

# Unsteady Triple-Shock Configurations in High Speed Flows past Combined Cylinder AD Bodies in Different Gas Media

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*Abstract:* - The paper is devoted to the control of supersonic flows past aerodynamic bodies under the action of external energy deposition. Mechanism of the interaction of a bow shock over a body with the oblique shock resulting from the refraction of the bow shock at the external energy source surface is revealed. Different types of shock waves intersection might appear in this process, among them are triple shocks configurations. Here unsteady triple-shock configurations in the vicinity of the surface of combined cylinder bodies "hemisphere-cylinder" and "hemisphere-cone-cylinder" are investigated under the action of external energy deposition at  $M=4$  for gaseous media with ratio of specific heats 1.4 and 1.2. The investigations have been conducted numerically using inviscid approach on a base of the Euler equations. Complex conservative difference schemes are used in the simulations. An effect of a triple-shock configuration on the body surface pressure has been studied. Generation of local space-time areas with increasing boundary pressure has been established. Mechanism of boundary pressure growth together with local front drag force increase is shown to be connected with a vortex action as well as the action of the arising shock segments in the vicinity of triple-shock configuration. Dependences of the angles of triple-shock configuration on the rarefaction degree in the energy source and on the incident shock angle have been obtained. Also, the comparison with the plane case has been conducted. The results can be used for organization of flow control via external energy deposition by means of laser, microwave or electrical discharge.

*Key-Words:* - Supersonic flow, gas media, external energy deposition, triple-shock configuration, complex conservative difference scheme

## 1 Introduction

Problems of control of supersonic flows past aerodynamic (AD) bodies constitutes a subject of numerous studies from the second half of the last century and are widely investigated now (see surveys [1-3]). Unsteady problems of flow control via external energy deposition were suggested in [4].

Different types of energy deposition were shown to be effective for the purpose of flow control: laser

[5], microwave [6], and DC discharge [7, 8] where the possibility of drag reduction was researched experimentally for the flow past a hemisphere-cylinder and for a cylindrically blunted plate. In [9] a problem of interaction of a bow shock over a sphere with the impinging oblique shock was considered and a scheme of the mechanism of this interaction is presented. In the interaction different types of shock waves intersections might appear. Among them

triple-shock configurations were observed.

Unsteady triple-shock configurations are the significant object of the investigation from the last century. Wide class of the reflection phenomena is considered [10-13]. A problem of vortex generation on slip stream in triple-shock structures were under an interest, as well [14,15]. Shock wave diffraction on a rounded corner in the presence of a supersonic flow together with following lines of vortices was analyzed in [16]. Pressure increase via the reflection of the cylindrically converged shock wave was established in [17].

Triple shock configurations are to be greatly influenced by the adiabatic index of the gas. Recently the research of generation and dynamics of triple shock configurations in the problem of supersonic flow control in different gas media has been suggested [18]. Possibility of the control of a shape of triple-shock structure in supersonic flows past AD bodies has been studied in [19]. Plane flow symmetry has been considered there.

In this paper unsteady triple-shock configurations in supersonic flow past combined cylinder AD bodies “hemisphere-cylinder” and “hemisphere-cone-cylinder” are investigated under the action of external energy deposition at  $M=4$ . Gaseous media with the ratio of specific heats  $\gamma=1.4$  and  $1.2$  are considered, the latter can describe ionized low temperature plasma medium or mixture of different gases. The comparison with the plane case is conducted, too.

The results can be useful for the study of details of flow over flying bodies in air and in upper layers of the Earth atmosphere or in the atmospheres of other planets. Additionally, the results can be used for the organization of flow control via external energy deposition by means of laser, microwave or electrical discharge.

## 2 Statement of the Problem

Simulations are based on the Euler system of equations for cylinder flow symmetry:

$$(\mathbf{U}r)_t + (\mathbf{F}r)_x + (\mathbf{G}r)_r = \mathbf{H}, \quad (1)$$

$$\begin{aligned} \mathbf{U} &= (\rho, \rho u, \rho v, E)^T, \mathbf{F} = (\rho u, p + \rho u^2, \rho uv, u(E + p))^T, \\ \mathbf{G} &= (\rho v, \rho uv, p + \rho v^2, v(E + p))^T, \mathbf{H} = (0, 0, p, 0)^T, \\ E &= \rho(\varepsilon + 0.5(u^2 + v^2)). \end{aligned}$$

The state equation for a perfect gas is used:

$$\varepsilon = p/(\rho(\gamma - 1)).$$

Here  $\rho$ ,  $p$ ,  $u$ ,  $v$  are the gas density, pressure and velocity  $x$ - and  $y$ -components,  $\varepsilon$  is the specific internal energy. Gas media with the ratio of specific heats  $\gamma=1.4$  and  $1.2$  are considered.

The initial conditions are the fields of parameters for the converged steady supersonic flow past AD bodies “hemisphere-cylinder” and “hemisphere-cone-cylinder” at the freestream Mach number  $M=4$ . Energy source is supposed to have a shape of longitudinal homogeneous heated filament which arises instantly in front of the bow shock [20] and interacts with it. Density in the energy source is supposed to be constant and equal to  $\rho_i = \alpha_p \rho_\infty$ , where the subscript “ $\infty$ ” refers to the freestream parameters. Pressure and velocity in the energy source are equal to their freestream values (so temperature in the energy source is increased). Normalizing values for density and pressure are  $\rho_n = 1.293 \text{ kg/m}^3$  and  $p_n = 5.06625 \times 10^5 \text{ Pa}$ , accordingly. So the dimensionless values  $p_\infty = 0.2$  and  $\rho_\infty = 1$  refer to the normal conditions in air.

