Adaptive Control of Fluid Inside CSTR Using Continuous-Time and Discrete-Time Identification Model and Different Control Configurations

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Abstract: An adaptive control is a kind of modern control methods with great theoretical background and various modifications. This control approach could be used for system with negative control properties such as nonlinearity, non-minimum phase behaviour etc. The adaptive control in this paper is based on the recursive identification of the external linear model as a linear representation of the originally nonlinear system. The control synthesis is based on the polynomial approach together with the spectral factorization and the pole-placement method. The identification model in the continuous-time uses differential filters and so called delta-models in the discrete-time. There were tested also two types of control configurations with the one degree-of-freedom (1DOF) that has controller only in the feedback part and with the two degrees-of-freedom (2DOF) where the controller is separated into two parts the first is in the feedback and the second is feedforward part of the control loop. Paper shows usability of this control approach by the simulations on the mathematical model of the continuous stirred-tank reactor with the cooling in the jacket as a typical nonlinear system with lumped parameters.

Key–Words: Simulation, Mathematical Model, Adaptive control, Continuous-time model, Delta-model, Continuous Stirred-tank Reactor, 1DOF, 2DOF.

1 Introduction

The continuous stirred-tank reactor (CSTR) is typical nonlinear equipment used in the chemical and biochemical industry for production of various chemicals, drugs etc. [1].

The modelling and simulation is great tool which helps with the observing of the systems behaviour and designing of the appropriate controller. The mathematical model of CSTR is usually described by the set of nonlinear ordinary differential equations (ODEs) which can be solved mathematically for example by the Runge-Kuttas method.

The adaptive control [2] used here for the control is one of the approaches used for the nonlinear systems because it produces good control results. Advantage of this method can be found in very good theoretical background and variety of modifications [3].

The control synthesis uses polynomial approach which satisfies basic control system requirements such as a stability of the control loop, a reference signal tracking and a disturbance attenuation. There will be used two control configurations. The first is basic control configuration with one degree-of-freedom (often denoted as ”1DOF”) that has controller only in the feedback part of the control loop. The second configuration, the two degrees-of-freedom (2DOF) configuration has controller separated into two parts - the first one is in the feedforward part as previous and the second one is in the feedforward part that produces good results in the reference signal tracking [10].

The approach used here is based on the choice of the External Linear Model (ELM) which describes controlled, originally nonlinear, process in the linear way for example by the discrete or the continuous transfer function (TF) [3]. Parameters of ELM are then identified recursively during the control and parameters of the controller are recomputed according to identified parameters of the system. The advantage of used polynomial synthesis is that it produces the structure and also relations for computing controllers parameters that reflect identified parameters of ELM.

Two identification models with the continuous-time (CT) model [4] and special type of the discrete-time (DT) model called delta/model [5] where discussed here. Parameters of input and output variables are related to the sampling period. It was proved that parameters of the delta model approach to parameters of the CT model for sufficiently small sampling period [6]. This combination of the continuous-time control synthesis with the discrete-time identification is called Hybrid adaptive control and some applications can be found for example in [7] and [8]. The recursive least-squares method is used for online identification. This method is widely used because it is easily programmable in standard programming languages at one hand but it produces sufficiently good identification results with various modifications.

All results in this paper are simulations made in the mathematical software Matlab, version 7.0.1.

2 Continuous Stirred-Tank Reactor

The system under the consideration is Continuous Stirred-Tank Reactor (CSTR) with so called Van der Vusse reaction inside [9]:

$$\begin{array}{c} A \to B \to C \\ 2A \to D \end{array} \quad (1)$$

The mathematical model of this system comes from material and heat balances inside the reactor and results in the set of four nonlinear ordinary differential equations (ODE):

$$\begin{aligned}
\frac{dc_A}{dt} &= \frac{q_r}{V_r}\left(c_{A0} - c_A\right) - k_1 c_A - k_3 c_A^2 \\
\frac{dc_B}{dt} &= -\frac{q_r}{V_r} c_B + k_1 c_A - k_2 c_B \\
\frac{dT_r}{dt} &= \frac{q_r}{V_r}\left(T_{r0} - T_r\right) - \frac{h_r}{\rho_r c_{pr}} + \frac{A_r U}{V_r \rho_r c_{pr}}\left(T_c - T_r\right) \\
\frac{dT_c}{dt} &= \frac{1}{m_c c_{pc}}\left(Q_c + A_r U\left(T_r - T_c\right)\right)
\end{aligned} \quad (2)$$

The mathematical model described by the set of ODE (2) have state variables concentrations c_A, c_B and temperatures of the reactant T_r and the cooling T_c. This system provides theoretically four input variables – a volumetric flow rate of the reactant, q_r, a heat removal of the cooling, Q_c, an input concentration c_{A0} and an input temperature of the reactant, T_{r0}. The last two are only theoretical and could not be used as an input variable from the practical point of view. The scheme of this chemical reactor is in Figure 1.

