

Validation of a Mathematical-Based Model for the Rheological Characterization of Asphalt Mixtures

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Abstract: - Asphalt mixtures are viscoelastic materials whose behavior is highly dependent on temperature and loading frequency. The influence of these factors is described through master curves constructed at a given reference temperature based on the principle of frequency-temperature superposition. These curves are used as inputs in asphalt pavement design procedures based on mechanistic principles and related to their in-service pavement performance. This paper proposes the application of the Kramers-Kronig (K-K) relations to characterize the rheological properties of asphalt materials using a mathematical approach. Due to the complexity of the integration of the K-K relations, an approximate solution of the K-K relations was used to develop a Mathematical-Based Model to predict the master curves for the Dynamic Modulus $|E^*|$ and the Phase Angle δ . This model was validated using the experimental results of two different asphalt mixtures with different characteristics. The results indicate that the model is accurate, and could be an effective approach to mathematically predict the master curves of the asphalt mixture viscoelastic properties in a wide range of temperatures and frequencies.

Key-Words: - Asphalt Mixtures, Rheology, Viscoelastic Properties, Master Curves, Kramers-Kronig Relations

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1 Introduction

The knowledge of the mechanical properties of asphalt mixtures is an important issue in the analysis of pavement structure response and the prediction of performance based on mechanistic principles.

Because of the complex nature of this kind of material resulting from the combination of an asphalt binder and the aggregate skeleton, linear viscoelastic (LVE) characterization is usually considered. For asphalt mixtures, the relationship between applied stress and resulting strain depends on loading frequency and temperature and can be fully described using the complex modulus E^* .

Two components of the complex modulus E^* are the Dynamic Modulus $|E^*|$ and the Phase Angle ϕ at a given temperature and loading frequency.

Laboratory measurements of $|E^*|$ and ϕ are commonly done at different temperature and frequency combinations using different experimental configurations, [1], [2], [3], [4], [5].

Several mathematical functions have been proposed to model the dynamic modulus master curve to be used in pavement design procedures.

Among them, the symmetrical or standard logistic sigmoidal, non-symmetrical or generalized

logistic sigmoidal, power, and polynomial functions are the most used by different researchers, [6], [7], [8], [9], [10]. Also, for the time-temperature shift factor a_T , several functions describing the temperature dependency of this parameter have been considered.

In addition, other models have been proposed to construct simultaneously both the dynamic modulus and phase angle master curves, [11], [12], [13], [14], [15].

These master curves are usually constructed by fitting the selected mathematical function to the measured data at various temperatures and frequencies, and the coefficients defining the function and the time-temperature shift factor a_T are simultaneously solved by performing a nonlinear minimization algorithm, [16].

The master curve construction of $|E^*|$ uses a mathematical model with a shift factor equation, without constructing the master curve of ϕ ; or constructing the master curves of both $|E^*|$ and ϕ utilizing two mathematical models, respectively, which are independent of each other with either different parameters and/or shift factor equations, [17].

An important LVE requirement is the Kramers-Kronig (K-K) relations to ensure that various LVE functions are equivalent and the model is causal, [18]. If this requirement is ignored, the parameters of the models or the time-temperature shift factor aT for various LVE functions may be different, which is not consistent with the LVE theory, [19]. Since physical causality induces relationships between $|E^*|$ and ϕ , these two rheological properties are mathematically interrelated at any frequency, and they share the same shift factor equation with the same parameters, which should be determined based on the test data.

Xi and Luo have used the Kramers-Kronig relations to construct the master curves of asphalt materials, [20]. In this paper, two forms of exact K-K relations were employed in numerical form to construct the master curves of the dynamic modulus and the phase angle of asphalt binders and asphalt mixtures.

Other authors have applied K-K relations to wave propagation in anelastic media, [21]. It is shown in this paper the consistency of the K-K relations applied to experimental data of rocks.

The effect of humidity on LVE properties of asphalt mixtures has been evaluated using K-K relations in constructing master curves, [22].

Researchers have developed a simple procedure for the validation of laboratory measurements of Young's modulus of solid specimens at seismic frequencies using the K-K relations, [23].

This paper proposes a Unified Model for constructing the master curves of $|E^*|$ and ϕ based on an approximate solution of the K-K relations that satisfies the LVE theory. This Unified Model was validated using the experimental $|E^*|$ and ϕ results of two different asphalt mixtures with different characteristics as is explained in the next sections.

The model will be applied in the Virtual Asphalt Mixtures project (VirAM) that is being carried out at the National University of Rosario, Argentina.

VirAM is a new and innovative concept for asphalt mixtures that is in development as a set of mathematical models, equations, and data combined and connected to mimic and represent real asphalt mixtures as a software application.

