Stability of Dual Solutions in Boundary Layer Flow and Heat Transfer on a Moving Plate in a Copper-Water Nanofluid with Slip Effect

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Abstract: - An analysis is performed to study the flow and heat transfer characteristics on a moving plate in a nanofluid. The governing nonlinear differential equations are transformed into a system of nonlinear ordinary equations using a similarity transformation which is then solved numerically using a shooting method. While, for the stability analysis, the unsteady problem has to be introduced by introducing new dimensionless time variable which is then solved numerically using solver bvp4c. The numerical results are presented in tables and graphs for the skin friction coefficient and the local Nusselt number as well as the velocity and the temperature profile for a range of various parameters such as nanoparticles volume fraction, first order slip parameter and velocity ratio parameter. It is observed that the skin friction coefficient and the local Nusselt number which represents the heat transfer rate at the surface are significantly influenced by these parameters. The results indicate that dual solutions (first and second solutions) exist when the plate and free stream move in the opposite direction. A stability analysis has been performed to show which solutions are stable and physically realizable. Based on the analysis, the results indicate that the first solution is linearly stable, while the second solution is linearly unstable.

Key-Words: - Boundary layer flow, Heat transfer, Velocity slip, Nanofluid, Dual solutions, Stability analysis

1 Introduction

Nowadays, nanofluid has become one of the most important subjects of research due to its numerous applications in engineering and biomedical area. Some kind of the applications are heat exchangers, automotive cooling applications, electronic cooling and in nano drug delivery. Choi [1] is the first who discovered the term nanofluid in the real world. In the study, he found that the addition of a small amount (< 1% volume fraction) of nanoparticles to conventional heat transfer fluid enhance the thermal conductivity of the fluid twice. Nanofluid is a fluid that contained nanometer-sized particles, called nanoparticles. This type of fluid shows significant enhancement of their properties. The convectional heat transfer fluid like water, ethylene-glycol and mineral oil has low thermal conductivity which inadequate to satisfy the qualification for the cooling rate. Hence, nanoscale particles are dispersed in a base fluid in order to form a nanofluid, which later used as an alternative option to enhance the heat transfer of such fluids. Recent years, many works featuring nanofluid have been found in the literature. The pioneering work on the boundary layer flow of a nanofluid over a moving plate has been studied by Bachok et al. [2]. The problem later was continued by Rohni et al. [3] and Bachok et al. [4] with a different nanofluid model which proposed by Tiwari and Das [5]. Later, Mabood et al. [6], Sheremet et al. [7] and Shaw et al. [8] considered a new aspect of studies involving nanofluid.

Couples years back, the researchers start to performed and analyzed the results which have dual or multiple solutions in order to conclude that the first solution is stable and physically relevant. The papers consisted of stability analysis in their study such as Mahapatra and Nandy [9], Merkin [10], Weidmen et al. [11], Ishak [12] and Merill et al. [13]. The objective of the present paper is to extend the work by Bachok et al. [4] with slip effect. The effects of the selected parameters and behavior of the slip condition will be studied numerically and the stability of dual solutions is performed to proof the first solution is in stable state while the second solution is unstable.

2 Problem Formulation

Consider a two-dimensional laminar boundary layer flow on a fixed or continuously moving flat surface in a water-based nanofluid containing copper (Cu) nanoparticles. It is assumed that the plate moves in the same or opposite direction to the free stream, both with constant velocities. The nanoparticles are assumed to have a uniform spherical shape and size. The boundary layer equations are given by Bachok et al. [4]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions

$$u=U_w+L\left(\frac{\partial u}{\partial y}\right), v=0, T=T_w \quad \text{at } y=0,$$
$$u\to U_\infty, T\to T_\infty \quad \text{as } y\to\infty, \tag{4}$$

where U_w and U_∞ are constants and correspond to the plate velocity and the free stream velocity, respectively. Further, u and v are the velocity components along the $x-$ and $y-$ directions, respectively, L denotes the slip length, T is the temperature of the nanofluid, μ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid and ρ_{nf} is the density of the nanofluid, which are given by (Oztop and Abu-Nada [14])

$$\begin{aligned}
&\alpha_{nf}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}},\ \mu_{nf}=\frac{\mu_f}{\left(1-\varphi\right)^{2.5}},\\
&\rho_{nf}=\left(1-\varphi\right)\rho_f+\varphi\rho_s,\\
&\left(\rho C_p\right)_{nf}=\left(1-\varphi\right)\left(\rho C_p\right)_f+\varphi\left(\rho C_p\right)_s,\\
&\frac{k_{nf}}{k_f}=\frac{\left(k_s+2k_f\right)-2\varphi\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\varphi\left(k_f-k_s\right)}.
\end{aligned} \tag{5}$$