Due to the simplification of the mathematical model, other variables are supposed to be constant during the control. The volume of the reactor is denoted as V_r, A_r is the heat exchange surface, ρ_r is used for the density of the reactant, U is the heat transfer coefficient, c_{pc} and c_{pr} are specific heat capacities of the cooling and the reactant a m_c is the weight of the cooling mass. Values of these fixed parameters are in Table 1 [9].

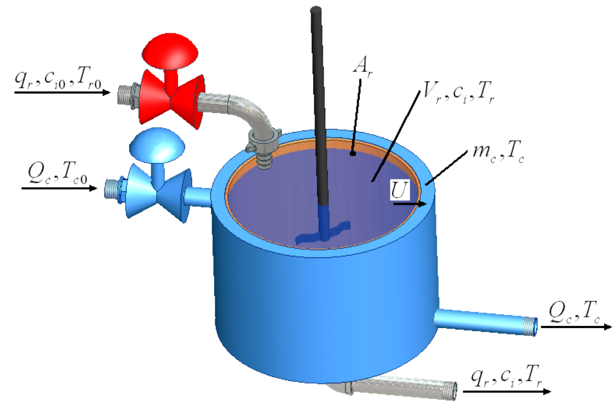

Figure 1: Scheme of the Continuous Stirred-tank Reactor

The first step of the simulation is the steady-state analysis which observes behaviour of the system in the steady-state where state variable does not change. This analysis can help us with the choice of the optimal working point for control. Experiments in [8] have shown working point defined by the volumetric flow rate of the reactant $q_r^s = 2.4 \cdot 10^{-3}\ m^3.min^{-1}$ and heat removal of the cooling liquid $Q_c^s = -18.56\ kJ.min^{-1}$.

The second, dynamic, analysis then observes the behaviour of the system after the step change of the input variable, in this case the heat removal of the coolant, ΔQ_c. The observed output is on the other hand the change of the reactants temperature, T_r, i.e.

$$\begin{array}{c} u(t) = \frac{Q_c(t) - Q_c^s}{Q_c^s} \cdot 100[\%] \\ y(t) = T_r(t) - T_r^s[K] \end{array} \quad (3)$$

Results for various step changes of the input variable, $u(t)$, which comes from the dynamic analysis for the range $\Delta u(t) =< -100\%; +100\% >$ and results are in Figure 2.

Results clearly shows, that all outputs could be described by the second order transfer function (TF) with relative order one in the polynomial form

$$G(s) = \frac{b(s)}{a(s)} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \quad (4)$$

Table 1: Fixed parameters of the reactor

Volume $V_r = 0.01\ m^{-3}$
Density of react. $\rho_r = 934.2\ kg.m^{-3}$
Heat capacity of react. c_{pr} = 3.01 $kJ.kg^{-1}.K^{-1}$
Hmotnost chladiva m_c = 5 kg
Tepeln kapacita chladiva c_{pc} = 2.0 $kJ.kg^{-1}.K^{-1}$
Transfer area A_r = 0.215 m^2
Heat transf. coef. U = 67.2 $kJ.min^{-1}.m^{-2}.K^{-1}$
Speed const. 1 $k_{01} = 2.145 \cdot 10^{10}\ min^{-1}$
Speed const. 2 $k_{02} = 2.145 \cdot 10^{10}\ min^{-1}$
Speed const. 3 $k_{03} = 1.5072 \cdot 10^{8}\ min^{-1}.kmol^{-1}$
Activ. energy 1 to R E_1/R = 9758.3 K
Activ. energy 2 to R E_2/R = 9758.3 K
Activ. energy 3 to R E_3/R = 8560 K
Enthalpy 1 h_1 = -4200 $kJ.kmol^{-1}$
Enthalpy 2 h_2 = 11000 $kJ.kmol^{-1}$
Enthalpy 3 h_3 = 41850 $kJ.kmol^{-1}$
Input concentration of A c_{A0} = 5.1 $kmol.m^{-3}$
Input temp. of reactant T_{r0} = 387.05 K

This will be later used for identification in the adaptive control.

3 Hybrid Adaptive Control

The control approach here is based on the term ”Adaptivity” known from the nature, where plant, animals or even human beings adapts their behaviour to the actual environment.

At first we will start with the control synthesis which uses advantages of the polynomial synthesis [10] that satisfies basic control requirements such as stability, disturbance attenuation and reference signal tracking. Moreover, this method produces not only the structure of the controller, but also the relations for computing of controllers parameters.

3.1 1DOF Control Configuration

The simplest, most common known control scheme with one degree of freedom (1DOF) [11] is shown in Figure 3. The controller is here represented by the TF $Q(s)$ and the controlled system is described by TF $G(s)$ from (4).

