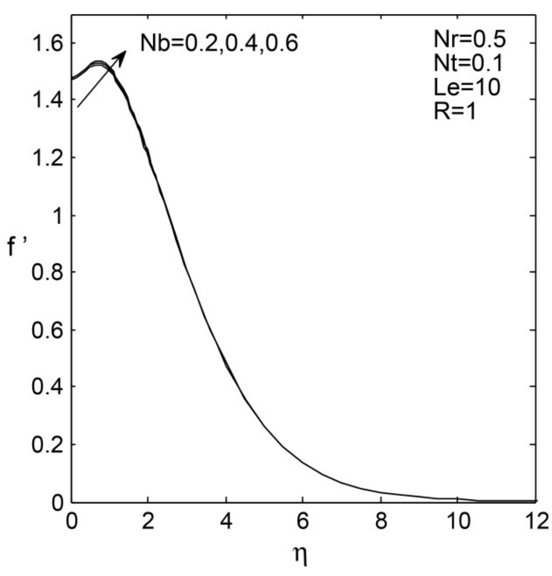
Fig. 2: Effects of Nr on dimensionless (a) velocity (b) temperature and (c) nanoparticle volume fraction profiles.

4.1 Effect of buoyancy-ratio parameter Nr

The effects of the buoyancy-ratio parameter Nr on the dimensionless velocity, temperature and volume fraction of nanoparticles profiles in the presence of internal heat generation are illustrated in Fig. 2. In order to discuss the effects of buoyancy-ratio parameter Nr the other parameters are kept constant at $Nb = 0.5$, $Nt = 0.1$, $Le = 10$ and $R = 1$. It can be seen that the dimensionless velocity profiles decrease near the plate when the buoyancy-ratio parameter Nr is increased. Meanwhile, no significance in the decrease of velocity is observed for the variations in Nr away from the plate. The temperature and (solid) nanoparticle volume fraction profiles increase with the increase in Nr. In fact, the increase of these profiles is not appreciable though Nr is varied.

4.2 Effect of Brownian motion parameter Nb

The random motion of nanoparticles within the base fluid is called Brownian motion which occurs due to the continuous collisions between the nanoparticles and the molecules of the base fluid. Fig. 3 depict the effects on velocity, temperature and nanoparticle volume fraction profiles in the presence of internal heat generation for the variations of Brownian motion parameter Nb. The effects of Brownian motion parameter Nb are explicated by setting the values of other parameters to be constant at $Nr = 0.5$, $Nt = 0.1$, $Le = 10$ and $R = 1$. It is observed that the velocity and temperature profiles are increased when Nb increases. The physics behind the rise in temperature is that the increased Brownian motion increases the thickness of thermal boundary layer. However, in the case of (solid) nanoparticle volume fraction, it results in a decrease while Nb is increased away from the plate.



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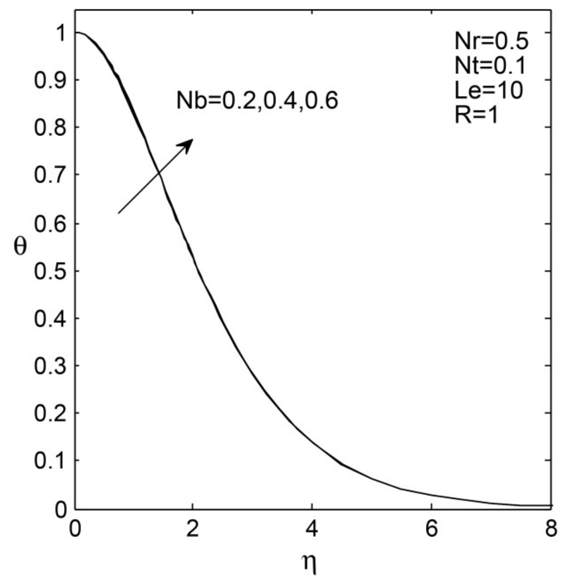

