Transient MHD Fluid Flow Past a Moving Vertical Surface in a Velocity Slip Flow Regime

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Abstract: - The problem of unsteady MHD fluid flow past a moving vertical surface in a slip flow regime is presented. The model is built on the assumption that the flow is naturally convective with oscillating time-dependent and exponentially decaying suction and permeability, double-diffusion, viscous dissipation, and temperature gradient-dependent heat source, and non-zero tangential velocity at the wall; the fluid is viscous, incompressible, Newtonian, chemically reactive, and magnetically susceptible; the surface is porous, and electrically conductive, and thermally radiative. The governing partial differential equations are highly coupled and non-linear. For easy tractability, the equations are reduced to one-dimensional using the one-dimensional unsteady flow theory. The resulting equations are non-dimensionalized and solved using the time-dependent perturbation series solutions, and the Modified Homotopy Perturbation Method (MHPM). The solutions of the concentration, temperature, velocity, rates of mass and heat diffusion, and wall shear stress are obtained, computed, and presented graphically and quantitatively, and analyzed. The results among others, show that the increase in the: Schmidt number increases the fluid concentration, velocity, the rate of heat transfer to the fluid, and the stress on the wall, but decreases the rate of mass transfer to the fluid; Magnetic field parameter decreases the fluid velocity and stress on the wall; Slip parameter increases the flow velocity, but decreases the stress on the wall; Permeability parameter increases the flow velocity and the stress on the wall. These results are benchmarked with the reports in existing literature and they agree.

Key-Words: - Chemically reacting, MHD, Slip flow, Thermally radiating, Thermo-diffusion, Temperature gradient-dependent heat source, Viscous dissipation.

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1 Introduction

Flow problems with the magnetic field, chemical reaction, and heat source/sink effects cut across nature, science, and engineering. They have applications in the chemical and petroleum industries, cooling of nuclear reactors, catalytic reactors, and the likes.

In fluid dynamics, the no-slip condition applies to viscous fluids, and therein it is assumed that at solid boundaries the fluid velocity is equal to zero. The exertion is based on the fact that fluid particles on the surface do not move along with the flow when the force of adhesion is stronger than the cohesion. At the fluid-solid interface, the force of attraction between the fluid particles and the solid particles (adhesive force) is stronger than that between the fluid particles (cohesive force). The force imbalance brings the fluid velocity to zero. The no-slip condition is a universal assertion\assumption or a mere ideology and does not apply to inviscid flows, where the effect of the boundary layer is neglected. However, for engineering applications, the concept of no-slip condition does not always hold. For example, at very low pressure (at high altitude) some of the fluid particles near the solid surface move along with the surface, thus bringing to bear the slip velocity condition. Furthermore, at very high altitudes fluid particles adjacent to the surfaces (of aircraft and rockets, say) no longer take the velocity of the solid surface but possess finite tangential velocities known as the slip velocities (non-zero) with which they slip along the surface. These slip velocities are linearly proportional to the shear stress on the plate, while the surface boundary with slip velocity condition is the velocity-slip regime. The slip factor is a function of mass flow rate, fluid entry condition, working fluid viscosity, boundary layer growth, flow separation, etc. A common approximation for fluid slip is $u-u_{wall}=\alpha\partial u/\partial y$, where $\alpha=(2-m_1)/m_1$ the slip length, m_1 is Maxwell's reflection coefficient [1]. Importantly, the Maxwell's reflection coefficient/transmission coefficient is a parameter that describes how the wave is reflected by impedance discontinuity in the transmission medium. It is the ratio of the amplitude of the reflected wave/ transmitted wave to the incident wave. The slip idealization was first conceived and presented in [2], as a flow model wherein the velocity normal to the boundary is set to zero, while the velocity parallel to the boundary is left free. Building on this, [3], formulated a slip model, which has been used extensively by researchers to date. Upon Maxwell's model, a lot of research has been carried out. For example, [4], studied the slip flow at the entrance region of a parallel plate channel; [5] considered the flow in rectangular and annular ducts; [6] studied the MHD steady flow in a channel with slip at the permeable boundaries.

Specifically, concerning the flow past moving vertical plates, [7] examined the MHD visco-elastic flow with velocity slip when the plate is oscillating; [8] investigated the transient flow under variable suction, periodic temperature, and slip conditions; [9] examined the effect of periodic heat and mass transfer on the unsteady natural convective flow in the slip-flow regime when the suction velocity oscillates with time; [10] examined the flow under a magnetic field influence when the plate is oscillating in a slip velocity regime. Furthermore, [11] studied the MHD flow under radiation and temperature gradient-dependent heat source in a slip flow regime; [12] looked into the slip boundary layer of non-Newtonian fluid with convective thermal boundary condition; [13] considered the MHD convective heat and mass transfer in a boundary layer slip flow over with thermal radiation and chemical reaction; [14] looked at the transient MHD flow of a third-grade fluid when the plate is insulated, and in the presence of thermo-diffusion, time-dependent suction, heat source, mass transfer and slip effects. [1], studied the unsteady MHD natural convective flow over a porous vertical plate in the presence of radiation and temperature gradient-dependent heat source, exponentially decaying suction and permeability in a slip flow regime using time-dependent perturbation method and numerical analysis, and observed that the velocity increases with the increase in the slip parameter and Grashof number, but decreases with the increase in the magnetic field, heat source, radiation, and chemical reaction rate parameters; the temperature decreases with the increase in the radiation and heat source parameters; the concentration decreases with the increase in the Schmidt number and chemical reaction rate parameter. [15], studied the MHD natural convective chemically reactive flow in the presence of thermo-diffusion, fluctuating wall temperature and concentration, thermal radiation, and free stream and slip velocities; [16], considered the MHD boundary layer flow with slip near a stagnation point; [17], considered the flow in the presence of heat generation/absorption, slip velocity, and temperature jump. [18], investigated thermal diffusion and chemical reaction effects on an unsteady flow in the presence of temperature-dependent heat source and velocity slip condition using the method of exponentially increasing small perturbation law, and saw that the velocity is enhanced by the increase in the slip and permeability parameters, and Grashof numbers; the temperature increases with the increase in the Prandtl number, but decreases with increase in the heat source parameter; the concentration increases with the increase in the Soret number, but decreases with the increase in the permeability parameter and Schmidt number. [19], examined the effects of variable viscosity and periodic boundary conditions on a free convective flow in a slip regime; [20], examined the flow of a micro-polar fluid over a radiating surface in the presence of variable viscosity in a slip regime; [21], investigated the flow under velocity slip and time-periodic boundary effects; [22], examined numerically the higher-order chemical reaction effects on MHD Nano-fluid flow with velocity slip boundary condition. More so, [23], considered the boundary layer flow in the presence of cross-diffusion effect in a velocity slip regime; [24], studied numerically the flow of a Newtonian fluid in the presence of buoyancy, order two thermal slip and entropy generation; [25], looked into the transient slip flow with ramped plate temperature and concentration, thermal radiation and buoyancy effects. Similarly, [26], considered the transient MHD natural convective flow in a slip regime with periodic movement, Hall currents, and rotation effects; [27], gave a Reynolds analogy for the flow at different regimes.

Other reports on the convective flow over a vertical plate with slip velocity conditions are found in [28], [29], [30], [31], [32], [33], [34], [35], [36], [37].