Here, φ is the nanoparticle volume fraction, $\left(\rho C_p\right)_{nf}$ is the heat capacity of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid, k_f and k_s are the thermal conductivities of the fluid and of the solid fractions, respectively, and ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively. The use of the above expression for k_{nf}/k_f is restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles [14,15]. Also, the viscosity of the nanofluid μ_{nf} has been approximated by Brinkman [16] as viscosity of a base fluid μ_f containing dilute suspension of fine spherical particles.

To obtain similarity solutions for the system of Eqs. (1) – (4), we introduce the following similarity variables

$$\eta=\left(\frac{U}{\nu_f x}\right)^{1/2} y, \quad \psi=\left(\nu_f xU\right)^{1/2} f(\eta),$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

where U is the composite velocity defined as $U=U_w+U_\infty$. This definition of U was first introduced by Afzal et al. [17]. Further, ψ is the stream function defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, which identically satisfies Eq. (1). Employing the similarity variables (6), Eqs. (2) and (3) reduce to the following ordinary differential equations:

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)}f'''+\frac{1}{2}ff''=0 \tag{7}$$

$$\frac{1}{Pr}\frac{k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f\right]}\theta''+\frac{1}{2}f\theta'=0 \tag{8}$$

subjected to the boundary conditions (4) which become

$$f(0)=0, \quad f'(0)=\lambda+\sigma f''(0), \quad \theta(0)=1$$
$$f'(\eta)\to 1-\lambda, \quad \theta(\eta)\to 0 \quad \text{as } \eta\to\infty. \tag{9}$$

In the above equations, primes denote differentiation with respect to η, Pr $(=\nu_f/\alpha_f)$ is the Prandtl number and λ is the velocity ratio parameter defined as

$$\lambda=\frac{U_w}{U}. \tag{10}$$

and the slip parameter is defined as

$$\sigma=LU\left(\frac{U}{\nu_f x}\right)^{1/2} \tag{11}$$

The case $0<\lambda<1$ is when the plate and the fluid move in the same direction, while they move in the opposite directions when $\lambda<0$, and when $\lambda>1$. If $\lambda<0$, the free stream is directed towards the positive $x-$direction, while the plate moves towards the negative $x-$direction. If $\lambda>1$, the free stream is directed towards the negative $x-$direction, while the plate moves towards the positive $x-$direction. However, in this paper we consider only the case $\lambda \le 1$, i.e. the direction of the free stream is fixed (towards the positive $x-$direction). It is worth mentioning that the present problem reduces to those considered by Ahmad et al. [18] when $\lambda = 0$ and $\lambda = 1$. Further, without the energy equation and when $\varphi = 0$ (regular fluid), the present problem reduces to those of Blasius [19] when $\lambda = 0$, and to those of Sakiadis [20] when $\lambda = 1$.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho_f U^2}, \quad Nu_x = \frac{x q_w}{k_f (T_w - T_\infty)}, \tag{12}$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = -k_{nf} \left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{13}$$

with μ_{nf} and k_{nf} being the dynamic viscosity and thermal conductivity of the nanofluids, respectively. Using the similarity variables (6), we obtain

$$C_f Re_x^{1/2} = \frac{1}{(1-\varphi)^{2.5}} f''(0), \tag{14}$$

$$Nu_x / Re_x^{1/2} = -\frac{k_{nf}}{k_f} \theta'(0), \tag{15}$$

where $Re_x = Ux / \nu_f$ is the local Reynolds number.