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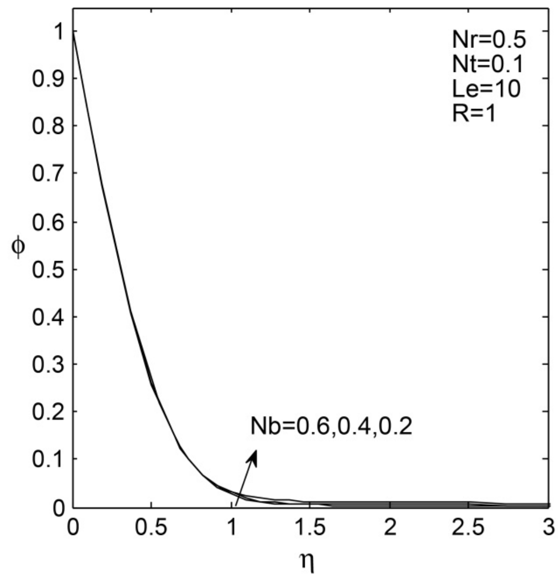
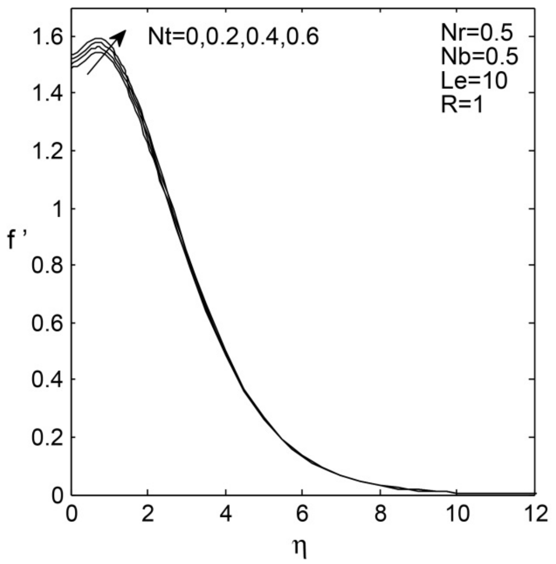


(c)

Fig. 3: Effects of Nb on dimensionless (a) velocity (b) temperature and (c) nanoparticle volume fraction profiles.

4.3 Effect of thermophoretic parameter Nt

The phenomenon in which the particles can diffuse under the effect of a temperature gradient is called thermophoresis. The influence of thermophoresis parameter Nt on velocity, temperature and nanoparticle volume fraction profiles can be viewed in Fig. 4 when the internal heat generation is present. The discussion for the effects of thermophoresis parameter Nt is carried out for the constant values of the other parameters such as $Nr = 0.5$, $Nb = 0.5$, $Le = 10$ and $R = 1$.



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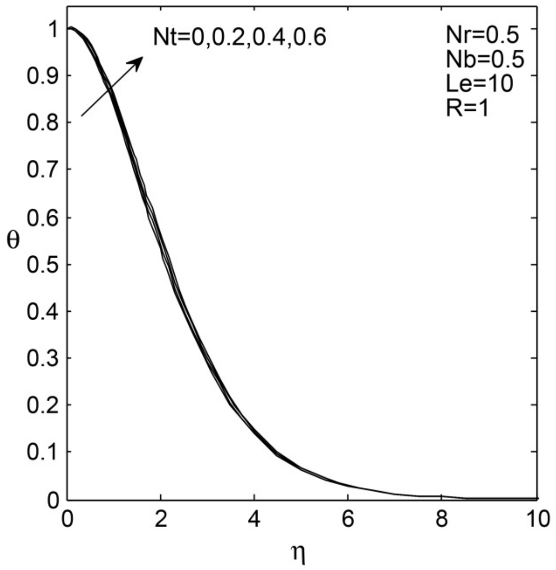
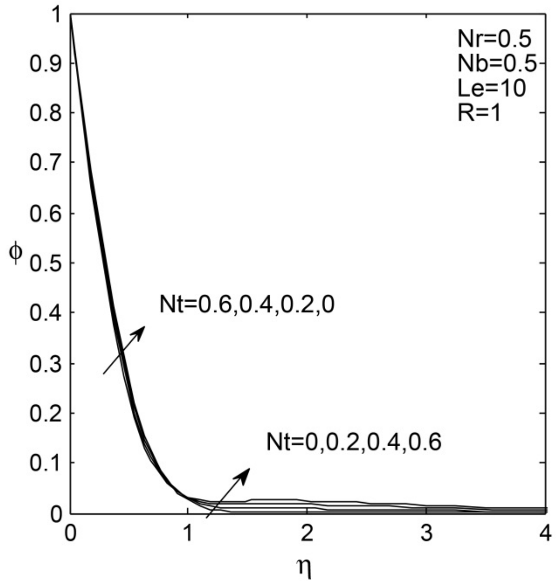


(b)

The velocity decreases while Nt is increased near the plate whereas the thermal boundary layer slightly decreases along with the increase in Nt.