More so, the interaction of electric and magnetic fields in a flow system results in many factors that influence the flow. By application, when a wire carrying alternating current is applied to a non-zero resistive plate/conductor a voltage difference is created between the ends of the conductor in the electric field. The electric field accelerates the charge carriers (electrons, ions, and holes) on the plate in the direction of the electric field to give kinetic energy. At collision with each other on the plate, the charged particles are scattered/randomized. The scattering motions of the charged particles cause the temperature of the plate/conductor to rise. This thermal effect is called the Joule/Ohmic heating. By this, electric energy is converted into thermal energy. Also, as the fluid flows past the plate a dissipating force that works mechanically to heat the fluid is produced. It is noteworthy that Joule heating is limited by viscosity, electric conductivity, and fouling deposits on the conductor. Furthermore, the varying alternating currents lead to the heating of the plate non-uniformly. Similarly, the heating of the plate to a high-temperature regime leads to the emission of thermal radiant rays. The effects of Joules/Ohmic heating, magnetic field, and viscous dissipation in the problem of convective heat and mass transfer on the flow past vertical plates have been investigated. For example, [38], examined the viscous and Joule heating effects on the MHD free convection flow with variable plate temperature; [39], studied the MHD natural convective flow of a radioactive fluid past an inclined plate in the presence of chemical reaction, temperature-dependent heat source, and Joule heating using the method of regular perturbation, and noticed that an increase in the magnetic field parameter decreases the velocity, whereas an increase in the permeability parameter increases it; the increase in the Prandtl and Schmidt numbers, respectively, condense the thermal and concentration boundary layers.

In highly interactive systems, where magnetic flux, convection, and chemical reaction are significant heat and mass transfer occur simultaneously. The simultaneous effect on the system called double-diffusion induces buoyancy. The differential in temperature produces Dufour (thermo-diffusion), while the differential in concentration produces Soret (diffusion-thermo). The double-diffusion phenomena were developed from the kinetic theory of gases, [40], [41]. They are smaller than the Fourier and Ficks effects, [42]. For their relevance, many reports bearing double-diffusion effects exist in the literature. Specifically, on the flow over vertical plates, [43], considered natural convective and mass transfer effects on a two-dimensional case using the similarity technique and Runge-Kutta sixth-order approach; [44], examined the effects of thermal radiation, Hall currents, Dufour and Soret numbers on the MHD mixed convective flow; [45] investigated the free convective flow with double-diffusive convection using the successive linearization method; [46] studied a mixed convective heat and mass transfer flow along a wavy surface in a Darcy porous medium in the presence of cross-diffusion effects using similarity transformation and numerical scheme for aiding flow, opposing flow, and for both aiding and opposing flows.

The problem of natural convective fluid flow over a vertical plate with chemical reaction, radiation, and temperature-dependent heat source in a slip flow regime using the time-dependent and exponentially decaying perturbation series solution approach was examined by [1]. In their work, the effects of thermo-diffusion and viscous dissipation were neglected. As an extension of [1], this present work investigates the flow problem in the presence of the aforementioned parameters using the time-dependent and exponentially decaying perturbation series solutions and the Modified Homotopy Perturbation Method.

This paper is presented in the following format: Section 2 gives the problem formulation; Section 3 holds the problem Solution; Section 4 holds the conclusion.

2 Problem Formulation

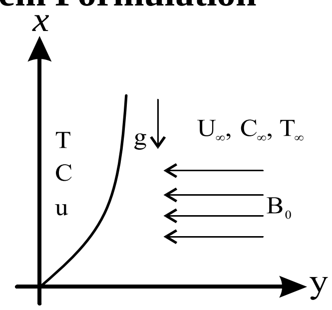

Fig. 1: The model of a vertically accelerating plate in a fluid

The transient MHD natural convective flow past a moving vertical surface in a slip flow regime is investigated. The schematic of the flow is shown in Figure 1. The model formulation is based on the assumptions that: the fluid is Newtonian, thermally radiating, chemically reacting and electrically conducting; the physical properties of the fluid such as the specific heat at constant pressure, thermal conductivity, and density remain constant throughout the fluid; the fluid is mixed with a chemical species at a higher concentration to initiate a chemical reaction; the plate is porous, its permeability and suction at the wall are oscillating, time-dependent and exponentially decaying; the plate is connected to a wire carrying an alternating current, which produces a voltage between the ends, and which in turn energizes the ions, electrons and holes on the plate to generate a Joule/Ohmic heating that produces a mechanical force/viscous dissipation; a magnetic field force of uniform strength and negligible induction effect is applied transversely to the plate; there is a convective temperature gradient between the bottom and upper surfaces of the plate with a heat source at the bottom and sink at the top; the plate is heated to a high temperature regime such that thermal rays are emitted into the fluid; the flow is naturally convective. In this model, the x-axis is taken to be in the vertical direction of the plate and the y-axis is normal to it. Therefore, if (u,v) are the velocity components in the spatial (x,y,t) coordinates; T_W and C_W are the temperature and concentration at the wall; v is the velocity along the y-axis and the suction at the wall; T_∞ and C_∞ are the fluid equilibrium temperature and concentration, T and C are the fluid temperature and concentration. Then, using the unsteady one-dimensional flow theory and Boussinesq's approximations the governing equations of continuity, momentum, energy, and mass diffusion are:

$$\frac{\partial v'}{\partial y'}=0 \Rightarrow v_O \tag{1}$$

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=\upsilon\frac{\partial^2 u'}{\partial y'^2}+g\beta_t(T'-T_\infty) +g\beta_C(C'-C_\infty)-\left(\frac{\sigma_e B_o{}^2}{\rho\mu_m}+\frac{\upsilon}{\kappa}\right)u' \tag{2}$$

$$\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}=\frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{\rho C_p}\frac{\partial q_r'}{\partial y'} +\frac{Q'}{\rho C_p}\frac{\partial}{\partial y}(T'-T_\infty)+\frac{\upsilon}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2-\frac{Dk_t}{\rho C_p C_s}\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}-k_r(C'-C_\infty) \tag{4}$$

with the boundary condition:

$$u'(t',0)=\alpha\frac{\partial u'}{\partial y'}, v'(t',0)=0,\ T'(t',0)=T_W,$$
$$C'(t',0)=C_W \text{ at } y'=0 \tag{5}$$
$$u'(t',\infty)=0, v'(t',\infty)=0, T'(t',\infty)\to T_\infty,$$
$$C'(t',\infty)\to C_\infty \text{ at } y'\to\infty \tag{6}$$

υ is the kinematic viscosity; ρ is the density; g is the acceleration due to gravity; β_t is the coefficient of volumetric expansion due to temperature; β_C is the coefficient of volumetric expansion due to concentration; k is the thermal conductivity of the fluid; μ_m is the magnetic field permittivity; Q' is the heat source/sink; σ_e is the electrical conductivity of the fluid; B_o^2 is the magnetic field flux; C_p is the specific heat capacity at constant pressure; C_S is the concentration susceptibility; k_t is the thermal diffusivity ratio; D is the coefficient of mass transfer/ mass diffusion coefficient; κ is the permittivity of the porous plate; k_r is the chemical reaction term of the species.