2 Theoretical Background

The Kramers-Kronig relations (K-K) for the viscoelastic parameters are presented in a couple of equations as follows, whose detailed derivations can be found in the literature, [24].

$$\ln|E^*(\omega)| = \frac{2}{\pi} \int_0^{+\infty} \frac{u \cdot \phi(u)}{\omega^2 - u^2} du \quad (1)$$

$$\phi(\omega) = \frac{2\omega}{\pi} \int_0^{+\infty} \frac{\ln|E^*(u)|}{u^2 - \omega^2} du \quad (2)$$

where $|E^*(\omega)|$, $\phi(\omega)$ are the dynamic modulus and phase angle, respectively; ω is the angular frequency; and u is the dummy variable.

Due to the difficulties in solving the integrals in an exact form, an approximate solution was proposed by, [25]:

$$\phi(\omega) \approx \frac{\pi d[\ln|E^*(\omega)|]}{2 d[\ln(\omega)]} \quad (3)$$

If the master curve for $|E^*|$ is constructed according to the conventional symmetrical or standard logistic sigmoidal, the following equation applies:

$$\log|E^*| = \delta + \frac{\alpha}{1 + e^{[\beta + \gamma \cdot \log(f \cdot aT)]}} \quad (4)$$

with f = loading frequency ($f = \omega/2\pi$), δ = value of the lower asymptote of the $|E^*|$ master curve; α = difference between the upper and lower asymptotes; β and γ = shape coefficients; and aT = time-temperature shift factor of $|E^*|$ and ϕ .

The time-temperature shift factor aT was adopted as an Arrhenius equation in the form:

$$\log(aT) = K \cdot \left(\frac{1}{T} - \frac{1}{T_R} \right) \quad (5)$$

with K = Arrhenius factor; T = test temperature ($^{\circ}\text{K}$); and T_R = reference temperature ($^{\circ}\text{K}$).

Based on Eq. (4), the master curve model of ϕ was derived according to the approximate Kramers-Kronig relations presented in Eq. (3):

$$\phi(f \cdot aT) \approx \frac{\pi}{2} \cdot \frac{\alpha \cdot \gamma}{(1 + e^{[\beta - \gamma \cdot \log(f \cdot aT)]})^2} \cdot e^{[\beta - \gamma \cdot \log(f \cdot aT)]} \quad (6)$$

Because the relationship between ϕ and $|E^*|$ proposed by Booij and Thoone in Eq. (3) is an approximation, a linear function was added to obtain potentially more accurate predictions. The new expression for ϕ results:

$$\phi(f \cdot aT) = (c_1 + c_2 \cdot \log(f \cdot aT)) \cdot \frac{\pi}{2} \cdot \frac{\alpha \cdot \gamma}{(1 + e^{[\beta - \gamma \cdot \log(f \cdot aT)]})^2} \cdot e^{[\beta - \gamma \cdot \log(f \cdot aT)]} \quad (7)$$

with c_1 and c_2 = adjustment factors.

The set of Eqs. (4) and (7) are the Mathematical-Based Model (M-B Model) proposed in this paper.

3 Materials and Procedures

3.1 Asphalt Mixtures

In this study, two types of hot asphalt mixtures (HMA) were designed according to the recommendations of the Road Authority in Argentina to be used as base or surface courses in flexible pavements. These mixtures are two dense-graded asphalt concretes with a Nominal Maximum Aggregate Size equal to 19 mm fabricated with the same type of granitic aggregates but two different asphalt binders, respectively.

One of these mixtures was elaborated using a conventional non-modified asphalt binder, and it was identified as AC30. The other one was fabricated using a modified asphalt binder with SBS, and it was identified as PmB. The samples were compacted according to the Marshall Mix Design procedure with 75 blows per face. In both cases, the asphalt content was 5.5%. Two samples of each mixture were tested. The average main volumetric properties of these mixtures are presented in Table 1.

3.2 Dynamic Tests

The dynamic tests were experimentally carried out with the Indirect Tension (IDT) mode with haversine loading under controlled stress conditions using a servo-pneumatic machine following a similar procedure as in the EN 12697-26 Standard, Annex F: Test applying cyclic indirect tension to cylindrical specimens (CIT-CY).

The test frame is enclosed in a temperature chamber where the temperature control system can achieve the required testing temperatures ranging from 0 °C to 50 °C as displayed in Figure 1.

Displacements were measured along the horizontal diameter on a gauge length equal to 60 mm using two loose core type miniature LVDTs mounted on both plane faces of the samples as shown in Figure 2.

The load capable of producing a measurable displacement by the recording device was adjusted at each testing temperature to minimize the induced damage to the sample. The $|E^*|$ and ϕ results were determined for seven frequencies (5, 4, 2, 1, 0.5, 0.25, and 0.1 Hz) and five temperatures (5, 10, 20, 30, and 40°C) having a full rheological characterization of the asphalt mixtures.