Increases in this parameter physically imply high temperature gradients. As for as, the nanoparticle volume fraction profile is considered, the nanoparticle volume fraction increases with the increase in Nt away from the plate.

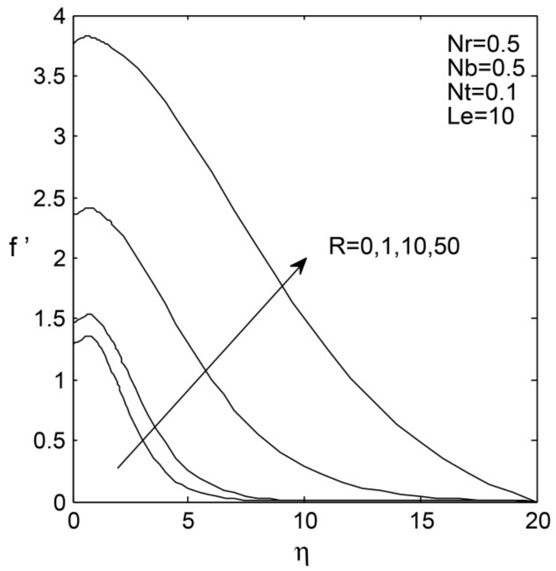


(c)

Fig. 4: Effects of Nt on dimensionless (a) velocity (b) temperature and (c) nanoparticle volume fraction profiles.

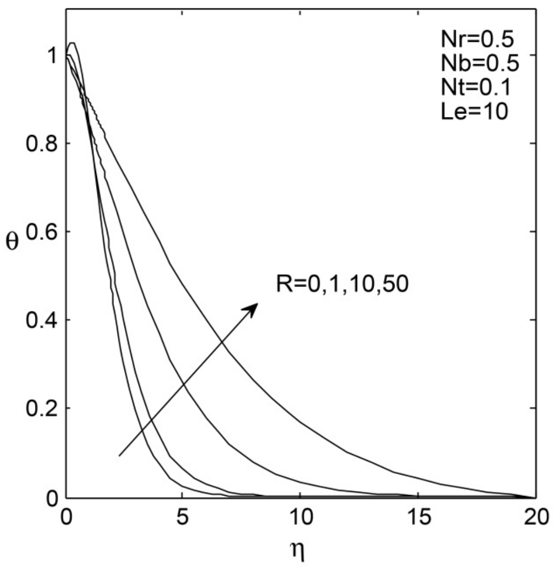
4.4 Effect of radiation parameter R

For the constant parameters $Nr = 0.5$, $Nt = 0.1$ and $Le = 10$, Fig. 5 illustrates the effects of radiation parameter in the presence of internal heat generation on the dimensionless velocity, temperature and nanoparticle volume fraction profiles. From Fig. 5(a) the significance of the addition of radiation parameter in the present work is confirmed.



(a)

The motion of the fluid increases within the flow region since the fluid absorbs heat while radiation imitates from the heated plate. As a result, it can be observed that the flow velocity strictly increases considerably when radiation is increased. Furthermore, similar effects are observed for the dimensionless temperature and nanoparticle volume fraction. In other words, the increase in the values of the radiation parameter results in an appreciable increase in the temperature and nanoparticle volume fraction profiles. Therefore higher values of radiation parameter imply higher surface heat flux which will increase the temperature within the boundary layer region.



