On practical gas and liquid leakage diameter analytic estimation for vacuum applications

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Abstract: This paper presents analytical adaptive expressions for the two distinct cases of tank leakage estimations for gas (sonic and subsonic) and liquid flows under specific measurements data that assists to evaluate a circular hole/slit/orifice (crack) diameter and area. The analytic process is performed by equalization between analytic reformulation of the traditional mass flow formulations and the test formulation for mass flow dependent driven pressure differential over time multiplied by volume. In case of uniform environment conditions, the slit diameter might also represent the total sum of numerous exit holes/slits possible existence. Finally, a qualitative agreement was found between literature and current results in the context of orifice diameter versus pressure differential.

Keywords: Gas leakage, Liquid leakage, Leakage orifice area, Leakage orifice diameter, Analytic model and solution.

Received: June 17, 2021. Revised: June 24, 2022. Accepted: July 15, 2022. Published: September 19, 2022.

1 Introduction

The case of gas and liquid leakage through a circular orifice (hole, crack, and pinhole), inside a general shaped tank geometry, is represented by mass flow equilibrium. In order to evaluate the orifice diameter and area parameters, data measurements of pressure difference through time in the tank are required. The original mass flow formulation is presented in number of references as will elaborated here.

In 1966 Roth [1] has published an intensive and comprehension book on vacuum sealing techniques which introduces in the first chapter the mass flow (for liquid and gases) dependent on the driven pressure differential, divided by the time difference and multiplied by the chamber volume. The expression of the mass flow might point on the leakage existence and magnitude. During the same year, Amesz [2] has also published an essay concerning flow rates expressions, while the test measurement expression is reported in his documental brief. Moreover, he [2] has also developed generalized expressions for the flow rate of gases capillaries (or pinholes are small hole size with diameter between a few micrometers and a hundred micrometers alongside neglected length size) using a combination between Knudsen's law (molecular flow) and viscous flow (dependent on the viscosities, pressure differences,

temperature, molecular weight, capillary diameter and capillary length). This formula can be applied on large and small diameters ($d<0.1\mu m$), respectively. He also introduced laminar liquid mass flow rate formulation. Finally, he has presented capillary average diameter depending internal and outside pressure ratio among other flow properties (temperature, molecular weight and gas viscosity). In similar way, he developed the cases of liquid and mixed gas-liquid capillaries mass flow and average diameters expressions. The current study will also suggest a generalized modified approach for different cases (including capillary).

In similar way to [2] (based on diffusive and viscous flow), Davy [3] suggested analytic formula to evaluate the gas leakage mass flow and diameter parameters for micro hermetic electronic packages. Based on aerosols molecular behavior, Keller [4] has evaluated the threshold-leak size of various packages loss of sterility due to microorganism penetration.

Some modification improvement of the mentioned mass flow dependent on the pressure differential has been suggested and developed by Gahzi [5] in the context of particles transport through narrow passages and particle entrapment in laminar flow at the passage entrance. In addition, Hou *et al*. [6] have developed modified analytic solution dependent on temperature difference, pressures and other gas molecular properties for natural gas pipeline mass flow, but without diameter estimations. Later, Yoshida *et al*. [7] have calculated air mass flow (including pinholes) using driven pressure differential extended formula compared to capillary samples and also presented all types of theoretical mass flow equations (laminar, turbulent, molecular, compressible, Knudsen and modified Knudsen equation), concentrated on capillary flows.

The test application of recognition leakage in laminar and molecular flow is well presented by Fojtášek *et al*. [8] and Leybold Catalogue [9]. Wu *et al*. [10] have performed numerical method, presenting the two-phase gas-liquid flow leakage.

Some alternatives methods based on finite element analysis are also exist to forecast the leakage behavior through crack in a pipe and other geometrical elements (Ndalila *et al*. [11], Moreira *et al*. [l2]). For liquid case, Ifran *et al*. [13] have performed boundary layer numerical analysis.

Alternative method to assess the orifice diameter through droplet size analytic estimation based on Weber number multiplied by the characteristic contained length, has been proposed by Plumecocq *et al*. [14].

An investigation considering plates with different orifices diameters geometry were examined experimentally regarding pressure drop and mass flow rates by Mincks [15] and Tomaszewski *et al*. [16]. Mincks [15] has concentrated on three types of gas flow (laminar, transition and turbulent) in relative to Euler number and Reynolds number. Tomaszewski *et al*. [16] have made experimentally and numerically investigation for a six-hole orifice flow meter with and without obstacle, while comparing their results to ISO 5167 single-hole orifice formulations. Tomaszewski *et al*. have found good agreement between all three methods (empirical, numerical and ISO formulations). Note that Rahman *et al*. [17] and Spaur [18] (who investigated also irregular orifices) investigated the discharge coefficient values versus the beta ratio (diameter of orifice to pipe).

However, in the current essay we will suggest an extension to the original work performed by Guthrie and Wakerling [19] and Yoshida *et al*. [7] by developing diameters expressions using equalization between mass flow expressions and test measurement mass flow expressions for various types of flows: gas (sonic and subsonic flows) and liquids, including generalized approach.

The prominent advantage of diameter expressions development might assist evaluating the source of impermeability severity in case where leaking occurs. In other words, an evaluation of the hole or even number of holes (sum of areas) could be estimated using these expressions. Although the leaking occurrence itself will be determined preceded by empirical measurements evaluation (pressure difference).

