Simulations of Frictional Losses in a Turbulent Blood Flow Using Three Rheological Models

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Abstract: - Blood flow rate is a crucial factor in transporting an oxygen and depends on several parameters like heart pressure, blood properties like density and viscosity, frictional loss and diameter and shape of vein. Frictional loss is a main challenge of current engineering. Therefore, simulation of dependence of blood properties on frictional loss is very important. When blood properties are considered the first step is to find proper rheological model. It is well known that human blood demonstrates a yield shear stress. Therefore, the research is focused on simulating frictional losses in a turbulent flow of human blood, which demonstrates a yield stress. Three arbitrarily chosen rheological models were considered, namely Bingham, Casson and Herschel-Bulkley. Governing equations describing turbulent blood flow were developed to axially symmetrical an aorta. The mathematical model constitutes three partial differential equations, namely momentum equation, kinetic energy of turbulence and its dissipation rate. The main objective of the research is examining influence of the yield shear stress on frictional losses in a human blood in an aorta when flow becomes turbulent. Simulation of blood flow confirmed marginal influence of a yield shear stress on frictional losses when flow becomes turbulent. Results of simulations are discussed and final conclusions are stated.

Key-Words: - Turbulent blood flow; simulation of frictional loss; blood yield shear stress

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1 Introduction

Simulation of blood flow rate is great challenge of fluid mechanics and biomechanics. Blood flow rate is a crucial factor in transporting an oxygen and depends on several parameters like heart pressure, blood properties like density and viscosity, frictional loss and diameter and shape of vein. Decreasing the frictional loss is a challenge of current engineering. We can decrease frictional loss using chemical or mechanical techniques. Chemical techniques include medications, which decrease a blood viscosity or act as deflocculant, while mechanical techniques can use stands in order to increase a vein diameter. The process of blood flow is extremely complex because flowing medium and its environment are very complex. Blood is not a liquid with uniform properties causing that interactions between cells and between cells and veins depend on many factors, mainly including blood flow rate, veins dimension and concentration of hematocrit [1–5]. Blood is a special liquid which contains about 55% of plasma and about 45% of cells. The plasma exists in close vicinity of a vein wall and contains about 90% of water and 10% of proteins, metabolites and ions. Density of plasma is about 1025 kg/m^3. Cells, which constitute about 45% of blood, are more complex. We recognize three types of cells, as: red cells, white cells, and platelets. Density of blood cells is about 1 125 kg/m^3. Density of blood is about 1060 kg/m^3 [6].

Simulation of a blood flow is extremely difficult, as red blood cells are deformable, have a complex shape, and play a leading role in blood rheology in contrast to white cells and platelets [7]. Concentration of red blood cells, called also hematocrit, has a substantial influence on blood flow phenomena together with plasma film [8]. As the phenomenon of blood flow is very complex, we can find different approaches in literature concerning development of mathematical models. Some of them treat a blood as Newtonian liquid [9] or as mixture of liquid and tissue (cells). Apart from that, we can treat red blood cells as flexible or non-flexible solid bodies. If we consider methods of blood flow modelling, we can recognize meso- and macroscopic approaches, as microscopic modelling refers to the scale of single atoms and molecules. If a blood flow environment is taken into account, we know that a constitutive model is effective at describing the anisotropic mechanical response of artery walls.

Literature review indicates that majority of mathematical approaches regard laminar flow, which is rather easy to modelling compared to a turbulent flow. Considering simulations of blood flow in micro channels at low and high concentrations of hematocrit one can mention the research of Fedosov et al. [2] McWhirter et al. [3],[4], Freund and Orescanin [5], Peng et al. [10], Dupin et al. [11], Doddi and Bagchi [12], and Krüger et al. [13]. However, their models deal with laminar blood flow. Therefore, this paper presents a mathematical model, which assumes that blood flow is turbulent, and the maximum Reynolds number does not exceed 5000. The mathematical model consists of averaged Navier-Stokes equations (RANS) and a turbulent stress tensor was calculated using the indirect method, which takes into account the Boussinesque hypothesis [14]. Such hypothesis utilizes turbulent viscosity, which is calculated based on the chosen two-equation turbulence model.

