Anomalous Filtration in a Rectangular Medium

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Abstract: - In this paper, the problem of anomalous filtration of a homogeneous fluid in a two-dimensional porous medium is considered. The conductivity equation is given by a fractional derivative in two dimensions. An anomalous model of fluid filtration in two dimensions is numerically analyzed. The problem of constant pressure at the boundary is considered. The effect of the anomaly on the pressure field distribution and the filtration rate is evaluated.

Key-Words: - Anomalous filtration, anomalous Darcy's law, filtration velocity, fractional derivative, one-dimensional, porous media.

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1 Introduction

Darcy's linear law is valid for homogeneous and Newtonian fluids; it has been established that phenomena in which friction and other effects occurring in a porous medium are significant cannot be explained by the linear Darcy's law, [1]. In classical models, for instance, using the linear Darcy's law or Fick's law, the change in velocity and concentration profiles over time is described by exponential-type solutions. Subsequently, experimental results from fluid flow studies have revealed processes that cannot be explained by these classical laws, [2].

The mathematical model proposed in [3] is suitable for describing anomalous processes observed in media with fractal geometry, as well as in disordered and non-uniform porous media. A numerical scheme based on an existing discretization method is applied to process the mathematical model based on modified memory. The accuracy of the numerical model is verified by the analytical solution of a simplified problem.

Various formulas describing non-Darcy flow have been applied to determine fluid flow in natural geological environments; however, no universal law or formula exists that can reliably describe the complex relationship between flow velocity and pressure gradient for single-phase fluid flow in natural oil and gas reservoirs. Model fitting shows that the spatial fractional Darcy's law can accurately reflect flow characteristics and corresponds well with the experimental data presented in the flow curves. The model parameters can also be significantly simplified to provide a physical interpretation that is closely related to the structure of the medium, [4].

A memory mechanism is presented that mathematically models the potential changes in the physical properties of a fluid due to alterations in its physical or chemical interaction, [5]. The elastic response of the matrix is disregarded, and the diffusion equations are decoupled from the elasticity equations, which are not considered here. The main focus is on the fluid pressure periodically observed in the flow, such as under periodic boundary conditions. At first glance, introducing a mathematical memory into the formulation of the governing equations may seem to complicate the problem; however, using the Laplace transform (LT) allows for a direct solution to be obtained for the given boundary conditions.

A fractional derivative model has been proposed for high-velocity flow in porous media that does not obey Darcy's law. Additionally, a model for low-velocity flow in a low-permeability medium has also been developed using a fractional derivative, based on the Swartzendruber model, [6].

In [7], both uniform and mixed media with different characteristic particle sizes were used. It was found that memory parameters, particularly the small value adopted in terms of an order of magnitude, have a multifaceted influence on the experiment. The data and theory have shown how mechanical compression can reduce the conductivity and consequently the rate of flow.

We consider a generalization of the governing equation for diffusion in the case where a phase memory mechanism is at work, [8]. The order of the medium is described by fractional-order differential equations covering the continuum within a given range; in this case, the governing equation is transformed into a distributed-order differential equation.

In this paper, on the basis of the filtration law [9], which accounts for changes in the permeability of the medium, a formulation of the filtration equation that depends on pressure is derived. For this equation, some finite-layer problems are posed and considered with constant boundary conditions. At one of the boundaries of the medium, constant pressure regimes are considered. The pressure and filtration velocity domains are determined for various values of a parameter α, which characterizes the memory effects in the filtration law.

2 Derivation of the Filtration Equation

The law of anomalous filtration is taken as [9], [10], [11]

$$\vec{u} = -\frac{k}{d}\nabla\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right), \tag{1}$$

where is $\vec{u}$ is the filtration velocity, k is the pseudo-permeability, i.e. fractal permeability with the dimension L^2T^α, L is the dimension of length, T is the dimension of time, d is the viscosity of the liquid, α is the order of the derivative, ∇ is the Hamilton operator, is the pressure.

Let us take the continuity equation in the form [11], [12]:

$$\frac{\partial(\rho n)}{\partial t} + div\left[\rho\vec{u}\right] = 0, \tag{2}$$

where ρ is the density of the liquid $\left(\frac{kg}{m^3}\right)$, n is the porosity of the medium $\left(\frac{m^3}{m^3}\right)$

Substituting (1) into (2) we obtain:

$$\frac{\partial(\rho n)}{\partial t} - div\left[\rho\frac{k}{d}\nabla\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right)\right] = 0. \tag{3}$$

We assume the porous medium and the fluid to be compressible, and the dynamic viscosity of the fluid to be constant. In addition, the conductivity of the medium is not uniform in all directions., etc. k is a function of coordinates.