Assuming the suction at the wall and permeability of the medium are oscillating, time-dependent, and exponentially decaying, then:

$$v=-v_O(1+\varepsilon e^{-m't'}) \tag{7}$$

$$\kappa=\kappa_O(1+\varepsilon e^{-m't'}) \tag{8}$$

where m' is a positive constant, v_O is the steady suction at the wall. Suction is a criterion for determining certain flow situations. For example, for $v<0$ suction (the fluid moves towards the plate), $v>0$ injection (the fluid moves from the plate), and $v=0$ the plate is impermeable. Similarly, κ_O is the steady permeability of the wall. Permeability is Darcian for $\kappa_O<1$, and non-Darcian for $\kappa_O>1$. While porosity is a measure of the voids in a material, permeability is a measure of the ease of flow of a fluid through a porous solid. In other words, porosity determines the number, sizes, and inter-connectedness of the voids in a solid material, while permeability determines the ease of fluid flow through the voids. Both porosity and permeability are related to the number, size, and connections of openings in solid materials, hence many a time they are used interchangeably.

More so, radiation is seen as a heat transfer from a high-temperature regime. In effect, it is comparable to convective heat transfer. It is described in terms of optical depths: depths at which photons travel/penetrate fluids. Optical depths can be thin or thick. A fluid is optically thin/transparent when its density is relatively low, and the depth of penetration/distance it allows long photon travel in it is far less than unity ($\alpha << 1$). Examples of optically thin environments include the non-participating media in which energy is emitted from the fluid but is not absorbed, as in gray gas. Also, a fluid is optically thick/non-transparent when its density is high enough to allow short photon travel in it. The optically thick fluid emits and absorbs radiation at the boundaries. The analysis of radiation is based on the optic limits: thin or thick. Importantly, the radiative heat flux is approximated by the Roseland diffusion approximations. Now, on the assumption that the fluid here is optically thin, we adopt the Roseland approximation:

$$q_r' = 3\alpha_1 q_r - 4\sigma\alpha_1 \frac{\partial}{\partial y}(T^4 - T_\infty^4)$$

such that

$$\frac{\partial q_r'}{\partial y} = -4\sigma\alpha_1 \frac{\partial^2}{\partial y'^2}(T^4 - T_\infty^4) \tag{9}$$

Taking the temperature difference within the flow to be sufficiently small such that $(T - T_\infty) = \xi$, and ξ is a non-constant small temperature correction factor, then T^4 can be expressed as a linear function of the temperature in the Taylor series about T_∞ :

$$T^4 = 4T_\infty^3 T - 3T_\infty^4$$

with the higher-order terms neglected.

Substituting this into equation (9) gives:

$$\frac{\partial q_r'}{\partial y} = -16\sigma\alpha_1 T_\infty^3 \frac{\partial^2 T'}{\partial y'2} \tag{10}$$

and by equations (7), (8) and (10), equations (2) - (4) become:

$$\frac{\partial u'}{\partial t'} - v_o(1+\varepsilon e^{-m't'})\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + g\beta_t(T' - T_\infty) + g\beta_c(C' - C_\infty) - \left(\frac{\sigma_e B_o{}^2}{\rho\mu_m} + \frac{\upsilon}{\kappa_o(1+\varepsilon e^{-m't'})}\right)u' \tag{11}$$

$$\frac{\partial T'}{\partial t'} - v_o(1+\varepsilon e^{-m't'})\frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r'}{\partial y'} + \frac{Q'}{\rho C_p}\frac{\partial}{\partial y}(T' - T_\infty) + \frac{\upsilon}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 - \frac{Dk_t}{\rho C_p C_s}\frac{\partial^2 C'}{\partial y'^2} \tag{12}$$

$$\frac{\partial C'}{\partial t'} - v_o(1+\varepsilon e^{-m't'})\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - k_r(C' - C_\infty) \tag{13}$$

Making the problem independent of particular units of measurement and geometry, and generating the necessary parameters that control the flow, we introduce the following non-dimensionalized quantities:

$$t = \frac{v_o^2 t'}{4\upsilon},\ y = \frac{v_o y'}{\upsilon}, u = \frac{u'}{v_o},\ m = \frac{4\upsilon m'}{v_o^2},\ \Theta = \frac{T' - T_\infty}{T_w - T_\infty},$$

$$\Phi = \frac{C' - C_\infty}{C_w - C_\infty}, Gr = \frac{g\beta_t(T_w - T_\infty)\upsilon}{v_o^2},$$

$$Gc = \frac{g\beta_c(C_w - C_\infty)\upsilon}{v_o^2}, M = \frac{\sigma_e B_o^2 \upsilon}{\mu_m v_o^2},\ \kappa_o = \frac{\upsilon}{\kappa},$$

$$\Pr = \frac{\upsilon}{k},\ Sc = \frac{\upsilon}{D},\ \delta = \frac{k_r \upsilon}{D}, N = \frac{Q'}{k\rho C_p v_o},$$

$$Ec = \frac{v_o^2}{C_p(T_w - T_\infty)},\ Ra = \frac{16\sigma\alpha_1 T_\infty^3}{\upsilon^2 k},$$

$$Dr = \frac{Dk_t(C_w - C_\infty)}{kC_s(T_w - T_\infty)} \tag{14}$$

where Θ and Φ are the non-dimensionalized temperature and concentration, respectively; N is the heat generation/absorption parameter; Gr is the Grashof number due to temperature difference; Gc is the Grashof number due to concentration difference; M is the magnetic field force; χ is the porosity parameter; $\Pr$ is the Prandtl number; Dr is the Dufour number; Sc is the Schmidt number; δ is the chemical reaction rate) into equations (11) – (13), (5) and (6), we have:

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1+\varepsilon e^{-mt})\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y'^2} + Gr\Theta + Gc\Phi - \left(M + \frac{1}{\kappa_o(1+\varepsilon e^{-mt})}\right)u \tag{15}$$

$$\frac{1}{4}\frac{\partial \Theta}{\partial t} - (1+\varepsilon e^{-mt})\Pr\frac{\partial \Theta}{\partial y} = \lambda\frac{\partial^2 \Theta}{\partial y^2} + \Pr N\frac{\partial \Theta}{\partial y}$$

$$+\mathrm{Pr}Ec\left(\frac{\partial u}{\partial y}\right)^2 - Dr\frac{\partial^2\Phi}{\partial y'^2} \quad (16)$$

$$\frac{1}{4}\frac{\partial\Phi}{\partial t} - (1+\varepsilon e^{-mt})Sc\frac{\partial\Phi}{\partial y} = \frac{\partial^2\Phi}{\partial y^2} - Sc\delta\Phi \quad (17)$$

where $\lambda = 1 + Ra$
and the boundary conditions

$$u = \alpha\frac{\partial u}{\partial y}, \Theta = 1, \Phi = 1 \text{ at } y = 0 \quad (18)$$

$$u = 0, \Theta = 0, \Phi = 0 \text{ at } y = \infty \quad (19)$$

Other factors influencing the flow are the Nusselt number (or thermal conductivity of the fluid), Sherwood number (or species conductivity of the fluid), and wall shear stress (the force the fluid exerts on the wall), and these are prescribed non-dimensionally as:

$$Nu = -\Theta'(y)\big|_{y=0} \quad (20)$$

$$Sh = -\Phi'(y)\big|_{y=0} \quad (21)$$

$$Cf = \mu u'(y)\big|_{y=0} \quad (22)$$

3 Problem Solution

3.1 Method of Solution

Equations (15) - (17) are reduced to ordinary differential equations and solved using time-dependent perturbation series solutions of the form, [11]:

$$u(y,t) = u_o(y) + \varepsilon u_1(y)e^{-mt} + O(\varepsilon^2) \quad (23)$$

$$\Theta(y,t) = \Theta_o(y) + \varepsilon\Theta_1(y)e^{-mt} + O(\varepsilon^2) \quad (24)$$

$$\Phi(y,t) = \Phi_o(y) + \varepsilon\Phi_1(y)e^{-mt} + O(\varepsilon^2) \quad (25)$$