The main objective of the research is examining dependence of the yield shear stress on frictional losses in human blood in an aorta when flow becomes turbulent.

The friction factor is a crucial parameter determining the resistance of blood flow in a vein or an aorta. The friction factor effects on frictional losses in a blood flow. Frictional losses depend on friction factor, blood density and viscosity, blood velocity and an aorta diameter. Therefore, atherosclerosis is a major cause of human mortality caused by decreasing cross section of blood flow and is localized mainly in aorta or middle-sized veins [15]. Higher friction factor causes higher flow resistance resulting in decreasing transport of oxygen. Taking into account the mathematical model of turbulent blood flow in the aorta, influence of a yield shear stress on the friction factor and frictional losses are examined.

2 Validation of Rheological Models

Wells and Merrill's experimental data were chosen, which presenter dependence of share rate on shear stress in human blood for concentration of hematocrit equal to 43% by volume [10]. Experimental data were used to validate three arbitrarily chosen rheological models, namely Bingham, Casson and Herschel- Bulkley. All rheological models were chosen arbitrarily and are described by equations (1), (2) and (3), respectively.

– The Bingham model [16]:

$$\tau = \tau_o + \mu_{PL}\gamma \qquad (1)$$

– The Casson model [17]:

$$\tau^{1/2} = \tau_o^{1/2} + (\mu_\infty\gamma)^{1/2} \qquad (2)$$

– The Herschel-Bulkley model [18]:

$$\tau = \tau_o + K\gamma^n \qquad (3)$$

Taking into account the apparent viscosity concept, one can determine the shear stress for a Newtonian liquid, as follows:

$$\tau = \mu_{app}\gamma \qquad (4)$$

The apparent viscosity concept means that for shear thinning blood, the apparent viscosity decreases as the shear rate increases [9], [19]. The apparent viscosity depends on rheological model therefore such viscosity will be developed for each of three rheological models. Taking into account equations (1) and (4), one can develop the equation for apparent viscosity using Binghm model, as follows:

$$\mu_{app} = \frac{\tau_o}{\gamma} + \mu_{PL} \qquad (5)$$

In an analogous way it is shown that the apparent viscosity for Casson and Herschel-Bulkley rheological models can be presented respectively, as follows [20]:

$$\mu_{app} = \frac{\mu_\infty}{\left[1-\left(\frac{\tau_o}{\tau_w}\right)^{1/2}\right]^2} = \left[\left(\frac{\tau_o}{\gamma}\right)^{1/2} + \mu_\infty^{1/2}\right]^2 \qquad (6)$$

$$\mu_{app} = \frac{\tau_o}{\gamma} + K\gamma^{n-1} \qquad (7)$$

Validation of the aforementioned rheological models has been performed for human blood data reported by Wells and Merrill's containing 43% of hematocrit with a density of 1060 kg/m^3 at a temperature of 37 °C [21]. The following rheological parameters were obtained based on the best fitting shear stresses measured and calculated using the above rheological models:

– The Bingham model, described by equation (1): τ_0=0.0588 [Pa]; μ_{PL}=0.00584 [Pa s];
– The Casson model, described by equation (2): τ_0=0.0144 [Pa]; μ_∞=0.0046 [Pa s];
– The Herschel-Bulkley model, described by equation (3): τ_0=0.0144 [Pa]; K=0.020 [Pa s^n]; n=0.75;

Experimental data of Wells and Merrill's presented in Fig.1 demonstrate the dependence of the shear rate on the shear stress of human blood, which contains 43% of hematocrit [21]. The Casson and Herschel-Bulkley rheological models, described by equations (2) and (3), provided similar results of simulated shear stresses of human blood, which is presented in Fig.1. Both models demonstrate tremendous increase of blood viscosity at low shear rates in contrast to the Bingham model. The Bingham model presents significant simplification of predicted shear stresses comparing to measurements, which is seen mainly in Fig.1.