The problem is solved in the dimensionless variables. The domestic code founded on complex conservative schemes [21] is used in the simulations. The comparison of experimental pressure values on a surface of hemisphere with the numerical ones obtained using this code is presented in [22]. Staggered Cartesian difference grid with the space steps equal to 0.0002 (1000 points per body’s diameter) is used.

## 3 Results

Effect of the energy source on supersonic flows past hemisphere-cylinder and hemisphere-cone-cylinder are studying at freestream Mach number 4 for  $\gamma=1.4$  and  $1.2$ . Rarefaction degree in the energy source changing in the interval  $0.18 < \alpha_p < 0.67$  is considered.

### 3.1 Flow past a hemisphere-cylinder

The first stage of generation of a triple-shock configuration near the surface of the hemisphere-cylinder as the result of interaction of the energy source with the bow shock is shown in Fig.1. Here the full calculation area is presented. A scheme of the flow in the vicinity of triple-shock configuration is presented in Fig.2. The considered angles are indicated.

One can see a triple-shock structure and two tangential discontinuities. The lower of these tangential discontinuities is generated as a consequence of the boundary of energy source-bow shock wave interaction. It forms the lower “internal”

vortex. The upper tangential discontinuity is a slip stream coming out from the triple point. This slip stream forms the upper vortex. For sufficiently large Mach numbers when a triple configuration is generated these two vortices are twisting together to form a complicated shock wave structure.

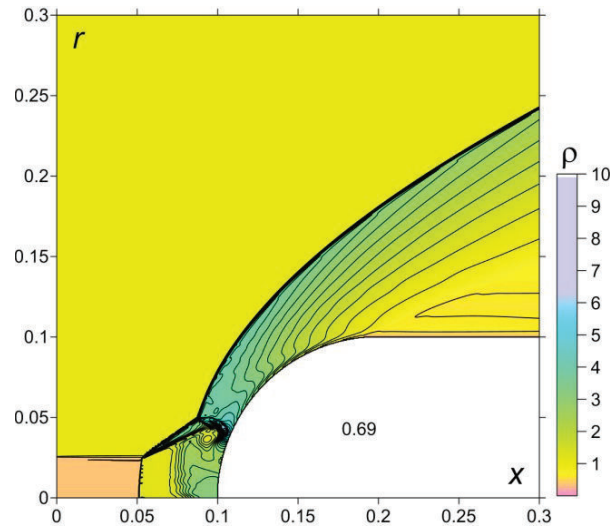


Fig.1. First stage of formation of triple-shock configuration,  $\gamma=1.4$ ,  $\alpha_p=0.33$

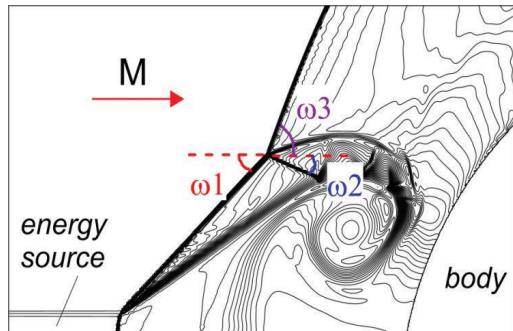


Fig.2. Generating vortices and considered angles (scheme)

Note that the lower vortex is generated independently of the formation of triple-shock configuration and it might present in the flow when the triple configuration doesn't form (for not large freestream Mach numbers). This vortex has been shown to be one of the reason of essential drag force reduction of AD body under the action of external energy deposition – “vortex drag reduction” (at  $M=1.89$ ) [6].

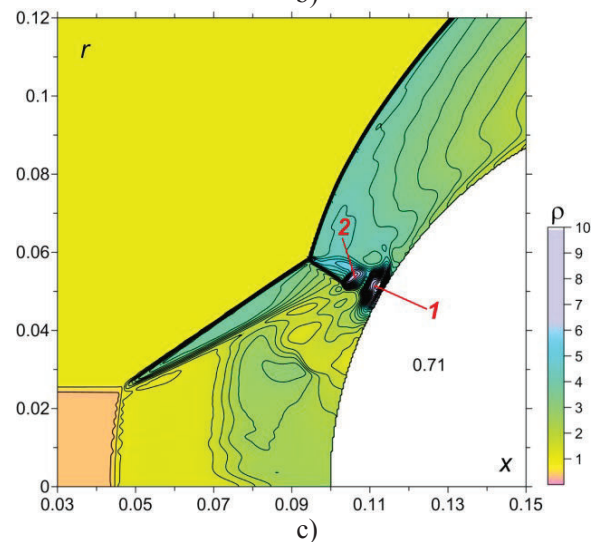
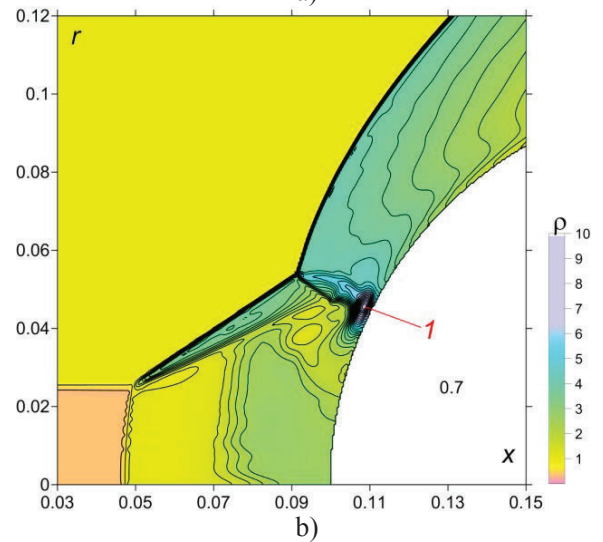
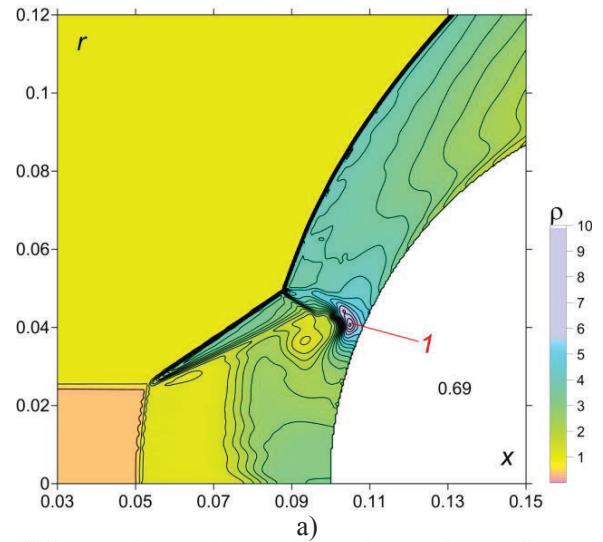