Fig. 1: Testing system



Fig. 2: Detailed view of the LVDT's

3.3 Calculation of Dynamic Viscoelastic Properties

Figure 3 shows the variation of the applied loads and the resulting measured displacements during a test.

Assuming the plane stress state, the linear viscoelastic solution for the dynamic modulus $|E^*|$ of an asphalt mixture under the IDT mode results:

$$|E^*| = \frac{P_0}{D_0 \cdot t} (K_1 + K_2 \cdot \mu) \quad (8)$$

with P_0 = amplitude of the applied haversine load; D_0 = amplitude of the resulting horizontal displacement; t = thickness of specimen; μ = Poisson's ratio; K_1 and K_2 = coefficients depending on the specimen diameter and gauge length.

For the adopted gauge length equal to 60 mm and for specimens with 100 mm diameter, the

coefficients K1 and K2 result: K1 = 0.256 and K2 = 0.869.

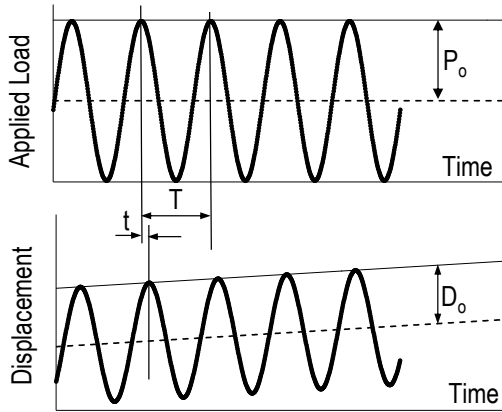


Fig. 3: Applied loads and resulting displacements

The Poisson's ratio was adopted as a function of the test temperature in the form:

$$\mu = 0.005 T + 0.25 \quad (9)$$

where T = testing temperature in °C, [26].

The phase angle ϕ results:

$$\phi = \frac{t}{T} \cdot 360^\circ \quad (10)$$

with t = time lag between peak loading and peak displacement and T = period of the haversine loading.

For materials with viscoelastic properties, the range of ϕ is between 0° and 90°. For those two extreme points, $\phi = 0^\circ$ corresponds to a purely elastic material and $\phi = 90^\circ$ corresponds to a purely viscous material.

3.4 Obtained Experimental Results

Table 2 shows the obtained experimental results of the dynamic modulus $|E^*|$ in MPa and the phase angle ϕ in Degrees for the AC30 mixture at the seven frequencies and the five temperatures for the two tested samples identified as Sample 1 and 2 respectively. In a similar form, Table 3 shows the same results for samples A and B of the PmB mixture respectively.

4 Application of the Proposed Model

The six model parameters considered in Eq. (4) and (7) ($\delta, \alpha, \beta, \gamma, c_1$ and c_2) and the parameter K of the time-temperature shift factor aT in Eq. (5) were simultaneously determined by performing a nonlinear minimization of the error function defined as:

$$\text{Error} = \sum_{i=1}^N \frac{(|E^*|_i^m - |E^*|_i^M)^2}{(|E^*|_i^m)^2} + \sum_{i=1}^N \frac{(\phi_i^m - \phi_i^M)^2}{(\phi_i^m)^2} \quad (11)$$

with N = number of test data points; $|E^*|_i^m$ = measured dynamic modulus; $|E^*|_i^M$ = modeled dynamic modulus; ϕ_i^m = measured phase angle; and ϕ_i^M = modeled phase angle.

The nonlinear minimization of the error was implemented using the Solver function in an Excel spreadsheet. The resulting values of those parameters are listed in Table 4.

For both asphalt mixtures, the coefficient c_1 is close to unity and c_2 is close to zero, so the entire proposed linear correction function is close to unity. Thus, the approximate solution proposed in Eq. (3) is a very good alternative to solve the mathematical difficulty for the exact integration of Eq. (2).

Figure 4 shows the master curve of $|E^*|$ and Figure 5 presents the master curve of ϕ , both at the reference temperature equal to 20°C for the AC30 mixture. Figure 6 and Figure 7 show the same master curves but for the PmB mixture.

