Numerical Simulation of the Localized Disturbance Development in a Supersonic Boundary Layer

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Abstract: - For the first time the development of localized disturbances of a small intensity in a supersonic boundary layer for Mach number M=2 was investigated numerically. Propagation speeds of localized disturbances are estimated. It was established that the speed of the forward front is more than the speed of the back front. Maximum mass flux disturbances were observed at $y_{\max}$ where $\rho U(y)/(\rho U)_\infty \approx 0.6$—$0.8$, that will be coordinated with experimental data. In the field of the forward front oscillations are arising in process of the wave packet movement down a stream, and its amplitude grows on time. For comparison of results of numerical modeling to the classical stability theory the wave package at $y_{\max}$ were expanded in a spectrum on frequencies and wave numbers. The maximum contribution to the total perturbation to belong to waves with inclination angles of a wave vector to the leading edge of the plate equal to forty degrees. The spatial amplification rates of oblique wave are in the good correspondence with data of the stability theory of locally nonparallel flows.

Key-Words: - **S**upersonic flow, boundary layer, hydrodynamic stability, laminar-turbulent transition, localized disturbances

1 Introduction

Due to the problem of laminar -turbulent transition of the boundary layer a specific place is held by a problem of origin of turbulent spots. Turbulent spots have been studied since Emmons discovered their existence in 1951 [1]. The review of early works on development of turbulent spots in boundary layers is given in [2]. Along with natural turbulent spots in works [3, 4] researches of the controlled turbulent spots generated by the localized initial disturbances have been begun. For the first time the development of linear wave packages in a subsonic boundary layer was studied in [5, 6]. The review of other works on experimental studies of development of the localized disturbances and a formation of turbulent spots can be found in [7]. In it on the basis of the photos taken from works [8-10] the attention is paid that longitudinal structures are watched in a turbulent spot trace. In the book [11] the extensive bibliography on the experimental study of longitudinal structures and their interactions with turbulent spots is provided in subsonic interfaces. In it much attention is paid to experimental investigations of the controlled localized disturbances in a subsonic boundary layer.

Besides in [11] the theory of the longitudinal structures formation on the basis of optimum indignations [12] is described.

Other approaches to generation of striate structures is based on researches of a continuous spectrum of a problem instability of a subsonic interface [13-15] and the resonant theory of interaction of vortex disturbances of an external stream with the boundary layer [16].

Along with researches of turbulent spots in a subsonic boundary layer in [17] their studying in supersonic streams has been begun. Rather full review of such works is available in [18]. Papers [19, 20] are devoted to experimental studying of localized indignations in a supersonic boundary layer.

Direct numerical simulation (DNS) of spot growth in supersonic boundary layers at M = 2, 4 and 6 were performed in [21], and for M = 6 in [22]. In these papers explored a large amplitude perturbations, i.e., in the face of strong nonlinearity. In these papers the disturbances with big amplitudes were studied, that is in the conditions of the strong nonlinearity.

In this paper direct numerical modeling of development of the small localized disturbances in the boundary layer of a is carried out at number M = 2, at conditions of experiments [19, 20].

2 Problem statement, basic equations, numerical method

The gas flow is described by the known Navier - Stokes, continuity, energy and state equations [22]:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho v_i)}{\partial x_i}=0$$

$$\frac{\partial\rho v_i}{\partial t}+\frac{\partial(\rho v_j v_i)}{\partial x_j}=\frac{\partial\tau_{ij}}{\partial x_j}-\frac{\partial p}{\partial x_i}$$

$$\frac{\partial(\rho E)}{\partial t}+\frac{\partial(\rho v_j E)}{\partial x_j}=\frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right)+\frac{\partial}{\partial x_j}(\tau_{ij}v_i)$$

$$\tau_{ij}=\mu\left[\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)-\frac{2}{3}\frac{\partial v_k}{\partial x_k}\right]$$

Here **v** –velocity with components $(u=v_1, v=v_2, w=v_3)$ in $x=x_1, y=x_2, z=x_3$ – directions, p,ρ,T –pressure, density and temperature, $\mu=\mu_r\left(\frac{T}{T_r}\right)^{3/2}\frac{T_r+T_s}{T+T_s}$ – dynamic viscosity.