Fig.3. Dynamics of density in flow area close to triple-shock configuration,  $\gamma=1.4$ ,  $\alpha_p=0.33$  (enlarged): a) –  $t=0.69$ , b) –  $t=0.7$ , c) –  $t=0.71$

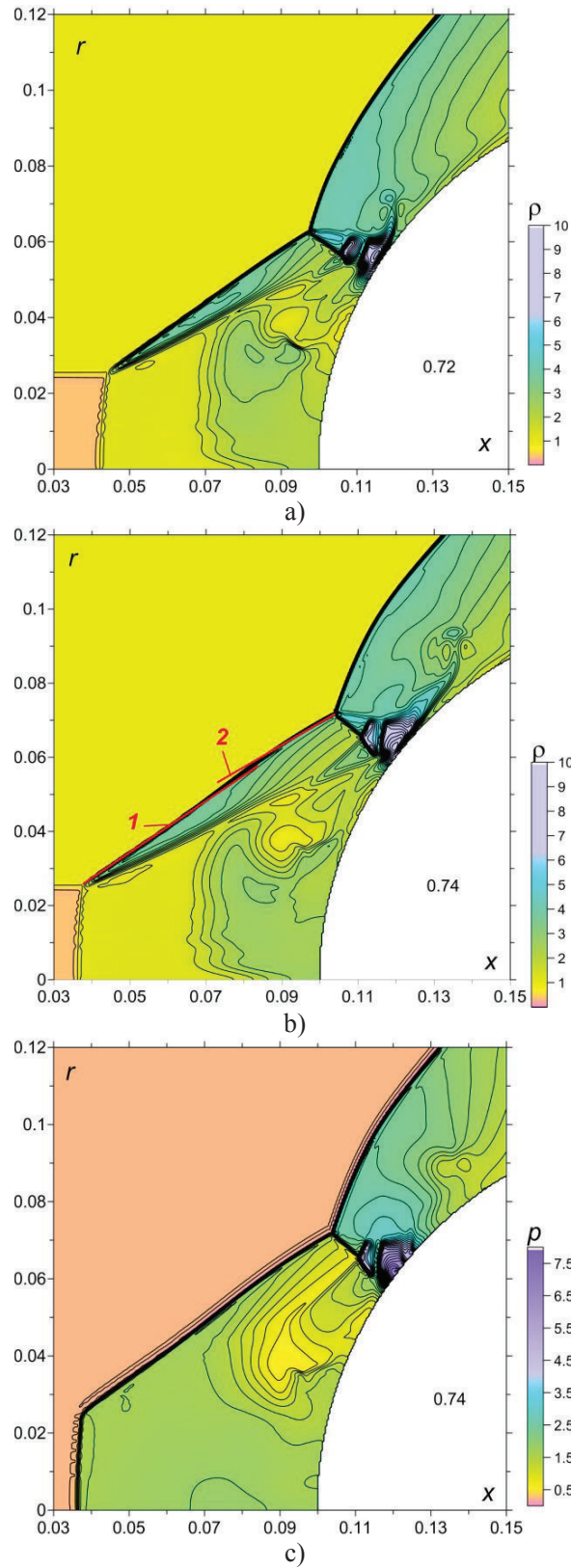


Fig.4. Density – a), b) and pressure – c) in flow area close to the triple-shock configuration,  $\gamma=1.4$ ,  $\alpha_p=0.33$  (enlarged): a) –  $t=0.72$ , b), c) –  $t=0.74$

### 3.1.1 Effect of triple-shock configuration on the surface pressure

Here the effect of triple-shock configurations on the flow area adjacent to the body’s boundary is studied which is local in space and in time. At the time stage of existence of triple configuration a flow region of small sizes with increased density and pressure is formed in front of the initial vortex (Figs.3a-3c, 1). Here the values of time are indicated. Another smaller area of increased pressure is formed near the reflected shock (Fig.3c, 2).