Substituting these into equations (15) - (19) appropriately, collecting and equating the cefficient of the powers ε, we have:

for the zeroth order

$$\frac{d^2u_o}{dy^2} + \frac{du_o}{dy} - M_1u_o = -Gr\Theta_o - Gc\Phi_o \quad (26)$$

$$\lambda\frac{d^2\Theta_o}{dy^2} + \gamma\frac{d\Theta_o}{dy} = -\mathrm{Pr}Ec\left(\frac{du_o}{dy}\right)^2 - Dr\frac{\partial^2\Phi_o}{\partial y^2} \quad (27)$$

$$\frac{d^2\Phi_o}{dy^2} + Sc\frac{d\Phi_o}{dy} - Sc\delta\Phi_o = 0 \quad (28)$$

with the boundary conditions

$$u_o = \alpha\frac{\partial u_o}{\partial y}, \Theta_o = 1, \Phi_o = 1 \text{ at } y = 0 \quad (29)$$

$$u_o \to 0, \Theta_o \to 0, \Theta_o \to 0 \text{ at } y = \infty \quad (30)$$

and for the first order

$$\frac{d^2u_1}{dy^2} + \frac{du_1}{dy} - M_2u_1 = -\frac{du_o}{dy} - Gr\Theta_1 - Gc\Phi_1 \quad (31)$$

$$\lambda\frac{d^2\Theta_1}{dy^2} + \gamma\frac{d\Theta_1}{dy} + \mathrm{Pr}\frac{m}{4}\Theta_1 = -\mathrm{Pr}\frac{d\Theta_o}{dy} - 2\mathrm{Pr}Ec\left(\frac{du_o}{dy}\frac{du_1}{dy}\right) - Dr\frac{\partial^2\Phi_1}{\partial y'^2} \quad (32)$$

$$\frac{d^2\Phi_1}{dy^2} + Sc\frac{d\Phi_1}{dy} - \psi\Phi_1 = -Sc\frac{d\Phi_o}{dy} \quad (33)$$

with the boundary conditions

$$u_1 = \alpha\frac{\partial u_1}{\partial y}, \Theta_1 = 0, \Phi_1 = 0 \text{ at } y = 0 \quad (34)$$

$$u_1 \to 0, \Theta_1 \to 0, \Theta_1 \to 0 \text{ at } y \to \infty \quad (35)$$

where $M_1 = M + \frac{1}{\kappa_o}, M_2 = M + \frac{1}{\kappa_o} - \frac{m}{4}$,

$\psi = \left(Sc\delta - \frac{m}{4}\right), \lambda = 1 - Ra, \gamma = \mathrm{Pr}(1+N)$

Equations (26), (27), (31), and (32) are still highly coupled. A second perturbation becomes necessary. We resort to using the Modified Homotopy Perturbation Method (MHPM) of solutions.

The analysis associated with the Homotopy Perturbation Method is as follows:
Consider the nonlinear equation

$$L(v) + N(v) = f(r), r\Xi\Omega \quad (36)$$

with the boundary condition

$$B\left(u, \frac{\partial u}{\partial y}\right) = 0, r\Xi\Gamma \quad (37)$$

where L is a linear operator, N is a nonlinear operator, B is a boundary operator, Γ is the boundary of the domain Ω, $f(r)$ is a known analytic function. For a Homotopy Perturbation technique, He (a Chinese) constructed a homotopy:
$v(r,p) = \Omega[0,1] \to R$

which satisfies

$$H(v,p)=(1-p)[L(v)-L(u_o)]+p[L(v)+N(v)-f(r)]=0 \quad (38)$$

where $p\Xi[0,1]$ is an impending parameter, u_o is an initial approximation that satisfies the boundary conditions. Clearly, from equation (38), we have:

$H(v,0)=[L(v)-L(u_o)]=0$,

$H(v,1)=[L(v)+N(v)-f(r)]=0$

Importantly, the process of changing p from zero to unity $v(r,p)$ is like that of changing from $u_o(r)$ to $u(r)$, and this is called a *deformation* in *Topology*; the $[L(v)-L(u_o)]=0$ and $[L(v)+N(v)-f(r)]=0$ are called homotopic.

Here, the basic assumption is that the solution of equation (38) can be expressed as a power series in p:

$$v=v_o+pv_1+p^2v_2+...$$

and the approximate solution of equation (36) is obtained as, [47]; [48]:

$$u=\lim_{p\to 1} v=v_o+v_1+v_2+...$$

The difference between HPM and MHPM is seen in their use of boundary conditions. In HPM, order zero takes the boundary conditions at t<0, order one takes the boundary conditions at t>0 for y=0, and order two takes that at t>0 for y=∞, while in MHPM all the orders use the boundary conditions at t>0: y=0 and y=∞ but with little modifications, as we shall see below.

Based on the given analysis, writing equations (26) - (35) in MHPM form, we have

For the Zeroth Order

$$(1-p)u_o''=p[-u_o''-u_o'+M_1u_o-Gr\Theta_o-Gc\Phi_o] \quad (39)$$

$$(1-p)\lambda\Theta_o''=p[-\lambda\Theta_o''-\gamma\Theta_o'+\Pr Ec(u_o'')^2-Dr\Phi_o''] \quad (40)$$

$$(1-p)\Phi_o''=p[-\Phi_o''-Sc\Phi_o'-Sc\delta\Phi_o] \quad (41)$$

such that

$$u_o''=p[-u_o'+M_1u_o-Gr\Theta_o-Gc\Phi_o] \quad (42)$$

$$\lambda\Theta_o''=p[-\gamma\Theta_o'+\Pr Ec(u_o'')^2-Dr\Phi_o''] \quad (43)$$

$$\Phi_o''=p[-Sc\Phi_o'-Sc\delta\Phi_o] \quad (44)$$

Expanding the dependent variables in terms of p, we have:

for the zeroth order:

$$u_o=u_{00}+pu_{01}+p^2u_{02}+.. \quad (45)$$

$$\Theta_o=\Theta_{00}+p\Theta_{01}+p^2\Theta_{02}+.. \quad (46)$$

$$\Phi_o=\Phi_{00}+p\Phi_{01}+p^2\Phi_{02}+.. \quad (47)$$

and for the first order

$$u_1=u_{10}+pu_{11}+p^2u_{12}+.. \quad (48)$$

$$\Theta_1=\Theta_{10}+p\Theta_{11}+p^2\Theta_{12}+.. \quad (49)$$

$$\Phi_1=\Phi_{10}+p\Phi_{11}+p^2\Phi_{12}+.. \quad (50)$$

Substituting equations (45)-(47) into equations (42)-(44) and (29) and (30), gives:

$$u_{00}''+pu_{01}''+p^2u_{02}''=p\left\{\begin{array}{l}-(u_{00}'+pu_{01}'\\+p^2u_{02}')\\+M_1(u_{00}+pu_{01}+p^2u_{02})\\-Gr(\Theta_{00}+p\Theta_{01}\\+p^2\Theta_{02})\\-Gc(\Phi_{00}+p\Phi_{01}\\+p^2\Phi_{02})\end{array}\right\} \quad (51)$$

$$\lambda\left(\Theta_{00}''+p\Theta_{01}''+p^2\Theta_{02}''\right)=p\left\{\begin{array}{l}-\gamma(\Theta_{00}'+p\Theta_{01}'+\\p^2\Theta_{02}')\\+\Pr Ec[(u_{00}'')^2\\+p(u_{01}'')^2\\+p^2(u_{02}'')^2]\\-Dr(\Phi_{00}''\\+p\Phi_{01}''+\\p^2\Phi_{02}'')\end{array}\right\} \quad (52)$$

$$\Phi_{00}''+p\Phi_{01}''+p^2\Phi_{02}''=p\left\{\begin{array}{l}-Sc(\Phi_{00}'\\+p\Phi_{01}'+\\p^2\Phi_{02}')\\-Sc\delta(\Phi_{00}+\\\Phi_{01}+p^2\Phi_{02})\end{array}\right\} \quad (53)$$

Collecting the coefficients of the of the powers of p in each case, we have:

$$u_{00}''=0 \quad (54)$$

$$\lambda\Theta_{00}''=0 \tag{55}$$

$$\Phi_{00}''=0 \tag{56}$$

$$u_{01}''=-u_{00}'+M_1 u_{00}-Gr\Theta_{00}-Gc\Phi_{00} \tag{57}$$

$$\lambda\Theta_{01}''=-\gamma\Theta_{00}'-\Pr Ec(u_{00}'')^2-Dr\Phi_{00}'' \tag{58}$$

$$\Phi_{01}''=-Sc\Phi_{00}'-Sc\delta\Phi_{00} \tag{59}$$

$$u_{02}''=-u_{01}'+M_1 u_{01}-Gr\Theta_{01}-Gc\Phi_{01} \tag{60}$$

$$\lambda\Theta_{02}''=-\gamma\Theta_{01}'-\Pr Ec(u_{01}'')^2-Dr\Phi_{01}'' \tag{61}$$

$$\Phi_{02}''=-Sc\Phi_{01}'-Sc\delta\Phi_{01} \tag{62}$$

with the boundary conditions

$$u_{00}=\alpha\frac{\partial u_{00}}{\partial y},\Theta_{00}=1,\Phi_{00}=1 \text{ at } y=0 \tag{63}$$

$$u_{00}=0,\Theta_{00}=0,\Phi_{00}=0 \text{ at } y=\infty \tag{64}$$

$$u_{01}=\alpha\frac{\partial u_{01}}{\partial y},\Theta_{01}=0,\Phi_{01}=0 \text{ at } y=0 \tag{65}$$

$$u_{01}=0,\Theta_{01}=0,\Phi_{01}=0 \text{ at } y=\infty \tag{66}$$

$$u_{02}=\alpha\frac{\partial u_{02}}{\partial y},\Theta_{02}=0,\Phi_{02}=0 \text{ at } y=0 \tag{67}$$

$$u_{02}=0,\Theta_{02}=0,\Phi_{02}=0 \text{ at } y=\infty \tag{68}$$

The First Order

Similarly, by expressing equations (31)-(35) in MHPM form, we substitute equations (48)-(50) into them, and collecting and equating the coefficients of the powers of p in the resulting equations to zero, we obtain:

$$u_{10}''=0 \tag{69}$$

$$\lambda\Theta_{10}''=0 \tag{70}$$

$$\Phi_{10}''=0 \tag{71}$$

$$u_{11}''=-u_{10}'+M_2 u_{10}-u_{00}'-Gr\Theta_{10}-Gc\Phi_{10} \tag{72}$$

$$\lambda\Theta_{11}''=-\gamma\Theta_{10}'-\frac{\Pr m}{4}\Theta_{10}-\Pr\Theta_{00}' -2\Pr Ec(u_{00}'u_{10}')^2-Dr\Phi_{10}'' \tag{73}$$

$$\Phi_{11}''=-Sc\Phi_{10}'+\psi\Phi_{10}-Sc\delta\Phi_{00} \tag{74}$$

$$u_{12}''=-u_{11}'+M_2 u_{11}-u_{01}'-Gr\Theta_{11}-Gc\Phi_{11} \tag{75}$$

$$\lambda\Theta_{12}''=-\gamma\Theta_{11}'-\frac{\Pr m}{4}\Theta_{11}-\Pr\Theta_{01}' -2\Pr Ec[(u_{00}'u_{11}')+(u_{01}'u_{10}')]-Dr\Phi_{10}'' \tag{76}$$

$$\Phi_{12}''=-Sc\Phi_{11}'+\psi\Phi_{11}-Sc\delta\Phi_{01} \tag{77}$$

with the boundary conditions,

$$u_{10}=\alpha\frac{\partial u_{10}}{\partial y},\Theta_{10}=0,\Phi_{10}=0 \text{ at } y=0 \tag{78}$$

$$u_{10}=0,\Theta_{10}=0,\Phi_{10}=0 \text{ at } y=\infty \tag{79}$$

$$u_{11}=\alpha\frac{\partial u_{11}}{\partial y},\Theta_{11}=0,\Phi_{11}=0 \text{ at } y=0 \tag{80}$$

$$u_{11}=0,\Theta_{11}=0,\Phi_{11}=0 \text{ at } y=\infty \tag{81}$$

$$u_{12}=\alpha\frac{\partial u_{12}}{\partial y},\Theta_{102}=0,\Phi_{12}=0 \text{ at } y=0 \tag{82}$$

$$u_{12}=0,\Theta_{12}=0,\Phi_{12}=0 \text{ at } y=\infty \tag{83}$$

Equations (54)-(68) and (69)-(83) are solved using the Mathematica 11.2 computational software, and their solutions are found in the Appendices.

3.2 Results and Discussion

The solutions of the concentration, temperature, velocity, Nusselt number, Sherwood number, and wall shear stress are computed and presented quantitatively and graphically. The effects of the rate of chemical reaction, Schmidt number, Grashof number, slip, magnetic field, and permeability parameters are considered. For physically realistic constant values of $Ec=0.1, Dr=0.01, \Pr=0.7, Ra=3, Gc=3, N=3, \varepsilon=0.01, p=0.1, t=1, m=0.5$ and varied values of $\delta, Sc, Gr, \alpha, M, \kappa_0$, we obtained the figures and table below.