The second term in (3) can be rewritten as follows:

$$\begin{aligned} div\left[\rho\nabla\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right)\right] &= \rho\Delta\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) + \nabla\rho\cdot\nabla\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) = \\ &= \rho\Delta\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) + \frac{\partial\rho}{\partial x}\cdot\frac{\partial}{\partial x}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) + \\ &+ \frac{\partial\rho}{\partial y}\cdot\frac{\partial}{\partial y}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) + \frac{\partial\rho}{\partial z}\cdot\frac{\partial}{\partial z}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) = \\ &= \rho\Delta\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) + \frac{\partial\rho}{\partial p}\cdot\frac{\partial p}{\partial x}\cdot\frac{\partial}{\partial x}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) + \\ &+ \frac{\partial\rho}{\partial p}\cdot\frac{\partial p}{\partial y}\cdot\frac{\partial}{\partial y}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) + \frac{\partial\rho}{\partial p}\cdot\frac{\partial p}{\partial z}\cdot\frac{\partial}{\partial z}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right), \end{aligned} \tag{4}$$

where Δ is the Laplace operator.

Considering that the fluid motion in the medium is slow and the pressure gradient is small, we can disregard the last three terms in expression (4). Considering this, with constant k from (3) we obtain:

$$\frac{\partial(\rho n)}{\partial t} - \frac{k}{d}\rho\Delta\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) = 0. \tag{5}$$

Let's transform the first term of (5) as

$$\begin{aligned} \frac{\partial(\rho n)}{\partial t} &= n\frac{\partial\rho}{\partial t} + \rho\frac{\partial n}{\partial t} = n\frac{\partial\rho}{\partial p}\cdot\frac{\partial p}{\partial t} + \\ &+ \rho\frac{\partial n}{\partial p}\cdot\frac{\partial p}{\partial t} = \rho\left(n\frac{1}{\rho}\frac{\partial\rho}{\partial p} + \frac{\partial n}{\partial p}\right)\frac{\partial p}{\partial t}. \end{aligned} \tag{6}$$

Let us take the laws of change of ρ and n with dependent to p in the form:

$$\rho = \rho_0 \cdot e^{\beta_f (p-p_0)}, \quad (7)$$

$$n = n_0 + \beta_r (p - p_0) \ , \quad (8)$$

where ρ_0 and n_0 – are the density and porosity at $p = p_0$, p_0 – is a certain reference pressure, β_f and β_r – are the coefficients of volumetric elasticity of the liquid and the medium, respectively.

When the fluid is weakly compressible, instead of (7) we take:

$$\rho = \rho_0 [1 + \beta_f (p - p_0)] \quad (9)$$

Taking into account (7), (8) from (6) we have

$$\frac{\partial(\rho n)}{\partial t} = \rho(m_0 \beta_f + \beta_r)\frac{\partial p}{\partial t}, \quad (10)$$

where in the parentheses on the right side, taking into account the smallness of the change in value, n_0 is taken instead of n.

Taking into account (10) from (5) we have:

$$\frac{\partial p}{\partial t} = \varkappa \ \Delta\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right), \quad (11)$$

where $\varkappa = \dfrac{k}{d\beta^*}$ is is the coefficient of piezoconductivity, $\beta^* = n_0 \beta_f + \beta_r$ is the elasticity coefficient of the medium.

For inhomogeneous media in terms of permeability, equation (11) has the form:

$$d\beta^* \frac{\partial p}{\partial t} = \Delta\left(k(x,y,z) \frac{\partial^\alpha p}{\partial t^\alpha} \right). \quad (12)$$

For homogeneous anisotropic media, the piezoconductivity equation is written as:

$$d\beta^* \frac{\partial p}{\partial t} = k_x \frac{\partial^2}{\partial x^2}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) + k_y \frac{\partial^2}{\partial y^2}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) + k_z \frac{\partial^2}{\partial z^2}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right), \quad (13)$$

where k_x, k_y, k_z – are the permeability coefficient in the directions, x, y, z respectively.

3 Statement of the Problem

We will consider the two-dimensional equation (11)

$$\frac{\partial p}{\partial t} = \chi\left[\frac{\partial^2}{\partial x^2}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right) \right] \quad (14)$$

in a rectangular region $D\{0 \le x \le L_1,\ 0 \le y \le L_2\}$.

