MHD Visco-Elastic Fluid Flow and Heat Transfer Around a Circular Cylinder

RITA CHOUDHURY [1] and BIBHASH DEKA [2]
[1,2]Department of Mathematics, Gauhati University, Guwahati-781014, Assam, INDIA.
[1]E-mail: rchoudhury66@yahoo.in
[2]E-mail: bibhashdeka66@gmail.com

Abstract:-An investigation is made on the temperature distribution within the thermal boundary layer region due to the flow of a second-grade fluid around a heated circular cylinder, maintained at a constant temperature higher than that of the fluid at infinity in presence of magnetic field applied transversely to the direction of the main flow. The problem has been solved by the application of steepest descent method used by Meksyn. The impact of various pertinent parameters on flow characteristics have been discussed through graphical illustrations. Newtonian results are found to emerge as limiting cases of the present analysis.

Key-words:-Visco-elastic, Boundary layer, MHD, Circular Cylinder, Heat transfer.

1 Introduction

The boundary layer concept has tremendous achievement in the interdisciplinary activities concerning engineering and technology developments. The mechanism of thermal boundary layer flow of magnetohydrodynamic visco-elastic fluids are used in different manufacturing processes such as extrusion of plastic sheets, fabrication of adhesive tapes, coating layers into rigid surfaces etc.

The influence of magnetic field on an electrically conducting viscous incompressible fluid past a cylinder in presence of heat transfer has practical significance in many engineering applications viz. solar power collectors, compact heat exchanges and nuclear reactors. Dennis and Chang [1] have presented the numerical solutions for steady flow past a circular cylinder at Reynolds numbers up to 100. An experimental investigation of the steady separated flow past a circular cylinder has been studied by Grove *et al.* [2]. Fornberg [3] has analysed the numerical study of steady viscous flow past a circular cylinder. Steady two-dimensional viscous flow of an incompressible fluid past a Circular cylinder has been investigated by Takami and Keller [4]. Also, the authors viz. Thoman and Szewczyk [5], Kawaguti [6], Hamielec and Raal [7], Gschwendter [8], Sen *et al.* [9] etc. have remarkable contribution in this field.

The study of visco-elastic fluid flow has been the objective of immense research due to its applications in industries of chemical processes such as food processing and polymer production etc. The differential models of visco-elastic fluid encountering the effects of shear thinning/thickening and normal stress differences are known as second-grade fluids which follow from generalized Rivlin-Ericksen's fluid. The viscous feature of second-grade fluid is caused by the transport phenomenon of the molecules of fluids whereas the elastic characteristic is assignable to the chemical structure and configuration of polymer molecules. Many scientists have contribution on this field but a few of them are mentioned here. The second-order thermal boundary layer equation for the flow of a second-order fluid past heated body has been investigated by Srivastava [10] using the order of magnitude approach. The temperature distribution for the flow near a two-dimensional stagnation point occurring on a flat plate maintained at a temperature higher than that of the fluid at infinity has been analysed. The thermal boundary layer on a steadily rotating sphere in infinitely extending second-order fluid has been discussed by Bhatnagar and Palekar [11]. Srivastava and Maiti [12] have analysed the flow of a second-order fluid past a cylinder by expanding the flow functions in series and obtaining the first four terms by Karman-Pohlhausen method. The heat transfer in a second-grade fluid for flow around a circular cylinder by Karman-Pohlhausen method has been studied by Srivastava and Saroa [13].The heat transfer in the boundary layer region of a second-order fluid past a plate by presenting a uniform constant suction and temperature at the plate has been studied by Agarwal and Bhatia [14]. Srivastava and Saroa [15] have investigated the heat transfer in a second-order fluid for flow around a circular cylinder. Numerical simulation of visco-elastic flow past a cylinder has been analysed by Hu and Joseph [16]. Chhabra *et al*. [17] has presented the steady non–Newtonian flow past a ci rcular cylinder: a numerical study.

In this paper, we have studied the problem of flow and heat transfer of a second-order fluid around a circular cylinder in presence of magnetic field by series expansion used by Meksyn [18] and found that the point of separation for the Newtonian case comes out to be 109.09° whereas the exact value is 109.6° [Schlichting,[19]]. Srivastava and Saroa [14] have obtained the separation point at 110.8° by taking the boundary layer thickness to be variable along the cylinder. By using Karman-Pohlhausen method, Srivastava and Maiti obtained the corresponding point of separation at 116.5°.

