On distribution of speeds and pressures near the surface of rotating discs

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Abstract: - A joint solution of the Navier-Stokes equations and the continuity equation for a disc of finite radius R rotating at a constant angular frequency of ω are found. The gas compressibility and, accordingly, the second viscosity in the Navier-Stokes equation are taken into account. An analytical solution of the problem is found, and velocity distributions, as well as density and pressure near the surface of the disk under conditions of purely laminar flow, are obtained as functions of the cylindrical coordinates r and z. It is shown that if a disk is a sandwich type structure consisting of two identical disks but rotating in opposite directions, a lifting force effect appears.

Key-Words: - equation of continuity, lifting force, Navier-Stokes equation, rotation frequency.

1 Introduction

The problem that will be discussed here is related to the general problems of theoretical gas dynamics, and will consist in clarifying the distribution of velocities, density, and pressure near a surface of a disk of finite size rotating at a constant angular velocity under steady-state conditions. This problem will be quite different from the classical problem of T. Karman [1] (also described in the monograph [2], p. 112) for the perfectly understandable reason that it is a question of gas, and not of an incompressible fluid. In addition, we will not consider an infinite disk, as in [1], [2], but a finite disk. When the gas compressibility is taken into account, the stationary equation of continuity does not allow us to take the gas density behind the sign of the divergence operator (see below). The latter circumstance complicates the problem quite considerably, however, an analytical solution, nevertheless, can be found by compensating for this complication by the condition of laminar flow near the surface of the disk. By this condition, we mean a completely simple assumption about the small Reynolds numbers, which is satisfied exactly in the region of the laminar flow, for which the solutions found below are valid. We should also pay attention to the fact that unlike the solution [1], where the solution was sought as a function of only one coordinate z. We will look for the dependencies of all physical parameters of interest to us, namely, the velocity projections v_r, v_φ, v_z, as well as the density ρ and pressure distribution P near the surface of the disk as functions of two independent variables r and z.

2 The solution of the problem

To solve the problem, we write down the general system of Navier-Stokes equations and the continuity equation under steady flow conditions in the form of the following system:

$$\begin{cases} \rho(\mathbf{v}\cdot\nabla)\mathrm{v}_r-\rho\dfrac{\mathrm{v}_\varphi^2}{r}=-\dfrac{\partial P}{\partial r}+\eta\left(\Delta\mathrm{v}_r-\dfrac{\mathrm{v}_r}{r^2}\right)+\varsigma\left[\dfrac{\partial}{\partial r}div\mathbf{v}+\dfrac{\partial}{\partial r}\left(\dfrac{\mathrm{v}_r}{r}\right)\right], \\ \rho(\mathbf{v}\cdot\nabla)\mathrm{v}_\varphi+\rho\dfrac{\mathrm{v}_r\mathrm{v}_\varphi}{r}=\eta\left(\Delta\mathrm{v}_\varphi-\dfrac{\mathrm{v}_\varphi}{r^2}\right), \\ \rho(\mathbf{v}\cdot\nabla)\mathrm{v}_z=-\dfrac{\partial P}{\partial z}+\eta\Delta\mathrm{v}_z+\varsigma\dfrac{\partial}{\partial z}div\mathbf{v}, \\ \dfrac{1}{r}\dfrac{\partial}{\partial r}(r\rho\mathrm{v}_r)+\dfrac{\partial}{\partial z}(\rho\mathrm{v}_z)=0, \end{cases} \tag{1}$$

where $\rho-$ a density of gas, $P-$ a pressure, $\eta-$ a dynamic viscosity, $\varsigma-$ the second viscosity, $\mathbf{v}=(\mathrm{v}_r,\mathrm{v}_\varphi,\mathrm{v}_z)$ - velocity in cylindrical coordinates. For the laminar flow region (in the case of small Reynolds numbers), we have the right to disregard the quadratic velocity terms on the left-hand side of the first three equations. Although such an approximation greatly simplifies the system (1), the resulting equations still remain nonlinear, since the dependence of the gas density in the immediate vicinity of the disk on the coordinates must be taken into account. Indeed, in the end we find:

$$\begin{cases} \dfrac{\partial P}{\partial r}=\eta\left(\Delta\mathrm{v}_r-\dfrac{\mathrm{v}_r}{r^2}\right)+\varsigma\left[\dfrac{\partial}{\partial r}div\mathbf{v}+\dfrac{\partial}{\partial r}\left(\dfrac{\mathrm{v}_r}{r}\right)\right], \\ \Delta\mathrm{v}_\varphi-\dfrac{\mathrm{v}_\varphi}{r^2}=0, \\ \dfrac{\partial P}{\partial z}=\eta\Delta\mathrm{v}_z+\varsigma\dfrac{\partial}{\partial z}div\mathbf{v}, \\ \dfrac{1}{r}\dfrac{\partial}{\partial r}(r\rho\mathrm{v}_r)+\dfrac{\partial}{\partial z}(\rho\mathrm{v}_z)=0. \end{cases} \tag{2}$$

The lower equation in (1) and (2) is the continuity equation written in a cylindrical coordinate system. During researching of the problem, due to its axial symmetry, we can assume that all the unknown functions are functions of only two independent arguments r and z, and do not depend on the angular variable φ, i.e:

$$\rho=\rho(r,z),\ P=P(r,z),\ \mathrm{v}_r=\mathrm{v}_r(r,z),\ \mathrm{v}_\varphi=\mathrm{v}_\varphi(r,z),\ \mathrm{v}_z=\mathrm{v}_z(r,z). \tag{3}$$

We formulate the boundary conditions here in this way:

$$\begin{aligned} &\mathrm{v}_z(r,z)\big|_{z=0}=0,\ \mathrm{v}_z(r,z)\big|_{r\to\infty}=0,\ \mathrm{v}_r(r,z)\big|_{r=0}=u_0, \\ &\mathrm{v}_r(r,z)\big|_{r\to\infty}=0,\ \mathrm{v}_\varphi(r,z)\big|_{z=0}=f(r); \\ &\mathrm{v}_\varphi(r,z)\big|_{r\to\infty}=0,\ \rho(r,z)\big|_{r=0}=\rho(r,z)\big|_{r\to\infty}=0, \\ &P(r,z)\big|_{r=0}=P(r,z)\big|_{r\to\infty}=0. \end{aligned} \tag{4}$$

where u_0- some small but finite value of radial velocity. The function $f(r)$ is well known one and it satisfies only the following requirement $f(r)\to 0$ if $r\to\infty$ (see below). Since laplacian operator in a cylindrical coordinate system is defined as $\Delta=\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$, then due to the linearity of the second equation of the system (2), it is completely solved using the Fourier method, and allows us to write the physical solution of interest to us, which satisfies the boundary conditions (4) as expansion of eigenfunctions:

$$\mathrm{v}_\varphi(r,z)=\sum_{n=1}^{\infty}C_n e^{-\lambda_n y}J_1(\lambda_n x), \tag{5}$$

where $J_1(a)-$ cylindrical functions of the first order, λ_n- eigenvalues of the Sturm-Liouville problem. From the condition that the gas "adheres" to the disk surface, according to which $\mathrm{v}_\varphi(r,z)\big|_{z=0}=\omega r$, and due to the solution (5), we obtain an equation for determining the constants C_n:

$$\omega r=\sum_{n=1}^{\infty}C_n J_1\left(\lambda_n r\sqrt{\frac{\omega}{\nu_1}}\right). \tag{6a}$$

Using here the orthogonality property of cylindrical functions, we obtain:

$$C_n=\frac{\omega}{J_1^2\left(\lambda_n\sqrt{\dfrac{\omega}{\nu_1}}\right)}\int_0^R r^2 J_1\left(\lambda_n\sqrt{\frac{\omega}{\nu_1}}r\right)dr. \tag{6b}$$

We return to solutions (5), (6) below. As for calculating the velocity components v_r and v_z, we shall find them in the form:

$$\mathbf{v}=rot\left(f(r,z)[\vec{\omega}\times\mathbf{r}]\right), \tag{7}$$

where $f(r,z)-$ the required function, depending on two independent arguments.