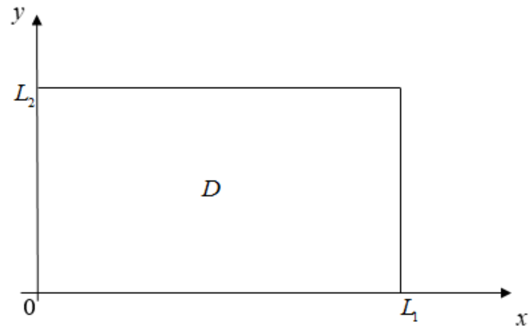

Fig. 1: Filtration area D
Source: created by the authors

Let the pressure in the environment initially be constant p_0. Starting from $t > 0$ At the origin, a constant pressure is created p_c. At , $p_c < p_0$ fluid is withdrawn from the medium, and at $p_c > p_0$, $[0, L_2]$ fluid is injected into the medium. The sides of the region D and $[0, L_1]$ are impermeable to liquid. We assume L_1 that and L_2 are sufficiently large, and in the time range used in this study, we assume that the pressure change does not reach the boundaries $x = L_1$, $y = L_2$. Consequently, the initial pressure is maintained on these sides p_0. It is also possible to specify the input pressure at the origin as a function that oscillates harmonically over time.

Based on the formulated conditions, the initial condition has the form:

$$p(0,x,y) = p_0 = \text{const} \ , \quad (15)$$

we consider the following boundary conditions:

$$p(t,0,0) = p_c = \text{const}, \quad (16)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right)(t,0,y) = 0, \ \ y > 0, \quad (17)$$

$$p(t, L_1, y) = p_0 \quad (18)$$

$$\frac{\partial}{\partial y}\left(\frac{\partial^\alpha p}{\partial t^\alpha}\right)(t,x,0) = 0, \ \ x > 0, \quad (19)$$

$$p(t, x, L_2) = p_0 \quad (20)$$

4 Numerical Solution Algorithm

Equation (14) under conditions (15)–(20) is solved using the finite difference method [13], [14]. To do this, $\{0 \le t \le T_{max}, 0 \le x \le L_1\, 0 \le y \le L_2\}$ we introduce a grid in the domain $\overline{\omega}_{h_1} = \{x = i \cdot h_1,\ i = 0,1,...N_1\}$, $\overline{\omega}_{h_2} = \{y = j \cdot h_2,\ j = 0,1,...N_2\}$, $\overline{\omega}_{\tau} = \{t_i = k \cdot \tau, k = 0,1,2,..M\}$, where h_1, h_2 are the grid steps in direction x and y, respectively, τ is the grid step in time. From these grids, we construct $\overline{\omega} = \overline{\omega}_{h_1} \times \overline{\omega}_{h_2} \times \overline{\omega}_{\tau}$ a three-dimensional grid. Its nodes consist of points (x_i, y_j, t_k), $i = \overline{0, N_1}$; $j = \overline{0, N_2}$; $k = \overline{0, M}$.

On this grid, equation (1) is approximated using the fractional step method of N.N. Yanenko, [15], [16]

$$\frac{p_{i,j}^{k+1/2} - p_{i,j}^{k}}{\tau} = \frac{\chi}{h_1^2}\frac{\tau^{1-\alpha}}{\Gamma(2-\alpha)}\left[\frac{p_{i-1,j}^{k+1/2} - p_{i-1,j}^{k}}{0,5\tau} + (S_1)_i^j - 2\left(\frac{p_{i,j}^{k+1/2} - p_{i,j}^{k}}{0,5\tau} + (S_2)_i^j\right) + \frac{p_{i+1,j}^{k+1/2} - p_{i+1,j}^{k}}{0,5\tau} + (S_3)_i^j\right] \tag{21}$$

$$\frac{p_{i,j}^{k+1} - p_{i,j}^{k+1/2}}{\tau} = \frac{\chi}{h_2^2}\frac{\tau^{1-\alpha}}{\Gamma(2-\alpha)}\left[\frac{p_{i,j-1}^{k+1} - p_{i,j-1}^{k+1/2}}{0,5\tau} + (S_4)_i^j - 2\left(\frac{p_{i,j}^{k+1} - p_{i,j}^{k+1/2}}{0,5\tau} + (S_5)_i^j\right) + \frac{p_{i,j+1}^{k+1} - p_{i,j+1}^{k+1/2}}{0,5\tau} + (S_6)_i^j\right], \tag{22}$$