The constitutive equation for the second-order incompressible fluid is taken in the form

$$\sigma = -PI + \mu_{(1)}A_{(1)} + \mu_{(2)}A_{(2)} + \mu_{(3)}A_{(1)}^2 \quad (1)$$

where σ is the stress tensor, $A_{(n)}$ $(n = 1,2)$ are the kinematic Rivlin-Ericksen tensors, $\mu_{(n)}$ $(n = 1,2,3)$ are the material co-efficients describing the viscosity, elasticity and cross viscosity respectively. The case $\mu_{(2)} = \mu_{(3)} = 0$ corresponds to an incompressible Newtonian fluid. On thermodynamic considerations $\mu_{(2)}$ is found to be negative whereas $\mu_{(1)}$ and $\mu_{(3)}$ are positive. Coleman and N oll [20] derived the equation from the simple fluid by assuming that stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past. Markovitz and Brown determined experimentally the material constants for solutions of poly-isobutylene in cetane of various concentrations.

2 Basic Equations

Consider an incompressible MHD second-grade fluid moving with a uniform velocity U_∞ at infinity in presence of a fixed circular cylinder of radius a maintained at a constant temperature T_w. Let the temperature of the fluid at infinity be T_∞ where $T_w > T_\infty$. Let (r, θ, z) be the cylindrical polar co-ordinates with z-axis coincides with the axis of the cylinder. The fluid flow is two-dimensional in r and θ directions.

The two-dimensional velocity boundary layer equations are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_1\frac{\partial^2 u}{\partial y^2} + \nu_2\left\{\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right\} + \frac{\sigma {B_y}^2}{\rho}(U-u) + U\frac{\partial U}{\partial x} \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

where $y = r - a, x = a\theta$ and u, v are velocity components along x and y-axis respectively within the viscous boundary layer region, U is the main stream velocity, ρ is the fluid density, B_y is the strength of the magnetic field and $\nu_i = \frac{\mu_i}{\rho}, (i = 1,2)$. Also, equations (2) and (3) are independent of the curvature of the wall and thus are applicable to the case of a flat wall.

Srivastava [1967] has derived the thermal boundary layer equation for the second-grade incompressible fluid against a heated wall as:

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \mu_1\left(\frac{\partial u}{\partial y}\right)^2 + \mu_2\left(v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right) + k\frac{\partial^2 T}{\partial y^2} \quad (4)$$

where C_p is the specific heat, k is the thermal conductivity and T is the temperature. This equation is valid within the boundary layer over both a curved wall and a flat wall when x-axis is taken in the tangential direction and y-axis along the normal to the surface.

The relevant boundary conditions are,

$$u = 0, v = 0, T = T_w \text{ at } y = 0 \quad (5)$$

$$u \to U, T \to T_\infty \text{ in } y \to \infty \quad (6)$$

The velocity distribution U outside the velocity boundary layer region created by the cylinder is given by,

$$U(\theta) = 2U_\infty \sin\theta$$

and the stream-function ψ is given by,

$$\psi = \sqrt{\frac{\nu_1 a}{2U_\infty}}\left\{2U_\infty\theta f_1(\eta) - \frac{8}{3!}U_\infty\theta^3 f_3(\eta) + \frac{12}{5!}U_\infty\theta^5 f_5(\eta) - \frac{16}{7!}U_\infty\theta^7 f_7(\eta) + \cdots\right\} \quad (7)$$

where $\eta = y\left(\frac{2U_\infty}{\nu_1 a}\right)^{\frac{1}{2}}$

Now, the velocity components u and v within the velocity boundary layer region are given by,

$$u = \frac{\partial\psi}{\partial y} = 2U_\infty\theta\left\{f_1'(\eta) - \frac{2}{3}\theta^2 f_3'(\eta) + \frac{1}{20}\theta^4 f_5'(\eta) - \frac{1}{630}\theta^6 f_7'(\eta) + \cdots\right\} \quad (8)$$

and $$v = -\frac{\partial\psi}{\partial x} = -\sqrt{\frac{2\nu_1 U_\infty}{a}}\left\{f_1(\eta) - 2\theta^2 f_3(\eta) + \frac{1}{4}\theta^4 f_5(\eta) - \frac{1}{90}\theta^6 f_7(\eta) + \cdots\right\} \quad (9)$$

Here prime denotes differentiations w.r.t η.
The temperature T within the thermal boundary layer region should be taken in the form

$$T^* = \frac{T - T_\infty}{T_w - T_\infty} = 4E\{T_1(\eta) - \theta^2 T_3(\eta) + \theta^4 T_5(\eta) - \theta^6 T_7(\eta) + \cdots\} \quad (10)$$

where E is the Eckert number and it is given by $E = \frac{U_\infty^2}{C_p(T_w - T_\infty)}$. For simplicity, we shall confine ourselves to terms up to $f_7(\eta)$ and $T_7(\eta)$ only.