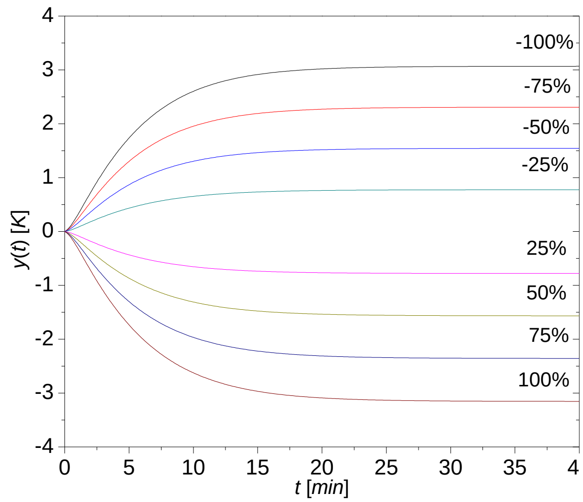

Figure 2: Results of the dynamic analysis for the various changes of the input variable $\Delta u(t)$

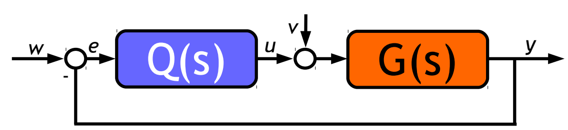

Figure 3: One degree-of-freedom (1DOF) control configuration

The signal w in Figure 3 is reference signal (i.e. wanted value), v denotes disturbance, u is an input and y an output variable. It can be seen, that controller is here only in the feedback part.

The TF of this controller $Q(s)$ is generally:

$$Q(s) = \frac{q(s)}{p(s)} \tag{5}$$

where degrees of polynomials $p(s)$ and $q(s)$ must hold properness condition:

$$\deg q(s) \leq \deg p(s) \tag{6}$$

The condition for the reference signal tracking is satisfied if the polynomial $p(s)$ in the denominator of the controllers transfer functions (7) is divided into

$$p(s) = f(s) \cdot \tilde{p}(s) \tag{7}$$

where $f(s)$ is a least common divisor of the reference and the disturbance transfer functions. If we have these TF in the form of the step function, $f(s) = s$ and (7) could be rewritten into

$$\tilde{Q}(s) = \frac{q(s)}{s \cdot \tilde{p}(s)} \tag{8}$$

Parameters of controllers polynomials and $q(s)$ are computed from Diophantine polynomial equation [10]

$$a(s) \cdot s \cdot \tilde{p}(s) + b(s) \cdot q(s) = d(s) \tag{9}$$

and they can be solved by the Method of uncertain coefficients. Polynomials $a(s)$ and $b(s)$ in (9) are known from the recursive identification which will be discussed in the next subchapter. The polynomial $d(s)$ on the right side of Diophantine equations (9) is stable optional polynomial which could affect the quality of the control.

Degrees of controllers polynomials $\tilde{p}(s)$ and $q(s)$ and the degree of the stable polynomial $d(s)$ are

$$\begin{aligned} \deg q(s) &= \deg a(s) \\ \deg \tilde{p}(s) &= \deg a - 1 \\ \deg d(s) &= 2 \cdot \deg a(s) \end{aligned} \tag{10}$$

3.2 2DOF Control Configuration

The second control configuration has two degrees-of-freedom often denoted as 2DOF control configuration. This configuration is displayed in Fig. 4 and it can be seen that it has controller divided into two parts - feedback $Q(s)$ and feedforward $R(s)$ parts.

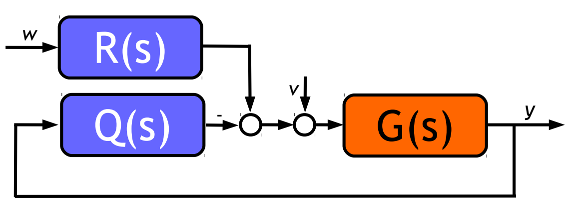

Figure 4: Two degrees-of-freedom (2DOF) control configuration

Signals are similar to those in Fig. 3 and transfer functions of the controller $Q(s)$ and $R(s)$ are in the general form:

$$Q(s) = \frac{q(s)}{p(s)}, R(s) = \frac{r(s)}{p(s)} \tag{11}$$

Degrees of polynomials $p(s)$, $q(s)$ and $r(s)$ must satisfied

$$\deg q(s) \leq \deg p(s), \deg q(s) \leq \deg r(s) \tag{12}$$

Transfer functions in (11) could be rewritten with respect to Equation (7) to

$$\tilde{Q}(s) = \frac{q(s)}{s \cdot \tilde{p}(s)}, \tilde{R}(s) = \frac{r(s)}{s \cdot \tilde{p}(s)} \tag{13}$$

Parameters of unknown polynomials $\tilde{p}(s)$, $q(s)$ and $r(s)$ are computed similarly to 1DOF control configuration from Diophantine equations

$$\begin{aligned} a(s) \cdot s \cdot \tilde{p}(s) + b(s) \cdot q(s) = d(s) \\ t(s) \cdot s + b(s) \cdot r(s) = d(s) \end{aligned} \tag{14}$$

The Method of uncertain coefficients was used again for computing of these parameters. Polynomial $t(s)$ in equation (14) is an auxiliary stable polynomial and coefficients of this polynomial are not used for computing of coefficients of the polynomial $r(s)$.