In generalized analytic perspective, the diameter of the leak hole is necessary to calculate the Knudsen number (*Kn* - the ratio of the mean free path to the diameter). Similarly, the diameter (the area) of leak hole is necessary to calculate the Reynolds number (*Re*) because the flow velocity is calculated from the flow rate *Q* divided by the area. In first view, this is a kind of circular relationships due to the flow parameters dependency (*Kn* and *Re*) which are also dictating the flow regime type. However, in the current essay, most applications might be solved by evaluating the driven pressure differential term multiplied by a given volume, which is assumed to be given by experimentally time dependent measurements that later compared to analytic expressions based on measured pressure ratio.

To sum it up, the motivation and importance to investigate the leakage diameters is because numerous industrial (reactor cooling) and academic applications based on vacuum and sealing elements.

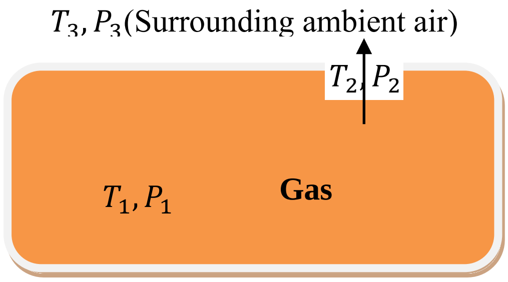


Figure 1. Gas flowing leakage illustration out of the container (tank) at flow rate Q_g through a hole (slit/crack) to the surrounding area.

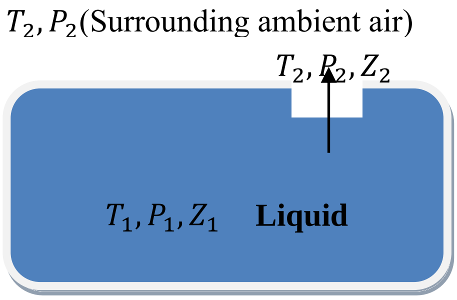


Figure 2. Liquid flowing leakage illustration out of the container (tank) at flow rate Q_{Liquid} through a hole (slit/crack) to the surrounding area.

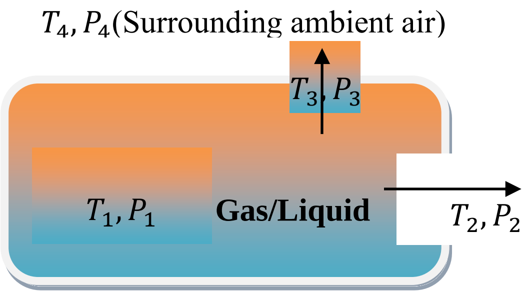


Figure 3. Generalized Gas/Liquid control volume.

2 Problem Formulation – Gas leakage orifice diameter/area estimation by analytical method

Consider a given volume depicted by a tank/container/package or a pipe geometry surrounding by gas (usually air). Suddenly, a leakage of internal gas with molecular weight M_w, subjected to temperature T_1 and pressure P_1 occurred through unknown crack/orifice/hole inlet (with constant geometry) location discharging from the container to outlet (outdoor ambient surrounding air (or other gas) conditions) as shown in Fig. 1. Depending temperature (inlet/outlet), gas pressure (inlet/outlet), gas properties (density, discharge and molecular weight) and section geometry, mass flow might occurred for the following cases; subsonic or sonic. Assuming one-dimensional isentropic ideal gas steady flow.

During discussion the terms orifice, crack, hole (similarly, as well as the terms container and tank), will be used or mentioned interchangeably.

In order to distinguish between sonic and subsonic flows, pressure conditions should be measured and fulfill (inlet/outlet) within defined time difference will be elaborated below for each type of flow ([1]-[2], [20] – [21]).

Moreover, in order to discern between viscous laminar flow, turbulent and molecular flows; Knudsen (*Kn*) and Reynolds (*Re*) number should be determined by measuring the flow velocity (or Mach number), geometry and fluid dynamic viscosity. Now, the differences will be prescribed using references [2] and [7].

- In case where $Kn < 0.01$, $1000 < Re < 2000$ the flow is viscous laminar for long circular pipe model.
- In case where $Kn < 0.01$, Re > (2000 - 4000) the flow is turbulent for long circular pipe.
- In case where $Kn > (0.5 – 1)$, the flow is molecular.

The suitable relations that include the length geometrical parameter will be developed using literature references [2] and [7] in Sec. 4. In the gas sonic, subsonic and liquid flows cases (Sec. 2.1, 2.2 and Sec. 3); it is assumed in those cases that the slit length is small enough (leak point) such as no dependency is exist between the flow rate and the slit length.

2.1. Subsonic flow

Now, alternative formulation to gas flow (Q_T) that derived from the continuity equation and used in many pumping test measurements methods, based on the pressure differential over specific time difference [1] – [2] will be brought as:

$$Q_T = V\frac{\Delta P}{\Delta t}, \tag{4}$$

whereas $V, \Delta p, \Delta t$ parameters are the given tank volume (V), internal tank volume pressure difference ($\Delta p = P_2 - P_1$) and the representative time difference ($\Delta t = t_2 - t_1$) between specified tank volume inlet pressures, respectively. Note that there are two states inside the tank chamber – before the leaking (P_1, t_1) and after the leaking (P_2, t_2) which are measured during the vacuum test.