Now, the boundary conditions of $f_i(\eta)$ and $T_i(\eta), (i = 1,3,5,7)$ are,

$$\text{At} \quad \eta = 0;\ f_i(\eta) = 0,\ f_i'(\eta) = 0,\ T_1 = \frac{1}{4E},\ T_3 = T_5 = T_7 = 0 \quad (11)$$

$$\text{In} \quad \eta \to \infty;\ f_1' \to 1,\ f_3' \to \frac{1}{4},\ f_5' \to \frac{1}{6},\ f_7' \to \frac{1}{8},\ T_i \to 0 \quad (12)$$

Now, substituting the values of u, v and T from equations (8) to (10) into (2) and (4) and equating the co-efficient of like powers of θ with the neglect of higher order terms on both sides of the equations we have,

$$f_1''' + f_1 f_1'' = -1 + M(f_1' - 1) + f_1'^2 + \alpha_1(f_1''^2 + f_1 f_1'''' - 2f_1' f_1''') \quad (13)$$

$$f_3''' + f_1 f_3'' = -1 + M\left(f_3' - \frac{1}{4}\right) + 4f_1' f_3' - 3f_3 f_1'' + \alpha_1[-4(f_1' f_3''' + f_3' f_1''') + 4f_1'' f_3'' + (f_1 f_3'''' + 3f_3 f_1'''')] \quad (14)$$

$$f_5''' + f_1 f_5'' = -\frac{8}{3} + M\left(f_5' - \frac{1}{6}\right) + 6f_1' f_5' - 5f_5 f_1'' + \frac{80}{3}(f_3'^2 - f_3 f_3'') + \alpha_1\Big[-6(f_1' f_5''' + f_5' f_1''') + 6f_1'' f_5'' + (f_1 f_5'''' + 5f_5 f_1'''') + \frac{80}{3}(-2f_3' f_3''' + f_3 f_3'''' + f_3''^2)\Big] \quad (15)$$

$$f_7''' + f_1 f_7'' = -8 + M\left(f_7' - \frac{1}{8}\right) + 8f_1' f_7' - 7f_7 f_1'' + 168f_3' f_5' - 63f_3 f_5'' - 105f_5 f_3'' + \alpha_1[-8(f_1' f_7''' + f_7' f_1''') + (f_1 f_7'''' + 7f_7 f_1'''') - 168(f_3' f_5''' + f_3''' f_5') + (105f_5 f_3'''' + 63f_3 f_5'''') + 168f_3'' f_5'' + 8f_1'' f_7''] \quad (16)$$

$$T_1'' + Prf_1 T_1' = 0 \quad (17)$$

$$T_3'' + Prf_1 T_3' = \Pr[2T_3 f_1' - 2f_3 T_1' + f_1''^2 + \alpha_1\{f_1''(f_1' f_1'' - f_1 f_1''')\}] \quad (18)$$

$$T_5'' + Prf_1 T_5' = \Pr\Big[4T_5 f_1' - 2f_3 T_3' + \frac{4}{3}T_3 f_3' - \frac{1}{4}f_5 T_1' + \frac{4}{3}f_1'' f_3'' + \frac{2}{3}\alpha_1\{f_1''(f_1'' f_3' + 4f_1' f_3'') - f_1''(f_1 f_3''' + 3f_3 f_1''') - f_1 f_1''' f_3''\}\Big] \quad (19)$$

$$T_7'' + Prf_1 T_7' = \Pr\Big[6T_7 f_1' - 2f_3 T_5' + \frac{8}{3}T_5 f_3' - \frac{1}{4}f_5 T_3' + \frac{1}{10}T_3 f_5' - \frac{1}{90}f_7 T_1' + \frac{1}{10}f_1'' f_5'' + \frac{4}{9}f_3''^2 + \alpha_1\Big\{\frac{1}{20}f_1''(-f_1 f_5''' + 6f_1' f_5'' + f_1'' f_5' - 5f_5 f_1''') - \frac{4}{9}f_3''(f_1 f_3''' + 3f_3 f_1''' - 4f_1'' f_3') + \frac{1}{60}(80f_1' f_3''^2 - 80f_1'' f_3''' f_3 - 3f_5'' f_1''' f_1)\Big\}\Big] \quad (20)$$

where $\alpha_1 = \frac{2U_\infty \nu_2}{a\nu_1}$ is the visco-elastic parameter, $Pr = \frac{\rho\nu_1 C_p}{k}$ is the Prandtl number and $M = \frac{\sigma B^2 a}{2U_\infty \rho}$ is the magnetic parameter.