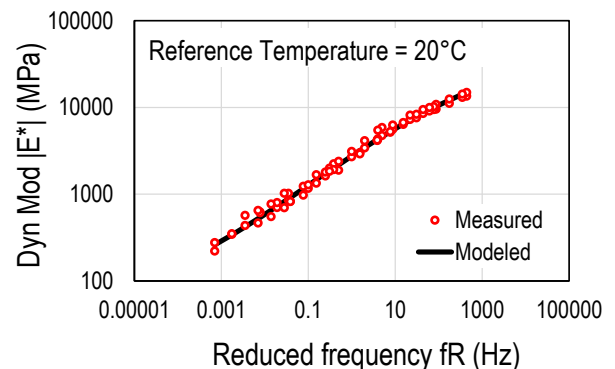


Fig. 4: Master curve of $|E^*|$ for the AC30 mixture

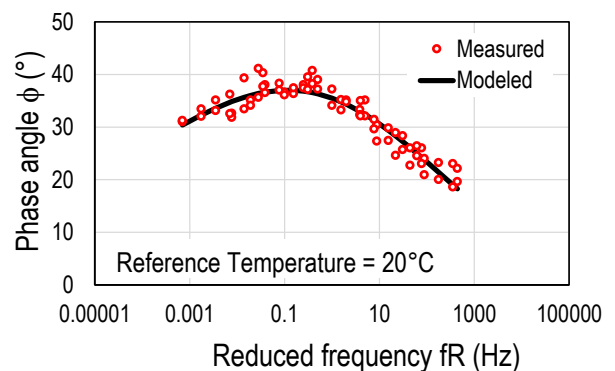


Fig. 5: Master curve of ϕ for the AC30 mixture

Table 1. Volumetric properties of the tested mixtures

	%AC	Gmb (g/dm ³)	Gmm (g/dm ³)	Vb (%)	Va (%)	VMA (%)	VFB (%)
AC30	5.5	2.376	2.477	13.07	4.08	17.15	76.22
PmB	5.5	2.374	2.476	13.06	4.12	17.18	76.02

AC: Asphalt Content; Gmm: Theoretical Maximum Specific Gravity; Gmb: Bulk Specific Gravity; Vb: Bitumen Volume; Va: Air Voids Volume; VMA: Voids in the Mineral Aggregate; VFB: Voids Filled with Bitumen

Table 2. Experimental $|E^*|$ and ϕ results for the AC30 mixture

		5°C		10°C		20°C		30°C		40°C	
		$ E^* $	ϕ	$ E^* $	ϕ	$ E^* $	ϕ	$ E^* $	ϕ	$ E^* $	ϕ
Sample 1	5 Hz	13480	22.1	9421	26.0	4730	35.1	2234	40.7	853	40.3
	4 Hz	13044	23.0	9082	26.4	4339	35.0	1992	39.5	698	41.1
	2 Hz	11135	23.2	7584	28.3	3414	35.1	1674	36.3	550	39.3
	1 Hz	9513	24.0	6347	29.8	2698	37.2	1229	37.0	465	36.2
	0.5 Hz	8479	26.0	5152	31.4	1891	39.0	824	38.0	436	33.1
	0.25 Hz	7234	28.9	4154	33.3	1620	38.0	700	35.1	344	32.0
	0.1 Hz	5570	30.4	2995	35.2	1167	36.6	625	32.6	221	31.0
Sample 2	5 Hz	14847	19.6	10174	23.0	5870	32.1	1919	38.2	1020	37.7
	4 Hz	14179	18.6	9951	24.5	5464	32.1	1831	37.1	1021	35.6
	2 Hz	12470	20.0	8295	25.7	4127	34.8	1338	37.4	767	33.4
	1 Hz	10823	20.9	6690	27.4	3096	34.1	970	38.3	648	32.5
	0.5 Hz	9413	22.7	5244	29.6	2382	37.2	822	36.5	569	35.1
	0.25 Hz	8122	24.6	4194	32.3	1798	37.4	800	34.1	349	33.4
	0.1 Hz	6235	27.3	2910	33.2	1280	36.1	612	31.8	276	31.2

$|E^*|$ in MPa, ϕ in Degrees

Table 3. Experimental $|E^*|$ and ϕ results for the PmB mixture

		5°C		10°C		20°C		30°C		40°C	
		$ E^* $	ϕ	$ E^* $	ϕ	$ E^* $	ϕ	$ E^* $	ϕ	$ E^* $	ϕ
Sample A	5 Hz	13551	12.9	9928	16.2	4855	23.1	2337	27.9	1154	28.0
	4 Hz	13272	13.6	9737	16.3	4736	23.1	2239	27.2	1063	31.8
	2 Hz	11738	13.8	8546	17.6	3942	24.5	1802	27.6	861	28.1
	1 Hz	10624	14.6	7513	18.6	3266	26.0	1574	26.7	642	28.6
	0.5 Hz	9343	15.7	6467	20.4	2724	27.1	1376	26.5	578	26.5
	0.25 Hz	8293	16.7	5623	21.2	2318	27.6	1150	26.9	456	26.3
	0.1 Hz	6926	18.1	4705	22.9	1772	29.3	876	27.4	361	24.4
Sample B	5 Hz	14352	13.2	10419	18.6	4754	24.6	3224	29.9	1424	31.6
	4 Hz	13946	13.3	10185	19.6	4631	24.1	3092	28.9	1355	29.1
	2 Hz	12502	14.4	8846	20.3	3684	24.6	2390	29.4	1192	28.1
	1 Hz	11223	15.2	7630	21.4	3029	25.9	1943	30.0	1078	26.7
	0.5 Hz	9994	15.7	6489	22.4	2489	26.3	1763	29.0	930	26.5
	0.25 Hz	8897	16.8	5567	23.8	2034	26.4	1298	30.3	690	26.8
	0.1 Hz	7430	18.4	4420	25.7	1696	27.4	890	29.8	567	25.2