$i = \overline{1, N_1 - 1}$, $j = \overline{1, N_2 - 1}$, $k = \overline{0, M-1}$,

where

$$(S_1)_i^j = \sum_{l=0}^{k-1}\frac{p_{i-1,j}^{l+1} - p_{i-1,j}^{l}}{\tau}\cdot\left((k-l+1)^{1-\alpha} - (k-l)^{1-\alpha}\right),$$

$$(S_2)_i^j = \sum_{l=0}^{k-1}\frac{p_{i,j}^{l+1} - p_{i,j}^{l}}{\tau}\cdot\left((k-l+1)^{1-\alpha} - (k-l)^{1-\alpha}\right),$$

$$(S_3)_i^j = \sum_{l=0}^{k-1}\frac{p_{i+1,j}^{l+1} - p_{i+1,j}^{l}}{\tau}\cdot\left((k-l+1)^{1-\alpha} - (k-l)^{1-\alpha}\right)$$

$$(S_4)_i^j = \sum_{l=1}^{k-1}\frac{p_{i,j-1}^{l+1/2} - p_{i,j-1}^{l-1/2}}{\tau}\cdot\left((k-l+1)^{1-\alpha} - (k-l)^{1-\alpha}\right)$$

$$(S_5)_i^j = \sum_{l=0}^{k-1}\frac{p_{i,j}^{l+1/2} - p_{i,j}^{l-1/2}}{\tau}\cdot\left((k-l+1)^{1-\alpha} - (k-l)^{1-\alpha}\right),$$

$$(S_6)_i^j = \sum_{l=0}^{k-1}\frac{p_{i,j+1}^{l+1/2} - p_{i,j+1}^{l-1/2}}{\tau}\cdot\left((k-l+1)^{1-\alpha} - (k-l)^{1-\alpha}\right).$$

Scheme (21) is reduced to a system of linear algebraic equations (SLAE) in the form

$$A_1 p_{i-1,j}^{k+1/2} - B_1 p_{i,j}^{k+1/2} + C_1 p_{i+1,j}^{k+1/2} = -(F_1)_{i,j}^{k}, \tag{23}$$

Where

$$A_1 = k_1;\ B_1 = 2k_1 + 1;\ C_1 = k_1;$$

$$k_1 = \frac{2\tau^{1-\alpha}\chi}{\Gamma(2-\alpha)h_1^2},$$

$$(F_1)_{i,j}^{k} = p_{i,j}^{k} + k_1\left(2p_{i,j}^{k} - p_{i-1,j}^{k} - p_{i+1,j}^{k} + 0.5\tau\left((S_1)_i^j - 2(S_2)_i^j + (S_3)_i^j\right)\right).$$

Scheme (22) is also reduced to SLAE

$$A_2 p_{i-1,j}^{k+1} - B_2 p_{i,j}^{k+1} + C_2 p_{i+1,j}^{k+1} = -(F_2)_{i,j}^{k+1/2} \tag{24}$$

where

$$A_2 = k_2;\ B_2 = 2k_2 + 1;\ C_2 = k_2;$$

$$k_2 = \frac{2\tau^{1-\alpha}\chi}{\Gamma(2-\alpha)h_2^2},$$

$$(F_2)_{i,j}^{k+1/2} = p_{i,j}^{k+1/2} + k_2\left(2p_{i,j}^{k+1/2} - p_{i,j-1}^{k+1/2} - p_{i,j+1}^{k+1/2} + 0.5\tau\left((S_4)_i^j - 2(S_5)_i^j + (S_6)_i^j\right)\right)$$

The initial condition is approximated as:

$$p_{i,j}^{0} = p_0,\ i = \overline{0, N_1},\ \ j = \overline{0, N_2}. \tag{25}$$