3 Stability Analysis

In order to perform a stability analysis, we consider the unsteady problem. Eq. (1) holds, while Eqs. (2) and (3) are replaced by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} \tag{16}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} \tag{17}$$

where *t* denotes the time. Based on the variables (6), we introduce the following new dimensionless variables:

$$\eta = \left(\frac{U}{\nu_f x}\right)^{1/2} y, \quad \psi = \left(\nu_f x U\right)^{1/2} f(\eta,\tau), \qquad \theta(\eta,\tau) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \tau = \frac{U}{x} \mathrm{t}, \tag{18}$$

so that Eqs. (2) and (3) can be written as

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)} \frac{\partial^3 f}{\partial \eta^3} + \frac{1}{2} f \frac{\partial^2 f}{\partial \eta^2} - \frac{\partial^2 f}{\partial \eta \partial \tau} = 0 \tag{19}$$

$$\frac{1}{Pr} \frac{k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f\right]} \frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{2} f \frac{\partial \theta}{\partial \eta} - \frac{\partial \theta}{\partial \tau} = 0 \tag{20}$$

and are subjected to the boundary conditions

$$\begin{gathered} f(0,\tau) = 0, \quad \frac{\partial f}{\partial \eta}(0,\tau) = \lambda + \sigma \frac{\partial^2 f}{\partial \eta^2}(0,\tau), \\ \theta(0,\tau) = 1, \\ \frac{\partial f}{\partial \eta}(\eta,\tau) \to 1-\lambda, \\ \theta(\eta,\tau) \to 0 \quad \text{as } \eta \to \infty \end{gathered} \tag{21}$$

To test the stability of the steady flow solution $f(\eta) = f_0(\eta)$ and $\theta(\eta) = \theta_0(\eta)$ satisfying the boundary value problem (1) – (4), we write

$$f(\eta,\tau) = f_0(\eta) + e^{-\gamma\tau} F(\eta,\tau), \qquad \theta(\eta,\tau) = \theta_0(\eta) + e^{-\gamma\tau} G(\eta,\tau), \tag{22}$$

where γ is an unknown eigenvalue, and $F(\eta,\tau)$ and $G(\eta,\tau)$ are small relative to $f_0(\eta)$ and $\theta_0(\eta)$. Solutions of the eigenvalue problem (19) – (21) give an infinite set of eigenvalues $\gamma_1 < \gamma_2 < \ldots$; if the smallest eigenvalue is negative, there is an initial growth of disturbances and the flow is unstable but when γ_1 is positive, there is an initial decay and the flow is stable. Introducing (22) into (19) and (20), we get the following linearized problem

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)}\frac{\partial^3 F}{\partial\eta^3} + \frac{1}{2}\left(f_0\frac{\partial^2 F}{\partial\eta^2}+\frac{\partial^2 f_0}{\partial\eta^2}F\right)+\gamma\frac{\partial F}{\partial\eta}-\frac{\partial^2 F}{\partial\eta\partial\tau}=0 \tag{23}$$

$$\frac{1}{Pr}\frac{k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f\right]}\frac{\partial^2 G}{\partial\eta^2} + \frac{1}{2}\left(f_0\frac{\partial G}{\partial\eta}+\frac{\partial\theta_0}{\partial\eta}F\right)+\gamma G-\frac{\partial G}{\partial\tau}=0 \tag{24}$$

along with the boundary conditions

$$\begin{aligned} &F(0,\tau)=0, \quad \frac{\partial F}{\partial\eta}(0,\tau)=\sigma\frac{\partial^2 F}{\partial\eta^2}(0,\tau),\\ &G(0,\tau)=0,\\ &\frac{\partial F}{\partial\eta}(\eta,\tau)\to 0, \quad G(\eta,\tau)\to 0 \quad \text{as } \eta\to\infty \end{aligned} \tag{25}$$

The solutions $f(\eta)=f_0(\eta)$ and $\theta(\eta)=\theta_0(\eta)$ of the steady equations (7) and (8) are obtained by setting $\tau=0$. Hence $F(\eta)=F_0(\eta)$ and $G(\eta)=G_0(\eta)$ in (23) and (24) identify initial growth or decay of the solution (22). In this respect, we have to solve the linear eigenvalue problem