$|E^*|$ in MPa, ϕ in Degrees

Table 4. Resulting parameters for the M-B Model

	K	δ	α	β	γ	c_1	c_2	Error
AC30	22730	1.442	3.379	-0.369	-0.404	1.196	-0.199	1.03
PmB	26592	1.531	3.340	-0.406	-0.307	1.151	-0.203	1.28

Figure 8 shows a comparison of measured and modeled values in the Black space ($|E^*| - \phi$) for the AC30 mixture, and Figure 9 presents the same comparison but for the PmB mixture. The measured values are tightly located around the model curve.

Therefore, the proposed model can characterize the Linear Viscoelastic behavior of both considered asphalt mixtures in the frequency domain quite accurately.

5 Overall Performance of the Model

Measured and modeled values were compared to examine the scattering of the data along the line of equality (LOE). For the AC30 mixture, Figure 10 shows the comparison of measured and modeled $|E^*|$ values in an arithmetic space, while Figure 11 shows the same comparison in a logarithmic space.

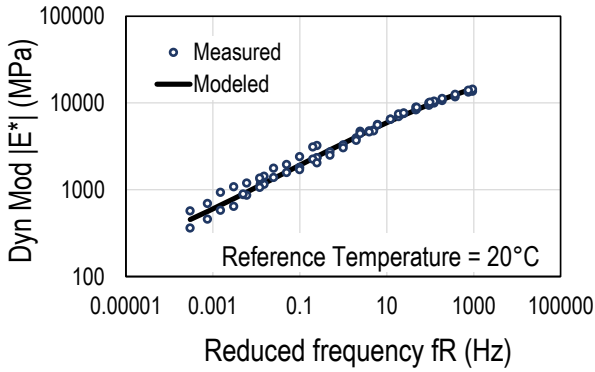


Fig. 6: Master curve of $|E^*|$ for the PmB mixture

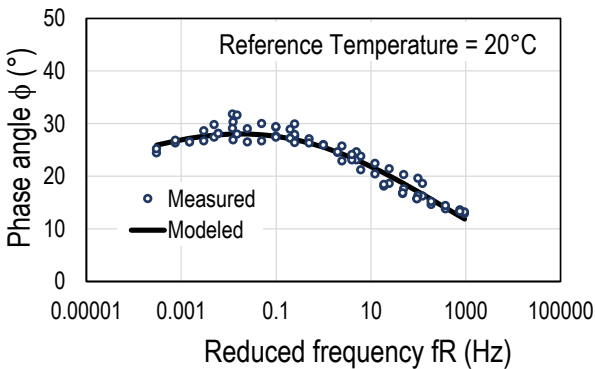


Fig. 7: Master curve of ϕ for the PmB mixture

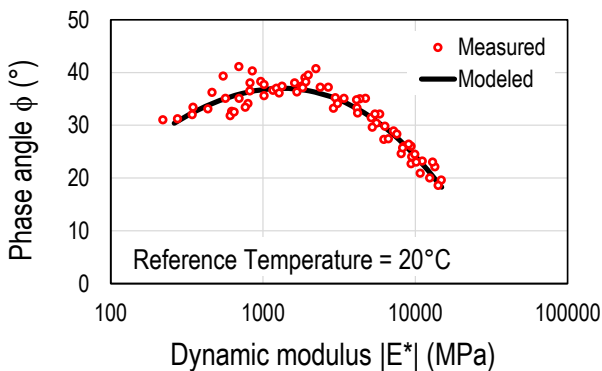


Fig. 8: Black space for the AC30 mixture

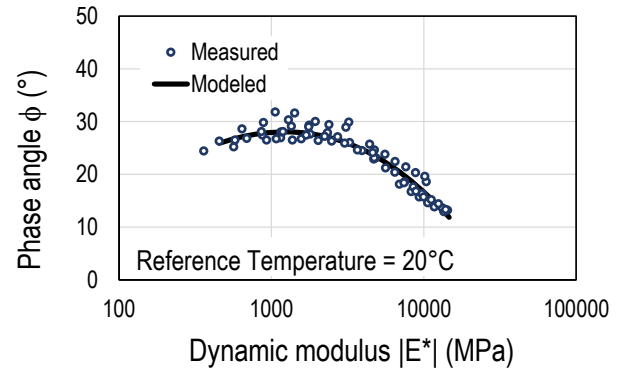


Fig. 9: Black space for the PmB mixture

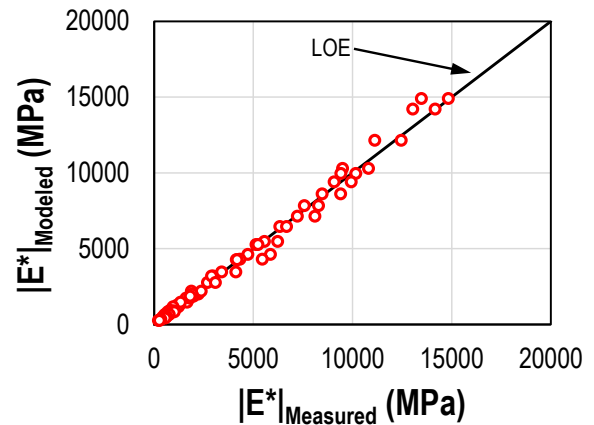


Fig. 10: Measured vs. Modeled $|E^*|$ values for the AC30 mixture (arithmetic space)

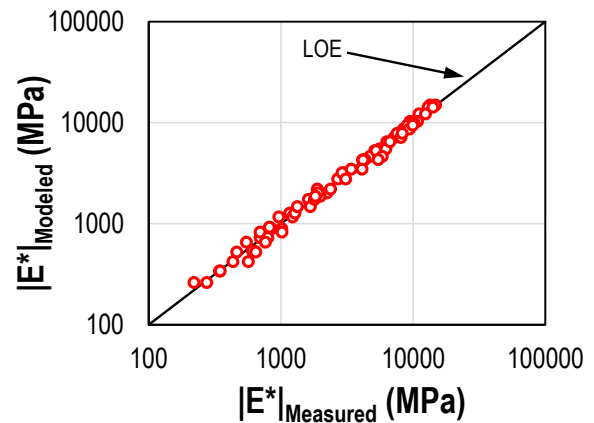


Fig. 11: Measured vs. Modeled $|E^*|$ values for the AC30 mixture (logarithmic space)

Figure 12 shows the comparison of measured and modeled ϕ values. Figure 13, Figure 14, and Figure 15 show the same comparisons for the PmB mixture.