3 Solution of the Problem

The equations (13) to (20) subject to the boundary conditions (11) and (12) have been solved by the application of steepest decent method used by Meksyn followed by the method of Laplace. In this method we express the functions $f_i(\eta)$ and $T_i(\eta)$ in power series of η as

$$f_i(\eta) = \frac{A_i}{2!}\eta^2 + \frac{B_i}{3!}\eta^3 + \frac{C_i}{4!}\eta^4 + \frac{D_i}{5!}\eta^5 + \frac{E_i}{6!}\eta^6 + \cdots \quad (21)$$

$$T_1(\eta) = \frac{1}{4E} + a_1\eta + \frac{b_1}{2!}\eta^2 + \frac{c_1}{3!}\eta^3 + \frac{d_1}{4!}\eta^4 + \frac{e_1}{5!}\eta^5 + \cdots \quad (22)$$

$$T_j(\eta) = a_j\eta + \frac{b_j}{2!}\eta^2 + \frac{c_j}{3!}\eta^3 + \frac{d_j}{4!}\eta^4 + \frac{e_j}{5!}\eta^5 + \cdots \quad (23)$$

where $i = 1,3,5,7$ and $j = 3,5,7$.

We have taken the above forms of f_i , T_1 and T_j for sufficiently small values of η and all of them satisfy the boundary conditions (11) at $\eta = 0$. Now substituting the expressions of f_i , T_1 and T_j from (21), (22) and (23) into (13) to (20) and equating the co-efficient of different powers of η to zero, we obtain the constants B_i , C_i , D_i , E_i ...; b_j , c_j , d_j , e_j ... as functions of A_i's and a_i's only. So, if A_i and a_i are known, the velocity and the temperature distributions are completely determined.

Now we write equations (13) to (16) and (17) to (20) in the following forms:

$$f_i''' + f_1 f_i'' = H_i(\eta)\,,\ i = 1,3,5,7 \quad (24)$$

and

$$T_i'' + Pr f_1 T_i' = M_i(\eta)\,,\ i = 1,3,5,7 \quad (25)$$

Here we have denoted the right-hand sides of the equations (13) to (16) and (17) to (20) by $H_i(\eta)$ and $M_i(\eta)$ respectively. Now integrating twice the equations (24) and (25) w.r.t 'η' from 0 to η, we get

$$f_i'(\eta) = \int_0^\eta e^{-F(\eta)}\phi_i(\eta)\,d\eta \quad (26)$$

$$T_i(\eta) = a_0 + \int_0^\eta e^{-G(\eta)}\psi_i(\eta)\,d\eta \quad (27)$$

where

$$F(\eta) = \int_0^\eta f_1(\eta)\,d\eta$$
$$G(\eta) = Pr\int_0^\eta f_1(\eta)\,d\eta$$
$$\phi_i(\eta) = A_i + \int_0^\eta e^{F(\eta)}H_i(\eta)\,d\eta$$
$$\psi_i(\eta) = a_i + \int_0^\eta e^{G(\eta)}M_i(\eta)\,d\eta$$

Now, taking $\eta \to \infty$ in the equations (26) and (27) we get,

$$\left.\begin{aligned}\int_0^\infty e^{-F(\eta)}\phi_1(\eta)\,d\eta &= 1\\ \int_0^\infty e^{-F(\eta)}\phi_3(\eta)\,d\eta &= \frac{1}{4}\\ \int_0^\infty e^{-F(\eta)}\phi_5(\eta)\,d\eta &= \frac{1}{6}\\ \int_0^\infty e^{-F(\eta)}\phi_7(\eta)\,d\eta &= \frac{1}{8}\end{aligned}\right\} \quad (28)$$

and

$$\left.\begin{aligned}\int_0^\infty e^{-G(\eta)}\psi_1(\eta)\,d\eta &= -\frac{1}{4\mathrm{E}}\\ \int_0^\infty e^{-G(\eta)}\psi_3(\eta)\,d\eta &= 0\\ \int_0^\infty e^{-G(\eta)}\psi_5(\eta)\,d\eta &= 0\\ \int_0^\infty e^{-G(\eta)}\psi_7(\eta)\,d\eta &= 0\end{aligned}\right\} \quad (29)$$