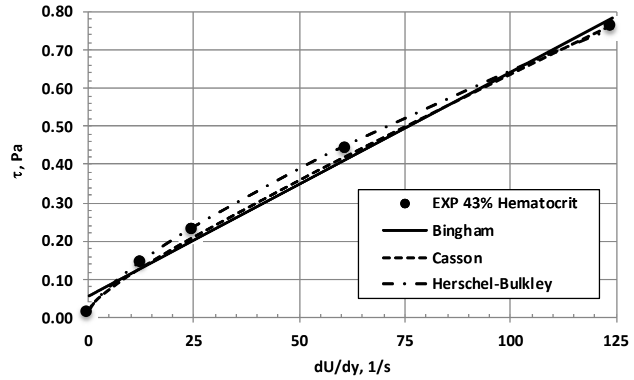

Fig.1 Wells' and Merrill's experiments compared with calculated wall shear stresses in human blood containing 43% of hematocrit.

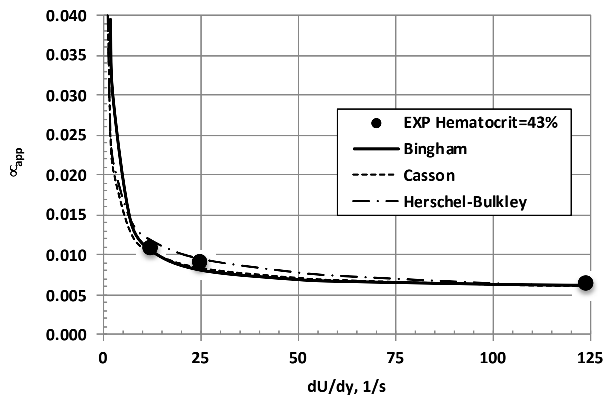

Fig.2 Measured and predicted apparent viscosity of human blood containing 43% of hematocrit.

Apparent viscosity of human blood calculated on the base of experimental data of Wells and Merrill [21] and results of simulations using equations (5), (6) and (7), which correspond to Bingham, Casson and Herschel-Bulkley rheological models, are presented in Fig.2. It is seen that all rheological models give almost the same results for high shear rates. For low shear rates, however, the Casson and Herschel-Bulkley models demonstrate small advantage comparing to Bingham model.

Concluding, one can say that two rheological models, namely Casson and Herschel-Bulkley, predict fairly well shear stresses and viscosity of human blood, however, the Herschel-Bulkley model seems to be slightly better comparing to the Casson model. It is seen that differences between results of calculations using three rheological models are not substantial. It is not very crucial which model is most suitable to predict the apparent viscosity. The apparent viscosity will be used in a momentum equation for a blood flow. The crucial point is developing mathematical model in which constitutive equations will take into account complex nature of a blood flow, especially when flow becomes turbulent. In further steps the Casson rheological model will be used, as simpler one comparing to Herschel-Bulkley model, and much adequate than the Bingham model.

3 Physical and Mathematical Models

The physical model assumes that human blood has a yield shear stress, which is in line with the aforementioned experiments of Wells and Merrill [21]. It is well known that transport of oxygen is strictly related to a blood flow rate, while blood flow rate depends on frictional losses, which depend on friction factor. For this reason, the research is focused on the influence of blood yield shear stress on the friction factor and the frictional losses when flow becomes turbulent. Looking for simplicity of the physical model, it is assumed that the blood is flowing in a rigid, smooth and horizontal aorta of constant diameter and the flow is fully developed, axially symmetrical, turbulent and homogeneous. It is also assumed that the flow is stationary. Therefore, the blood is treated as a single-phase liquid with increased density and viscosity. The blood has a constant temperature equal to 37 °C.

In order to develop mathematical model of a turbulent blood flow, the starting point are the time-averaged Navier-Stokes equations, continuity equation and boundary conditions. In order to build the mathematical model, the Random Averaged Navier-Stokes approach (RANS) has been used.