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)}F_0''' + \frac{1}{2}\left(f_0F_0''+f_0''F_0\right)+\gamma F_0'=0 \tag{26}$$

$$\frac{1}{Pr}\frac{k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f\right]}G_0'' + \frac{1}{2}\left(f_0G_0'+F_0\theta_0'\right)-G_0'+\gamma G_0=0 \tag{27}$$

along with the new boundary conditions

$$\begin{aligned} &F_0(0)=0, \quad F_0'(0)=\sigma F_0''(0), \quad G_0(0)=0,\\ &F_0'(\eta)\to 0, \quad G_0(\eta)\to 0 \quad \text{as } \eta\to\infty \end{aligned} \tag{28}$$

It should be stated that for particular values of *Pr* and γ, the stability of the corresponding steady flow solutions $f_0(\eta)$ and $\theta_0(\eta)$ are determined by the smallest eigenvalue γ. As it has been suggested by Harris et al. [21], the range of possible eigenvalues can be determined by relaxing a boundary condition on $F_0(\eta)$ or $G_0(\eta)$. For the present problem, we relax the condition that $F_0'(\eta)\to 0$ as $\eta\to\infty$ and for a fixed value of γ we solve the system (26, 27, 28) along with the new boundary condition $F_0''(0)=1$.

4 Results and Discussion

Numerical solutions to the governing ordinary differential equations (7) and (8) with the boundary conditions (9) are obtained using shooting method in Maple software. The dual solutions are obtained by setting different initial guesses for the missing values of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$, where all profiles satisfy the far field boundary conditions (9) asymptotically but with different shapes of profiles and then being illustrated in graphs.

Figures 1 and 2 illustrate the variation of skin friction $f''(0)$ and local Nusselt number $-\theta'(0)$ for different values of first order slip parameter σ. The variations of λ_c has been shown in Table 2. From the figures and table, when no slip (σ = 0) occur in between the boundary layer, the result will be equal to Bachok et al. [4]. Thus, our computational results are in good agreement with the previous researches and hence prove that other solutions are correct. When the slip occur in between the boundary layer, the boundary layer separation is drag to happen and hence the λ_c will become larger (if σ > 0). The skin friction coefficient and the heat loss from the surface will increases as the slip parameter σ is increasing. Therefore, the presence of slip will accelarates the separation of the boundary layer.

The variations of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ for Cu-water for different values of nanoparticle volume fraction φ were shown in Figs. 3 and 4, respectively. It is seen that the solution is unique when $\lambda\geq 0$, while multiple (dual) solution exist up to $\lambda_c<\lambda<0$, where the plate and free stream moves in the opposite direction. However, no solutions are found to exist when $\lambda>\lambda_c$. From the figures, as the nanoparticles volume fraction φ is increasing, then the skin friction coefficient and heat transfer rate at the surface will increased.

The validity of these numerical solutions and dual nature solutions is supported by the velocity and temperature profiles presented in Figs. 5 – 10. These profiles are satisfied the boundary conditions (9) assymptotically with different shapes of graphs.

The solid line represents the first solution while the dash line represents second solution. As we can see, the boundary layer thickness for second solution will always greater then the first solution.

A stability analysis is performed using bvp4c function in Matlab software to determine which solution is stable (first or second solution). The linear eigenvalue problem (26) and (27) is used to find the unknown eigenvalue γ subjected to the new boundary condition (28). If the smallest eigenvalue is negative, there is an initial growth of disturbance and the flow is unstable while for the positive smallest eigenvalue, there is an initial decay and the flow is stable. The smallest eigenvalue γ for some values of σ at selected values of λ are stated in Table 3 which shows that γ is positive for the first solution and negative for second solution. The eigenvalue γ is approaching 0 as λ is approaching λ_c ($\gamma \to 0$ as $\lambda \to \lambda_c$) either from positive or negative sign. Thus, the first solution is stable, while the second solution is unstable.

Table 1. Thermophysical properties of fluid and nanoparticles (Oztop and Abu-Nada, [14]).