Opening the expression (7), we are finding:

$$\mathbf{v}=\omega r\left(\mathbf{n}\frac{\partial f}{\partial z}-\mathbf{k}\frac{\partial f}{\partial r}\right), \qquad (8)$$

where $\mathbf{n}=\frac{\mathbf{r}}{r},\ \mathbf{k}=\frac{\vec{\omega}}{\omega}$ - unit vectors. For further calculations, we need a projection of the vector $rot\mathbf{v}$ onto a moving unit vector $\vec{\tau}_\varphi$ from a moving orthogonal unit basis $\vec{\tau}_\varphi,\mathbf{n},\mathbf{k}$. Hence, $(rot\mathbf{v})_i=e_{ikl}\left(\frac{\partial \mathrm{v}^k}{\partial x^l}+\Gamma^k_{nl}\,\mathrm{v}^n\right)$, where $e_{ikl}-$ antisymmetric due to all the indices, the unit tensor of the third rank, $\Gamma^i_{kl}-$ Christoffel symbol of the second type. In a cylindrical coordinate system, we easily find for the projection onto the axis $\vec{\tau}_\varphi$, which $(rot\mathbf{v})_\varphi=\frac{\partial \mathrm{v}_r}{\partial z}-\frac{\partial \mathrm{v}_z}{\partial r}$, according to (8), we have from here:

$$(rot\mathbf{v})_\varphi=\omega r\Delta f\,, \qquad (9)$$

where $\Delta=\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$ - laplacian operator. If we rewrite the system of equations (1) in vector form, we are getting:

$$\frac{\nabla P}{\rho}=\nu_1\Delta\mathbf{v}+\nu_2 graddiv\mathbf{v}, \qquad (10)$$

where $\nu_1=\frac{\eta}{\rho},\ \nu_2=\frac{\varsigma}{\rho}$, so taking an operation rot from both of its parts, we find:

$$\frac{1}{\rho^2}\left[\nabla\rho\times\nabla P\right]=\nu_1\Delta rot\mathbf{v}\ . \qquad (11)$$

Due to equation of condition, we can assume $P=P(\rho)$, hence $\nabla P=\frac{\partial P}{\partial\rho}\nabla\rho$ and therefore the substitution in (11) because of the disappearance of the left-hand side, leads us to a much simpler equation (compare with the results of [5], see also [6]):

$$\Delta rot\mathbf{v}=0.$$

Substituting here the solution (8) allows us to obtain the following equation:

$$\Delta rot\left[r\left(\mathbf{n}\frac{\partial f}{\partial z}-\mathbf{k}\frac{\partial f}{\partial r}\right)\right]=0\cdot \qquad (12)$$

After spatial derivation, we find:

$$\Delta\left\{\left[\mathbf{n}\times\mathbf{k}\right]\left[r\frac{\partial^2 f}{\partial z^2}+\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right)\right]\right\}=0\cdot \qquad (13)$$

Projecting this equation on the direction of the mobile basis $\mathbf{e}_\varphi$, we will have as a result:

$$\Delta_2\left(r\Delta_2 f\right)-\frac{\Delta_2 f}{r}=0\,, \qquad (14)$$

where $\Delta_2=\frac{\partial^2}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}$ - two-dimensional laplacian operator. To solve equation (14) it is convenient to introduce a new function:

$$u=u(r,z)=r\Delta_2 f\ . \qquad (15)$$

Then the equation (14) will be:

$$\Delta_2 u-\frac{u}{r^2}=0\cdot \qquad (16)$$

or in the opened form:

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2}=0\cdot \qquad (17)$$

We will find the solution of equation (17) in a factorized form, setting:

$$u(r,z)=E(r)F(z)\,. \qquad (18)$$

As a result of the substitution in (17), we find:

$$\frac{1}{E}\left(\frac{d^2E}{dr^2}+\frac{1}{r}\frac{dE}{dr}-\frac{E}{r^2}\right)=-\frac{1}{F}\frac{d^2F}{dz^2}=-\mu^2$$

where $\lambda=const$. The resulting two equations

$$\begin{cases}\dfrac{d^2F}{dz^2}-\mu^2F=0,\\[2mm] \dfrac{d^2E}{dr^2}+\dfrac{1}{r}\dfrac{dE}{dr}-\dfrac{E}{r^2}+\mu^2E=0,\end{cases} \qquad (19)$$

have two solutions that are finite at zero and at infinity:

$$F \sim e^{-\mu z},\ E \sim J_1(\mu r),$$

where $J_1(x)-$ cylindrical function of the first order and $\mu = \lambda\sqrt{\frac{\omega}{\nu_1}}$ - the parameter, where $\lambda-$ some constant of separation of variables, which we calculate later. Therefore, according to (15) and (18), representing laplacian operator in an explicit form, we obtain:

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{\partial^2 f}{\partial z^2} = \frac{C_2}{r} e^{-\mu z} J_1(\mu r), \qquad (20)$$

where $C_2 -$ integration constant. We find a solution of this equation also in a factorized form, setting

$$f(r,z) = e^{-\mu z}\Phi(r). \qquad (21)$$

In the result, we find such equation for the function Φ:

$$\Phi'' + \frac{\Phi'}{r} + \mu^2\Phi = \frac{C_2}{r} J_1(\mu r). \qquad (22)$$

The homogeneous equation (22) has the following fundamental solution:

$$\bar{\Phi} = C_3 J_0(\mu r) + C_4 Y_0(\mu r), \qquad (23)$$

where $C_{3,4}-$integration constants and $J_0(x),\ Y_0(x)-$ zero-order cylindrical functions of the first and the second type. Solving the equation (22) by using the method of variation of parameters, we are finding:

$$\begin{cases} C_3' J_0(\mu r) + C_4' Y_0(\mu r) = 0, \\ C_3' J_0'(\mu r) + C_4' Y_0'(\mu r) = \frac{C_2}{r} J_1(\mu r). \end{cases}$$

Hence,

$$\begin{cases} C_3(r) = C_2 \int \frac{J_1(\mu r) Y_0(\mu r)\, dr}{r\left[J_0'(\mu r) Y_0(\mu r) - J_0(\mu r) Y_0'(\mu r)\right]} + C_3, \\ C_4(r) = -C_2 \int \frac{J_1(\mu r) J_0(\mu r)\, dr}{r\left[J_0'(\mu r) Y_0(\mu r) - J_0(\mu r) Y_0'(\mu r)\right]} + C_4. \end{cases}$$

(24)

Therefore, substituting (24) into (23), and then (21), we find the required solution for the determining function in the form:

$$\begin{aligned} f(r,z) = e^{-\mu z}\Bigg[& C_3 J_0(\mu r) + C_4 Y_0(\mu r) + \\ & + C_2\Bigg(J_0(\mu r) \int \frac{J_1(\mu r) Y_0(\mu r)\, dr}{r\left[J_0'(\mu r) Y_0(\mu r) - J_0(\mu r) Y_0'(\mu r)\right]} \\ & - Y_0(\mu r) \int \frac{J_1(\mu r) J_0(\mu r)\, dr}{r\left[J_0'(\mu r) Y_0(\mu r) - J_0(\mu r) Y_0'(\mu r)\right]} \Bigg) \Bigg] \end{aligned}$$

(25)

The obtained solution allows us to find the velocity components in the laminar layer. In fact, according to the solution of (7), we have for them:

$$\begin{aligned} \mathrm{v}_r &= \omega r \frac{\partial f}{\partial z}, \\ \mathrm{v}_z &= -\omega r \frac{\partial f}{\partial r}. \end{aligned} \qquad (26)$$