In this paper $c_p = 1006.43$ j/kg-k and thermal conductivity was taken in according to the kinetic theory, the temperature of the main flow in the wind tunnel $T_r = 164^\circ K$, (c_p –specific heat at a constant pressure), $T_s = 110^\circ K$, velocity in wind tunnel $U_\infty \approx 500$m/s, pressure was equal to 3600 Pa and it corresponded to unit Ruynolds number $Re_1=(U\rho/\mu)_\infty=3.6\cdot10^6$/m.

The calculated domain is presented schematically in Fig.1. A`D`BC is the plate with the disturbance source. Length of a plate equaled 140 mm, before a plate the area of 5 mm was set. On a plate conditions of an adiabatic wall were set. Height of the calculated domain corresponded to about 20 mm and on the upper bound of EFGH nonreflecting boundary conditions were laid down. Width of the domain was set equal 40 mm and it was enough that disturbances were equal to zero on lateral borders. Conditions of the running stream were laid down on border AEFD. Exit conditions were set on the surface BCGH.

In this calculated domain the structured grid was constructed with condensation on y-coordinate closer to a plate. The quantity of cells on x was equal 750, on z —200 and on y —400. For the solution of this task the program complex ANSYS was used.

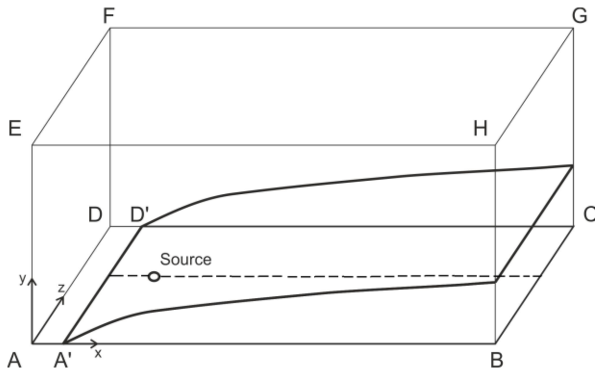

Fig.1. Computational domain.

The problem was solved in two stages. At first stationary problem was solved. At the second phase the task was solved in the presence of a localized disturbance, which was created by the air injection through a hole with a diameter of 1 mm, and which is located on the center of the plate at a distance of 30 mm from the front edge. During the first 25μs the normal velocity over a hole was accepted equal 25m/s then it was accepted equal to zero on all surface of a plate. Duration of calculation equaled 1000 microseconds, the time step was equal 10^{-2} μs.

3 Discussion of the results

As a result of calculations values of disturbances depending on the time and coordinates including mass flow perturbations, $m' = m'(t,x,y,z)$, have been received.

Both numerically and experimentally [20] (Fig.2) it was obtained that maximum disturbances were observed at y_{max}, where a dimensionless mass flux of a main flow $\rho U(y)/(\rho U)_\infty \approx 0.6$—$0.8$. Below calculations results of mass flux disturbances only at y_{max} will be shown.

Dependences of mass flux disturbances on time at z=0 and two values of longitudinal coordinate are shown in Fig.3: a—*x=50mm* ($Re_x=xRe_1=0.195\cdot10^6$), b—*x=80mm* ($Re_x=0.31\cdot10^6$). It is visible that the total intensity of disturbances decreases with a growth of the longitudinal coordinate. At the same time, in the field of the forward front oscillations are arising in process of the wave packet movement down a stream, and its amplitude grows on time. It has been established that the speed of the forward front is more than the speed of the back front, as well as in case of turbulent spots.