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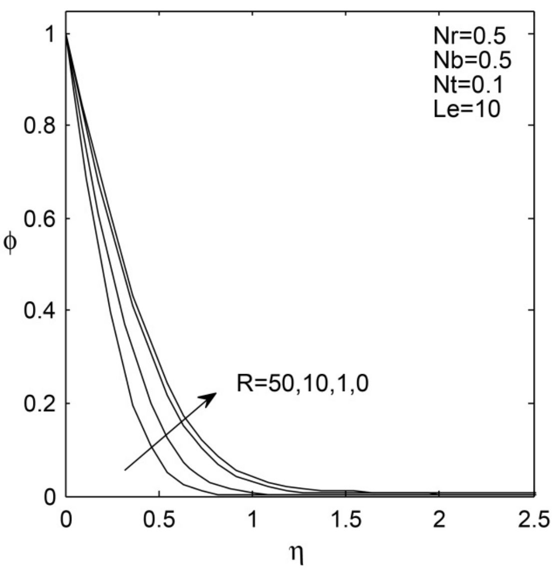
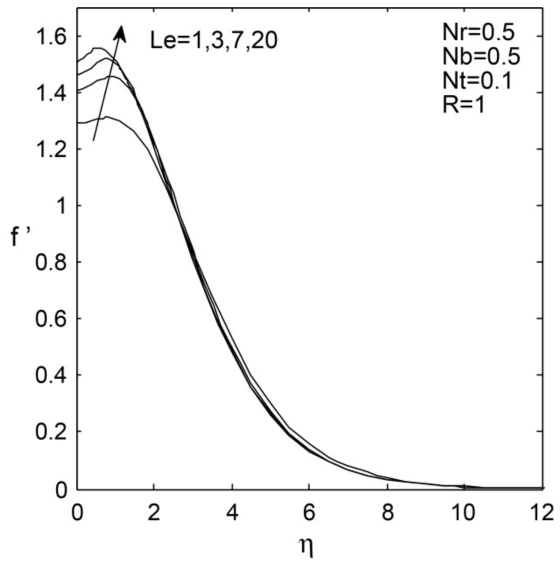


(c)

Fig. 5: Effects of R on dimensionless (a) velocity (b) temperature and (c) nanoparticle volume fraction profiles.

4.5 Effect of Lewis number Le

The Lewis number is defined as the ratio of thermal diffusivity to mass diffusivity. In the presence of internal heat generation, the effects of Lewis number are presented in Fig. 6 by fixing the parameters constantly at $Nr = 0.5$, $Nb = 0.5$, $Nt = 0.1$ and $R = 1$. The velocity is increased near the plate as Le increases and no remarkable variation is observed away from the plate though Le is varied such as $Le = 1, 3, 7$ and $Le = 20$. Meanwhile, the dimensionless temperature decreases as Le is increased. While the nanoparticle fraction profile is considered, it strongly decreases along with the increase of Lewis number.



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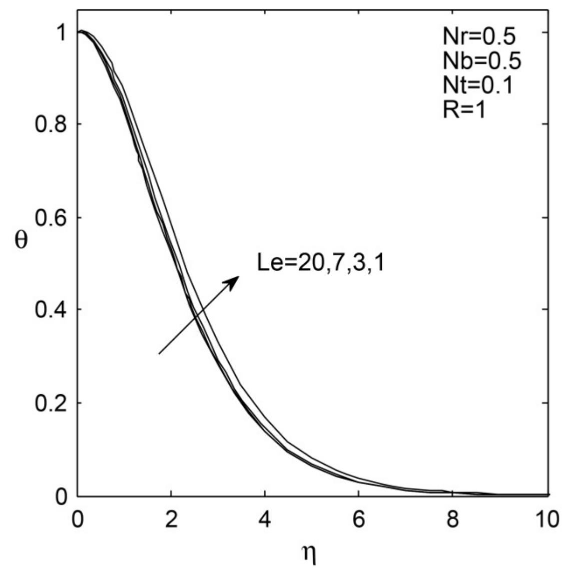

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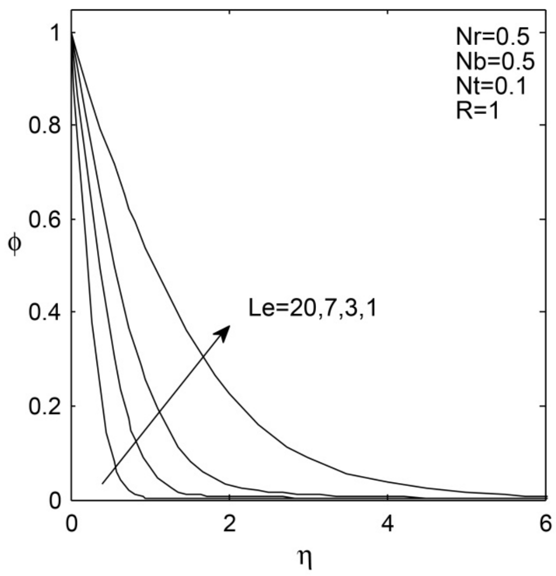

(c)

Fig. 6: Effects of R on dimensionless (a) velocity (b) temperature and (c) nanoparticle volume fraction profiles.