The above integrals can be evaluated asymptotically by Laplace's method. Putting $F(\eta) = G(\eta) = \delta$, transforming the equations (28) and (29) to the variable δ and integrating in the gamma functions, we have,

$$\left.\begin{aligned}&m_{10}\Gamma_{\frac{1}{3}} + m_{11}\Gamma_{\frac{2}{3}} + m_{12}\Gamma_1 + m_{13}\Gamma_{\frac{4}{3}} + m_{14}\Gamma_{\frac{5}{3}} + \cdots = 1\\ &m_{30}\Gamma_{\frac{1}{3}} + m_{31}\Gamma_{\frac{2}{3}} + m_{32}\Gamma_1 + m_{33}\Gamma_{\frac{4}{3}} + m_{34}\Gamma_{\frac{5}{3}} + \cdots = \frac{1}{4}\\ &m_{50}\Gamma_{\frac{1}{3}} + m_{51}\Gamma_{\frac{2}{3}} + m_{52}\Gamma_1 + m_{53}\Gamma_{\frac{4}{3}} + m_{54}\Gamma_{\frac{5}{3}} + \cdots = \frac{1}{6}\\ &m_{70}\Gamma_{\frac{1}{3}} + m_{71}\Gamma_{\frac{2}{3}} + m_{72}\Gamma_1 + m_{73}\Gamma_{\frac{4}{3}} + m_{74}\Gamma_{\frac{5}{3}} + \cdots = \frac{1}{8}\end{aligned}\right\} \quad (30)$$

$$\left.\begin{aligned}&n_{10}\Gamma_{\frac{1}{3}} + n_{11}\Gamma_{\frac{2}{3}} + n_{12}\Gamma_1 + n_{13}\Gamma_{\frac{4}{3}} + n_{14}\Gamma_{\frac{5}{3}} + \cdots = -\frac{1}{4E}\\ &n_{30}\Gamma_{\frac{1}{3}} + n_{31}\Gamma_{\frac{2}{3}} + n_{32}\Gamma_1 + n_{33}\Gamma_{\frac{4}{3}} + n_{34}\Gamma_{\frac{5}{3}} + \cdots = 0\\ &n_{50}\Gamma_{\frac{1}{3}} + n_{51}\Gamma_{\frac{2}{3}} + n_{52}\Gamma_1 + n_{53}\Gamma_{\frac{4}{3}} + n_{54}\Gamma_{\frac{5}{3}} + \cdots = 0\\ &n_{70}\Gamma_{\frac{1}{3}} + n_{71}\Gamma_{\frac{2}{3}} + n_{72}\Gamma_1 + n_{73}\Gamma_{\frac{4}{3}} + n_{74}\Gamma_{\frac{5}{3}} + \cdots = 0\end{aligned}\right\} \quad (31)$$

where the constants are not presented here due to sake of brevity. Now solving the equations (30) and (31) using MATLAB, we get the values of A_i's and a_i's ($i = 1,3,5,7$) for different flow parameters involved in the equations (Tables 1 to 5). Throughout the computations, we use different values of magnetic parameter M, Prandtl number Pr

and visco-elastic parameter α_1 with fixed value of Eckert number $E = 0.1$. In the study, the Newtonian fluid flow phenomenon is illustrated by $\alpha_1 = 0$ and $\alpha_1 \neq 0$ characterizes the visco-elastic fluid. Now using the values of A_i's and a_i's in equations (21) to (23), we get the expressions for f_i , T_1 and T_j . Finally putting the values of f_i , T_1 and T_j in equations (8) to (10) we get the analytical expressions for velocity components u, v within the velocity boundary region and temperature T^* within the thermal boundary layer region.

4 Results and Discussion

Knowing the velocity and temperature fields, we obtain some important flow characteristics of the problem viz. wall shear stress and local heat flux.

The non-dimensional shearing stress τ at the wall $\eta = 0$ is given by,

$$\tau = \frac{\sigma_{xy}}{2\rho U_\infty \left(\frac{2\nu_1 U_\infty}{l}\right)^{\frac{1}{2}}} = \left\{\theta f_1''(\eta) - \frac{4}{3!}\theta^3 f_3''(\eta) + \frac{6}{5!}\theta^5 f_5''(\eta) - \frac{8}{7!}\theta^7 f_7''(\eta) + \cdots\right\}_{\eta=0} \quad (32)$$

We get the location of the point of separation if the shearing stress at the wall vanishes. The conditions for which the shearing stress τ vanishes at the surface of the wall for $\alpha_1 = 0, -0.01,$ and -0.03 with magnetic parameter $M = 0.08$ are given by,

$$0.022009\,X^3 - 0.14664\,X^2 + 0.481624\,X - 0.8664 = 0 \quad (33)$$

$$0.024521\,X^3 - 0.14839\,X^2 + 0.492758\,X - 0.8692 = 0 \quad (34)$$

$$0.030386\,X^3 - 0.15226\,X^2 + 0.516892\,X - 0.875 = 0 \quad (35)$$

respectively with $X = \theta^2$.