Lines of the constant mass-flow pulsations of the wave packet in the plane (z,t) is shown in Fig.4 at y_{max}: a—DNS ($Re_x=0.39\cdot10^6$), b— experimental data

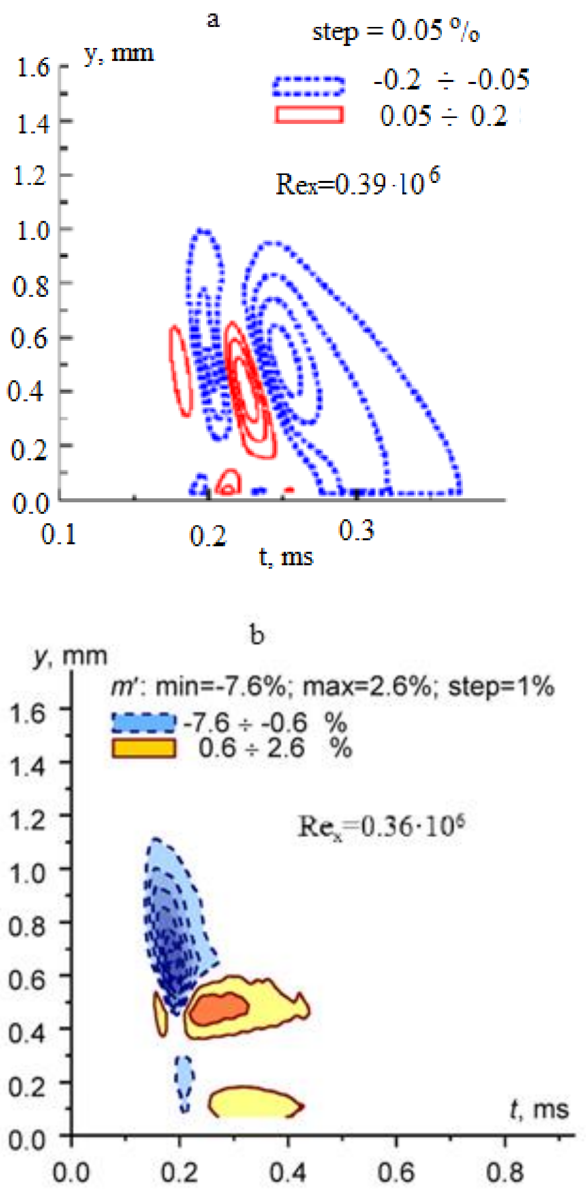

Fig.2 Lines of the constant mass-flow pulsations of the wave packet in the plane at z=0:
a—DNS, b — experimental data

[20] (Re_x=0.36·10^6). From Fig.4 it is possible to notice a qualitative similarity of numerical and experimental data. The quantitative distinction is explained easily by difference in levels of disturbances, Reynolds numbers values and initial wave packets.

Mass-flow pulsations at the fixed y_{max} and x were expanded in a spectrum on frequencies and wave numbers according to transformation:

$$A_{f\beta}(x) = A_{f\beta}(x, y_{\max})\exp(i\Phi_{f\beta}(x)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m'(x, y_{\max}, t, z) e^{(-i(\beta z - 2\pi f t))} dtdz.$$

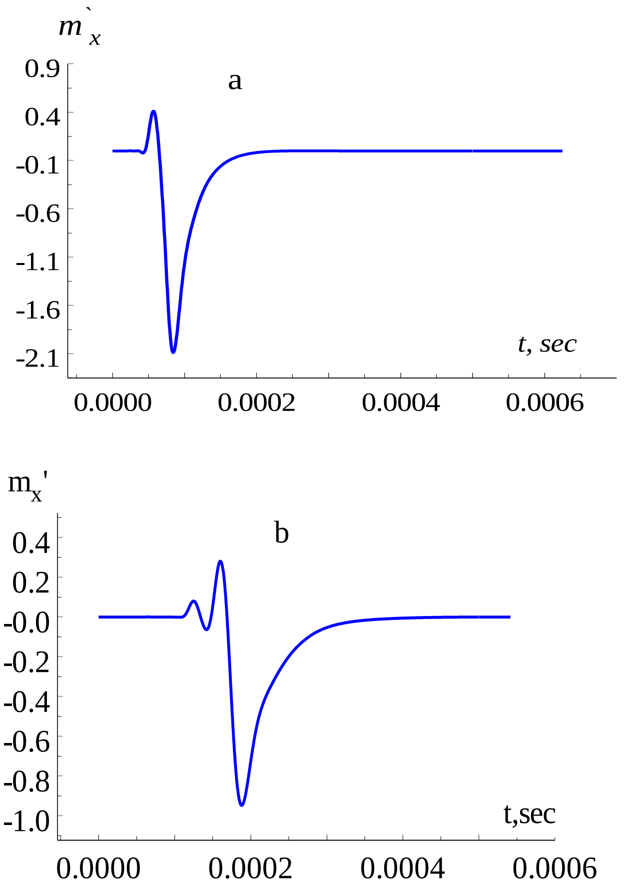

Fig.3 Dependences of mass flux disturbances on time at z=0: *x=50mm* (a), *80 mm* (b)