Subsonic flow occurs for cases where Mach number fulfils $M < 1$ and the pressure near the outlet is equal to the outdoor pressure according to the following condition [20] – [21]:

$$P_2 = P_3,\ \frac{P_3}{P_1} > \left(\frac{P_2}{P_1}\right)_C, \tag{1}$$

with the appropriate gas mass flow (Q_{Subsonic}) derivation [20] – [21]:

$$Q_{Subsonic} = CAP_1\left\{\frac{2k}{k-1}\frac{M_w}{RT_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}}\right]\right\}^{\frac{1}{2}}, \tag{2}$$

where $A = \frac{\pi}{4}d^2$, C are defined as the orifice area (A) with the appropriate diameter (d) and gas discharge coefficient (C) that intended to include gas streamline flow losses through the slit [22] - [23], respectively. A reasonable value for the discharge coefficient is in between the range of 0.7 – 0.9 for most channel/tank/chamber vacuum/leakage applications whereas $0 < C \leq 1$. Also, M_w represents the molecular weight, T_1 – the inlet temperature and R is the universal gas constant. The gas specific heat ratio (k) which defined by the ratio $k = C_p/C_v$ (C_p – specific heat at a constant pressure, C_v - specific heat at a constant volume), could also expressed by the following pressure ratio ([20], [23] – [24]):

$$\left(\frac{P_2}{P_1}\right)_C = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}. \tag{3}$$

Next step, we will multiply the measured gas flow (4) in the ratio ρ_g/P_1 to accommodate the appropriate units as similar to [2]:

$$Q_{T,avg} = V\frac{\Delta p}{\Delta t}\frac{\rho_g}{P_1}\ , \qquad (5)$$

where ρ_g is the gas density that will be determined by the simple linear average:

$$\rho_g = \frac{\rho_1+\rho_3}{2}, \qquad (6)$$

that is divided by the internal tank pressure in the first state (P_1) as appear in expression (5).

Alternative accurate methods to evaluate the flow rate achieved by using the following relation that considering the density difference in the control volume (inside the tank – before the slit minus after the slit, multiplied by the appropriate pressure difference) as:

$$Q_{T,accurate} = \frac{\rho_1 P_1-\rho_3 P_3}{P_1}\frac{V}{\Delta t}. \qquad (7)$$

Although the gas densities (ρ_1, ρ_3) before (inlet) /after (outlet) the slit will be calculated by the ideal-gas equation $\rho_{1,3} = \frac{P_{1,3}M_w}{RT_{1,3}}$. Equalization between relations (2) and (7) yields the orifice area:

$$A_{Subsonic} = \frac{V}{CP_1^2}\frac{\rho_1 P_1-\rho_3 P_3}{\Delta t}\left\{\frac{2k}{k-1}\frac{M_w}{RT_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}}\right]\right\}^{-\frac{1}{2}}. \qquad (8)$$

Hence, the circular diameter will be given by:

$$d_{Subsonic} = \left(\frac{4}{\pi}\frac{V}{CP_1^2}\frac{\rho_1 P_1-\rho_3 P_3}{\Delta t}\right)^{1/2}\left\{\frac{2k}{k-1}\frac{M_w}{RT_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}}\right]\right\}^{-\frac{1}{4}} \qquad (9)$$

2.2. Sonic flow

Sonic flow occurs for cases where Mach number fulfils $M = 1$ as the gas flows out from the hole (chock state) and the pressure near the outlet will supply the following condition [1] – [2]:

$$\frac{P_3}{P_1} \le \left(\frac{P_2}{P_1}\right)_C, \quad P_2 = P_1\left(\frac{P_2}{P_1}\right)_C, P_2 \ge P_3 \quad (10)$$

with the appropriate gas mass flow (Q_{Sonic})

$$Q_{Sonic} = CAP_1\left[\frac{kM_w}{RT_1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]^{\frac{1}{2}}. \quad (11)$$

In similar way to the Sub - sonic case, the following area and diameter parameters are

Such as the obtained hole diameter would be in the form:

$$d_{Sonic} = \left(\frac{4}{\pi}\frac{V}{CP_1^2}\frac{\rho_1P_1-\rho_3P_3}{\Delta t}\right)^{1/2}\left\{\left[\frac{kM_w}{RT_1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]\right\}^{-\frac{1}{4}}. \quad (13)$$

3 Problem Formulation – Liquid leakage orifice diameter/area estimation by analytical method

In similar way to the previous case, suppose we have a tank or pipe filled with liquid that is located at Z_1 height, under pressure P_1 and velocity v_1. Suddenly, the liquid exits the hole with the appropriate pressure (P_2) velocity (v_2) and height (Z_2) as shown in Fig. 2. Then, by using Bernoulli's equation (or by Torricelli's law) without neglecting the height difference ($\Delta Z = Z_2 - Z_1$), we have [24]:

$$\frac{v_2^2-v_1^2}{2g} + \frac{P_2-P_1}{S\rho_{liquid}g} + Z_2 - Z_1 = 0\,. \quad (14)$$

Accordingly,

$$Q_{Liquid} = CA\sqrt{\frac{2\Delta P}{S\rho_{liquid}} + 2g\Delta Z + v_1^2}\,. \quad (15)$$

$$A_{Liquid} = \frac{\Delta P}{P_1}\frac{V\rho_{liquid}}{\Delta tC}\left(\frac{2\Delta P}{S\rho_{liquid}}\right)^{-1/2}, \quad (18)$$

derivation [6] – [7]:

derived/obtained by making an equalization between relations (11) and (7):

$$A_{Sonic} = \frac{V}{CP_1^2}\frac{\rho_1P_1-\rho_3P_3}{\Delta t}\left[\frac{kM_w}{RT_1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]^{-\frac{1}{2}}. \quad (12)$$

whereas fluid Bernoulli's assumptions are incompressibility, inviscid and steady flow along a streamline. Symbolic representation: S – the liquid specific gravity, g – gravity acceleration and ρ_{liquid} – liquid mass density. v_1 , v_2 are the inlet and outer velocities before and after the slit, respectively. Characteristic discharge values are in the range 0.6 – 0.7. In liquid, it also fulfilled that $\rho_1 = \rho_2 = \rho_{liquid}$.