Taking into account the aforementioned assumptions, the continuity equation of incompressible blood flow can be described in the following form:

$$\frac{\partial \bar{U}}{\partial x} = 0 \qquad (8)$$

while the momentum equation consists of the Random Averaged Navier-Stokes equation, the final form of which for the aforementioned assumptions in cylindrical coordinates is as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu_{app}\frac{\partial \bar{U}}{\partial r} - \bar{\rho}\,\overline{u'v'}\right)\right] = \frac{\partial \bar{p}}{\partial x} \qquad (9)$$

where the upper dash means the time averaged quantity.

The component of turbulent stress tensor, which appears in equation (9), can be designated through an indirect method using the Boussinesque hypothesis, as follows [14]:

$$-\bar{\rho}\overline{u'v'} = \mu_t \frac{\partial \bar{U}}{\partial r} \qquad (10)$$

Several turbulence models are available in literature, which make it possible to describe the turbulent stress tensor. In this research, the Launder and Sharma turbulence model was used [22]. This particular turbulence model has a great capacity for predicting solid-liquid flows [23]. The Launder and Sharma turbulence model assumes that the turbulent viscosity, which appears in equation (10), can be designated using dimensionless analysis, as follows [22]:

$$\mu_t = f_\mu \frac{\bar{\rho}}{\varepsilon} k^2 \tag{11}$$

where the kinetic energy of turbulence and its dissipation rate are derived from Navier-Stokes equations using the time-average procedure and are as follows:

– The kinetic energy of turbulence:

$$\frac{1}{r}\left[r\left(\mu_{app} + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial r}\right] + \mu_t\left(\frac{\partial \bar{U}}{\partial r}\right)^2 = \bar{\rho}\varepsilon + 2\mu_{app}\left(\frac{\partial k^{1/2}}{\partial r}\right) \tag{12}$$

– The rate of dissipation of the kinetic energy of turbulence:

$$\frac{1}{r}\left[r\left(\mu_{app} + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial r}\right] + C_1\frac{\varepsilon}{k}\mu_t\left(\frac{\partial \bar{U}}{\partial r}\right)^2 = C_2[1 - 0.3\, exp(-Re_t^2)]\frac{\bar{\rho}\varepsilon^2}{k} - 2\frac{\mu_{app}}{\bar{\rho}}\mu_t\left(\frac{\partial^2 \bar{U}}{\partial r^2}\right) \tag{13}$$

The turbulence damping function (f_μ) in equation (11) and the turbulent Reynolds number in equation (13) were defined by Launder and Sharma in the turbulence model [22], as follows:

$$f_\mu = 0.09\, exp\left[\frac{-3.4}{\left(1+\frac{Re_t}{50}\right)^2}\right] \tag{14}$$

$$Re_t = \frac{\bar{\rho}}{\varepsilon}\frac{k^2}{\mu_{app}} \tag{15}$$

The mathematical model of turbulent human blood flow in an aorta consists three partial differential equations (9), (12) and (13) together with complimentary equations (6), (10), (11), (14) and (15). The model assumes non slip velocity at the aorta wall. The boundary conditions at the aorta wall assume that U=0, k=0 and ε=0, while in symmetry axis it is assumed that ∂U/∂r=0, ∂k/∂r=0, ∂ε/∂r=0. Constants in the Launder and Sharma turbulence model are the same like for Newtonian flow and are following: C_1=1.44; C_2=1.92; σ_k=1.0; σ_ε=1.3 [22]. The mathematical model has been solved for 80 nodal points not uniformly distributed on the aorta radius R=0.004 [m]. Most of the nodal points were localized in close vicinity of the aorta wall with expansion coefficient equal to 1.10. The number of nodal points was set experimentally to provide nodally independent simulations. Computations were made using own computer code [22]. The set of partial differential equations (9), (12) and (13) were solved by taking into account TDMA approach, with iteration procedure, using control volume method [24]. Iteration cycles were repeated until criterion of convergence, defined by equation (16), was achieved.

$$\sum_j \left|\frac{\emptyset_j^n - \emptyset_j^{n-1}}{\emptyset_j^n}\right| \leq 0.0005 \tag{16}$$

The $\emptyset_j^n$ is the value of $\emptyset$ at the j^{th} grid node after the n^{th} iteration cycle while $\emptyset_j^{n-1}$ is for the $(n-1)^{th}$ iteration cycle.