These two areas give rise to two shock segments which sequentially promote growth of the pressure on the body’s boundary (Fig.4). The second triple point is formed on the reflected wave (Fig.4a, 4b). It looks like double Mach reflection [15]. In addition,

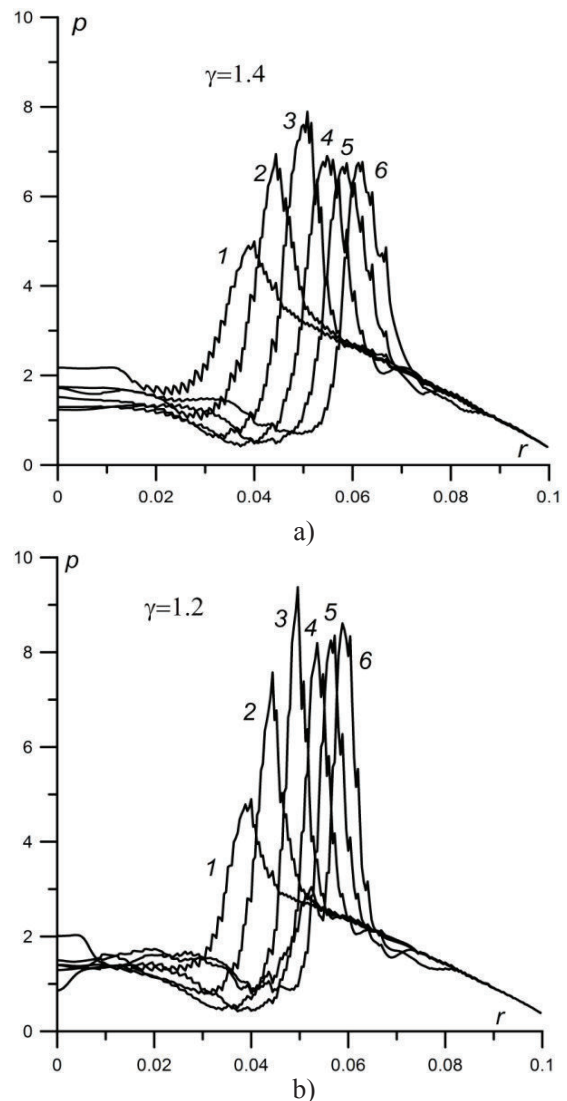


Fig.5. Dynamics of pressure on body’s boundary,  $\alpha_p=0.33$ , curve 1 –  $t=0.69$ , curve 2 –  $t=0.7$ , curve 3 –  $t=0.71$ , curve 4 –  $t=0.72$ , curve 5 –  $t=0.73$ , curve 6 –  $t=0.74$ : a) -  $\gamma=1.4$ , b) -  $\gamma=1.2$

one can see the curved stream arising near the surface which contributes some flexion of the bow shock. According field of pressure demonstrates significant growth of the boundary pressure (Fig.4c).

In Fig.5 the dynamics of the pressure profiles on the body's boundary are presented for  $\gamma=1.4$  (Fig.5a) in comparison with the case of  $\gamma=1.2$  (Fig.5b), the boundary pressure being stronger in this case.

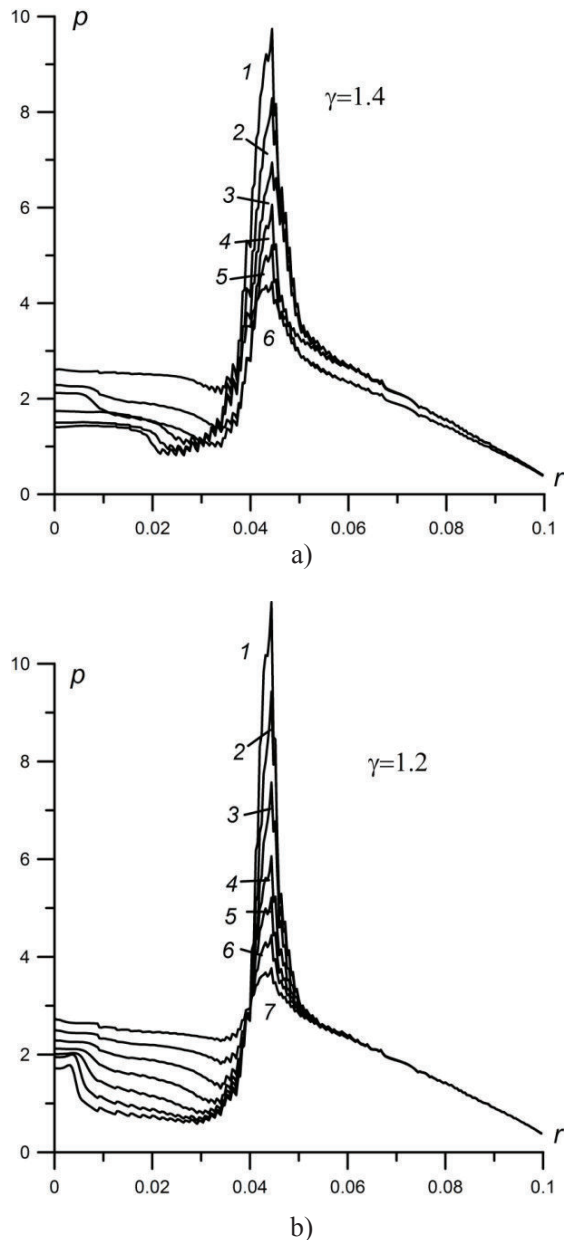


Fig.6. Pressure on body's boundary,  $t=0.7$ , curve 1 –  $\alpha_p=0.18$ , curve 2 –  $\alpha_p=0.25$ , curve 3 –  $\alpha_p=0.33$ , curve 4 –  $\alpha_p=0.41$ , curve 5 –  $\alpha_p=0.5$ , curve 6 –  $\alpha_p=0.59$ , curve 7 –  $\alpha_p=0.67$ : a) -  $\gamma=1.4$ , b) -  $\gamma=1.2$

In Fig.6 the pressure profiles on the body's boundary at  $t=0.7$  are presented for different values of rarefaction degree  $\alpha_p$  for  $\gamma=1.4$  (Fig.6a) and  $\gamma=1.2$  (Fig.6b). It is seen that the growth of boundary pressure is stronger for smaller  $\alpha_p$ .