For the $|E^*|$ values and the two mixtures considered in this study, the data points are located along the line of equality in a narrow band and distributed on both sides without a remarkable bias.

Phase angle data points are also located along the line of equality but are slightly more scattered.

However, in all cases, the modeled values are tightly clustered around the line of equality, which indicates that these modeled dynamic modulus and phase angle values are in good match with the measured values.

To evaluate the overall performance of the model, the correlation of the measured and modeled values was assessed using goodness-of-fit statistics according to the subjective criteria proposed by, [27], [28], as shown in Table 5. The statistics include the correlation coefficient, R^2 , and Se/Sy relationship (standard error of estimate values/standard deviation of measured values). Se/Sy is a measure of the accuracy of the estimates, and R^2 represents the accuracy of the model. Both parameters have been used to standardize the results in a subjective goodness classification.

Table 5. Criteria for goodness-of-fit statistical parameters

Criteria	R^2	Se/Sy
Excellent	≥ 0.90	≤ 0.35
Good	0.70 – 0.89	0.36 – 0.55
Fair	0.40 – 0.69	0.56 – 0.75
Poor	0.20 – 0.39	0.76 – 0.89
Very Poor	≤ 0.19	≥ 0.90

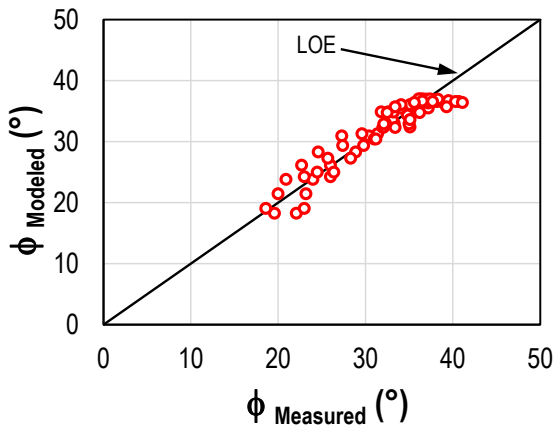


Fig. 12: Measured vs. Modeled ϕ values for the AC30 mixture

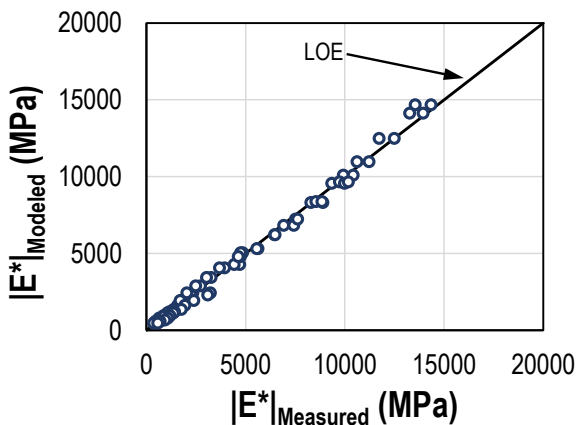


Fig. 13: Measured vs. Modeled $|E^*|$ values for the PmB mixture (arithmetic space)

Table 6 presents the evaluation of the model in modeling the $|E^*|$ and ϕ values according to these criteria for the results of the AC30 and the PmB mixtures.