The acceptable roots after solving these cubic equations (33), (34) and (35) are $X = 3.6218$, $X = 3.2282$ and $X = 2.6753$ respectively. Thus the points of separation for $\alpha_1 = 0, -0.01,$ and -0.03 occur at $\theta = 109.09^\circ$, $\theta = 102.99^\circ$ and $\theta = 93.76^\circ$ respectively. Thus, we observe that the separation point diminishes with the enhancement of absolute value of visco-elastic parameter. Again, if we increase the value of magnetic parameter, ($M = 0.15$) then we get the separation point for $\alpha_1 = 0$, -0.01, and -0.03 at $\theta = 111.09^\circ$, $\theta = 104.32^\circ$ and $\theta = 94.42^\circ$ respectively. So, it can be remarked that the enhancement of magnetic parameter increases the value of separation point in both Newtonian and non-Newtonian cases.

The heat generation flux g from the cylinder to the fluid is given by,

$$g = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\frac{4kE\sqrt{Re}}{a}(T_w - T_\infty)\{T_1'(\eta) - \theta^2 T_3'(\eta) + \theta^4 T_5'(\eta) - \theta^6 T_7'(\eta)\}_{\eta=0} \quad (36)$$

where $Re = \frac{2U_\infty \rho}{a\mu_1}$ is the Reynolds number.

Defining the Nusselt number $Nu = \frac{ag}{k(T_w - T_\infty)}$ we have,

$$Nu = -4E\sqrt{Re}\{T_1'(\eta) - \theta^2 T_3'(\eta) + \theta^4 T_5'(\eta) - \theta^6 T_7'(\eta)\}_{\eta=0} \quad (37)$$

Figures 1 to 5 demonstrate the variation of shearing stress τ and $\frac{Nu}{\sqrt{Re}}$ against θ with various values of other flow parameters. The various angles (in degrees) are converted in radians ($1^\circ = 0.0175$ radian) while plotting the graphs in different cases. The graphs reveal that the shearing stress τ and $\frac{Nu}{\sqrt{Re}}$ gradually diminish for Newtonian and non-Newtonian fluids. Also, it is observed that the growth of absolute value of visco-elastic parameter α_1, decelerate the shearing stress τ and $\frac{Nu}{\sqrt{Re}}$ in comparison with Newtonian fluid flow phenomenon. The rising values of magnetic parameter M accelerate the shearing stress for both types of fluids (Figures 1 and 2). Again from the figures 3, 4 and 5, it can be revealed that the growth of magnetic parameter increases $\frac{Nu}{\sqrt{Re}}$ but an opposite trend is demonstrated during the growing behaviour of Prandtl number Pr. It has been observed that the Meksyn method is useful and depicts better result when skin friction, heat flux etc. are calculated at the wall of the solid body.

5 Conclusion

The flow and heat transfer of a v isco-elastic fluid around a circular cylinder in presence of a magnetic field has been analysed using the application of steepest descent method used by Meksyn. This study leads to the following conclusions:

- The velocity and temperature fields are significantly affected by visco-elasticity.

- The point of separation for Newtonian case comes to be 109.09° whereas the exact value is 109.6°.
- The point of separation has been found to shift towards the forward stagnation point due to the elasticity of the fluid.
- The enhancement of the value of separation point is noticed for both Newtonian and visco-elastic fluids with the increasing value of magnetic parameter.
- With the growth of the absolute value of the visco-elastic parameter, the shearing stress diminishes and the identical result is seen for Nusselt number.
- Both the shearing stress and Nusselt number show diminishing trends with the increase of the angle θ in both types of fluids.

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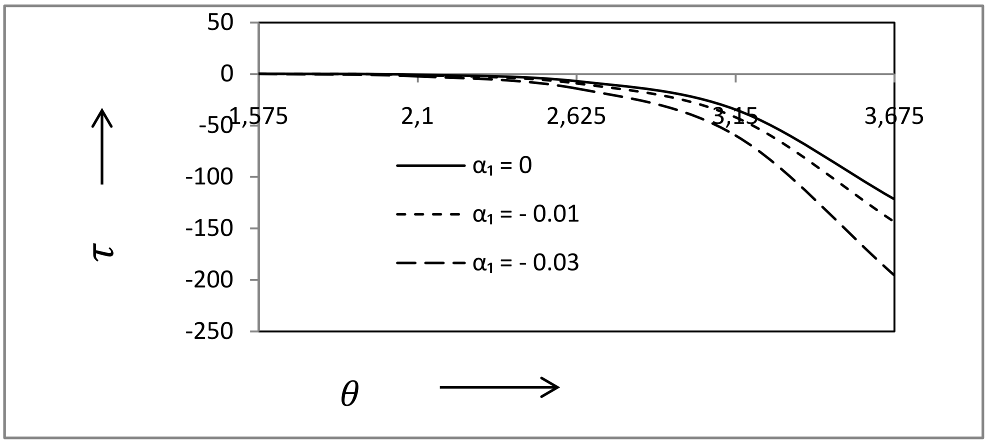

Fig. 1 Variation of τ against θ for $M = 0.08$.