Equalization between relations (15) and (7) leads to the following expressions for the slit diameter and area parameters:

$$A_{Liquid} = \frac{\rho_{liquid}\Delta P}{P_1}\frac{V}{\Delta tC}\left(\frac{2\Delta P}{S\rho_{liquid}} + 2g\Delta Z + v_1^2\right)^{-1/2}, \quad (16)$$

$$d_{Liquid} = \left(\frac{4}{\pi}\frac{\rho_{liquid}\Delta P}{P_1}\frac{V}{\Delta tC}\right)^{1/2}\left(\frac{2\Delta P}{S\rho_{liquid}} + 2g\Delta Z + v_1^2\right)^{-1/4}. \quad (17)$$

In cases where the height difference is small ($\Delta Z \approx 0$) and the pipe/channel/chamber/tank diameter are larger compared to the slit hole the liquid velocity might be small enough ($v_1 \ll 1$) such as Eqs. (16) – (17) approximations are:

$$d_{Liquid} = \left(\frac{4}{\pi}\frac{\Delta P}{P_1}\frac{V\rho_{liquid}}{\Delta tC}\right)^{1/2}\left(\frac{2\Delta P}{S\rho_{liquid}}\right)^{-1/4}. \quad)19)$$

It is assumed that length of leak hole is short enough such as the viscosity of liquid might be ignored in the flow rate calculation.

4 Problem Formulation – Generalized formulation

In general, alternative derivation for the area and diameters parameters provided by using equation (7) and equalizing it to the general analytic Q which might represent liquid, gas or mixing as appear in many variations and cases exhibiting by [2] and [7]. Q represents the mass flow towards inside or outside container, package, capillary or any other geometrical volume, such as:

$$Q\ (L,d) = \frac{\rho_1 P_1 - \rho_3 P_3}{P_1}\frac{V}{\Delta t}\ \text{[kg/s]}. \quad (20)$$

Or alternatively as propose by [2] and [7],

$$Q\ (L,d) = V\frac{\Delta P}{\Delta t}\ [\text{Pa } m^3/\text{s}]. \quad (21)$$

Whereas (21) with the unit of [Pa m^3/s] is easily converted to (20) with [kg/s] by multiplying with a ratio of M_w/RT.

Each formulation should be used according to the measurement set that given in the intended design. Now, in order to find the diameter, one should extract it from the Q equation which dependent on the diameter. In cases where the mass flow dependent on the diameter by polynomial form, one should solve it numerically, approximately or analytic quadratic equation.

Some examples to use the general formulation (21) will be given in the context of laminar viscous, turbulent and molecular flows based on studies of Amesz [2] and Yoshida *et al*. [7].

In case of viscous laminar gas mass flow for long circular pipe model ($Kn < 0.01$, $1000 < Re < 2000$) [7] with (21):

$$Q_{VL} = \frac{\pi d^4}{128\eta L}\frac{P_1^2 - P_3^2}{2};\ d_{VL} = \sqrt[4]{\frac{256\eta L}{\pi}\frac{V}{p_1^2 - p_3^2}\frac{\Delta P}{\Delta t}} \quad (22)$$

where η is the dynamic viscosity.

In case of turbulent mass flow for long circular pipe model ($Kn < 0.01$, Re > (2000 - 4000)) [7] with (21):

$$Q_{TB} = 1.015 d^{19/7}\left(\frac{\bar{v}^6}{\eta}\right)^{1/7}\left(\frac{P_1^2 - P_3^2}{L}\right)^{4/7};\ d_{TB} = \sqrt[7/19]{1.015\left(\frac{\eta}{\bar{v}^6}\right)^{1/7}\frac{V}{\{L/(P_1^2 - P_3^2)\}^{4/7}}\frac{\Delta P}{\Delta t}} \quad (23)$$

where the molecules arithmetic mean velocity [7] is defined by $\bar{v} = \sqrt{8RT/\pi M_w}$.

In case of molecular gas mass flow $Kn > (0.5 - 1)$:

$$Q_M = \frac{\pi d^2}{16}\sqrt{\frac{8RT}{\pi M_w}}\frac{4(L/d)+14}{3(L/d)^2+18(L/d)+14}(P_1 - P_3);$$

$$d^2\frac{4(L/d)+14}{3(L/d)^2+18(L/d)+14} = \frac{16}{\pi}\frac{\Delta P}{P_1 - P_3}\sqrt{\frac{\pi M_w}{8RT}}\frac{V}{\Delta t} \quad (24)$$

whereas the diameter appearing in (23) will be solved numerically. All other variations that appear in [2] and [7] should be solved (numerically implicit) similarly (Knudsen modified equation, semi-empirical viscous laminar & molecular flow equation).

For the compressible sonic and subsonic flows, the appropriate forms that fit relation (21) units will be using [7]:

$$Q_{Subsonic} = CAP_1 \left\{ \frac{2k}{k-1} \frac{RT_1}{M_w} \left[\left(\frac{P_2}{P_1} \right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1} \right)^{\frac{k+1}{k}} \right] \right\}^{\frac{1}{2}},$$

$$d_{Subsonic} = \left(\frac{4}{\pi} \frac{V}{CP_1} \frac{\Delta P}{\Delta t} \right)^{1/2} \left\{ \frac{2k}{k-1} \frac{RT_1}{M_w} \left[\left(\frac{P_2}{P_1} \right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1} \right)^{\frac{k+1}{k}} \right] \right\}^{-\frac{1}{4}} \quad (25)$$

$$Q_{Sonic} = CAP_1 \left[\frac{kRT_1}{M_w} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \right]^{\frac{1}{2}}, \quad d_{Sonic} = \sqrt{\frac{4\Delta P}{\pi C \Delta t} VP_1} \left[\frac{kRT_1}{M_w} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \right]^{-\frac{1}{4}} \quad (26)$$

Note that analysis of length L crucial for the viscous laminar, turbulent and molecular flows since those relations (22) – (24) are dependent on this parameter. Continually, this case will be examined numerically for the viscous laminar and turbulent flows. Although above sonic, subsonic and liquid cases are not dependent on the length since their model does not rely on the pipe model. Technically, in many cases the length L is constant and is not controllable.