Table No. 1-3 displays the values of the slip velocity, the Nusselt number and the Sherwood number for the variations of the parameters Brownian motion (Nb), thermophoresis (Nt), buoyancy-ratio (Nr), Lewis number (Le) and radiation parameter (R) in the presence of internal heat generation when the heated plate is facing upward.

Table 1 Values of slip velocity for different values of Nr, Nb, Nt, Le and R in the presence of internal heat generation.

Nr	Nb	Nt	Le	R	$f'(0)$
0.2	0.5	0.1	10	1	1.56156568
0.4	0.5	0.1	10	1	1.51770845
0.6	0.5	0.1	10	1	1.47301354
0.5	0.2	0.1	10	1	1.48758833
0.5	0.4	0.1	10	1	1.49321428
0.5	0.6	0.1	10	1	1.49757248
0.5	0.5	0.2	10	1	1.50317387
0.5	0.5	0.4	10	1	1.51891341
0.5	0.5	0.6	10	1	1.53507619
0.5	0.5	0.1	3	1	1.42431339
0.5	0.5	0.1	7	1	1.47822801
0.5	0.5	0.1	20	1	1.52225698
0.5	0.5	0.1	10	0	1.32837711
0.5	0.5	0.1	10	10	2.36033421
0.5	0.5	0.1	10	100	4.51733579

It can be understood from the table that the slip velocity, the local heat transfer rate and the mass transfer rate decrease for the increase of the parameter Nr. On increasing the values of Nb and Nt, an increase is observed in the slip velocity and local heat transfer rate is decreased. In other words, the Brownian motion and the thermophoresis of nanoparticles increase the effective thermal conductivity of the nanofluid. The Sherwood number decreases due to the increase of Nb. In other words, the Brownian motion decreases the mass transfer rate. However, the mass transfer rate increases for the increased thermophoresis Nt.

Table 2 Values of Nusselt number for different values of Nr, Nb, Nt, Le and R in the presence of internal heat generation

Nr	Nb	Nt	Le	R	$-\theta'(0)$
0.2	0.5	0.1	10	1	-0.02894865
0.4	0.5	0.1	10	1	-0.03051756
0.6	0.5	0.1	10	1	-0.03213822
0.5	0.2	0.1	10	1	-0.01995181
0.5	0.4	0.1	10	1	-0.02766023
0.5	0.6	0.1	10	1	-0.03479679
0.5	0.5	0.2	10	1	-0.03518518
0.5	0.5	0.4	10	1	-0.04265761
0.5	0.5	0.6	10	1	-0.04980731
0.5	0.5	0.1	3	1	-0.04529580
0.5	0.5	0.1	7	1	-0.03499552
0.5	0.5	0.1	20	1	-0.02512259
0.5	0.5	0.1	10	0	-0.22403821
0.5	0.5	0.1	10	10	0.11392136
0.5	0.5	0.1	10	100	0.08318467

Table 3 Values of Sherwood number for different values of Nr, Nb, Nt, Le and R in the presence of internal heat generation

Nr	Nb	Nt	Le	R	$-\phi'(0)$
0.2	0.5	0.1	10	1	1.85384153
0.4	0.5	0.1	10	1	1.83313842
0.6	0.5	0.1	10	1	1.81180210
0.5	0.2	0.1	10	1	1.86156352
0.5	0.4	0.1	10	1	1.82851842
0.5	0.6	0.1	10	1	1.81892282
0.5	0.5	0.2	10	1	1.85893365
0.5	0.5	0.4	10	1	1.93270716
0.5	0.5	0.6	10	1	2.00763129
0.5	0.5	0.1	3	1	0.99310713
0.5	0.5	0.1	7	1	1.52360390
0.5	0.5	0.1	20	1	2.58049278
0.5	0.5	0.1	10	0	1.75216932
0.5	0.5	0.1	10	10	2.24865048
0.5	0.5	0.1	10	100	3.10009118

The slip velocity, the local Nusselt number and Sherwood number significantly increase when the Lewis number Le and radiation R are increased. When the values of the radiation parameter is increased from $R=0$ to $R=10$ and then to $R=100$, the heat transfer rate increases. The negative sign in the values of local Nusselt number indicates that the plate is cooled by the fluid. In other words, the heat is transferred from the fluid to the plate. The CPU time of calculation for various values of R and fixed values of other controlling parameters is given in Table 4.