Physical properties	Fluid phase (water)	Cu
C_p (J/kg K)	4179	385
ρ (kg/m^3)	997.1	8933
k (W/mK)	0.613	400
$\alpha \times 10^7$ (m^2/s)	1.47	11163.1
$\beta \times 10^{-5}$ (1/K)	21	1.67

Table 2. Variations of λ_c with φ = 0.1 for different values of σ.

σ	Bachok et al. [4]	Present Result
0	-0.5482	-0.5482
0.2		-0.6821
0.4		-0.8999

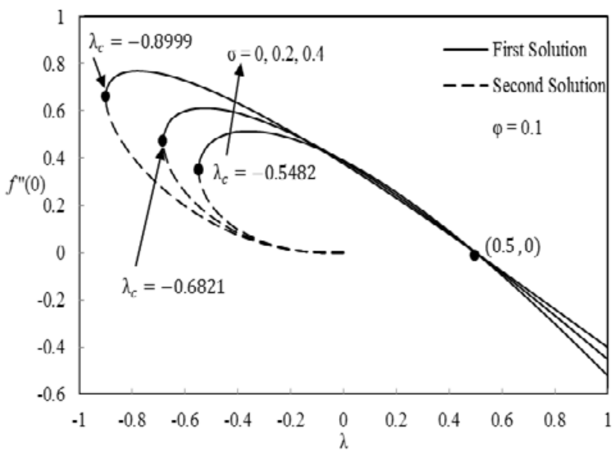

Fig. 1. Skin friction coefficient $f''(0)$ as a function of λ for various values of σ.

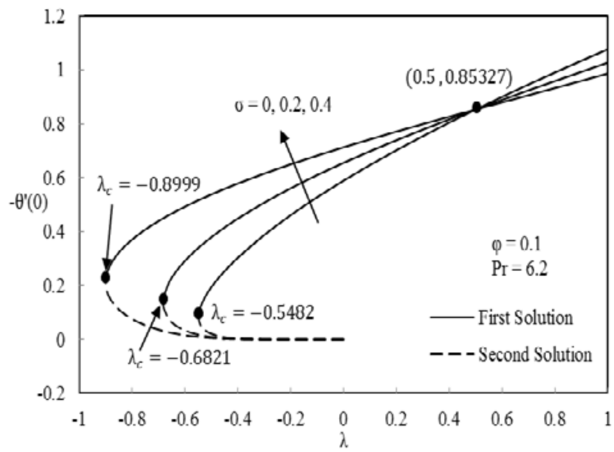

Fig. 2. Wall temperature gradient $-\theta'(0)$ as a function of λ for various values of σ.

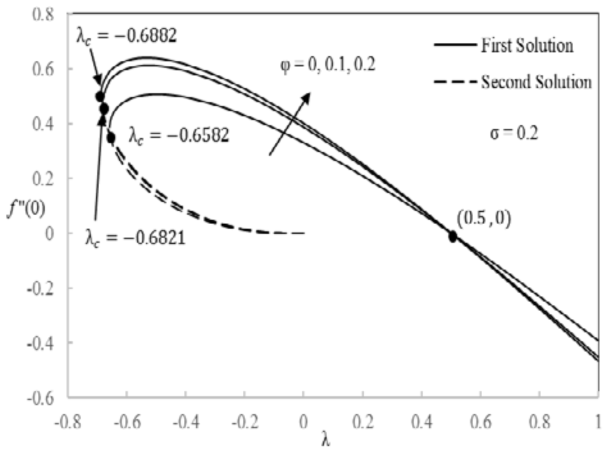

Fig. 3. Skin friction coefficient $f''(0)$ as a function of λ for various values of φ.

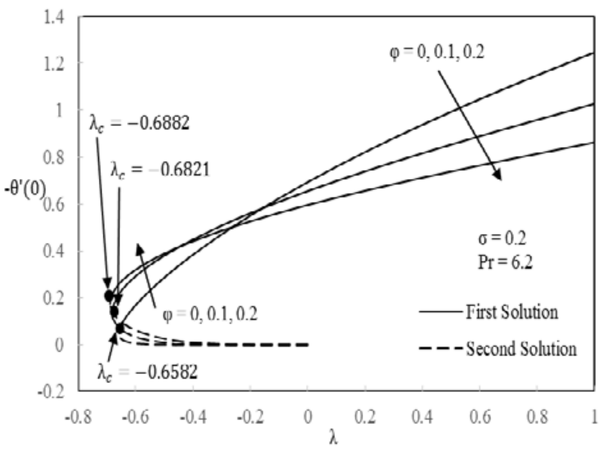

Fig. 4. Wall temperature gradient $-\theta'(0)$ as a function of λ for various values of φ.