5 Analytic examination

Examining Eqs. (8) – (9) (gas subsonic flow), (12) – (13) (gas sonic flow), (16) – (19) (liquid flow) and the generalized forms (20) – (21) lead to the following comprehensions:

1. In case of uniform environment conditions, the slit diameter might represent the total sum of numerous tiny or small sub-holes.
2. Otherwise, if the surrounding conditions outside the tank/package are non-uniform, then the problem should be solved using different control volumes regions dependent on the outer conditions alongside different flow rates that represent different slits area (or total sum) using the above relations.
3. In addition, a utilization could be performed for liquid-gas mixtures inside a container with appropriate leakage holes/slits nature to separate gas or liquid as shown in Fig. 3.
4. The case of mixing gas-liquid flow rate through the same hole is not concerned here and might occur in various cases, i.e., if the gas contains water vapor, freezing state might be occurring during subsonic flow through a hole.
5. Note that in all above formulations, the slit diameters are dependent on the volume square root ($\sqrt{V}$). Therefore, in case of constant flow rate, the increasing volume might cause to the diameter increase.
6. In similar way to the previous deduction, the same behavior also communicated for the pressure difference and the gas density parameters under constant flow rate. An inverse phenomenon in relating the orifice diameter (square root dependency) occurs for the time difference ($1/\sqrt{\Delta t}$), the discharge coefficient ($1/\sqrt{C}$) and the internal liquid pressure before the leaking ($1/\sqrt{P_1}$). Although in the case of gas the dependency ratio between the orifice diameter and the pressure P_1 is $1/P_1$,
7. The case of capillary might be solved using Amesz [2] and Yoshida *et al.* [7] mass flow formulations equalized to the measurement mass flow (21) formulation as exemplified in Eqs. (22) – (26).

6 Results and comparison

In this section, an illustrative results concerning Eqs. (8) – (9), (12) – (13) and (16) – (17) will be plotted as shown in Figs. 4(a) – (d) and compared qualitatively with literature Refs. [6], [25]. Later, investigation of sonic, subsonic, viscous - laminar and turbulent flows will be exhibited through comparison between Figs. 4(a) – (c) and Figs. 5 (a) – (b). Finally, formulations (22) – (26) (including the cases of sonic and subsonic) will be compared with Yoshida *et al*. numerical results [7] through Fig. 5(c).

Illustrations Fig. 4(a)-(b) present the driven pressure differential (ΔP) versus the orifice diameter (d) and the orifice area for gas (sonic and subsonic) and liquid flow, respectively. Where the input data for Nitrogen and water, respectively, is summarized in Table 1. Also, the gas pressure function will be assumed to behave according to the OCTAVE/MATLAB program distributed function:

$$P_1 = linspace(0.2,2,1000) * 101325\ [Pa]$$
$$P_2 = linspace(0.2,2,1000) * 101325 * 0.5\ [Pa]$$
$$P_3 = linspace(0.1,2,1000) * 101325 * 0.9\ [Pa]$$

(27)

whereas the gas densities are calculated by $\rho_{1,2,3} = \frac{P_{1,2,3} M_w}{RT_1}$. In the case of liquid $\rho_{water} = 998 [kg/m^3]$ at $293\,K$ temperature and $v_1 = 0\ [\frac{m}{\sec}]$.

One might observe that the liquid diameter case is about 150 times greater than the average values between the sonic and the subsonic cases as appear in Figs. 4(a) – (c) and is mainly derive due to the large liquid (water) density ${\rho_{liquid}}^{3/4} = {\rho_{water}}^{3/4} \approx 177$. It was found that the orifice diameter and area parameters decrease with the pressure differential increase for all type of flows as appear in Figs.4 (a) – (d) which qualitatively fits with Hou *et al*. [6] (Fig. 9 there).

Also, since the flow rate (Q) is proportional to the area (A) for all liquid, sonic and subsonic cases, then, the flow rate is parabolic proportional to the squared diameter as $Q \approx d^2$. The last conclusion also agrees qualitatively with Hou *et al.* [6] (natural gas, Fig. 3). Observing Fig. 3 at Hou *et al*. [6] teaches that pipe model is different than small hole model due to the length effect that is significantly decreases the leakage rate effect regarding to the current models (sonic and sub-sonic gas flows). In order to understand its importance one should also observe Fig. 6 at Hou *et al*. [6] for the different lengths effect on the orifice diameter. Similar behaviour might also been observed at Mu and Zhang [25] Figs. 12 – 13.

Furthermore, it can be observed from illustrations Figs. 4(a)-(c), that the subsonic flows have larger diameter values in relative to the sonic flows at the same pressure difference. Logically, since sonic flows are required small area in relative to subsonic flows for the same mass flow rate.

Moreover, one might deduce from Figs. 5(a) – (b) that turbulent gas flow requires the smallest pipe diameter per same conditions (one place before the viscous flow) as to the other sonic and subsonic flows (compared to Figs. 4(a)-(c)) due to its fast Nitrogen gas molecules velocity (v = 467 [m/sec]). The length parameter is crucial in the laminar viscous and turbulent flows, such as the orifice diameter increases as the length parameter increasing per pressure difference conditions. Qualitatively, on the one hand, the case of sonic flow (Fig. 4(a)) has the same order of magnitude as the viscous laminar flow (Fig. 5(a)), since they have similar flow characteristics, on the other hand, the viscous laminar flow is dependent on the orifice length (pipe model vs. leak point model).