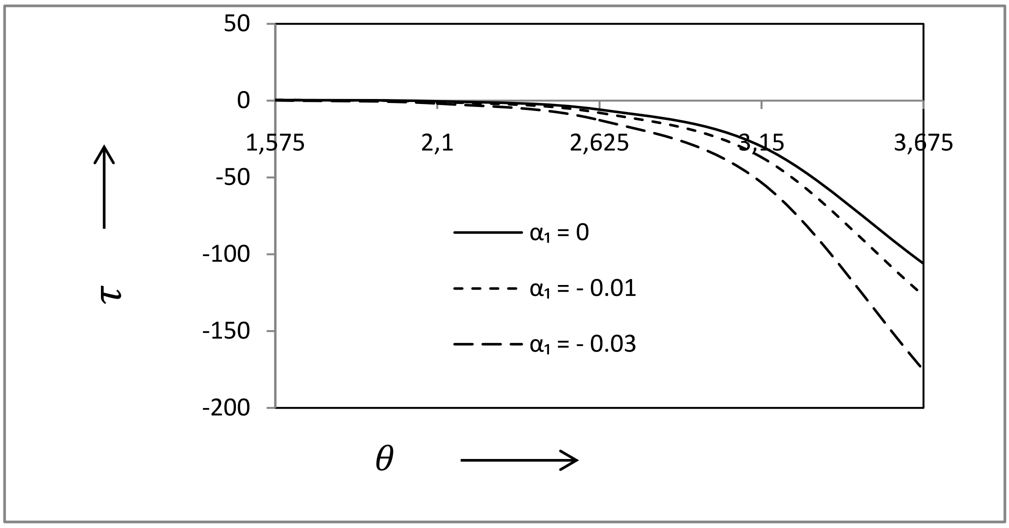

Fig. 2 Variation of τ against θ for $M = 0.15$.

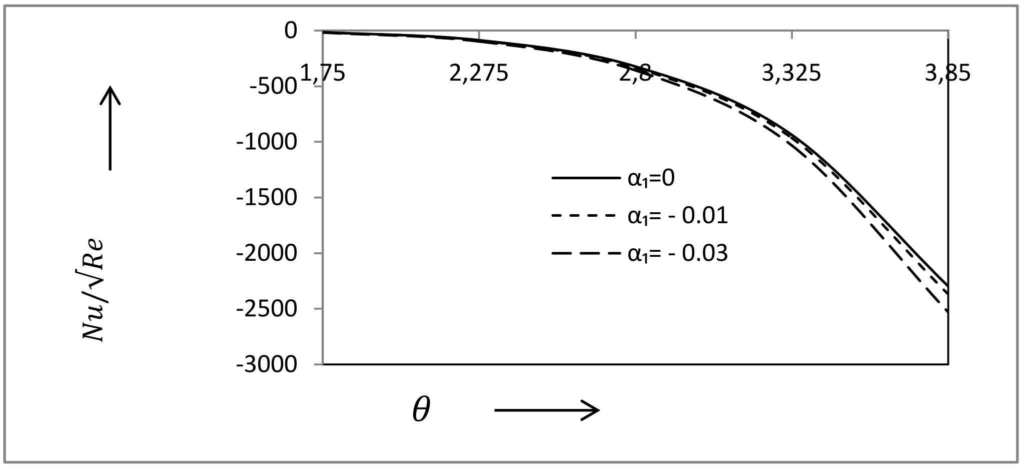

Fig. 3 Variation of $\frac{Nu}{\sqrt{Re}}$ against θ for $M = 0.08, Pr = 3, E = 0.1$.