Degrees of the unknown polynomials $\tilde{p}(s)$, $q(s)$ and $r(s)$ are

$$\begin{aligned} \deg q(s) = \deg a(s), \quad \deg \tilde{p}(s) = \deg a - 1 \\ \deg d(s) = 2 \cdot \deg a(s), \quad \deg r(s) = 0 \end{aligned} \tag{15}$$

Polynomials $a(s)$ and $b(s)$ in (14) are known from the recursive identification and the polynomial $d(s)$ on the right side of Diophantine equations (14) is again stable optional polynomial which could affect the quality of the control.

3.3 Desing of the Polynomial $d(s)$

As it was already mentioned, polynomial $d(s)$ is optional polynomial which could be designed for example by the Pole-placement method, generally

$$d(s) = \prod_{i=1}^{\deg d(s)} (s - s_i) \tag{16}$$

where roots si are generally in the complex form $s_i = \alpha_i + \omega_i \cdot j$ and the stability is satisfied for $\alpha_i < 0$. If we want to obtain an aperiodic output response, ω_i must hold 0 and (16) is then

$$d(s) = (s + \alpha)^{\deg d} \tag{17}$$

The equation (17) is very general which could be disadvantage of this method there is no recommendation for the choice of roots in polynomial $d(s)$. Our previous experiments [8] have shown that is good co connect the choice of this polynomial somehow with the controlled system. The Spectral factorization could be used for this task and it means that the polynomial $d(s)$ is divided into two parts

$$d(s) = n(s) \cdot (s + \alpha)^{\deg d - \deg n} \tag{18}$$

where one part is classic pole-placement method and $n(s)$ comes from the Spectral factorization of the polynomial $a(s)$ in the denominator of the controlled systems transfer function (4):

$$n^*(s) \cdot n(s) = a^*(s) \cdot a(s) \tag{19}$$

Advantage of the Spectral factorization can be also find in the feature, that the polynomial $n(s)$ is always stable even if the polynomial $a(s)$ is unstable. This could happen for example by inaccurate estimation at the beginning of the control when an estimator does not have enough information about the system.

4 Identification Models

It was already mentioned that the controller is based on the adaptivity. There are several adaptive approaches used in the control theory. The one used here is based on the online recursive identification of the External Linear Model (ELM) of the originally nonlinear system. Parameters of the controller are then recomputed according to identified parameters of the ELM. ELM could be for example TF in the form of (4).

There will be discussed two types of identification models continuous-time (CT) and discrete-time (DT) in the next subchapters.

4.1 Continuous-Time Identification Model

The ELM of the controlled system is described by continuous-time TF $G(s)$ (4) and this relation is also described to the fraction of the the Laplace transform of the output variable, $Y(s)$, to the input variable, $U(s)$, the ELM in the (4) could be also rewritten to the form

$$a(\sigma) \cdot y(t) = b(\sigma) \cdot u(t) \tag{20}$$

where $u(t)$ denotes the input variable, $y(t)$ is the output variable and σ is the differentiation operator.

The identification of CT model in (20) is problem because the derivatives of the input and the output variables are immeasurable but they could be replaced by the filtered ones denoted by u_f and y_f and computed from

$$\begin{aligned} c(\sigma) \cdot u_f(t) &= u(t) \\ c(\sigma) \cdot y_f(t) &= y(t) \end{aligned} \tag{21}$$

for a new stable polynomial $c(\sigma)$ that fulfils condition $\deg c(\sigma) \geq \deg a(\sigma)$, the Laplace transform of (21) is then

$$\begin{aligned} c(s) \cdot U_f(s) &= U(s) + o_1(s) \\ c(s) \cdot Y_f(s) &= Y(s) + o_1(s) \end{aligned} \tag{22}$$

where polynomials $o_1(s)$ and $o_2(s)$ includes initial conditions of filtered variables. If we substitute (22) into the Laplace transform of the Equation (20), the relation for the Laplace transform of the filtered output variable, $Y_f(s)$ is

$$Y_f(s) = \frac{b(s)}{a(s)} \cdot U_f(s) + \Psi(s) \tag{23}$$

and $\Psi(s)$ is a rational function which contains initial conditions of both filtered and unfiltered variables.

The dynamics of the differential filters $c(s)$ in (20) must be faster than the dynamics of the controlled system [12] which is satisfied if parameters of this polynomial sufficiently small.

The values of filtered values are taken in the discrete time moment $t_k = k \cdot T_v$ for $k = 0, 1, 2, \ldots N$. T_v is sampling period and the regression vector has $n+m$ parts where $\deg a = n = 2$ and $\deg b = m = 1$, i.e.

$$\phi_{CT}(t_k) = \left[-y_f(t_k), -y_f^{(1)}(t_k), u_f(t_k), u_f^{(1)}(t_k)\right]^T \tag{24}$$

The vector of parameters

$$\theta_{CT}(t_k) = [a_0, a_1, b_0, b_1]^T \tag{25}$$

is computed from the differential equation

$$y_f^n(t_k) = \theta_{CT}^T(t_k) \cdot \phi_{CT}(t_k) + \Psi_{LQ}(t_k) \tag{26}$$

where $\Psi(t_k)$ includes immeasurable errors.