Substituting here the solution of (26), we obtain:

$$\begin{aligned} \mathrm{v}_r = -\omega r e^{-\mu z}\Bigg[& C_3 J_0(\mu r) + C_4 Y_0(\mu r) + \\ & + C_2\Bigg(J_0(\mu r) \int \frac{J_1(\mu r) Y_0(\mu r)\, dr}{r\left[J_0'(\mu r) Y_0(\mu r) - J_0(\mu r) Y_0'(\mu r)\right]} - \\ & - Y_0(\mu r) \int \frac{J_1(\mu r) J_0(\mu r)\, dr}{r\left[J_0'(\mu r) Y_0(\mu r) - J_0(\mu r) Y_0'(\mu r)\right]} \Bigg) \Bigg], \\ \mathrm{v}_z = -\omega r e^{-\mu z}\Bigg[& C_3 J_0'(\mu r) + C_4 Y_0'(\mu r) + \\ & + C_2\Bigg(J_0'(\mu r) \int \frac{J_1(\mu r) Y_0(\mu r)\, dr}{r\left[J_0'(\mu r) Y_0(\mu r) - J_0(\mu r) Y_0'(\mu r)\right]} - \\ & - Y_0'(\mu r) \int \frac{J_1(\mu r) J_0(\mu r)\, dr}{r\left[J_0'(\mu r) Y_0(\mu r) - J_0(\mu r) Y_0'(\mu r)\right]} \Bigg) \Bigg], \end{aligned}$$

(27)

The above general solution, both qualitatively and quantitatively, correctly describes the velocity distribution near the surface of revolution. In this case, as can be seen from the solutions (27), all four terms for each of the velocity components both at zero and at infinity behave approximately identically. This circumstance allows us to rewrite the solutions of (27) in a simpler and much more compact form, without restricting, however, the generality of the solutions found. Therefore, the constants C_2 и C_4 has been set to zero. As a result, we shall have:

$$\begin{aligned} v_r &= -C_3\omega r e^{-\mu z} J_0(\mu r), \\ v_z &= C_3\omega r e^{-\mu z} J_1(\mu r), \end{aligned} \qquad (28)$$

where we used the well-known property of cylindrical functions, according to which the derivative was replaced by $J_0'(x)$ for $-J_1(x)$. With the help of solutions (28), we can now easily find the distribution of the gas density near the surface of revolution. According to the continuity equation, which we have not yet used, we have $\frac{\partial(\rho v_z)}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) = 0$. Substituting here the solutions (28), we find:

$$\frac{J_0'}{J_0}\frac{\partial \rho}{\partial z} + 2\frac{\rho}{r} + \frac{\partial \rho}{\partial r} = 0. \qquad (29)$$

The equation (29) is a first-order linear differential equation whose solution can be found in general form, for example, using the method of characteristics [3]. Since we are interested in a purely physical solution, the general form of the solution is of little interest to us, and we can integrate this equation in order to establish exactly the particular solution of interest to us with the help of the remarkable method of separation of variables. Because of simple actions, the solution can be represented in the form of quadratures:

$$\rho(r,z) = -\frac{\rho_0 b e^{-bz}}{r^2}\int r^2 \frac{J_1(\mu r)}{J_0(\mu r)} dr, \qquad (30)$$

where $b = a\sqrt{\frac{\omega}{\nu_1}}$, $a-$ the constant of separation of variables. It is easy to verify that by substituting dependence (30) in equation (29), it really is a solution of this equation. Thus, with the help of solution (30) and due to the equation of perfect-gas low [4], which can be written as как $P = \frac{\rho}{\overline{m}}T$, where $\overline{m}-$ is the average mass of the gas molecules, and Boltzmann constant k_B is equal to unity, the pressure dependence of interest to us can be written in this way:

$$P(r,z) = -\frac{P_0 b e^{-bz}}{r^2}\int r^2 \frac{J_1(\mu r)}{J_0(\mu r)} dr, \qquad (31)$$

where $P_0 = \frac{\rho_0}{\overline{m}}T$. To calculate the force acting on the surface of the disk from the gas side, we write down the general formula for viscous stress tensor [2]:

$$\sigma'_{ik} = \eta\left(v_{i,k} + v_{k,i} - \frac{2}{3}\delta_{ik} div\mathbf{v}\right) - P\delta_{ik}, \qquad (32)$$