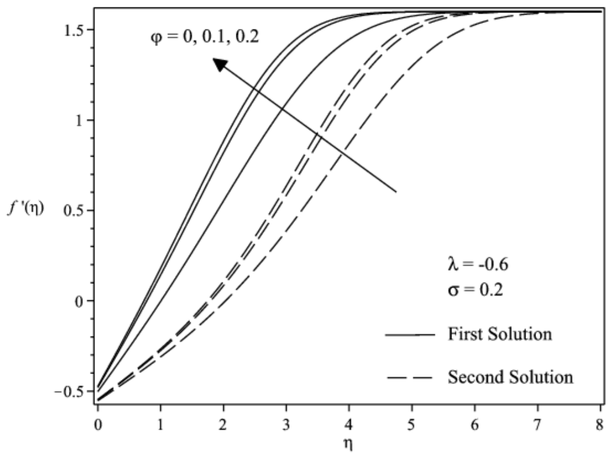

Fig. 7. Velocity profiles $f'(\eta)$ for several values of φ.

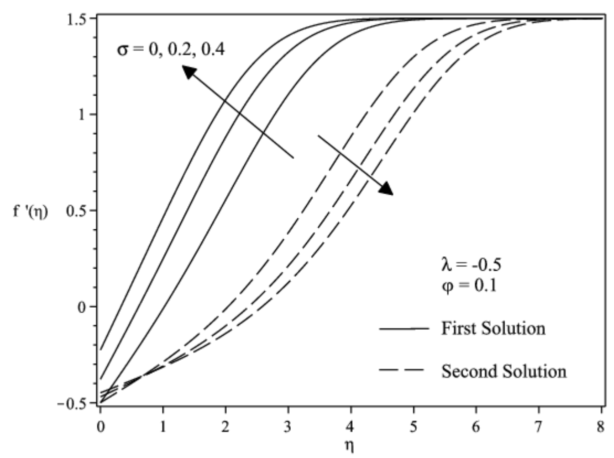

Fig. 5. Velocity profiles $f'(\eta)$ for various of σ.

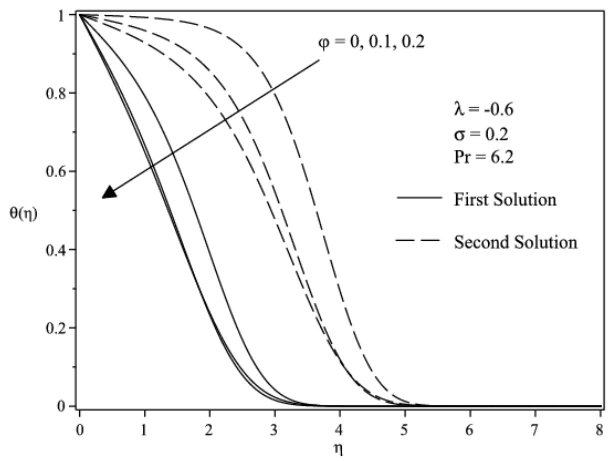

Fig. 8. Temperature profiles $\theta(\eta)$ for several values of φ.

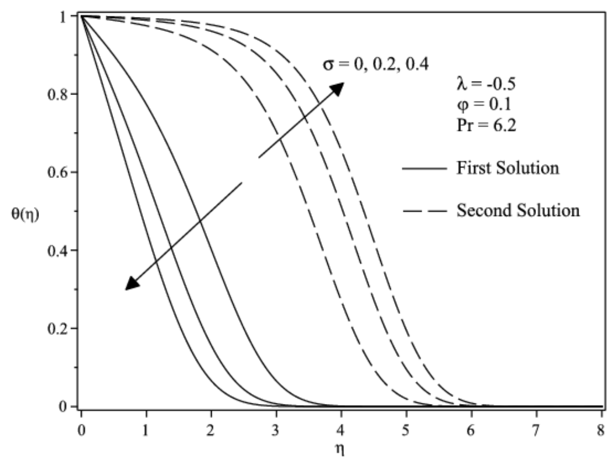

Fig. 6. Temperature profiles $\theta(\eta)$ for various of σ.