4.2 Discrete-Time Identification Model

The discrete-time identification model is better for practical purposes – it is more simple to read input and output variables in the defined time intervals then continuously. We can find also compromise between practically better DT model and more accurate CT model in so called delta-models [8] that are special types of DT models where input and output variables are related to the sampling period.

A new complex variable γ is defined generally as [13]

$$\gamma = \frac{z-1}{\beta \cdot T_v \cdot z + (1-\beta) \cdot T_v} \tag{27}$$

for T_v as a sampling period and β an optional parameter which holds $0 \leq \beta 1$. It is clear, that there could be an infinite number of delta-models as there is modifiability of β. One of the most used model is Forward delta-model for $\beta = 0$ was used here.

The complex variable γ is then

$$\gamma = \frac{z-1}{T_v} \tag{28}$$

It was proved for example in [6], that parameters of the delta-model approaches to the CT ones for sufficiently small sampling period T_v.

In delta-models, the CT model (20) can be rewritten to the form

$$a'(\delta) \cdot y(t') = b'(\delta) \cdot u(t') \tag{29}$$

where $a(\delta)$ and $b(\delta)$ are discrete polynomials and their coefficients are different from those in CT model but we suppose, that they are close to them because the sampling period is sufficiently small.

The regression vector is in this case for TF (4):

$$\begin{aligned} \phi_\delta(k-1) = [&-y_\delta(k-1), -y_\delta(k-2), \, , \\ &u_\delta(k-1), \ldots, u_\delta(k-2)]^T \end{aligned} \tag{30}$$

and the vector of parameters is then

$$\theta_\delta(k) = [a'_1, a'_0, b'_1, b'_0]^T \tag{31}$$

Parameters of this vector are computed again from the differential equation

$$y_\delta(k) = \theta_\delta^T(k) \cdot \phi_\delta(k-1) + e(k) \tag{32}$$

for $e(k)$ as a general random immeasurable component.

4.3 Identification Method

The last what needs to be described is the online identification method which satisfies the adaptivity of the controller. The Recursive Least-Squares Method [14] could be used because it is simple, accurate with modifications and it is also easily programmable.

This method is described in detail for example in [7] of [8].

Advantage of this method can be found also in the fact, that it could be easily modified with some kind of forgetting factor - for example exponential or directional. The constant exponential forgetting was used in this work for online identification.

5 Simulation Experiment

Proposed adaptive controller with two identification models was tested by simulation on the mathematical model of CSTR presented in Chapter 2. Also, both control configurations with one degree-of-freedom (1DOF) and two degrees-of-freedom (2DOF) mentioned in chapters 3.1 adn 3.2 were tested.

Due to comparability, both simulations were performed for the same simulation parameters. The sampling period was $T_v = 0.3\ min$, the initial covariance matrix $\mathbf{P}(0)$ has on the diagonal $1 \cdot 10^6$ and starting vectors of parameters for the identification was chosen $\theta_{CT}(0) = \theta_\delta(0) = [0.1; 0.1; 0.1; 0.1]^T$. The simulation was performed for 750 min and there were done 5 changes of the reference signal $w(t)$ during this time.

The controller needs some time for adaptation and our previous experiments have shown that it is good to insert the first change of the reference signal as an exponential function instead of the step function. The next changes were step functions The input signal $u(t)$ was limited to the values $u(t) = < -75\%; +75\% >$ due to physical limitations.

As it was mentioned, the tuning parameter for this adaptive controller is the position of the root α. There were observed courses of the output variable y for three values of $\alpha = 0.05; 0.08$ and 0.4 for both identification models and results are shown in the following figures.

The first analysis was done for the CT identification model where the filtered polynomial $c(\sigma)$ was $c(\sigma) = s^2 + 1.4s + 0.49$ and both control configurations.

Results in Fig. 5 - 8 show that the tuning parameter α affect mainly the speed of the control. Increasing value of α produces quicker output response but

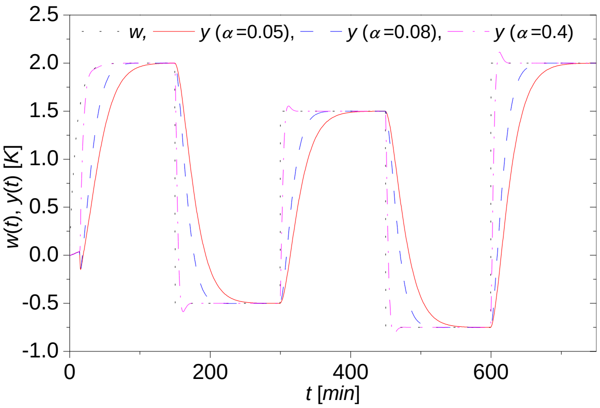

Figure 5: The course of the reference signal $w(t)$ and the output variable $y(t)$ for CT identification model and various parameter α, 1DOF