The registered increase of the boundary pressure (in which a triple-shock configuration plays a prominent role) is reflected in the behavior of the front drag forces (Fig. 7) (here  $F_0$  is the value of front drag force in the absence of energy deposition).

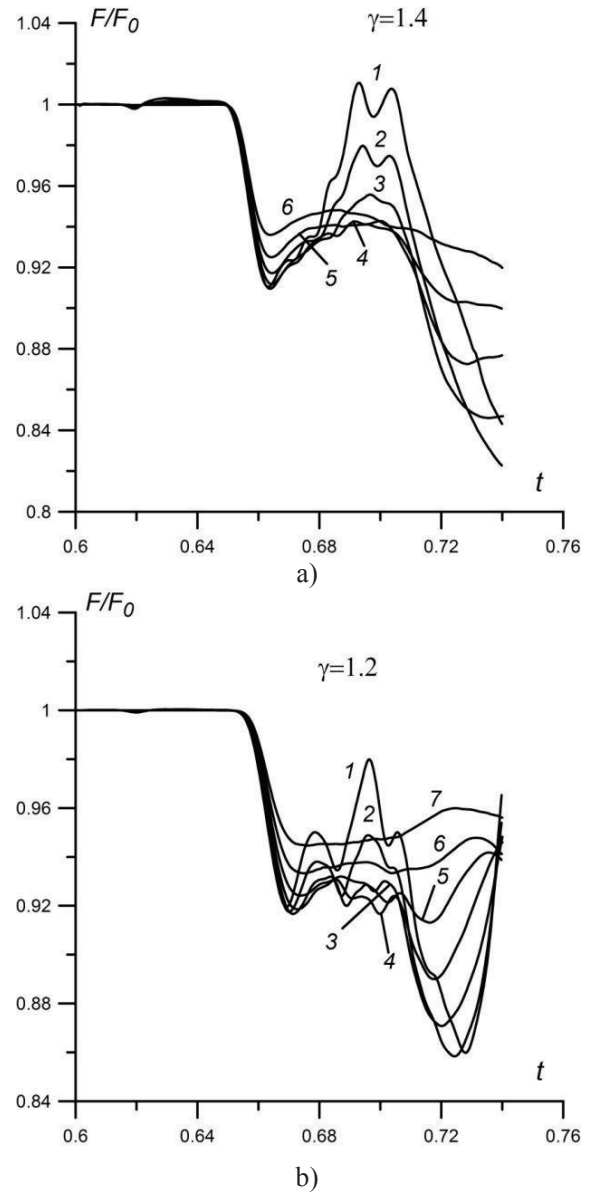


Fig.7. Dynamics of relative front drag force curve 1 –  $\alpha_p=0.18$ , curve 2 –  $\alpha_p=0.25$ , curve 3 –  $\alpha_p=0.33$ , curve 4 –  $\alpha_p=0.41$ , curve 5 –  $\alpha_p=0.5$ , curve 6 –  $\alpha_p=0.59$ , curve 7 –  $\alpha_p=0.67$ : a) -  $\gamma=1.4$ , b) -  $\gamma=1.2$

It is seen that the influence is stronger for smaller  $\alpha_p$ . Additionally, it can be concluded that the larger growth of the boundary pressure at  $\gamma=1.2$  is compensated by the processes of decreasing pressure in the vicinity of the axis of symmetry, in particular by the action of the initial vortex mentioned above.

The considered processes have a local increasing effect on the front surface drag force and take place on the background of the essential drag force decrease connected with the flow reconstruction under the action of the energy deposition (Fig.8).

It should be underlined that the results have been obtained within the inviscid approach. The comparison of viscous and inviscid approaches in the case of interaction of two shocks in the vicinity of a body surface is presented in [23]. It was shown that the viscous affects the process of shock waves interaction. Here we consider the influence of a vortex on the body surface together with the additional shocks, the studied shocks being in some distance from the body. So here the inviscid approach is justified in our view. Nevertheless the consideration of the influence of the viscous effects could be planned for the future work.

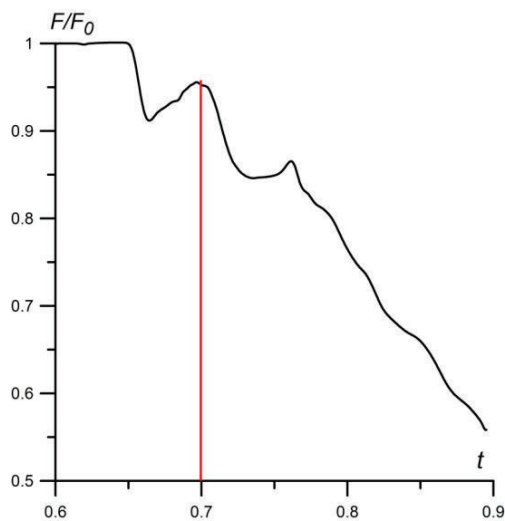


Fig.8. Dynamics of relative front drag force,  $\gamma=1.4$ ,  $\alpha_p=0.33$

### 3.1.2 Dynamics of angles in a triple-shock configuration

The considered stage of existence of triple configuration is self-similar [19]. Therefore a triple configuration develops without changing its shape and the angles in triple configuration do not change at this stage. Here the quantitative evaluation of these angles is performed. Additionally, the comparison of their values for cylinder bodies with

the data obtained earlier for plane AD body “hemi-cylinder-plate” (accordingly, with a plane shape of an energy source) [18, 19] has been done.

Considered angles are presented in Fig.2, absolute values of the angles are analyzed. Here in accordance with [10]  $\omega_1$  can be characterized as an incidence angle,  $\omega_2$  as a reflection angle and  $\omega_3$  as a Mach wave angle.