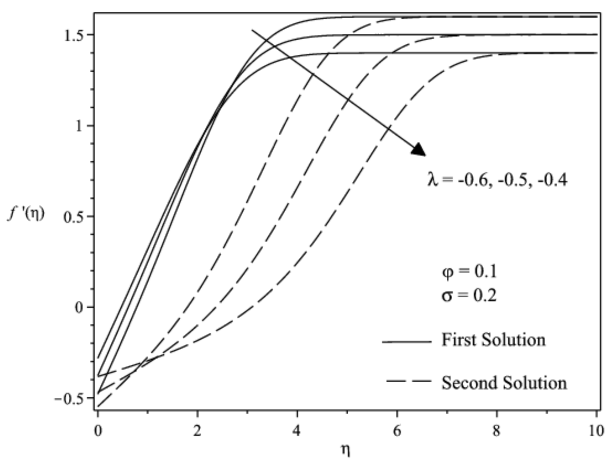

Fig. 9. Velocity profiles $f'(\eta)$ for various of λ.

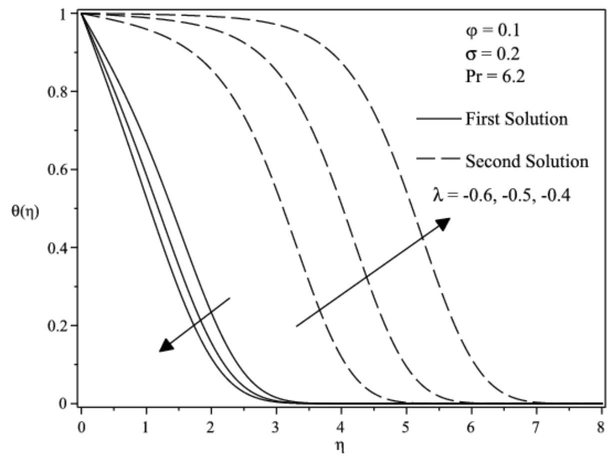

Fig. 10. Temperature profiles $\theta(\eta)$ for various of λ.

Table 3. Smallest eigenvalues γ at selected values of λ for φ = 0.1 and σ = 0, 0.2 and 0.4

σ	λ	Present result First Solution	Present result Second Solution
0	-0.54	0.0406	-0.0350
	-0.52	0.0788	-0.0597
	-0.5	0.1059	-0.0733
	-0.4	0.1992	-0.0982
	-0.3	0.2662	-0.0949
0.2	-0.68	0.0186	-0.0175
	-0.64	0.0901	-0.0689
	-0.6	0.1299	-0.0884
	-0.5	0.2018	-0.1085
	-0.4	0.2569	-0.1097
0.4	-0.89	0.0375	-0.0342
	-0.85	0.0879	-0.0712
	-0.8	0.1278	-0.0938
	-0.7	0.1862	-0.1168
	-0.6	0.2320	-0.1253

5 Conclusion

This paper considers the steady boundary layer flow and heat transfer on a moving plate in Cu-water nanofluid with presence of slip effect. The stability analysis is also performed to determine which solution is stable an unstable. The effects of nanoparticle volume fraction φ and first order slip parameter σ on skin friction coefficient and heat transfer rate at the surface were investigated and discussed. The results indicate that the presence of slip parameter σ will widen the range of velocity ratio parameter λ. Lastly, the first solution is linearly stable and can be realize physically while the second solution is linearly unstable and would not be realize physically.

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References:

[1] S.U.S. Choi, Enhancing thermal conductivity of fluids with nanoparticles, in: Developments and Applications of Non-Newtonian Flows, *FED-vol. 231/MD-vol*, 66, 1995, pp. 99-105.