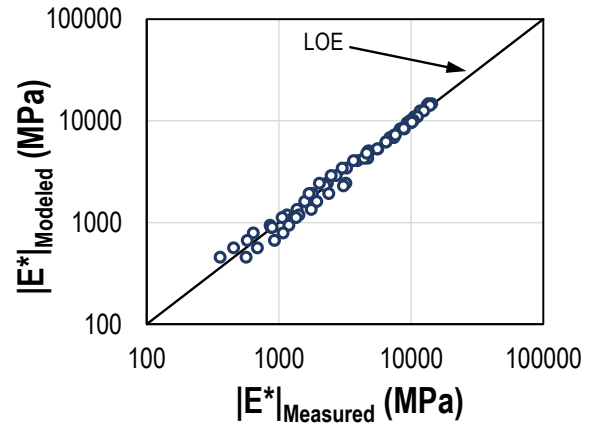


Fig. 14: Measured vs. Modeled $|E^*|$ values for the PmB mixture (logarithmic space)

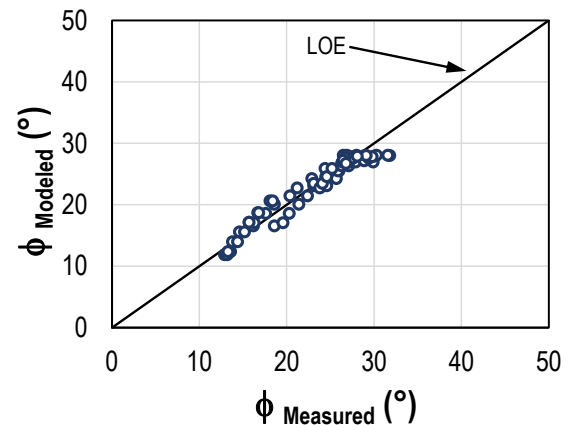


Fig. 15: Measured vs. Modeled ϕ values for the PmB mixture

The resulting evaluation according to this classification is Excellent, with R^2 greater than 0.945 in all cases and Se/Sy values significantly smaller than 0.35.

Table 6. Evaluation of the goodness-of-fit for the model

	Parameter	R^2	Se/Sy	Evaluation
AC30 mixture	$ E^* $	0.990	0.105	Excellent
	$\log E^* $	0.996	0.095	Excellent
	ϕ	0.945	0.270	Excellent
PmB mixture	$ E^* $	0.993	0.085	Excellent
	$\log E^* $	0.992	0.127	Excellent
	ϕ	0.968	0.237	Excellent

6 Model Sensitivity

Model sensitivity is an important part of understanding the accuracy and uncertainty of the proposed Mathematical-Based Model (MBM).

The ability of the model to capture the different rheological behavior of the two asphalt mixtures related to the different asphalt binders used in each case was investigated by comparing the master curves of $|E^*|$ and ϕ as shown in Figure 16 and Figure 17 respectively.

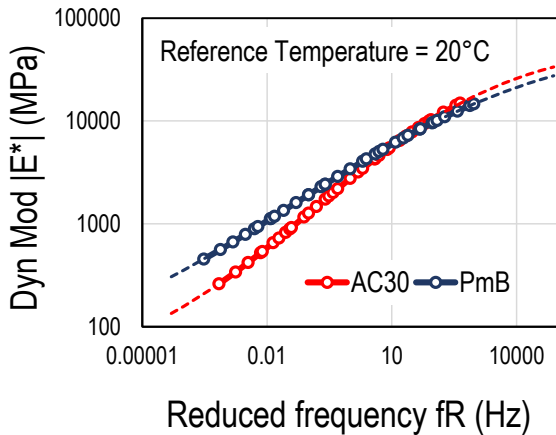


Fig. 16: Compared $|E^*|$ master curves

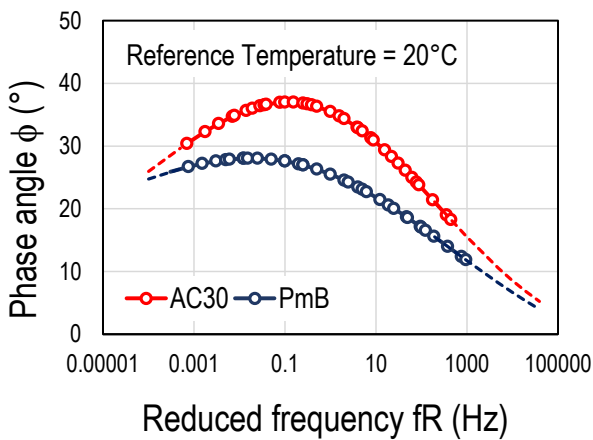


Fig. 17: Compared ϕ master curves

The PmB mixture is softer at high frequencies/low temperatures and stiffer at low frequencies/high temperatures with a smaller thermal susceptibility compared to the conventional AC30 mixture. In addition, the PmB mixture shows a more elastic behavior compared to the AC30 mixture, with smaller phase angles for the full frequency range.