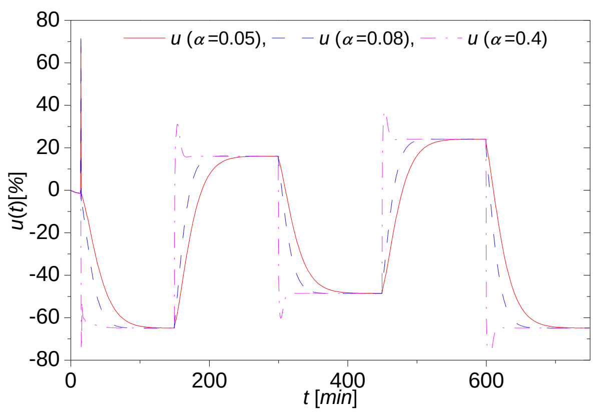

Figure 6: The course of the the input variable $u(t)$ for CT identification model and various parameter α, 1DOF

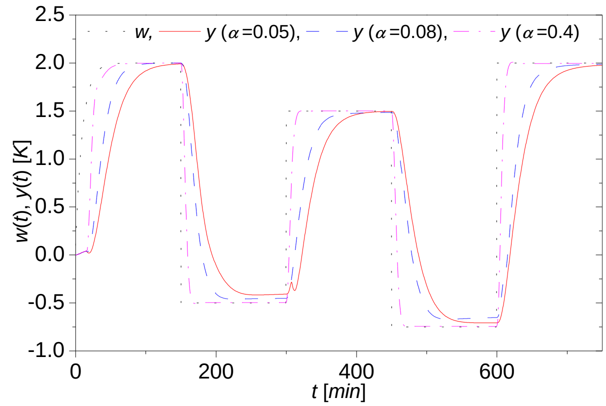

Figure 7: The course of the reference signal $w(t)$ and the output variable $y(t)$ for CT identification model and various parameter α, 2DOF

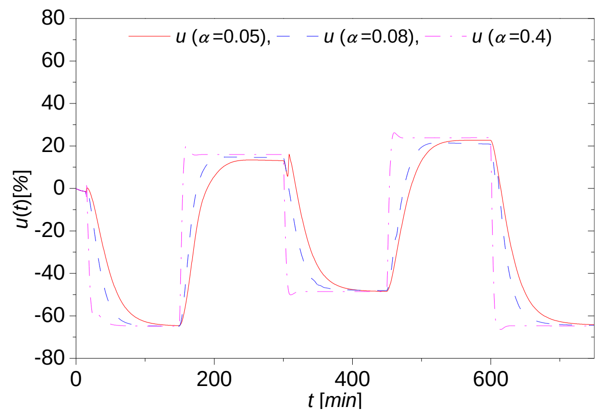

Figure 8: The course of the the input variable $u(t)$ for CT identification model and various parameter α, 2DOF

could end with small overshoots which is evident for $\alpha = 0.4$. On the other hand, a smaller value of α has smoother course of the input variable $u(t)$ in Figure 5 which is better from practical point of view.

The comparison of 1DOF and 2DOF control configuration shows very similar results, the only difference can be found in the course fo the biggest value of $\alpha = 0.4$, where 2DOF control configuration reduces overshoots after the step change of the reference signal $w(t)$.

The second analysis for DT delta model was performed for Forward delta model and the same tuning parameters α and both 1DOF and 2DOF control configurations.

Obtained results are very similar to those in the previous analysis. The biggest value of $\alpha = 0.4$ has again the quickest course but overshoots and their value depends on the height of the change. The course of the input variable is smoother for the lower values of tuning parameter. We can also say, that 2DOF control configuration has better control results mainly in the suppression of the overshoot of the output variable which can be compared in Figures 9 and 11.

As we want to compare and discuss the results more in detail, not only from the visual results, the control quality criteria S_u and S_y were introduced. These criteria took resulted input and output variables and compute values of criteria from the relations:

$$\begin{array}{ll} S_u = \sum_{i=2}^{N} (u(i) - u(i-1))^2 & [-] \\ S_u = \sum_{i=1}^{N} (w(i) - y(i))^2 & [K^2] \end{array} \quad (33)$$

where $N = \frac{T_f}{T_v}$ and final time $T_f - 450\ min$.