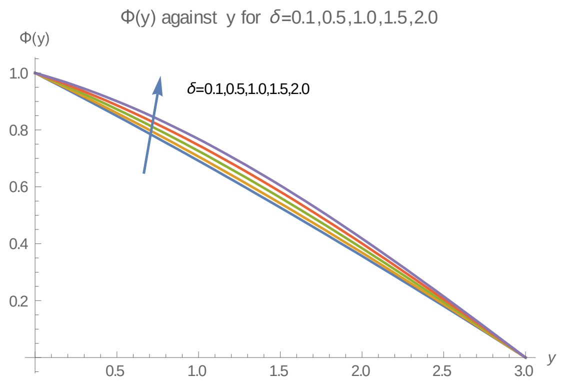

Fig. 2: Concentration-Chemical Reaction Rate Profiles

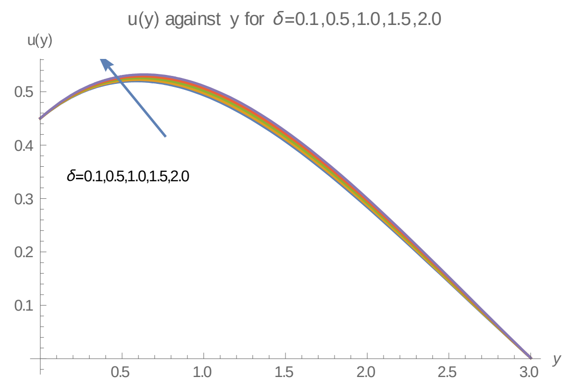

Fig. 3: Velocity-Chemical Reaction Rate Profiles

Table 1. Some Parameters-Sherwood Number, Nusselt Number, and Wall Shear Stress Relations

Parameters	$-\Phi'(0)$	$-\Theta'(0)$	$u'(0)$
δ			
0.1	0.294755	0.327439	0.256117
0.5	0.267821	0.327442	0.260611
1.0	0.233099	0.327447	0.266285
1.5	0.197205	0.327451	0.271934
Sc			
0.1	0.323176	0.327744	0.250048
0.5	0.280969	0.327446	0.258193
1.0	0.224703	0.327449	0.268373
1.5	0.164533	0.327451	0.278554
M			
0.1			0.293304
0.5			0.260064
1.0			0.220136
1.5			0.179636
α			
0.1			0.509107
0.5			0.382889
1.0			0.260636
1.5			0.165565
κ_O			
0.1			0.152264
0.5			0.260631
1.0			0.341636
1.5			0.368636

The effects of the chemical reaction rate on the flow are seen in Figure 2, Figure 3 and Table 1. They show that the increase in the rate of chemical reaction increases the fluid concentration, velocity, the rate of heat transfer to the fluid, the force exerted on the wall, and the rate of mass transfer to the fluid. A chemical reaction may lead an increase in the interaction of fluid particles, and the production of new species. More so, a chemical reaction may be exothermic or endothermic, and therein heat is generated or absorbed. Phenomenally, this ought to increase the velocity, thus accounting for what is seen in Figure 2, Figure 3 and Table 1.

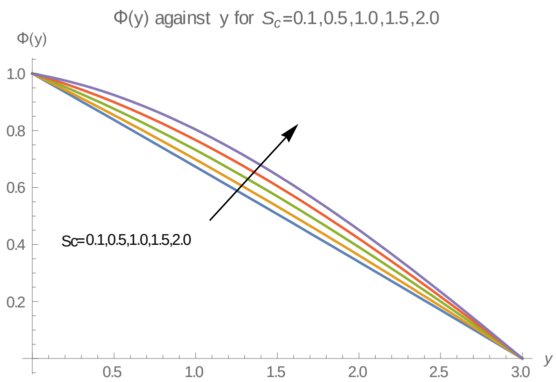

Fig. 4: Concentration-Schmidt Number Profiles

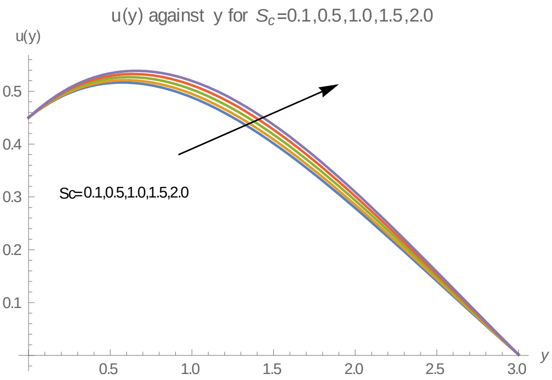

Fig. 5: Velocity-Schmidt number Profiles

The effects of Schmidt number on the flow are shown in Figure 4, Figure 5 and Table 1. They depict that the increase in the Schmidt number increases the fluid concentration, velocity, the rate of heat transfer to the fluid, and stress on the wall, but decreases the rate of mass transfer to the fluid. Schmidt number is the ratio of momentum diffusion to mass diffusion. When the mass diffusion increases the concentration increases. Similarly, when the momentum diffusion dominates the system, the velocity increases. More so, with the increase in the concentration, the velocity, as a function of concentration increases.

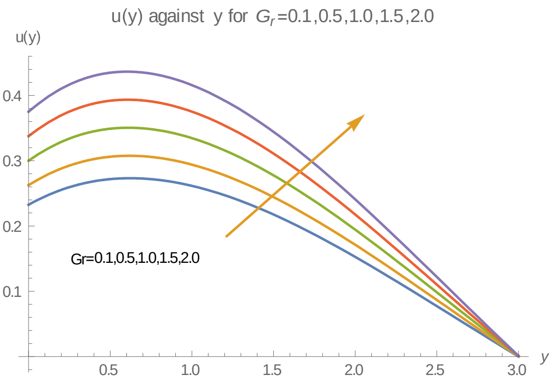

Fig. 6: Velocity-Grashof Number Profiles

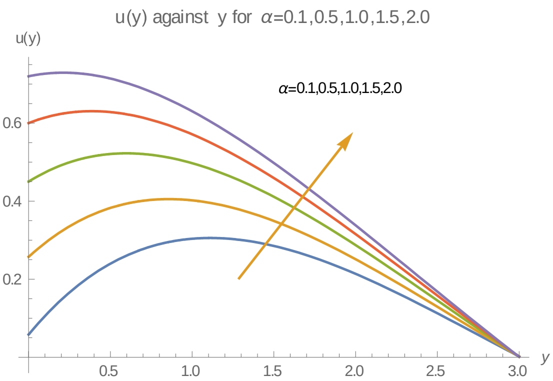

Fig. 7: Velocity-Slip Parameter Profiles

Furthermore, the effect of the Grashof number on the flow is shown in Figure 6. It depicts that the increase in the Grashof number increases the velocity of the flow. When the temperature at the plate/environmental temperature is higher than that of the fluid at equilibrium heat is transferred from the plate to the fluid. Now, a differential exists between the temperature at the plate and that of the fluid at equilibrium. In the presence of volumetric expansion due to heat exchange and gravity, convection currents are generated. The convection currents induce a buoyancy force which reduces the fluid viscosity, thus enhancing the fluid velocity; as seen in Figure 6. This result aligns with [1] and [18].

Additionally, the effects of the slip velocity parameter on the flow are shown in Figure 7 and Table 1. They show that the increase in the slip parameter increases the flow velocity, but decreases the stress on the wall. The slip length is a function of Maxwell's reflection/transmission coefficient, which describes the way wave reflection/transmission affects the flow velocity. A higher reflection/transmission increases the slip length, and vice versa. Therefore, an increase in the wave reflection/transmission increases the fluid velocity. This result is in agreement with [1] and [18].