where the symbol of $v_{i,k}$ means taking of covariant derivative, $\delta_{ik}-$ delta symbol. Hence, $v_{i,k} = \left(g_{is} v^s\right)_{,k} = g_{is,k} v^s + g_{is} v^s_{,k} = g_{is} v^s_{,k} = g_{is}\left(\frac{\partial v^s}{\partial x^k} + \Gamma^s_{kl} v^l\right)$, then it follows from (32) for the solutions (28) that $\sigma'_{zz} = 2\eta\left(\frac{\partial v_z}{\partial z} - \frac{1}{3r}\frac{\partial}{\partial r}(r v_r)\right) - P$. Hence,

$$\begin{aligned} F_z &= \int_S \sigma'_{zz} dS_z = \\ &= 4\pi\nu_1\int_0^R \rho\left(\frac{\partial v_z}{\partial z} - \frac{1}{3r}\frac{\partial}{\partial r}(r v_r)\right) r dr - 2\pi\int_0^R P r dr \end{aligned} \qquad (33)$$

Because of substituting the found solutions (28) and (31) in none-dimensionless variables, we have:

$$\begin{aligned} &F_z = 4C_3\pi\nu_1^2\rho_0 a e^{-(\mu+b)z} \times \\ &\times\int_0^{x_0}\frac{dx}{x}\left(\lambda x J_1(\lambda x) + \frac{1}{3x}\frac{\partial}{\partial x}\left(x^2 J_0(\lambda x)\right)\right)\int_0^x \frac{J_1(\lambda x')}{J_0(\lambda x')}dx' + \\ &+2\pi P_0 b e^{-bz}\left(\frac{\nu_1}{\omega}\right)^{\frac{3}{2}}\int_0^{x_0}\frac{dx}{x}\int_0^x x'^2\frac{J_1(\lambda x')}{J_0(\lambda x')}dx', \end{aligned} \qquad (34)$$

where $x_0 = R\sqrt{\frac{\omega}{\nu_1}}$. The force acting directly on the surface of the disk must be determined by the expression (34) taken at the coordinate value $z = 0$. Therefore, for it we get:

$$\begin{aligned} &F_z(0) = 4C_3\pi\nu_1^2\rho_0 a \times \\ &\times\int_0^{x_0}\frac{dx}{x}\left(\lambda x J_1(\lambda x) + \frac{1}{3x}\frac{\partial}{\partial x}\left(x^2 J_0(\lambda x)\right)\right)\int_0^x \frac{J_1(\lambda x')}{J_0(\lambda x')}dx' + \\ &+2\pi P_0 a\left(\frac{\nu_1}{\omega}\right)\int_0^{x_0}\frac{dx}{x}\int_0^x x'^2\frac{J_1(\lambda x')}{J_0(\lambda x')}dx', \end{aligned} \qquad (35)$$

The value of $F_z(0)$ force can be easily estimated if we take into account the rapid decay of cylindrical functions with distance. This allows us, in a crudely approximation, to assume that

$J_1(\lambda x) \approx \lambda x$, and $J_0(\lambda x) \approx 1$. As a result, a simple calculation leads us to the following estimate for the strength $F_z(0)$.

$$F_z(0) \approx 2C_3\pi\nu_1^2\rho_0 a\lambda x_0^2\left(\frac{1}{3}+\frac{\lambda^2 x_0^2}{4}\right)+\frac{\pi}{8}P_0 a\lambda\left(\frac{\nu_1}{\omega}\right)x_0^4$$

Substituting here $x_0 = R\sqrt{\frac{\omega}{\nu_1}}$, we finally find:

$$F_z(0) \approx \approx 2C_3 a\lambda\pi\nu_1\omega\rho_0 R^2\left(\frac{1}{3}+\frac{\lambda^2 R^2\omega}{4\nu_1}\right)+\frac{\pi}{8}P_0 R^4 a\lambda\left(\frac{\omega}{\nu_1}\right) \tag{36}$$

Numerical integration of the general expression (35) allows us to obtain an exact dependence for the force F_z, as a function of the rotational speed (see Fig. 1). It should be noted that the a, λ, C_3 constants in (35) and (36) do not influence on the qualitative picture of the dependence $F_z(\omega)$, illustrated in Fig. 1.