[2] N. Bachok, A. Ishak, I. Pop, Boundary-layer flow of nanofluids over a moving surface in a flowing fluid. *Int. J. of Thermal Sci*, 49, 2010, pp. 1663-1668.

[3] A.M. Rohni, S. Ahmad, I. Pop, Boundary layer flow over a moving surface in a nanofluid beneath a uniform free stream, *Int. J. Numer. Methods Heat Fluid Flow*, 21, 2011, pp. 828-846.

[4] N. Bachok, A. Ishak, I. Pop, Flow and heat transfer characteristics on a moving plate in a nanofluid, *Int. J. Heat Mass Trans*, 55, 2012, pp. 642-648.

[5] R.K. Tiwari, M.K. Das, Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids, *Int. J. Heat Mass Trans*, 50, 2007, pp. 2002-2018.

[6] F. Mabood, W.A. Khan, A.I.M. Ismail, MHD boundary layer flow and heat transfer of nanofluids over a nonlinear stretching sheet: a

numerical study, *J. Magn. Magn. Mater*, 374, 2015, pp. 569-576.

[7] M.A. Sheremet, I. Pop, N. Bachok, Effect of thermal dispersion on transient natural convection in a wavy-walled porous cavity filled with a nanofluid: Tiwari and Das' nanofluid model. *Int. J. Heat Mass Transfer*, 92, 2016, pp. 1053-1060.

[8] S. Shaw, P.K. Kameswaran, P. Sibanda, Effects of slip on nonlinear convection in nanofluid flow on stretching surfaces, *Boundary Value Problems*, 2016(2), 2016, pp. 1-11.

[9] T.R. Mahapatra, S.K. Nandy, Stability of dual solutions in stagnation point flow and heat transfer over a porous shrinking sheet with thermal radiation, *Meccanica*, 48, 2013, pp. 23-32.

[10] J.H. Merkin, On dual solutions occurring in mixed convection in a porous medium, *J. Eng. Math*, 20, 1985, pp. 171-179.

[11] D. Weidman, D.G. Kubitschek, A.M.J. Davis, The effect of transpiration on self-similar boundary layer flow over a moving surface, *Int. J Eng. Sci*, 44, 2006, pp. 730-737.

[12] A. Ishak, Flow and heat transfer over a shrinking sheet: A stability analysis, *Int. J. Mech. Indust. Sci. Eng*, 8(5), 2014, pp. 850-854.

[13] K. Merill, M. Beauchesne, J. Previte, J. Paullet, P. Weidman, Final steady flow near a stagnation point on a vertical surface in a porous medium, *Int. J Heat Mass Trans*, 49, 2006, pp. 4681-4686.

[14] H.F. Oztop, E. Abu-Nada, Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids, *Int. J Heat Fluid Flow*, 29, 2008, pp. 1326-1336.

[15] E. Abu-Nada, Application of nanofluids for heat transfer enhancement of separated flow encountered in a backward facing step, *Int. J Heat Fluid Flow*, 29, 2008, pp. 242-249.

[16] H.C. Brinkman, The viscosity of concentrated suspensions and solutions, *J Chem. Phys*, 20, 1952, pp. 571-581.

[17] N. Afzal, A. Badaruddin, A.A. Elgarvi, Momentum and transport on a continuous flat surface moving in a parallel stretam, *Int. J Heat Mass Trans*, 36, 1993, pp. 3399-3403.

[18] S. Ahmad, A.M. Rohni, I. Pop, Blasius and Sakiadis problems in nanofluids, *Acta Mech.*, 218, 2011, pp. 195-204.

[19] H. Blasius, Grenzschichten in Flussigkeiten mit kleiner Reibung, *Z. Math. Phys.*, 56, 1908, pp. 1-37.

[20] B.C. Sakiadis, Boundary layer behaviour on continuous solid surfaces: Boundary layer equations for two-dimensional and axisymmetric flow, *AIChE J.*, 7, 1961, pp. 26-28.

[21] S.D. Harris, D.B. Ingham, I. Pop, Mixed convection boundary-layer flow near the stagnation point on a vertical surface in a porous medium: Brinkman model with slip, *Trans. Porous Media*, 77, 2009, pp. 267-285.