The boundary conditions are approximated as:

$$p_{0,0}^{k+1/2} = p_c, \tag{26}$$

$$\frac{\tau^{1-\alpha}}{h_1\Gamma(2-\alpha)}\cdot\left((S_3)_0^j + \frac{p_{1,j}^{k+1/2} - p_{1,j}^{k}}{0.5\tau} - (S_2)_0^j - \frac{p_{0,j}^{k+1/2} - p_{0,j}^{k}}{0.5\tau}\right) = 0,\ j = \overline{1, N_2}, \tag{27}$$

$$p_{N_1,j}^{k+1/2} = p_0, \tag{28}$$

$$\frac{\tau^{1-\alpha}}{h_2\Gamma(2-\alpha)}\cdot\left(\left(S_6\right)_i^0+\frac{p_{i,1}^{k+1/2}-p_{i,1}^k}{0.5\tau}-\right. \\ \left.-\left(S_5\right)_i^0-\frac{p_{i,0}^{k+1/2}-p_{i,0}^k}{0.5\tau}\right)=0 \quad , \; i=\overline{1,N_1} \,, \quad (29)$$

$$p_{i,N_2}^{k+1/2}=p_0\,. \quad (30)$$

The following calculation sequence is established.
1. By solving equation (24) for $i=0$ on $(k+1/2)$ the layer and we find $p_{0,j}^{k+1/2}$ $\left(j=\overline{1,N_2-1}\right)$. Then equation (23) is solved for $j=\overline{0,N_2}$ on $(k+1/2)$the -layer and we determine $p_{i,j}^{k+1/2}$ $\left(i=\overline{1,N_1-1}\right)$. Thus, the pressure on $(k+1/2)$the -layer is completely determined.
2. By solving equation (23) for $j=0$ on $(k+1)$the -layer and we find $p_{i,0}^{k+1}$ $\left(i=\overline{1,N_1-1}\right)$. After this, equation (24) is solved for $i=\overline{0,N_1}$ on $(k+1)$the -layer and we determine $p_{i,j}^{k+1}$ $\left(j=\overline{1,N_2-1}\right)$.

Equations (23) and (24) are solved by sweeping along the directions i and j, respectively. For this, we use the following relations:

$$p_{i,j}^{k+1/2}=\delta_{i+1}^{(1)}\cdot p_{i+1,j}^{k+1/2}+\eta_{i+1}^{(1)},\, i=\overline{1,N_1-1},\; j=\overline{0,N_2}\,, (31)$$

$$p_{i,j}^{k+1}=\delta_{j+1}^{(2)}\cdot p_{i,j+1}^{k+1}+\eta_{j+1}^{(2)},\, i=\overline{0,N_1},\; j=\overline{1,N_2-1}\,, (32)$$

where $\delta_{i+1}^{(1)},\eta_{i+1}^{(1)},\;\delta_{i+1}^{(2)},\eta_{i+1}^{(2)}$ are the running coefficients.

For problem a), the starting values of the coefficients $\delta_{i+1}^{(1)},\eta_{i+1}^{(1)},\;\delta_{i+1}^{(2)},\eta_{i+1}^{(2)}$ are determined from conditions (26)–(30)

$$\begin{cases}\delta_1^{(1)}=0, & \eta_1^{(1)}=p_c \quad \text{при} \quad j=0,\\ \delta_1^{(1)}=0, & \eta_1^{(1)}=p_{0,j}^{k+1/2} \quad \text{при} \quad j=\overline{1,N_2},\end{cases} \quad (33)$$

$$\begin{cases}\delta_1^{(2)}=0, & \eta_1^{(2)}=p_c \quad \text{при} \; i=0,\\ \delta_1^{(2)}=0, & \eta_1^{(2)}=p_{i,0}^{k+1} \; \text{при} \; i=\overline{1,N_1}.\end{cases} \quad (34)$$

5 Discussion and Results

Some of the computational results are shown in Figure 1, Figure 2, Figure 3 and Figure 4 for different model parameters. The following parameter values are used: $k=10^{-13}\,\mathrm{m}^2\cdot\mathrm{s}^{\alpha}$, $d=10^{-2}\,Pa\cdot s$, $\beta^*=10^{-10}\,Pa^{-1}$, $p_c=5\cdot10^5\,Pa$, $p_0=0$, $L_1=2\;m, L_2=2\;m$. The following grid parameters were used: $h_1=0.05$, $h_2=0.05\;\tau=1$.

The pressure surfaces p for two time values t and various α are shown in Figure 1 and Figure 2. As the figures show, an increase in the value of α from 0.4 to 0.6 causes a decrease in the rate of pressure propagation in the medium. On the pressure surfaces, a deceleration in the development of the pressure profiles can be clearly observed with an increase in the value of α.