Finally, analysing Table 2 based on Yoshida *et al*. [7] numerical data (Fig. 9 there) alongside the given pressures [7] $P_1 = 101[kpa]$, $P_2 = 6.33[kpa]$. Hence, substituting those values into the modified Eqs. (22) – (26) yields for the pinhole sonic, subsonic, viscous laminar (VL), turbulent (TB) and molecular (M) flows diameters at the size of $d_{Subsonic} = 12\ [\mu m]$, $d_{Sonic} = 5.9\ [\mu m]$ $d_{VL} = 2.5\ [\mu m]$, $d_{TB} = 3.7\ [\mu m]$, $d_M = 16.5[\mu m]$ as the maximum value of the curves illustrated in Fig. 5(c) (the molecular case was only calculated for the maximum ΔP value without illustrating), respectively. Hence, all results has the same magnitude (5-50[μm]) as Yoshida *et al*. [7] result ($d = 5.46[\mu m]$), but the sonic and turbulent diameter values are most approaching to the specified numerical value (8% and 32% errors, respectively). In addition, it can be observed at Fig. 5(c) that the orifice diameter maximum value is achieved for the maximum pressure difference, while the subsonic flow has the highest profile values and the lower profile values are connected with the viscous laminar flow profile.

Note that the reason the flow profiles represented in Fig. 5(c) increase alongside the pressure difference while the opposite phenomenon has been achieved in Figs. 4 and 5(a)-(b) is derived due different flow rate formulation (21) [Pa m^3/s] compared to (20) [kg/s].

Remark that the reason that in some cases only qualitatively comparison was made is because the given data in the relevant literature was insufficient to make a comparable quantitative examination.

Table 1: Gas and liquid data

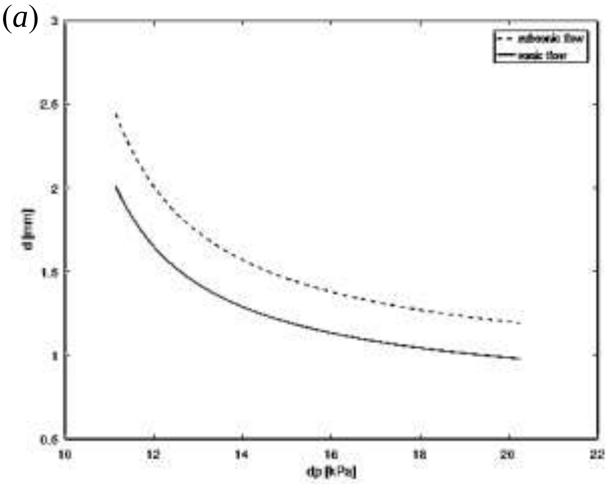
Fluid Type/parameter	V [m^3]	Δt [sec]	ΔZ [mm]	C	k	M_w [g/mole]	T_1 [K]	η [Pa s]	S
Gas Nitrogen (N_2)	0.2	200	--	0.85	1.4	28.0134	293	10^{-5}	0.97
Liquid – water (H_2O)	same	same	10	0.6	--	--	same	--	1

Table 2: Numerical data from Yoshida *et al*. [7] (Fig. 9 there) pinholes/capillaries

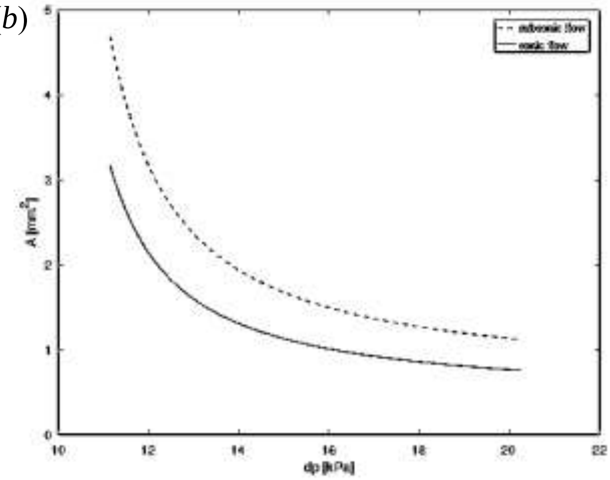
Fluid Type/parameter	V [m^3]	Δt [sec]	ΔP [Pa]	d* [μm]	k	M_w [kg/ mole]	T_1 [K]	L [μm]	η [Pa s]
Air Flow ($0.22O_2$+$0.78N_2$)	$5 \cdot 10^{-5}$	1	30 - 1600	5-50	1.4	0.029	293	0.35	$1.83 \cdot 10^{-5}$

*d – Measured

(a)



(b)