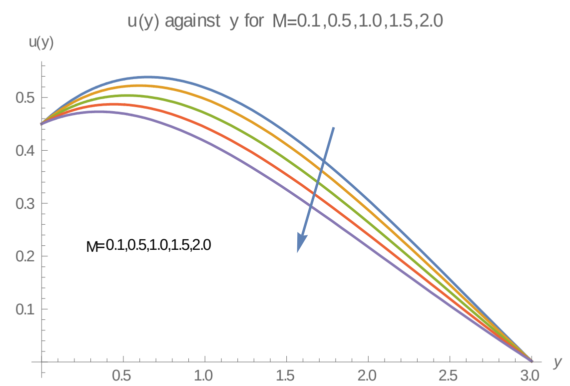

Fig. 8: Velocity-Magnetic Field Profiles

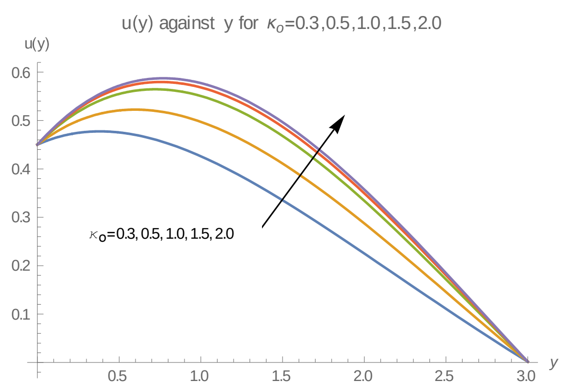

Fig. 9: Velocity-Permeability Profiles

Similarly, the effects of the magnetic field on the flow are shown in Figure 8 and Table 1. They show that the increase in the magnetic field parameter decreases the fluid velocity and the stress on the wall. The particles of a chemically reactive fluid exist as charges or ions and generate electric currents in the presence of an applied magnetic force. Again, the interaction of the electric currents with the magnetic field force produces a mechanical force called the Lorentz force ($F = j \wedge B_o$, where j is the electric current density, and B_o is the magnetic field flux). The Lorentz force has the potency of freezing up the flow velocity. More so, the decrease in the velocity must decrease the stress on the wall. The results are in consonant with [1].

Also, the effects of permeability/porosity are shown in Figure 9 and Table 1. They show that the increase in the permeability factor of the porous media increases the flow velocity and the stress on the wall. In addition to the hydraulic conductivity effect of the porous plate, the permeability factor determines the ease of flow of a fluid passing through a porous medium. The higher the permeability parameter the easier the fluid flows through the medium. Therefore, the velocity will increase with the increase in the permeability parameter. This result aligns with [18].

4 Conclusion

Transient MHD fluid flow past a moving vertical surface in a slip flow regime is investigated. The analysis of results shows that the increase in the:

- rate of chemical reaction increases the fluid concentration, velocity, rate of heat transfer to the fluid, and stress on the wall, but decreases the rate of mass transfer to the fluid.
- Schmidt number increases the fluid concentration, velocity, rate of heat transfer to the fluid, stress on the wall, and the rate of mass transfer to the fluid.
- Grashof number increases the fluid velocity.
- magnetic field parameter decreases the fluid velocity and stress on the wall.
- slip parameter increases the flow velocity, but decreases the stress on the wall
- porosity parameter increases the flow velocity and stress on the wall.

These results are benchmarked with some reports in existing literature, and they are in consonance.

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APPENDICES

$\Phi_{00}(y) = (A-y)/A$

$\Phi_{01}(y) = (3\ A\ y\ S_c-3\ y^2\ S_c+2\ A^2\ y\ \delta\ S_c-3\ A\ y^2\ \delta\ S_c+y^3\ \delta\ S_c)/(6\ A)$

$\Phi_{02}(y) = (30\ A^2\ y\ -90\ A\ y^2\ +60\ y^3\ +30\ A^3\ y\ \delta\ -60\ A^2\ y^2\ \delta\ +8\ A^4\ y\ \delta^2\ -20\ A^2\ y^3\ \delta^2\ +15\ A\ y^4\ \delta^2\ -3\ y^5\ \delta^2\ )/(360\ A)$

$\Phi_{10}(y) = 0$

$\Phi_{11}(y) = (2\ A^2\ y\ \delta\ S_c-3\ A\ y^2\ \delta\ S_c+y^3\ \delta\ S_c)/(6\ A)$

$\Phi_{12}(y) = 1/(360\ A\ \lambda)\ (-15\ A^3\ y\ \gamma\ \psi+30\ A\ y^3\ \gamma\ \psi-15\ y^4\ \gamma\ \psi+30\ A^2\ y\ \gamma\ S_c-90\ A\ y^2\ \gamma\ S_c+60\ y^3\ \gamma\ S_c+15\ A^3\ y\ \delta\ \lambda\ -30\ A\ y^3\ \delta\ \lambda\ +15\ y^4\ \delta\ \lambda\ +8\ A^4\ y\ \delta^2\ \lambda\ -20\ A^2\ y^3\ \delta^2\ \lambda\ +15\ A\ y^4\ \delta^2\ \lambda\ -3\ y^5\ \delta^2\ \lambda\ );$

$\Theta_{00}(y) = (A-y)/A$

$\Theta_{01}(y) = -(((-A\ y+y^2)\ \gamma)/(2\ A\ \lambda))$

$\Theta_{02}(y) = 1/(12\ A^2\ \lambda^2)\ (A^3\ y\ \gamma^2-3\ A^2\ y^2\ \gamma^2+2\ A\ y^3\ \gamma^2+3\ A^3\ y\ \lambda\ E_c\ P_r-6\ A^2\ y^2\ \lambda\ E_c\ P_r+4\ A\ y^3\ \lambda\ E_c\ P_r-y^4\ \lambda\ E_c\ P_r+6\ A^3\ y\ \lambda\ E_cG_c\ G_r\ P_r-12\ A^2\ y^2\ \lambda\ E_cG_c\ G_r\ P_r+8\ A\ y^3\ \lambda\ E_cG_c\ G_r\ P_r-2\ y^4\ \lambda\ E_cG_c\ G_r\ P_r+3\ A^3\ y\ \lambda\ E_c\ P_r-6\ A^2\ y^2\ \lambda\ E_c\ P_r+4\ A\ y^3\ \lambda\ E_c\ P_r-y^4\ \lambda\ E_c\ P_r-6\ A^2\ y\ \lambda\ D_r\ S_c+6\ A\ y^2\ \lambda\ D_r\ S_c-4\ A^3\ y\ \delta\ \lambda\ D_r\ S_c+6\ A^2\ y^2\ \delta\ \lambda\ D_r\ S_c-2\ A\ y^3\ \delta\ \lambda\ D_rS_c)$