Flow images obtained here for the cylinder symmetry have been compared with the analogous results for the plane case [19] (Fig.9). In spite of the difference in the whole flow modes the angles between the shock fronts in the triple structures were precisely coincided under overlapping, as it can be expected.

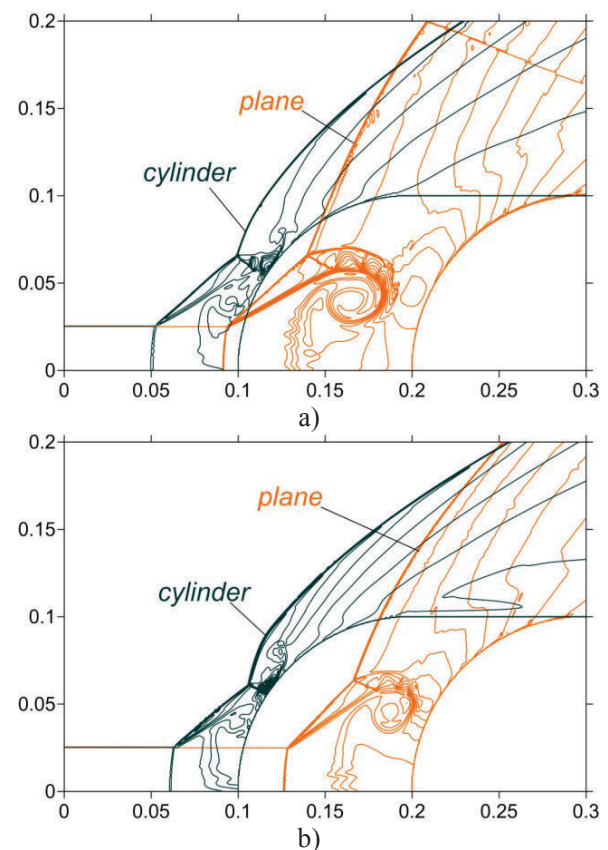


Fig.9. Comparison of density images for cylinder and plane cases,  $\alpha_p=0.41$ (image overlay): a) -  $\gamma=1.4$ , b) -  $\gamma=1.2$

Dependences of the considered angles on an incident angle  $\omega_1$  are presented in Fig.10. Here the comparison with the plane case is presented, too. It is seen that the dependences are almost the same. The reason is that the measurements were performed in the vicinity of a triple point where the shock fronts were close to straight lines. The difference of the dependences for the angles from the plane case is

rather caused by the error in the angles measuring (which is up to 3° for  $\omega_2$  and  $\omega_3$  and 1-2° for  $\omega_1$ ).

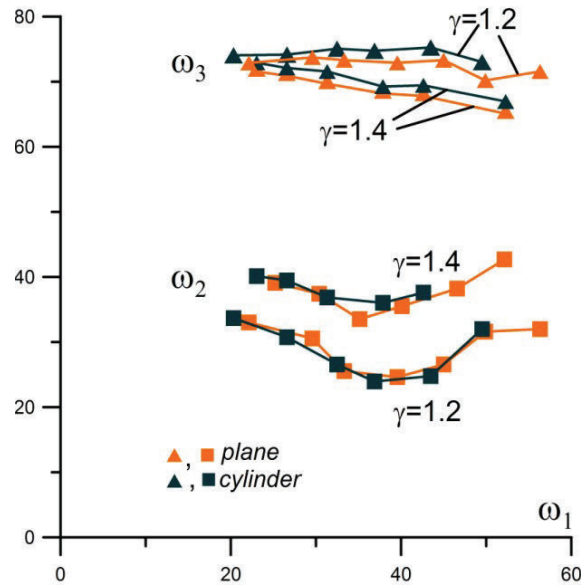


Fig.10. Angles vs  $\omega_1$ , cylinder and plane cases,  $\gamma=1.4$  and  $\gamma=1.2$

Change of  $\gamma$  effects essentially onto flow details at the neighborhood of a triple configuration. Comparison of the flow images and angle dependences on  $\alpha_p$  for  $\gamma=1.4$  and  $1.2$  for the cylinder case is presented in Figs.11, 12. It is seen that the

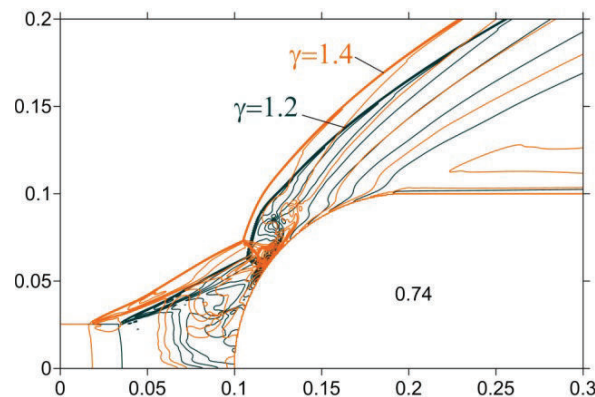


Fig.11. Comparison of density images for  $\gamma=1.4$  (orange) and  $\gamma=1.2$  (dark),  $\alpha_p=0.25$ , cylinder case (image overlay)

flow areas and the values of angles  $\omega_2$  and  $\omega_3$  differ essentially but the values of  $\omega_1$  are practically the same (Fig.11). These results confirm our previous results for the plane AD body “hemi-cylinder-plate” [18, 19].