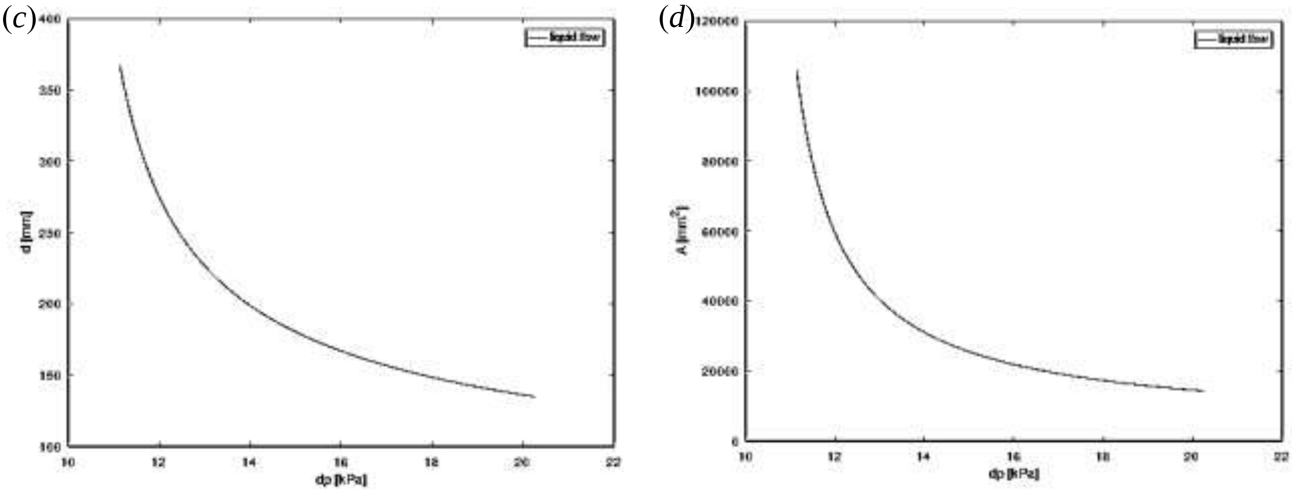


Figure 4. (*a*) Orifice diameter vs. the pressure difference for different gas flows. (*b*). Orifice area vs. the pressure difference for different gas flows. (*c*) Orifice diameter vs. the pressure difference for water flow. (*d*) Orifice area vs. the pressure difference for water flow.

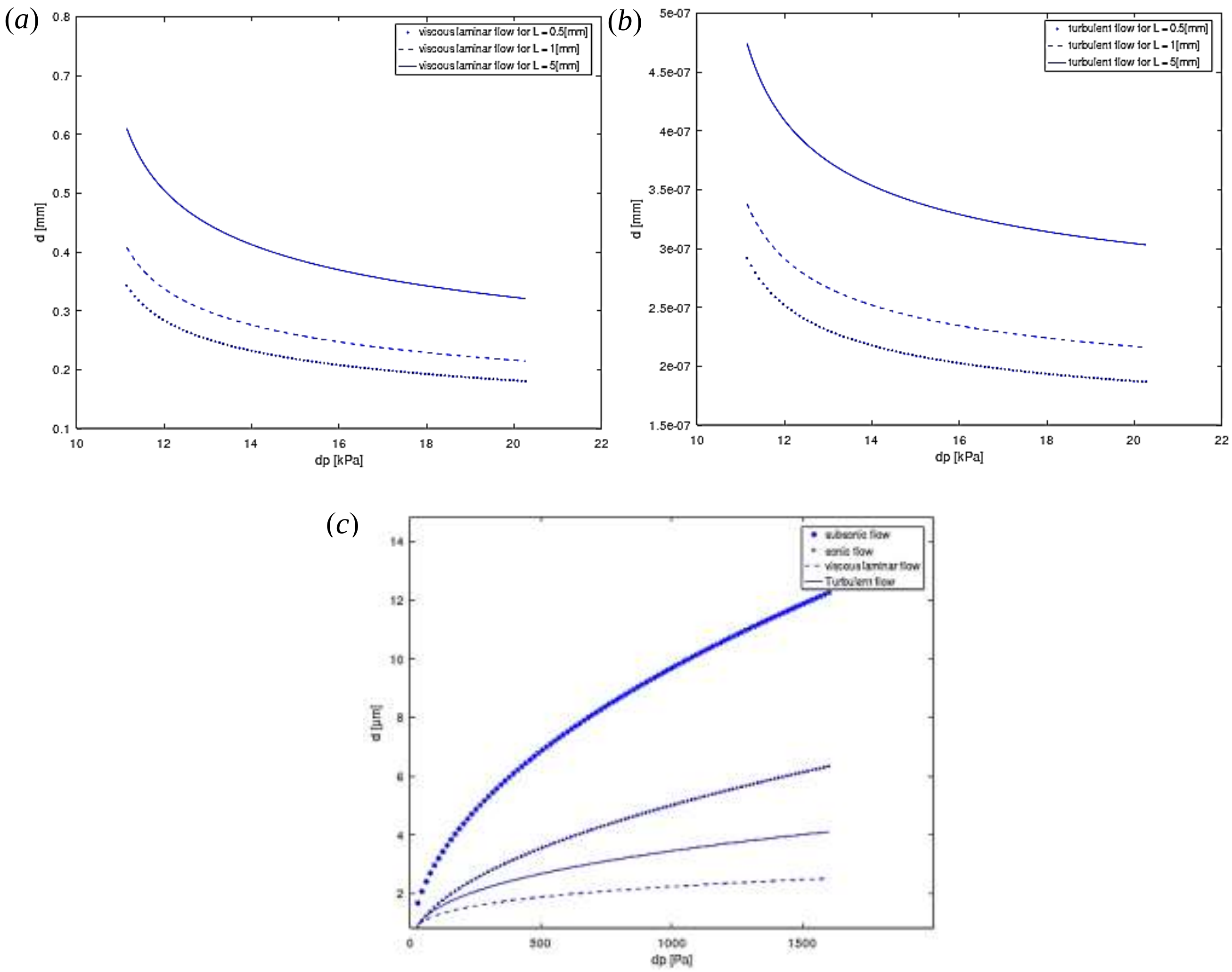


Figure 5. (*a*) Orifice diameter vs. the pressure difference for viscous laminar gas flows for different length values. (*b*). Orifice diameter vs. the pressure difference for turbulent gas flow for different length values. (*c*) Orifice diameter vs. the pressure difference for sonic, subsonic, viscous laminar and turbulent type gas flows, based on data given by [7] (Fig. 9 there).