4 Results of Simulations

The yield shear stress of human blood is an indicator of cells aggregation. The yield shear stress of a human blood describes a critical stress below which no flow takes place. Several researchers confirmed the importance of the yield shear stress in a flow. Some of them explained of its nature. Michaels and Bolger provided a comprehensive explanation of phenomenon of yield shear stress [25], [26]. They reasoned that a yield shear stress has two components: a true network strength, which must be overcome for motion to occur at all, and a creep energy dissipation effect accompanying the collisions between flocs. They considered the flocs to be the basic unit of the suspension and that aggregates of flocs formed at low shear rates. The flocs were smaller than the aggregates and shear tends to produce more dense flocs. If the blood flow rate in the aorta starts from zero and is increasing, we go through regimes of laminar, transient and turbulent flow.

Non-Newtonian behavior of a blood flow indicates that changes of wall shear stress resulting in changes of apparent viscosity, which is expressed by equation (6). Fig.3 presents calculated apparent viscosity using Casson model for different wall shear stress in the range from 5 to 50 Pa for human blood flow containing 43% of hematocrit. In the range of wall shear stress from 5 to 50 [Pa] there exists laminar, transient and turbulent flow in the aorta with a radius of R=0.004 [m]. For example, if the wall shear stress equals τ_w=30 [Pa], the Reynolds number, defined by equation (17), is Re=4000, which means that blood flow for $\tau_w \geq 30$ Pa is fully turbulent.

$$\mathrm{Re_{ap}} = \frac{\rho_b\, U_b\, D}{\mu_{app}} \tag{17}$$

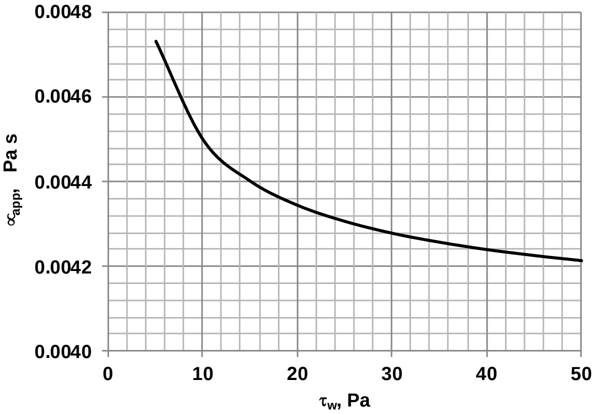

Fig.3 Simulation of the influence of human blood shear stress at the aorta wall containing 43% of hematocrit, on apparent viscosity.

After analyzing equation (6) and Fig.3, one can say that influence of the yield shear stress on the apparent viscosity of human blood increases when the wall shear stress decreases. It is well known that in laminar flow, the wall shear stress is low compared to a turbulent one. In order to demonstrate this phenomenon, simulation of the influence of the yield shear stress on the apparent viscosity of human blood for different values of wall shear stresses was made, which is presented in Fig.4. As an example, let us assume that the yield shear stress equals τ_o =0.03 [Pa]. In such a case, the apparent viscosity of human blood is μ_{app}=0.0047 [Pa s] or μ_{app}=0.00418 [Pa s] depending on the wall shear stress, which is respectively τ_w=5 [Pa] and τ_w=60 [Pa] – see Fig.4. In such a case, decrease of relative apparent viscosity is about 11%. If the wall shear tress equals τ_w=5 [Pa] the flow becomes laminar, while for τ_w=60 [Pa], it is turbulent. This clearly means that the importance of the yield shear stress in a turbulent blood flow is lower compared to its importance in a laminar flow.