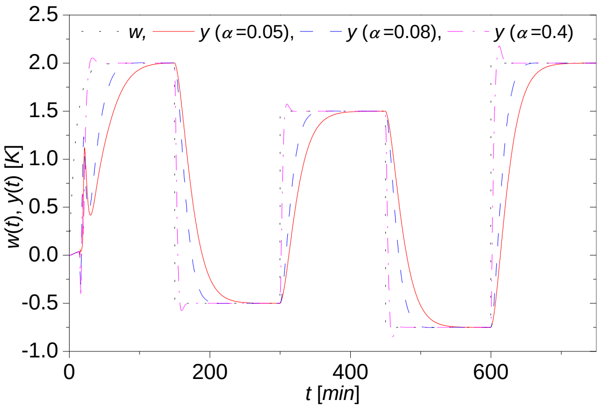

Figure 9: The course of the reference signal $w(t)$ and the output variable $y(t)$ for delta identification model and various parameter α, 1DOF

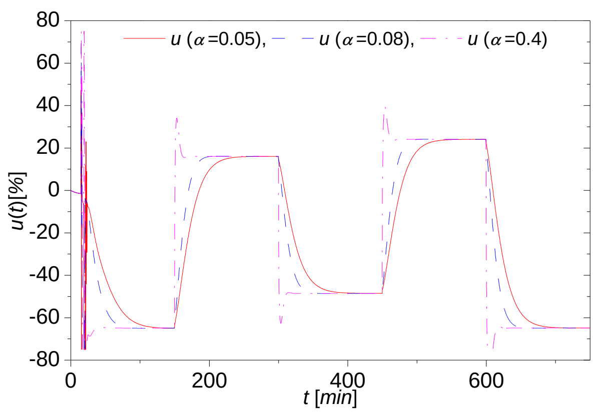

Figure 10: The course of the the input variable $u(t)$ for delta identification model and various parameter α, 1DOF

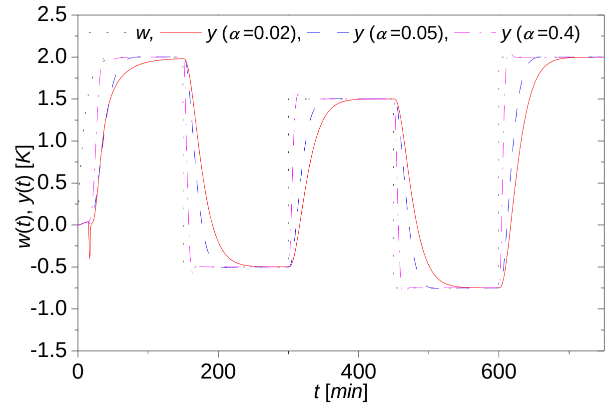

Figure 11: The course of the reference signal $w(t)$ and the output variable $y(t)$ for delta identification model and various parameter α, 2DOF

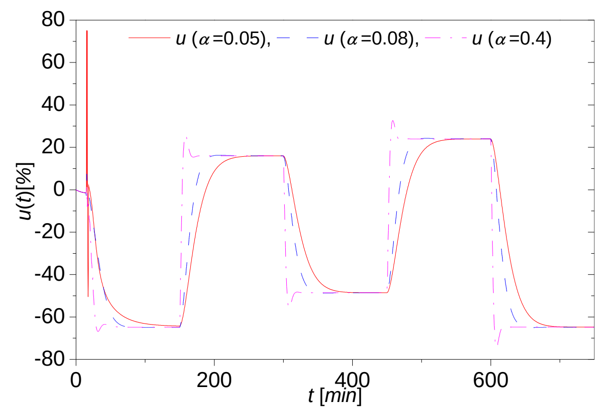

Figure 12: The course of the the input variable $u(t)$ for delta identification model and various parameter α, 2DOF

Values of these criteria for all simultions can be found in tables 2 and 3. Results are also displayed in bar graps in Fig. 13 and 14.

Values of both control quality criteria validate our previous assumptions. Bigger value of tuning parameter α produces better control results from the output point of view - i.e. value of S_y is , which display difference between actual output $y(t)$ and desired (wanted) value $w(t)$, is lower for bigger α. Oppositely, lower value of α is better from the input variable point of view because it produces lower values of S_u.

Both studies have very good control results except the beginning of the control. This is caused by the inaccurate identification which starts from the general point and it needs some time for adaptation. On the other hand, after initial 50 min the controller does not have problem with the online identification.

Courses of the identified parameters are in Figures 15 – 18. These figures show that recursive least squares method used for identification has no problem with the identification except the beginning of the control in the adaptation part.

6 Conclusion

The paper shows one approach for controlling of the nonlinear process represented by the continuous stirred-tank reactor with the cooling in the jacket. The mathematical model of this reactor is described by the set of four nonlinear ODE that are easily solvable by the numerical methods. Proposed control strategy is

Table 2: Values of control quality criteria S_u [-] for various α and both 1DOF and 2DOF control configurations

	Delta-model		*CT-model*	
α	1DOF	2DOF	1DOF	2DOF
0.05	60 522	23 939	10 001	277
0.08	34 476	390	13 056	373
0.4	67 757	1 704	23 435	2 202

Table 3: Values of control quality criteria S_y [K^2] for various α and both 1DOF and 2DOF control configurations

	Delta-model		*CT-model*	
α	1DOF	2DOF	1DOF	2DOF
0.05	1 513	1 732	1 006	1 254
0.08	994	1 232	632	887
0.4	266	453	134	312

based on the choice of the ELM parameters of which are identified recursively during the control and parameters of the controller are recomputed according to the identified ones.