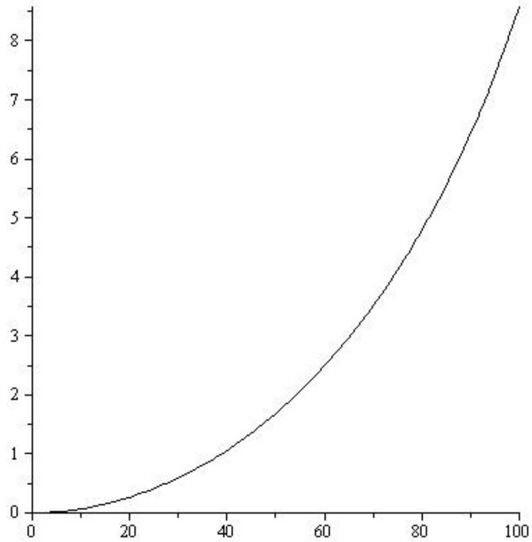

Fig.1. The dependent of the force (35) on the rotational frequencies with . Along the horizontal axis is got the value $x = R\sqrt{\omega/\nu_1}$, where ω - the rotational frequencies and the kinematic viscosity ν_1 is equal $\nu_1 = 10^{-1}\left(cm^2/s\right)$. The air density is $\rho = 10^{-2}\left(g/cm^3\right)$.

In the case when we have a system of two wheels rotating in parallel with different angular velocities $\vec{\omega}_1$ and $\vec{\omega}_2$ (see Fig. 2), the forces arising from above and below create a certain force difference ΔF_z, which, in accordance with the solution (36), can be estimated as:

$$\Delta F_z \approx 2C_3 a\lambda\pi\nu_1\rho_0 R^2 \times \times\left(\frac{\omega_1-\omega_2}{3}+\frac{\lambda^2 R^2\left(\omega_1^2-\omega_2^2\right)}{4\nu_1}\right)+\frac{\pi}{8}P_0 R^4 a\lambda\left(\frac{\omega_1-\omega_2}{\nu_1}\right) \tag{37}$$

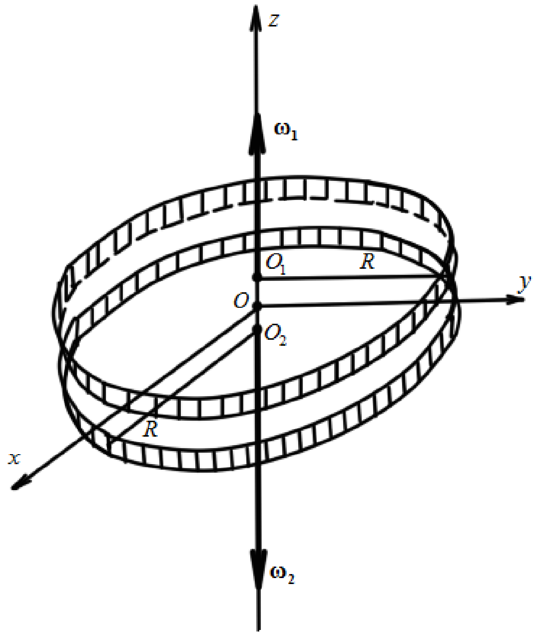

Fig.2. The graphic illustration of rotating discs with different angular velocities.

As we can see from (37), the force can be either pressing against the surface or lifting, depending on the relationship between the rotational frequencies. It is quite obvious that $\Delta F_z \sim \omega_1 - \omega_2$, the higher the speed of rotation, the higher the lifting force. This effect, by the way, is similar to a spin of a top: if it turns in one direction, its weight increases, and if it goes to the other, it decreases. The above calculations prove this purely physical effect by the example of two rotating coaxial disks. A great curiosity on our part is the experimental verification of such an opportunity, which is dcaused not so much by academic interest as by purely scientific one, since the practical importance of the task considered above is beyond doubt.

Conclusion

In conclusion, we should note that:

1. A solution of the stationary hydrodynamic problem on the rotational dynamic motion of a disk in a compressible gas is found;
2. A steady-state velocity distribution is obtained in the laminar layer immediately adjacent to the flat disk surface;
3. The pressure difference is calculated to provide the effect of a lifting force, provided that there are two coaxial discs rotating in opposite directions.

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