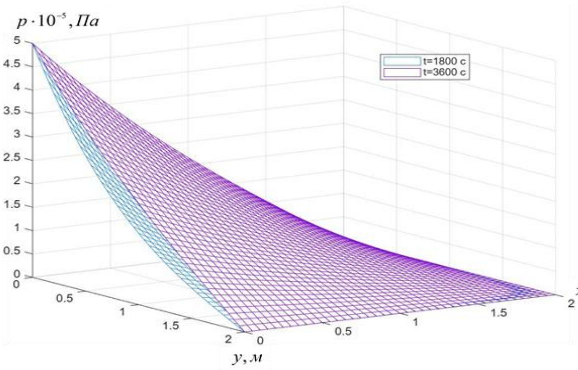

Fig. 2: Pressure profiles at $\alpha=0{,}4$.
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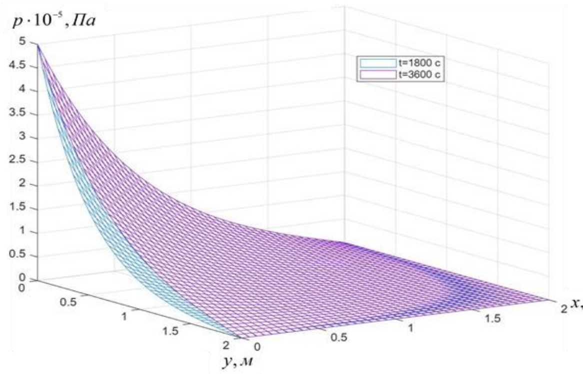

Fig. 3: Pressure profiles at $\alpha=0{,}6$.
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The filtration velocity surfaces for the cases analyzed above are shown in Figure 4 and Figure 5. Similar to pressure, the filtration velocity distribution u also decreases as α increases (Figure 4 and Figure 5)

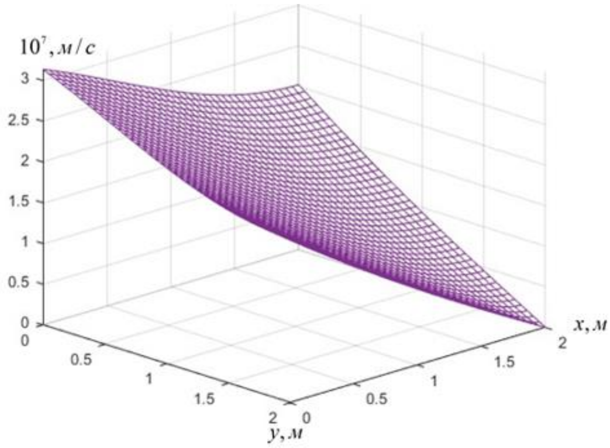

Fig. 4: Profiles of filtration velocity $\alpha = 0{,}4$.
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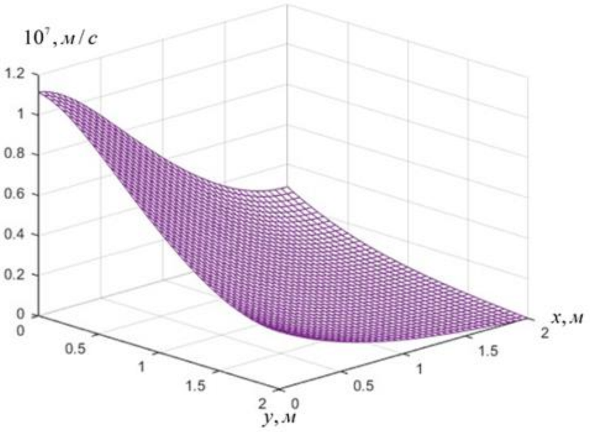

Fig. 5: Profiles of filtration velocity $\alpha = 0{,}6$.
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6 Conclusions

The piezoconductivity equation, based on the generalized Darcy's law, is augmented by the fractional derivative of the pressure gradient, which allows for a more accurate modeling of filtration processes in porous media involving time delays and memory effects. The use of fractional derivatives makes it possible to describe more complex processes that cannot be described using traditional differential operators. This results in a slower change in pressure and filtration rate, especially given that significant lag effects were observed as the order of the fractional derivative increased. This means that the higher the order of the fractional derivative, the greater the effect of time delay on pressure dispersion and filtration rate. These results show that the fractional derivative of Darcy's law allows for the effects of memory to be taken into account, which is important for modeling filtration in complex, porous, or anisotropic media, such as oil reservoirs or groundwater aquifers.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- MAXMUDOV J has derived the mathematical model and interpreted the results.
- USMONOV A approximated the model.
- The results were obtained in Python using the model and its approximation, and the overall framework was structured by Sh. Akramov and Gaybulov Yu.

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Conflict of Interest

The authors declare that there is no conflict of interest.