It can be seen in Fig.4 that under the laminar flow regime, which exists for the wall shear stress τ_w=5 [Pa], the apparent viscosity substantially increases with the yield stress increase. In such a case, for the yield shear stress τ_0=0 [Pa], the apparent viscosity equals μ_{app}=0.004 [Pa s], while for the yield shear stress τ_0=0.05 [Pa], the relative increase of the apparent viscosity is about 23.5%. However, for higher wall shear stress than 5 Pa, which is due to transient and turbulent flow regimes, the rate of increase of apparent viscosity drops down, which is seen in Fig.4 for τ_w=15; 30; 60 [Pa]. To clarify this, let us consider a blood flow at τ_w=30 Pa, which corresponds to a Reynolds number of Re=4000. For such a case, the relative increase of apparent viscosity equals to about 9% at τ_0=0.05 [Pa], comparing to its value at τ_0=0 [Pa]. This phenomenon is even more pronounced if the wall shear stress equals to 60 [Pa]. Concluding, one can say that the influence of the yield shear stress on the apparent viscosity of human blood is significant when flow becomes laminar, and is less important in a turbulent flow.

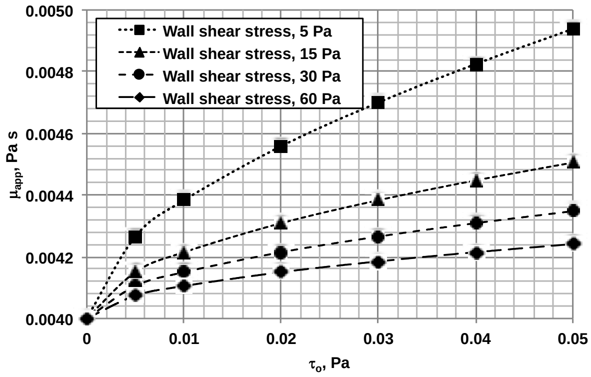

Fig.4 Simulations of the influence of yield shear stress on apparent viscosity in human blood flow containing 43% of hematocrit for different shear stresses at the aorta wall.

Lee et al. [27] examined two rheological models, namely the Casson and the Herschel-Bulkley models, looking for best fit for the experiments on human blood. They concluded that the yield shear stress value is τ_0=14.4 [mPa] for the Casson model and τ_0=32.5 [mPa] for the Herschel-Bulkley model. Their study showed that the Casson model is more suitable than the Herschel-Bulkley model for representing the non-Newtonian characteristics of blood viscosity. Taking into account the achievements of Lee at al. [27], the numerical simulations of friction factor λ were performed. Simulations were made for turbulent blood flow in the aorta with a radius R =0.004 [m] using Casson rheological model. Two values of yield shear stresses proposed by Lee at al. [27] were chosen: τ_0=0.0144 [Pa] and τ_0=0.0325 [Pa]. Of course, the value of the yield shear stress equals to τ_0=0.0325 [Pa] is about 125% higher than it should be for the Casson model, as Lee at al. [27] concluded. This was made intentionally in order to examine importance of the yield shear stress on the friction factor and the frictional losses. Simulations of friction factor λ for turbulent blood flow for two different yield shear stresses are presented in Fig.5. Simulations were made for Reynolds numbers from 2900 to 5000. Results clearly demonstrate there are no differences of friction factor for the two different values of the yield shear stress (τ_0=0.0144 and 0.0325 [Pa]), as both predictions lie on the same curve. The results confirmed that the influence of the yield shear stress on the friction factor in turbulent human blood flow containing 43% of hematocrit can be neglected.

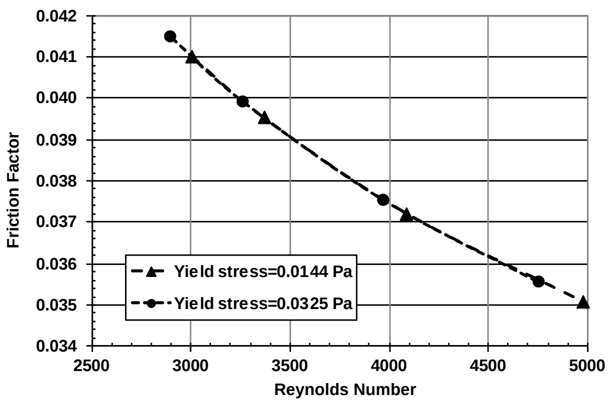

Figure 5. Simulation of the influence of Reynolds number on friction factor for human blood containing 43% of hematocrit, for two different yield stresses.