The controller could be tuned by the parameter α as a position of the root in the pole-placement method. The simulation experiments have shown that the increasing value of this parameter affect the speed of the control and overshoots – bigger value of α results in quicker output response but overshoots. Results are also qualified with the control quality criteria S_u and S_y.

Two control configurations with one degree-of-freedom (1DOF) and two degrees-of-freedom (2DOF) were tested. 2DOF control configuration which has one part of the controller in the feedback loop and the second one in the feedforward part works better for bigger values of parameter α.

Two identification models with delta-models and differential filters were also discussed and compared. We can say, that both models has good and comparable results and they are suitable for this type of model with negative properties such as strong nonlinear behaviour etc.

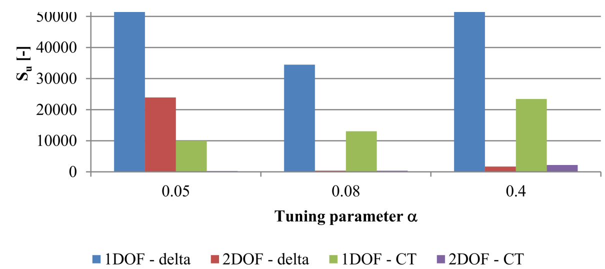

Figure 13: The course of the the input variable $u(t)$ for delta identification model and various parameter α, 2DOF

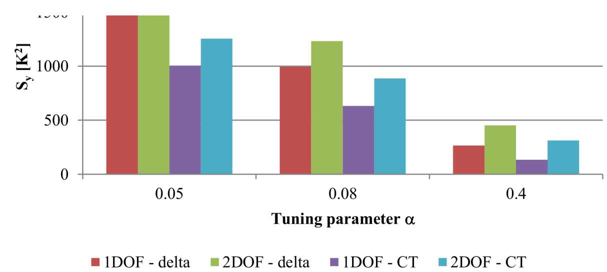

Figure 14: The course of the the input variable $u(t)$ for delta identification model and various parameter α, 2DOF

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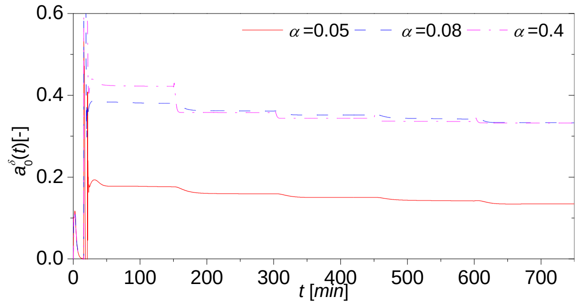

Figure 15: The course of the identified parameter $a_0^\delta(t)$ for various α in delta identification model, 1DOF

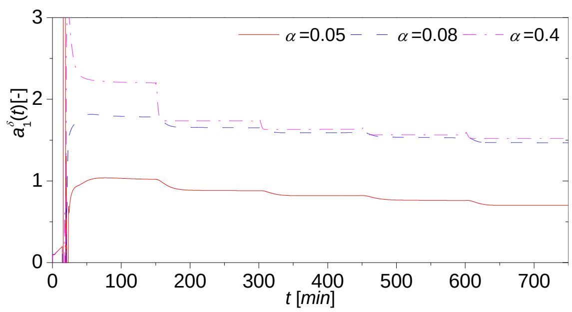

Figure 16: The course of the identified parameter $a_1^\delta(t)$ for various α in delta identification model, 1DOF

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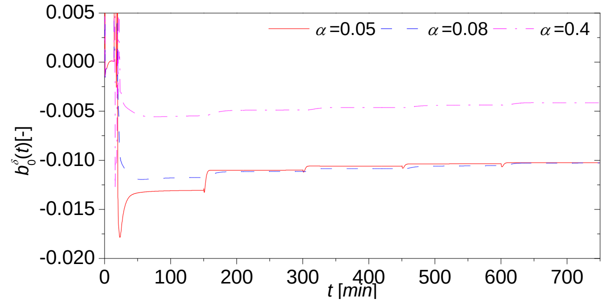

Figure 17: The course of the identified parameter $b_0^\delta(t)$ for various α in delta identification model, 1DOF

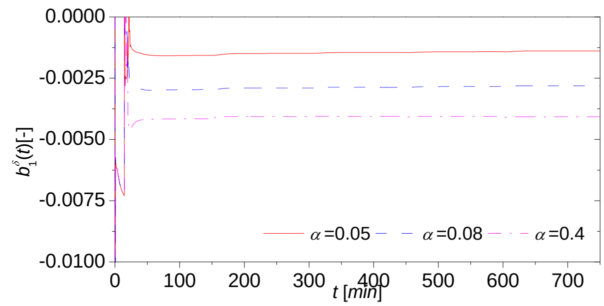

Figure 18: The course of the identified parameter $b_1^\delta(t)$ for various α in delta identification model, 1DOF

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