It has been obtained that in the cylinder case the precursor front is not a quite straight line (unlike the

plane case). In Fig.4b the straight lines 1 and 2 are drawn for evaluation of the precursor front angle and for the incident angle  $\omega_1$  which underline the difference between them. It is connected with the complicated processes which take place at refraction of the bow shock at the boundary of a heated filament. As a consequence the incident angle differs from the angle of the precursor front (up to 5°), unlike the plane case.

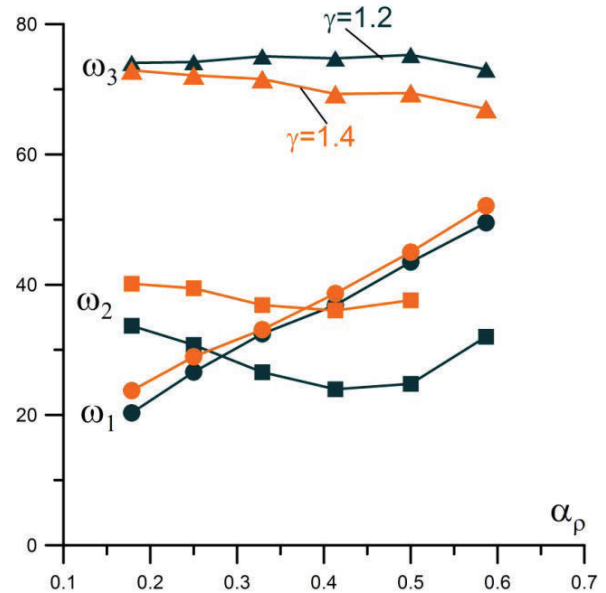


Fig.12. Comparison of angle dependences vs  $\alpha_p$  for  $\gamma=1.2$  (dark) and  $\gamma=1.4$  (orange), cylinder case

In the plane case the precursor front was shown to be a straight line for moderate values of  $\alpha_p$  [19]. For the small  $\alpha_p$  the precursor front is more complicated and it is characterized by new  $\lambda$ -waves forming on it.

The angle values for  $\alpha_p=0.59$  correspond to the situation without a triple structure formation. Here the precursor front is a strain line in the cylinder case, too.

### 3.2 Flow past a hemisphere-cone-cylinder

Formation of a triple-shock configuration near the surface of hemisphere-cone-cylinder for  $\gamma=1.4$  is presented in Fig.13. The angle between the cone generatrix and the axis of symmetry is 15.3°. Here the triple configurations are forming only for the small values of  $\alpha_p$ , up to  $\alpha_p=0.35$ . For greater  $\alpha_p$  a simple kink of bow shock is taken place (without generation of the reflected wave) [11] (Fig.14).

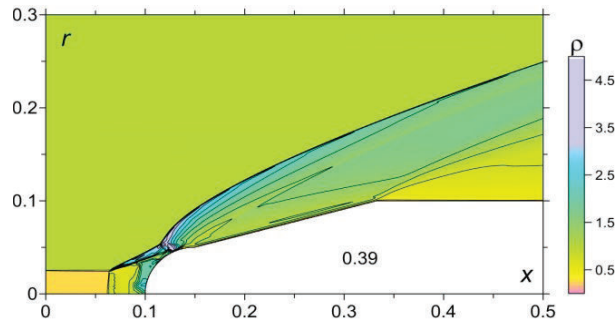


Fig.13. First stage of formation of a triple-shock configuration (full computation domain),  $\gamma=1.4$ ,  $\alpha_p=0.25$

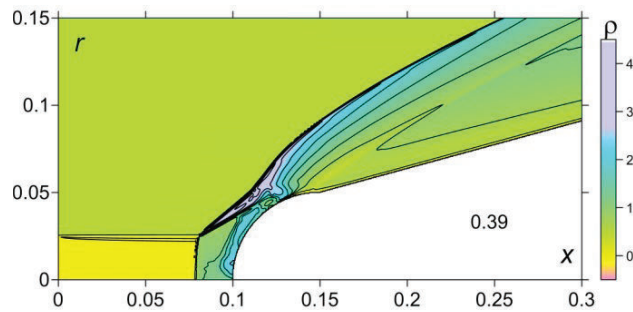


Fig.14. A kink in the bow shock without formation of a triple configuration,  $\gamma=1.4$ ,  $\alpha_p=0.41$  (enlarged)

The pressure change is obtained near the body's surface connected with the presence of triple-shock configuration (under the action of the mechanism described above for hemisphere-cylinder) (Fig.15).

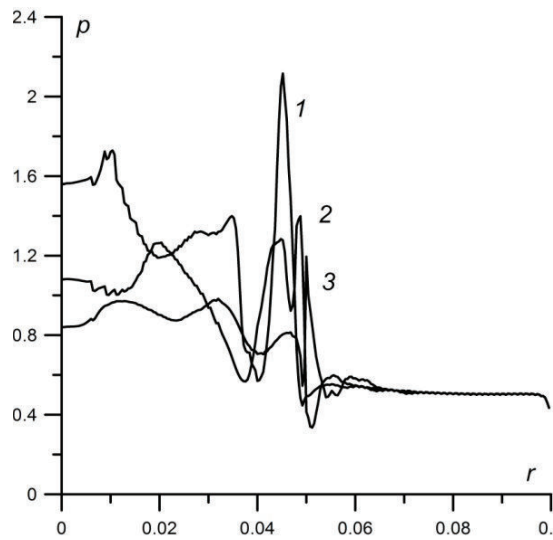
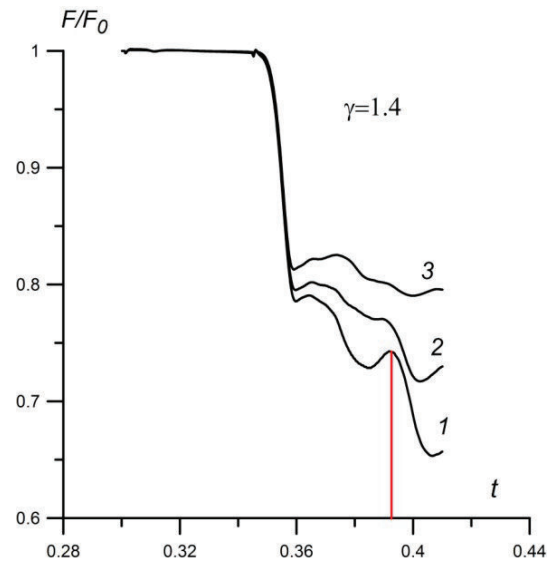