Frictional loss is the loss of pressure or "head" that occurs in a pipe or duct flow on length L, due to the effect of the fluid's viscosity near the pipe or duct wall. Considering an aorta of inner diameter D and length L the force responsible for a blood flow can be expressed as follows:

$$F_1 = \Delta p\, A = \Delta p\, \frac{\pi D^2}{4} \tag{18}$$

while the force responsible for a blood resistance is following:

$$F_2 = \pi\, D\, L\, \tau_w \tag{19}$$

Considering the equilibrium state, which means that the flow is steady, both forces F_1 and F_2 should be equal, therefore the equilibrium equation can be expresses as follows:

$$\Delta p\, \frac{\pi D^2}{4} = \pi\, D\, L\, \tau_w \tag{20}$$

or in other form as follows:

$$\frac{\Delta p}{L} = \frac{4\tau_w}{D} \tag{21}$$

The term $\Delta p/L$ is the same as dp/dx and is known as frictional loss or pressure gradient and demonstrates pressure losses during a blood flow on length L or dx.

Simulations of frictional losses for two different yield shear stresses equal to τ_0=0.0144 and 0.0325 [Pa] and for the range of Reynolds numbers from 2900 to 5000 are presented in Fig.6. It is seen that results of calculations using mathematical model presented by equations (6) and (9) – (15) are almost the same. Results confirmed again that the importance of the yield shear stress in turbulent human blood flow, which contain 43% of hematocrit is marginal.

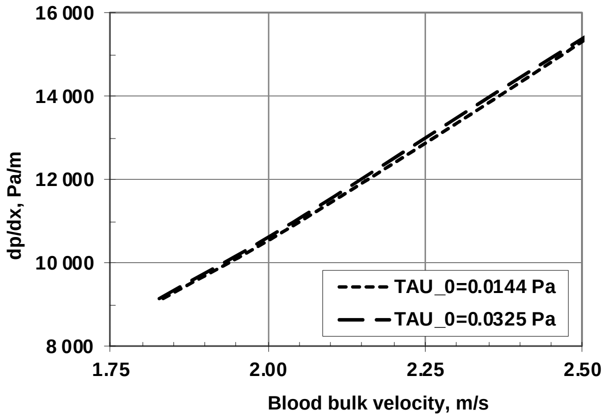

Fig.6 Simulation of the frictional losses for human blood containing 43% of hematocrit, for two different yield stresses.

5 Discussion

When the flow of human blood in an aorta is considered, it is usually assumed that such flow is laminar. However, it is known that under some circumstances, like physical activities, the flow of human blood in an aorta can be turbulent. Of course, the blood flow is pulsating in its nature, which means there exist acceleration and deceleration phases in a flow. It is well known that during deceleration phase the human blood demonstrates increase of turbulence. This clearly means that if we consider a blood flow as a laminar, we should consider that in some short period of a heart beat the flow could be turbulent. Therefore, assuming that a blood flow in an aorta is turbulent, it is interesting to know if blood yield shear stress plays an important role in transporting oxygen. If the yield shear stress will decrease the blood flow rate the transport of oxygen will be reduced too. For this reason, the mathematical model of fully developed and stationary blood flow in the aorta was developed. Of course, the mathematical model is simplified and does not take into account aorta flexibility, pulsation, and the complex nature of a blood, especially that red blood cells are deformable. Individual red blood cells experience severe deformation and transient folded conformations, which model does not include. Nevertheless, numerical simulations confirmed that under the turbulent regime of a human blood flow the influence of the yield shear stress on the blood friction factor is not important. This could be in contrast to a laminar blood flow because analyzing equation (6) and Fig.3, one can say that influence of the yield shear stress on the apparent viscosity of human blood increases when the wall shear stress decreases.

Considering rheology of a human blood one can say that Herschel-Bulkley and Casson models are fully adequate. Nevertheless, it was proved in Fig.1 and Fig.22 there are not substantial differences between all three chosen rheological models.