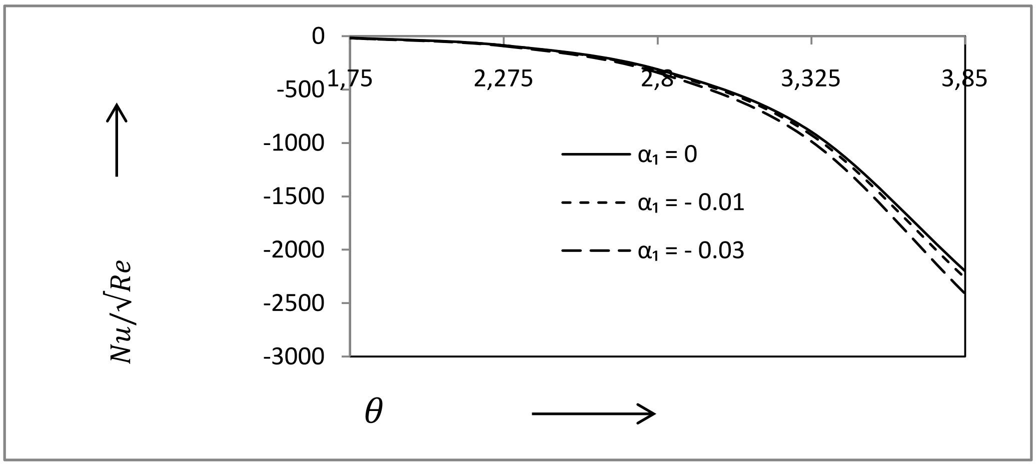

Fig. 4 Variation of $\frac{Nu}{\sqrt{Re}}$ against θ for $M = 0.15, Pr = 3, E = 0.1$.

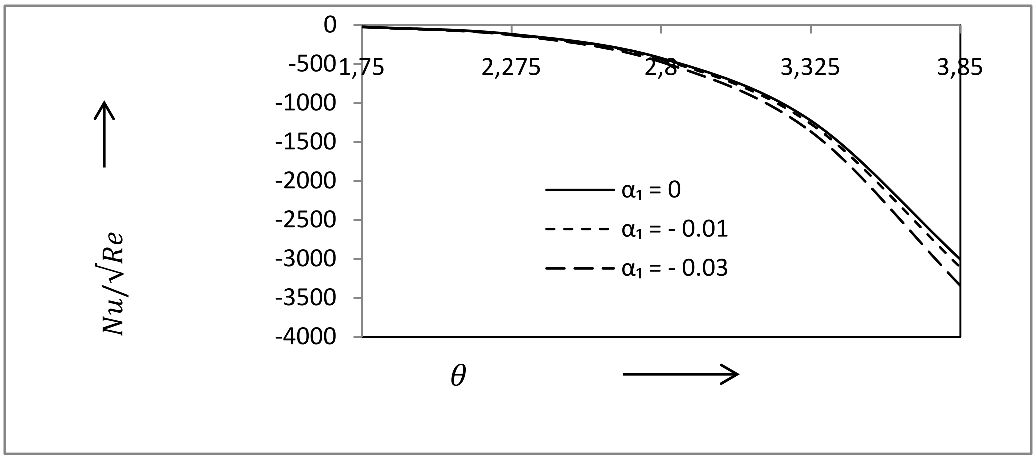

Fig. 5 Variation of $\frac{Nu}{\sqrt{Re}}$ against θ for $M = 0.08, Pr = 5, E = 0.1$.

α_1	0	-0.01	-0.03
A_1	0.8664	0.8692	0.875
A_3	0.7224	0.7391	0.7753
A_5	2.9328	2.9678	3.0452
A_7	13.756	15.3262	18.9918

Table 1. Values of A_i($i = 1,3,5,7$) at $M = 0.08$

α_1	0	-0.01	-0.03
A_1	0.8664	0.8692	0.875
A_3	0.7224	0.7391	0.7753
A_5	2.9328	2.9678	3.0452
A_7	13.756	15.3262	18.9918

Table 2. Values of A_i($i = 1,3,5,7$) at $M = 0.15$.

α_1	0	-0.01	-0.03
a_1	-0.0166	-0.0167	-0.0167
a_3	-0.6608	-0.6632	-0.6681
a_5	-1.5347	-1.5552	-1.6006
a_7	-1.8638	-1.9205	-2.0488

Table 3. Values of a_i($i = 1,3,5,7$) at $M = 0.08$, $Pr = 3$ and $E = 0.1$.

α_1	0	-0.01	-0.03
a_1	-0.0167	-0.0167	-0.0167
a_3	-0.8783	-0.8825	-0.891
a_5	-1.3362	-1.3544	-1.3928
a_7	-1.7736	-1.8257	-1.9417

Table 4. Values of $a_i(\, i = 1,3,5,7)$ at $M = 0.15, Pr = 3$ and $E = 0.1$

α_1	0	-0.01	-0.03
a_1	-0.0257	-0.0257	-0.0257
a_3	-1.0603	-1.0641	-1.0715
a_5	-2.021	-2.0518	-2.1195
a_7	-2.4416	-2.5225	-2.7049

Table 5. Values of $a_i(\, i = 1,3,5,7)$ at $M = 0.08, Pr = 5$ and $E = 0.1$.