From amplitude spectra (Fig.5) it is possible to notice that the maximum value of the amplitude corresponds to wave number β=bta ≈ 1rad/mm. Besides, it has been obtained that at accepted parameters of present calculations the maximum value is reached at *x=100mm* for the frequency *f=22kHz* (Fig.5b). For all frequencies *f<22kHz* (Fig.5a, *f=16kHz*) disturbances with β≈1rad/mm grow with increase *x*. In the area *f>22 kHz* (Fig.5c, *f=24kHz*) the corresponding disturbances have maxima in an interval *x<100 mm*.

On the dependence of the phase on longitudinal coordinates, $\Phi_{f\beta}(x)$ it is possible to define wave number in the x-direction and the angle between the direction of the main stream and the vector $\mathbf{k} = \alpha\mathbf{i}_x + \beta\mathbf{i}_z$ where $\mathbf{i}_x$, $\mathbf{i}_z$ — unit vectors of the corresponding coordinates. The corresponding recalculation of results of Fig.5a and Fig.5b it is shown in Fig.6a and Fig.6b. By calculations it was established that the largest disturbance amplitudes correspond to angles χ=kappa>45° that, in general, is coordinated with data of the stability theory of parallel flows [24].

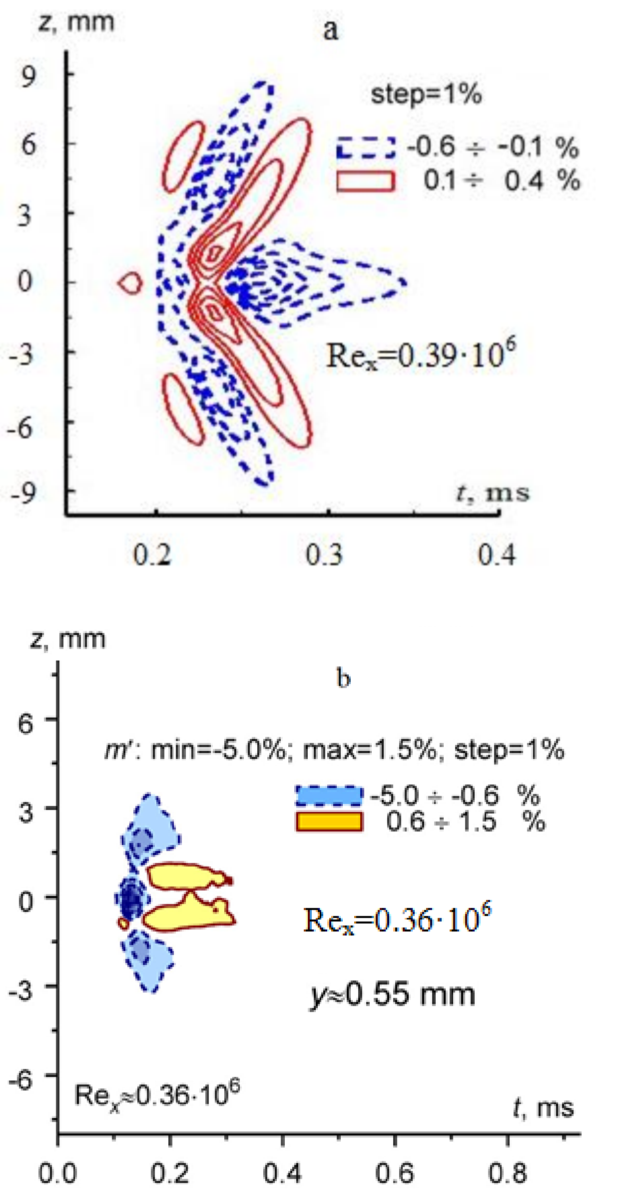

Fig.4 lines of the constant mass-flow pulsations in the plane (*z*,*t*), a–DNS, b – experiment [20].