Table 4 Performance of bvp4c measured in CPU time for fixed values of Nr =0.5, Nb =0.5, Nt =0.1, Le =10 and various values of R with internal heat generation.

R	$f'(0)$	$-\theta'(0)$	$-\phi'(0)$	CPU time [s]
0	1.32837	-0.22403	1.75216	0.9421
10	2.36033	0.11392	2.24865	1.0054
100	4.51733	0.08318	3.10009	1.0409

In order to verify the accuracy of the obtained solutions, comparisons have been made with the available results of Gorla and Chamkha [8], in the literature, which are presented in Table No. 5. For these comparisons, the numerical solutions are computed in the absence of internal heat generation and radiation. It can be viewed that the results in the present work show a good agreement with the previous results.

Table 5 Comparison values of Nusselt number and Sherwood number for different values of Nr, Nb, Nt and for $Le=10$,without internal heat generation and radiation.

Nr	Nb	Nt	Gorla and Chamkha [8]		Present study	
			$-\theta'(0)$	$-\phi'(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.3	0.1	0.326	1.484	0.326	1.483
0.3	0.3	0.1	0.322	1.453	0.322	1.452
0.5	0.3	0.1	0.319	1.419	0.318	1.419
0.5	0.1	0.1	0.368	1.328	0.367	1.326
0.5	0.5	0.1	0.273	1.448	0.272	1.446
0.5	0.3	0.3	0.293	1.417	0.293	1.416
0.5	0.3	0.5	0.271	1.429	0.271	1.428

5 Conclusion

In this paper, the effects of radiation and internal heat generation on the heat and mass transfer characteristics over an upward facing horizontal flat plate embedded in a porous medium filled with a nanofluid are numerically examined. The governing boundary layer equations for the considered flow model are transformed into non-linear ordinary differential equations by applying similarity transformations. The numerical results are computed by using MATLAB BVP solver bvp4c. The obtained solutions are graphically presented for the controlling parameters such as radiation parameter R, buoyancy-ratio parameter Nr, Brownian motion parameter Nb and thermophoresis parameter Nt and Lewis number Le. The following points are the conclusions drawn by a thorough observation from the analysis of the present work.

1. The increase in the values of the radiation parameter leads to a significant increase in flow velocity, the temperature and nanoparticle volume fraction profiles.
2. On increasing the values of Nb and Nt, an increase is observed in the slip velocity whereas the local heat transfer rate is decreased.
3. Mass transfer rate decreases with the increase in Nb whereas it increases for the increased thermophoresis Nt.
4. The slip velocity, the heat and mass transfer rates significantly increase when the Lewis number Le and radiation R are increased.

Nomenclature

C	Nanoparticle volume fraction
C_w	Nanoparticle volume fraction at the plate
C_∞	Ambient nanoparticle volume fraction
D_B	Brownian diffusion coefficient
D_T	Thermophoretic diffusion coefficient
f	Dimensionless stream function
g	Acceleration due to gravity
k	Permeability of the porous medium
k_e	Mean absorption coefficient
Le	Lewis number
Nb	Brownian motion parameter
Nt	Thermophoresis parameter
Nr	Buoyancy Ratio
Nu_x	Local Nusselt number
q_r	Radiative heat flux
q_W	Internal heat generation
R	Thermal radiation parameter

Ra_x Local Rayleigh number
Sh_x Local Sherwood number
T Temperature
T_w Wall temperature
T_∞ Temperature of the uniform flow
u, v Velocity components
x, y Co-ordinate system

Greek symbols

α_m Thermal diffusivity of the porous medium
β Volumetric expansion coefficient of fluid
ε Porosity
η Dimensionless similarity variable
θ Dimensionless temperature
μ Dynamic viscosity of the base fluid
υ Kinematic viscosity
$(\rho C)_p$ Effective heat capacity of nanoparticle material
$(\rho C)_f$ Heat capacity of the fluid
ρ_f Density of the base fluid
ρ_p Nano-particle mass density
σ_s Stephen Boltzmann constant
τ Ratio of nanoparticle heat capacity and the base fluid heat capacity
ϕ Nanofluid (solid) volume fraction
ψ Dimensionless stream function

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