7 Conclusion

In this study, we present a general framework for calculating orifice areas and diameters inside containers/packages filled with liquid or gas (sonic/subsonic) in various cases. Analytical examination for the obtained expression has been performed including numerical analysis for the orifice area and diameter parameters versus the pressure difference.

It was found that the orifice diameter and area parameters decreases with the driven pressure differential increase for all type of flows that agree with the relevant literature references. In case of uniform environment conditions, the slit diameter might also represent the total sum of numerous exit holes/slits possible existence. Otherwise, if the surrounding conditions outside the tank are non-uniform, then the problem should be solved using different control volumes regions dependent on the outer conditions alongside different flow rates that represent different slits area (or total sum) using the developed formulations. In addition, a utilization could be performed for liquid-gas mixtures inside a container with appropriate leakage holes/slits nature to separate gas or liquid. The slit diameters relations are dependent on the volume square root ($\sqrt{V}$). Therefore, in case of constant flow rate, the increasing volume might cause to the diameter increase. In similar way to the previous deduction, the same behaviour was found for the pressure difference and the gas density parameters under constant flow rate. An inverse phenomenon in relating the orifice diameter occurs for the dependency on time difference ($1/\sqrt{\Delta t}$), the discharge coefficient ($1/\sqrt{C}$) and the internal liquid pressure before the leaking ($1/\sqrt{P_1}$), respectively. Although in the case of gas the dependency ratio between the orifice diameter and the pressure P_1 is $1/P_1$, respectively. Specified formulations were suggested to use for various cases (i.e., capillary, pinhole) using Yoshida *et al.* [7] relations.

The fluid density was found to be meaningful regarding the orifice diameter, e.g., for the liquid fluid, the obtained diameter is about 150 times greater than the average values between sonic and subsonic flows. In addition, a qualitative agreement with literature was found in relative to the orifice diameter and area parameters decrease with the pressure differential increase for all type of flows. Another agreement was found in relative to the flow rate (Q) parabolic proportionality to the squared diameter (d^2) for all liquid, sonic and subsonic cases.

Furthermore, examination has led to conclusion that the subsonic flows have larger diameter values regarding to the sonic flows at the same pressure difference. Logically, since sonic flows required small area in relative to subsonic flows for the same mass flow rate. Moreover, numerical deduction has shown that turbulent gas flow requires the smallest pipe diameter per same conditions (one place before the viscous flow) as to the other sonic and subsonic flows due to its fast Nitrogen gas molecules velocity. The length parameter was found to play main role in the laminar viscous and turbulent flows, such as the orifice diameter increases as the length parameter increasing per pressure difference conditions. Qualitatively, on the one hand, the case of sonic flow has the same order of magnitude as the viscous laminar flow, since they have similar flow characteristics; on the other hand, the viscous laminar flow is dependent on the orifice length (pipe model vs. leak point model).

Finally, numerical estimation and comparison based on the data and relations given by Yoshida *et al*. [7] concerning pinhole orifice diameters values for various flow types has shown same magnitude fitness.

In future, better leakage mechanism approximations should be further study in the context of suspensions as aerosols and hydrogels including package sterility. In addition, cooling processes models of reactors based on orifices and pinholes, used for various needs in the energy industry should be developed.

Nomenclature

v_1	[m/sec]	Liquid velocity before the slit (inlet)
v_2	[m/sec]	Liquid velocity after (outer) the slit
$\rho_{1,2,3}$	[kg/m^3]	Gas densities before (inlet)/at the slit throat/after (outlet) the slit.
ρ_g	[kg/m^3]	Gas density
ρ_{liquid}	[kg/m^3]	Water liquid density
η	[Pa-sec]	Dynamic viscosity
A	[m^2]	Orifice area
C	--	Gas/Liquid discharge coefficient
C_p	[J·kg^{-1}·K^{-1}]	Specific heat at a constant pressure
C_v	[J·kg^{-1}·K^{-1}]	Specific heat at a constant volume
M	--	Mach Number
M_w	[kg/kmol]	Gas molecular weight
P_1	[Pa]	Internal gas pressure before the leaking
P_2	[Pa]	Internal gas pressure after the leaking
P_3	[Pa]	Outlet gas pressure
ΔP	[Pa]	Driven pressure differential
Q	[kg/sec]	Mass flow
Q_L		Liquid mass flow
Q_M	[kg/sec]	Molecular gas mass flow

Q_{Sonic}	[kg/sec]	Sonic mass flow
$Q_{Subsonic}$	[kg/sec]	Subsonic mass flow
Q_T	[kg/sec]	Gas mass flow continuity formulation
$Q_{T,accurate}$	[kg/sec]	Accurate mass flow expression
$Q_{T,avg}$	[kg/sec]	Average mass flow
Q_{VL}	[kg/sec]	Viscous laminar gas mass flow
Q_{TB}	[kg/sec]	Turbulent gas mass flow
R	8.3144 [$J \cdot K^{-1} mol^{-1}$]	Universal gas constant
S	--	Gas or Liquid specific gravity
T_1	[K]	The inlet temperature
V	[m^3]	The container volume
Z_1	[m]	Liquid height inside the container
Z_2	[m]	Liquid height outside the container or slit height
d	[m]	Orifice diameter
g	[m/sec^2]	Gravity acceleration
k	--	The gas specific heat ratio
Δt	[sec]	Time difference

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Conflict of interest statement

On behalf of all authors, the corresponding author states that there is no conflict of interest with his own fully creation, whereas NO scientific funding is involved.

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