$\Theta_{10}(y) = 0$

$\Theta_{11}(y) = -(((-A\ y+y^2)\ \gamma)/(2\ A\ \lambda))$

$\Theta_{12}(y) = 1/(96\ A\ \lambda^2)\ (8\ A^2\ y\ \gamma^2-24\ A\ y^2\ \gamma^2+16\ y^3\ \gamma^2-A^3\ m\ y\ +2\ A\ m\ y^3\ -m\ y^4\ -32\ A^2\ y\ \delta\ \lambda\ D_r\ S_c+48\ A\ y^2\ \delta\ \lambda\ D_r\ S_c-16\ y^3\ \delta\ \lambda\ D_r\ S_c+8\ A^2\ y\ \lambda\ P_r\ S_c-24\ A\ y^2\ \lambda\ P_r\ S_c+16\ y^3\ \lambda\ P_r\ S_c+4\ A^3\ y\ \delta\ \lambda\ P_r\ S_c-16\ A^2\ y^2\ \delta\ \lambda\ P_r\ S_c+16\ A\ y^3\ \delta\ \lambda\ P_r\ S_c-4\ y^4\ \delta\ \lambda\ P_rS_c)$

$u_{00}(y) = 0$

$u_{01}(y) = (2\ A^3\ y\ G_c-3\ A^2\ y^2G_c+A\ y^3\ G_c+2\ A^3\ \alpha\ G_c-3\ A\ y^2\ \alpha\ G_c+y^3\ \alpha\ G_c+2\ A^3\ y\ G_r-3\ A^2\ y^2G_r+A\ y^3\ G_r+2\ A^3\ \alpha\ G_r-3\ A\ y^2\ \alpha\ G_r+y^3\ \alpha\ G_r)/(6\ A\ (A+\alpha));$

$u_{02}(y) = 1/(360\ A\ (A+\alpha)\ \lambda)\ (15\ A^4\ y\ \lambda\ G_c-60\ A^3\ y^2\ \lambda\ G_c+60\ A^2\ y^3\ \lambda\ G_c-15\ A\ y^4\ \lambda\ G_c-45\ A^3\ y\ \alpha\ \lambda\ G_c+60\ A\ y^3\ \alpha\ \lambda\ G_c-15\ y^4\ \alpha\ \lambda\ G_c+15\ A^4\ y\ \gamma\ G_r-30\ A^2\ y^3\ \gamma$

$G_r+15\ A\ y^4\ \gamma\ G_r+15\ A^3\ y\ \alpha\ \gamma\ G_r-30\ A\ y^3\ \alpha\ \gamma\ G_r+15\ y^4\ \alpha\ \gamma\ G_r+15\ A^4\ y\ \lambda\ G_r-60\ A^3\ y^2\ \lambda\ G_r+60\ A^2\ y^3\ \lambda\ G_r-15\ A\ y^4\ \lambda\ G_r-45\ A^3\ y\ \alpha\ \lambda\ G_r+60\ A\ y^3\ \alpha\ \lambda\ G_r-15\ y^4\ \alpha\ \lambda\ G_r-8\ A^5\ y\ \lambda\ G_c\ M_1+20\ A^3\ y^3\ \lambda\ G_c\ M_1-15\ A^2\ y^4\ \lambda\ G_c\ M_1+3\ A\ y^5\ \lambda\ G_c\ M_1-48\ A^4\ y\ \alpha\ \lambda\ G_c\ M_1+60\ A^3\ y^2\ \alpha\ \lambda\ G_c\ M_1-15\ A\ y^4\ \alpha\ \lambda\ G_c\ M_1+3\ y^5\ \alpha\ \lambda\ G_c\ M_1-8\ A^5\ y\ \lambda\ G_r\ M_1+20\ A^3\ y^3\ \lambda\ G_r\ M_1-15\ A^2\ y^4\ \lambda\ G_r\ M_1+3\ A\ y^5\ \lambda\ G_r\ M_1-48\ A^4\ y\ \alpha\ \lambda\ G_r\ M_1+60\ A^3\ y^2\ \alpha\ \lambda\ G_r\ M_1-15\ A\ y^4\ \alpha\ \lambda\ G_r\ M_1+3\ y^5\ \alpha\ \lambda\ G_r\ M_1+15\ A^4\ y\ \lambda\ G_c\ S_c-30\ A^2\ y^3\ \lambda\ G_c\ S_c+15\ A\ y^4\ \lambda\ G_c\ S_c+15\ A^3\ y\ \alpha\ \lambda\ G_c\ S_c-30\ A\ y^3\ \alpha\ \lambda\ G_c\ S_c+15\ y^4\alpha\ \lambda\ G_c\ S_c+8\ A^5\ y\ \delta\ \lambda\ G_c\ S_c-20\ A^3\ y^3\ \delta\ \lambda\ G_c\ S_c+15\ A^2\ y^4\ \delta\ \lambda\ G_c\ S_c-3\ A\ y^5\ \delta\ \lambda\ G_c\ S_c+8\ A^4\ y\ \alpha\ \delta\ \lambda\ G_c\ S_c-20\ A^2\ y^3\ \alpha\ \delta\ \lambda\ G_c\ S_c+15\ A\ y^4\ \alpha\ \delta\ \lambda\ G_c\ S_c-3\ y^5\ \alpha\ \delta\ \lambda\ G_cS_c)$

$u_{10}(y) = 0$

$u_{11}(y) = 0$

$u_{12}(y) = 1/(360\ A\ (A+\alpha)^2)\ (15\ A^5\ y\ G_c-60\ A^4\ y^2\ G_c+60\ A^3\ y^3\ G_c-15\ A^2\ y^4\ G_c+15\ A^5\ \alpha\ G_c-45\ A^4\ y\ \alpha\ G_c-60\ A^3\ y^2\ \alpha\ G_c+120\ A^2\ y^3\ \alpha\ G_c-30\ A\ y^4\ \alpha\ G_c-45\ A^4\ \alpha^2\ G_c+60\ A\ y^3\ \alpha^2\ G_c-15\ y^4\ \alpha^2\ G_c+15\ A^5\ y\ G_r60\ A^4\ y^2\ G_r+60\ A^3\ y^3\ G_r-15\ A^2\ y^4\ G_r+15\ A^5\ \alpha\ G_r-45\ A^4\ y\ \alpha\ G_r-60\ A^3\ y^2\ \alpha\ G_r+120\ A^2\ y^3\ \alpha\ G_r-30\ A\ y^4\ \alpha\ G_r-45\ A^4\ \alpha^2\ G_r+60\ A\ y^3\ \alpha^2\ G_r-15\ y^4\ \alpha^2\ G_r+8\ A^6\ y\ \delta\ G_c\ S_c-20\ A^4\ y^3\ \delta\ G_c\ S_c+15\ A^3\ y^4\ \delta\ G_c\ S_c-3\ A^2\ y^5\ \delta\ G_c\ S_c+8\ A^6\ \alpha\ \delta\ G_c\ S_c+8\ A^5\ y\ \alpha\ \delta\ G_c\ S_c-40\ A^3\ y^3\ \alpha\ \delta\ G_c\ S_c+30\ A^2\ y^4\ \alpha\ \delta\ G_c\ S_c-6\ A\ y^5\ \alpha\ \delta\ G_c\ S_c+8\ A^5\ \alpha^2\ \delta\ G_c\ S_c-20\ A^2\ y^3\ \alpha^2\ \delta\ G_c\ S_c+15\ A\ y^4\ \alpha^2\ \delta\ G_c\ S_c-3\ y^5\ \alpha^2\ \delta\ G_cS_c)$

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