Besides, it has been received that with increase in frequency the shown on Fig. 6 maxima are displaced towards to large angles χ, for example, for *f=16kHz* (a) it is at χ=65° and for *f=22kHz* (b) it is at kappa=80°. In last case the wave length in the *x*-direction is approximately in 6 times more of wavelength in the z-direction, and the front of a wave almost parallel to the direction of a main flow.

Dependence of amplitude on the longitudinal coordinate, Fig. 6, at the fixed value β allows to calculate the spatial increment $-\alpha_i = \dfrac{1}{2}\dfrac{d\ln(A_{\beta f})}{d\,\mathrm{Re}}$ of the corresponding wave, where $\mathrm{Re} = \sqrt{\mathrm{Re}_x}$.

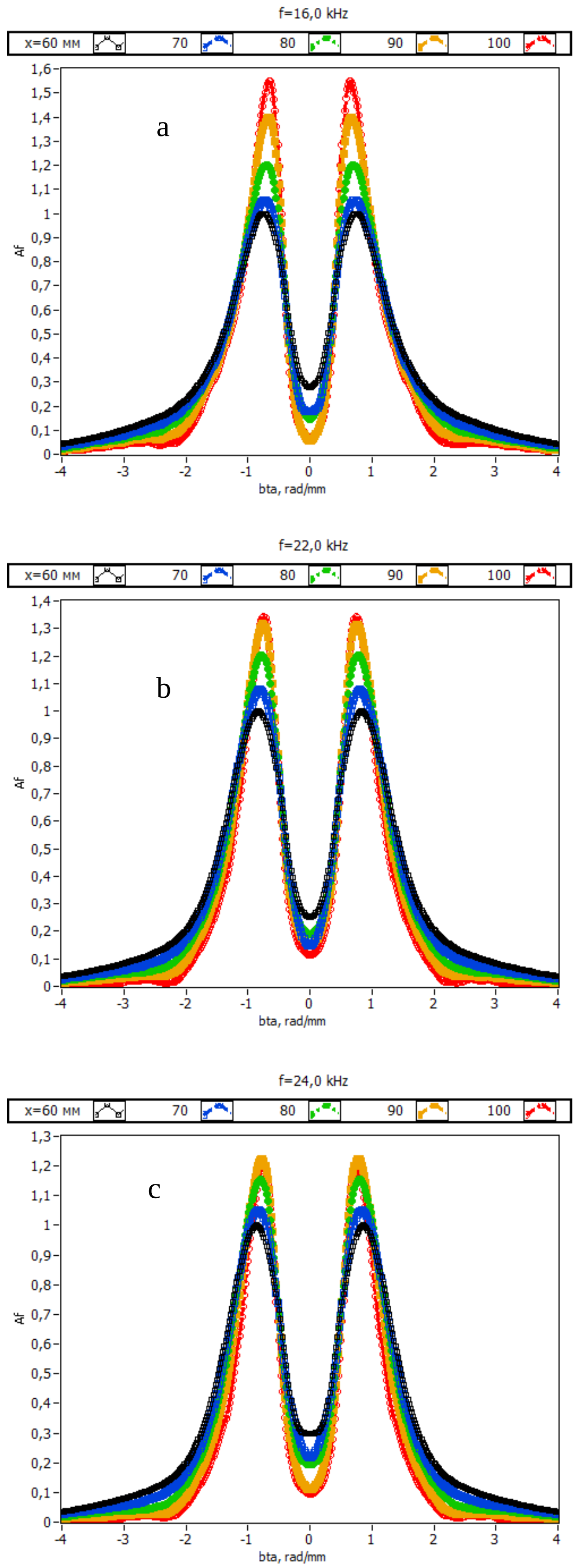

Fig. 5 Amplitude β-spectra for *f=16*, *22* and *24* кГц

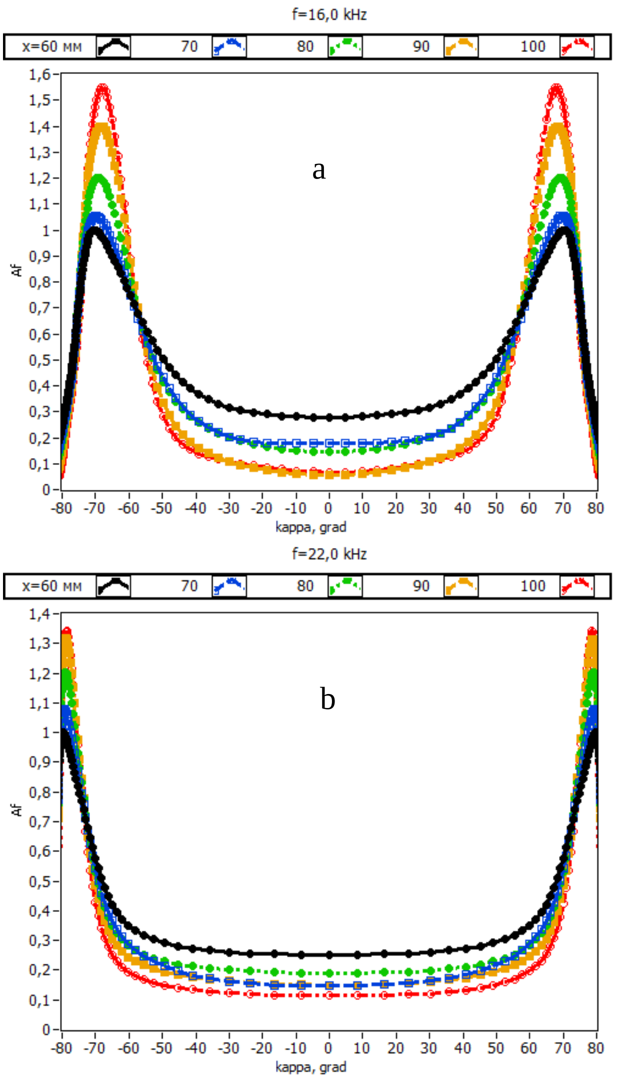

Fig. 6. Dependences of disturbances amplitudes on the angle kappa.

For example, it is possible to show that at $Re=(Re_x)^{1/2}=600$, $\chi=60°$ and $f=16kHz$ $-\alpha_i\approx 2.0\cdot 10^{-3}$. The obtained amplification rate is slightly higher than the predicted stability theory of parallel flows, $-\alpha_i\approx 1.5\cdot 10^{-3}$, and completely coincides with $-\alpha_i=1.96\cdot 10^{-3}$ of the local nonparallel flow [24], when the normal velocity and derivatives on x of the velocity and temperature of a main flow are taken account.

The beginning of three-dimensional disturbances amplification, $\chi\approx 65°$, (Fig.6a) is observed at Re≈480 (x≈60mm) that also is coordinated with Re≈450 of the local nonparallel flow. The two-dimensional wave behavior of the localized packet does not correspond