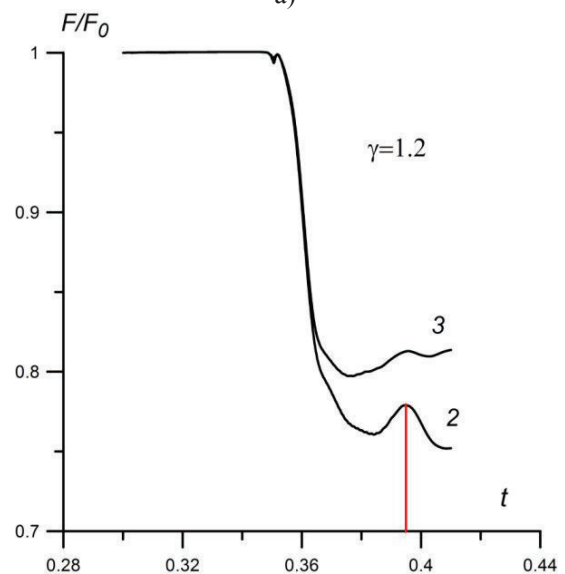
Fig.15. Dynamics of pressure on body's boundary,  $\gamma=1.4$ ,  $\alpha_p=0.25$ , curve 1 -  $t=0.39$ , curve 2 -  $t=0.4$ , curve 3 -  $t=0.41$

This pressure growth is significantly less than that obtained above for the hemisphere-cylinder.

The behavior of the relative front drag forces for  $\gamma=1.4$  and 1.2 is shown in Fig. 16. It is seen that the effect in front drag force is of local character in space and in time and it takes place only up to  $\alpha_p=0.33$ .



a)



b)

Fig.16. Dynamics of relative front drag force, curve 1 -  $\alpha_p=0.25$ , curve 2 -  $\alpha_p=0.33$ , curve 3 -  $\alpha_p=0.41$ : a) -  $\gamma=1.4$ , b) -  $\gamma=1.2$

Dependences of the considered angles on the incident angle  $\omega_1$  for the cases of existence of the triple-shock configurations are presented in Fig.17. Here the angles differ from those for hemisphere-cylinder which can be explained by the flow



inhomogeneity near the reflected shock and greater curvature of the bow shock wave.

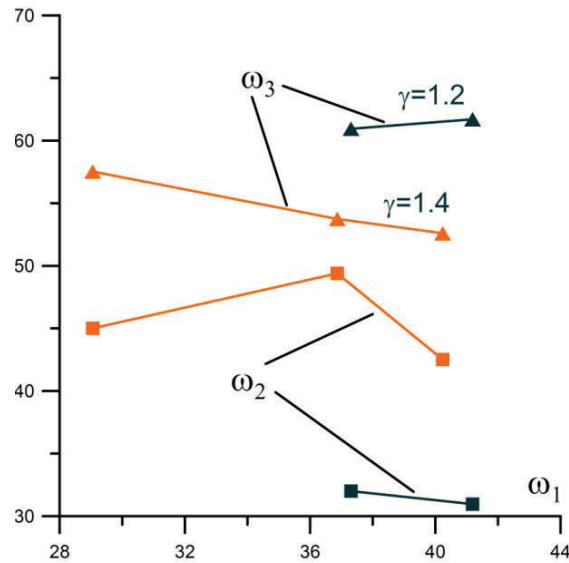


Fig.17. Comparison of angle dependences vs  $\omega_1$  for  $\gamma=1.2$  (dark) and  $\gamma=1.4$  (orange)

#### 4 Conclusion

Effect of a triple-shock configuration generated as the result of energy source – bow shock interaction on supersonic flow past hemisphere-cylinder and hemisphere-cone-cylinder has been studied. Comparison of flow details has been conducted for different gas media with  $\gamma=1.4$  (air) and  $\gamma=1.2$  for a set of the energy source characteristics at  $M=4$ .

Local growth of the boundary pressure in the vicinity of triple configuration has been established, this growth being stronger for  $\gamma=1.2$  and smaller gas rarefaction (or higher temperature) in the energy source. Mechanism of boundary pressure growth together with front drag force increase is shown to be connected with a vortex action as well as the action of the arising shock segments in the vicinity of triple-shock configuration which sequentially promote the pressure growth on the body's boundary. This effect has local space-time character.

Quantitative evaluation of the angles in triple structure for  $\gamma$  equal to 1.2 and 1.4 has been done. Dependences of the angles on the rarefaction degree in the energy source and on the incident shock angle are presented. Additionally, the triple-shock structures have been shown to be responsible for some curvature of the precursor front which forced an incident angle to differ slightly from the

precursor angle. Comparison of cylinder and plane flow modes in the vicinity of the triple-shock structures has been conducted.

Thus the possibility of control of a shape of triple-shock configuration together with the local pressure increase on the curved AD body surface via external energy deposition has been shown. In future it is planned to take into account the influence of gas viscosity on the established effects on the base of the full Navier-Stokes equations.

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