Rutting and fatigue cracking performance of asphalt mixtures can be evaluated using the dynamic modulus $|E^*|$ and phase angle ϕ , [29].

A Rutting Factor (RF) and a Fatigue Factor (FF) can be defined as:

$$RF = |E^*| / \sin(\phi) \quad (12)$$

$$FF = |E^*| \cdot \sin(\phi) \quad (13)$$

Higher values of RF indicate stiffer and more elastic mixtures that have good rutting resistance. Lower FF is an indication of better performance against fatigue with softer and elastic asphalt mixtures.

Rutting usually occurs during summer at high-temperature conditions and slow vehicle speeds, and fatigue cracking is a common phenomenon at intermediate pavement service temperature and vehicle speed of about 90 Km/h.

For the RF calculation, average values of $|E^*|$ and ϕ at 40°C and 0.1 Hz were selected from Table 2 and Table 3 as extreme summer conditions with very low vehicle speed. For the FF calculation, average values of $|E^*|$ and ϕ at 20°C and 5 Hz were selected from Table 2 and Table 3 as intermediate pavement service conditions.

Table 7 shows the RF and FF values calculated for both asphalt mixtures.

Table 7. RF and FF values for the asphalt mixtures

	RF (MPa)	FF (MPa)
AC30 mixture	481	2933
PmB mixture	1106	1943

The PmB mixture has a higher Rutting Factor than the AC30 asphalt, which implies that the PmB mixture has better resistance to rutting as compared to the AC30 asphalt mixture. The Fatigue Factor of the PmB asphalt mixture is smaller as compared to the AC30 and hence, with a better fatigue resistance.

This behavior is in good agreement with the well-documented effect of SBS modification on binders and mixtures with greater resistance to fatigue and permanent deformation as well as reducing thermal susceptibility, [30], [31], [32], [33].

7 Conclusions

A model has been proposed to establish the master curves of the viscoelastic properties $|E^*|$ and ϕ of asphalt mixtures using the approximate Kramers-Kronig relations and the sigmoidal function with the same time-temperature shift factor and fitting parameters.

Dynamic modulus $|E^*|$ and phase angle ϕ values at different temperatures and frequencies were experimentally determined using the IDT mode with haversine loading.

Modeled and measured $|E^*|$ results were compared in arithmetic and logarithmic spaces. Phase angle values were compared in an arithmetic space.

The correlation between modeled and measured data R^2 was greater than 0.945 and the Se/Sy ratio was smaller than 0.27 for both $|E^*|$ and ϕ values and for the two asphalt mixtures considered in this study. Therefore, the model could be evaluated as Excellent according to the adopted goodness-of-fit criteria. The adjustment coefficient $c1$ is close to unity and $c2$ is close to zero so that the entire proposed linear correction function is close to unity. Thus, this approximate solution is a very good alternative to solve the mathematical difficulty for the exact integration of the Kramers-Kronig relations.

The ability of the model to distinguish the different rheological behaviors of different asphalt mixtures was evaluated by the definition of the Rutting and Fatigue Factors. The mixture formulated with a polymer-modified asphalt binder has a higher Rutting Factor than the asphalt mixture with the conventional asphalt binder, which implies that the PmB mixture has better resistance to rutting as compared to the AC30 asphalt mixture. The Fatigue Factor of the PmB asphalt mixture is smaller as compared to the AC30 and hence, with a better fatigue resistance.

Hence, it can be concluded that the model is able to capture adequately the different rheological behavior of the PmB mixture compared to the AC30 mixture. The proposed model is enough accurate, and it could be an effective approach to mathematically predict the viscoelastic properties of master curves in a wide range of temperatures and frequencies for asphalt mixtures.

This model will be included in the Virtual Asphalt Mixtures project (VirAM) that is being carried out at the National University of Rosario, Argentina. VirAM is a new and innovative concept for asphalt mixtures that is in development as a set of mathematical models, equations, and data combined and connected to mimic and represent real asphalt mixtures as a software application. Virtual Asphalt Mixtures could be used to design, test, and analyze different mixture compositions and to model the behavior of asphalt pavements under different traffic and temperature conditions.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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