to the results of the classical stability theory. Almost in all explored area, 480(x=60mm) < Re <630(x=100mm), they fade, while in according to the stability theory of the local nonparallel flow they have to amplify ($0<-\alpha_i<0.55\cdot 10^{-3}$), though their maximum amplification rate is approximately four times less in comparison with a case of the oblique wave, $\chi=60°$, at Re=600. The reason of such discrepancy can consist in nonlinear influence of waves with big amplitudes ($\chi=60°$) on waves of small amplitudes ($\chi\approx 0$).

4 Conclusion

Direct numerical modeling of linear development of a wave package at supersonic flow (Mach number M=2) of a plate is for the first time carried out.

As in [20] it is established that the location of the maximum disturbances of a mass flux at distance of y_{max}, where a dimensionless mass flux of a main flow $\rho U(y)/(\rho U)_\infty\approx 0.6$—$0.8$.

In process of the wave packet movement to downstream the total amplitude of a disturbance decreases, and oscillations are observed in it.

The maximum contribution to the total perturbation to belong to waves with wave numbers $\beta\approx$1rad/mm, that corresponded to an inclination angle of a wave vector to the leading edge of the plate kappa>40°.

The maximum location in dependences of the wave amplitude on the wave number β is displaced to the area of greater values χ with increase in frequency.

The received spatial amplification rates of oblique wave are in the good correspondence with data of the stability theory of locally nonparallel flows.

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