We know that viscosity affects shearing stress, which increases blood friction. Higher blood friction results in lower blood flow rate and, as a consequence, lower transportation of oxygen. However, if blood flow becomes turbulent, the importance of viscosity decreases, as turbulence is a major player affecting blood flow properties. There are two main reasons affecting such behavior of a blood. Firstly, blood yield shear stress is relatively low, especially for low concentration of hematocrit. Secondly, the importance of apparent viscosity in turbulent flow is low, as turbulence plays a crucial role in a blood transportation. If turbulence is taken into consideration, the turbulent viscosity, described by equation (11), plays dominant role. Taking into account Fig.3 and Fig.4 it is clear that as the wall shear stress increases, the blood apparent viscosity decreases. However, if the yield stress increases, the apparent viscosity increases too (assuming that the wall shear stress is constant). In conclusion, one can say that the wall shear stress and the yield shear stress affect blood apparent viscosity oppositely. Fig.5 and Fig.6 explicitly show that the blood friction factor and the frictional losses lie on the same line for two different yield shear stresses equal to 14.4 [mPa] and 32.5 [mPa]. The presented results confirmed that if turbulent human blood flow is taken into consideration, the importance of the yield shear stress is marginal.

The research was carried out for human blood containing 43% of hematocrit. We can anticipate that for lower concentrations of hematocrit, the influence of the yield shear stresses on the human blood friction factor can be neglected too. However, for concentration of hematocrit higher than 43%, it is difficult to anticipate if the influence of the yield shear stress on the friction factor and the frictional losses is still marginal. Such simulations are important if the influence of medications on blood flow transportation, for known concentration of hematocrit, are considered.

6 Conclusions

On the base of numerical simulations, it is possible to formulate following conclusions:

1. All three rheological models, namely Bingham, Casson and Herschel-Bulkley, describing dependence of the shear rate on the shear stress, give similar results when human blood is considered. Nevertheless, the Casson model, which is simpler comparing to the Herschel-Bulkley model, and more accurate than the Bingham one, seems to be adequate to describe human blood rheology.
2. There is no influence of the yield shear stress on the friction factor in a turbulent human blood flow.
3. Influence of the yield shear stress on the frictional losses in a turbulent human blood flow is marginal.
4. When human blood flow becomes turbulent the influence of the yield shear stress on the apparent viscosity is marginal. However, when a flow becomes laminar the importance of the yield shear stress should not be marginalized.

Nomenclature:

A – cross section of an aorta [m^2]
C_i – constants in the Launder and Sharma turbulence model, i=1, 2
D – inner aorta diameter [m]
f_μ – turbulence damping function
F_i – force acting on a blood [N], i=1, 2
j – number of nodal points
k – kinetic energy of turbulence [m^2/s^2]
K – coefficient in the Herschel-Bulkley rheological model [Pa s^n]
L – length of an aorta [m]
n – power exponent in the Herschel-Bulkley rheological model or number of iterations cycles
p – static pressure [Pa]
r – distance from symmetry axis [m]
R – inner aorta radius [m]
Re – Reynolds number
u', v' – fluctuating components of blood velocity [m/s]
U – blood velocity component in direction x [m/s]
x – axial coordinate [m]
y – distance from the aorta wall [m]

Greek symbols:

Δ – difference
γ – shear rate, du/dy (shear deformation rate) [1/s]
Φ – general dependent variable Φ=U, k, e
ε – rate of dissipation of kinetic energy of

turbulence [m^2/s^3]
λ – friction factor
μ – blood viscosity [Pa s]
μ_∞ – coefficient in Casson rheological model
ρ – blood density [kg/m^3]
σ_k – effective Prandtl-Schmidt number for k
σ_ε – effective Prandtl-Schmidt number for ε
τ – shear stress [Pa]
τ_o – yield shear stress [Pa]

Subscripts:
app – apparent viscosity
b – bulk (cross sectional averaged value)
PL – plastic
t – turbulent
w – wall

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