Some Aspects of Oscillatory Visco-elastic Flow Through Porous Medium in a Rotating Porous Channel

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Abstract:-This paper presents the study of three-dimensional flow and the injection/suction on an oscillatory flow of a visco-elastic incompressible fluid through a highly porous medium bounded between two infinite horizontal porous plates. The fluid injected with constant velocity through the lower stationary plate and is being sucked simultaneously with same constant velocity through the upper plate oscillating in its own plane about a nonzero constant mean velocity. On the basis of certain simplifying assumptions,closed form analytical solutions are therefore constructed and the important properties of the overall structure of the flow are discussed. Emphasis has been given on the effects of the visco-elastic parameter with the combination of other physical parameters.

Key-words: Visco-elastic, oscillatory , porous medium, porous channel , injection/suction, rotational parameter

1.Introduction:

The study of three-dimensional unsteady flow has been the object of extensive research due to its possible applications in many branches of science and Technology. The analysis of oscillatory fluid flow through porous medium in a rotating channel is of considerable interest because of its applications in different areas of aeronautics, missiles, aerodynamics etc. The unsteady oscillatory flows play an important role in chemical engineering, Turbo machinery and Aerospace Technology. In view of such applications, Rott and Lewellen [1] have investigated free convective flow through a porous medium. Free convection flow through a porous medium bounded by a vertical surface has been investigated by Raptis *et al*.[2]. Raptis [3] has also investigated a steady free convection and mass transfer through a porous medium bounded by an infinite vertical plate. The study of oscillatory flow through porous medium in presence of free convection flow was investigated by Raptis and Perdikis [4]. Singh and Kumar [5] have analysed unsteady two-dimensional free convection through porous medium bounded by an infinite vertical plate with permeability fluctuates in time about a constant mean. Baghel *et al*.[6] have analysed incompressible two-dimensional unsteady free convection through a rotating porous medium. Under different aspects Raptis and Singh [7] have extended their studies in free convective unsteady fluid flow through porous medium. Singh and Cowling [8] have studied the effect of free convection on magnetic electrically conducting fluids past a semi-infinite flat plate. The combined effects of forced and free convection through vertical surface was investigated by Loyed and Sparrow[9]. In recent years , a tremendous development has occurred in the modelling of non-Newtonian fluid flow mechanics. Non-Newtonian fluids, unlike Newtonian fluids present characteristics which can not be described by the classical linear viscous model. In non-Newtonian fluid flow mechanics, the analysis of visco-elastic fluid lies on the fact that these fluids possess certain degree of elasticity in addition to viscosity and also can exhibit normal stress and relaxation effects.

The Walters liquid (Model $B^{'}$) [10] is a visco-elastic fluid which was developed to simulate viscous fluids possessing short memory elastic effects and can simulate accurately many complex polymetric and biotechnological fluids. This model has therefore been studied extensively in many flow problems. A number of researchers like Soundalgekar and Puri [11], Nanousis [12], Choudhury and Das [13], Murthy *et al*.[14], Mustafa *et al*.[15], Choudhury and Dey[16], Cheng *et al.*[17], Choudhury and Das[18, 19] etc. have shown their interest in this dynamic and engineering field for its application in Geo-Physics, soil-Physics, Bio Physics, Chemical and Petroleum engineering, Hydrology, Paper and Pulp Technology etc.

In this paper , we investigate the effects of the flow of oscillatory Walters liquid (Model $B^{'}$) through porous medium in a rotating porous channel. The governing equations have been solved by using multi-parameter perturbation technique as discussed by Nowinski and Ismail [20]. To apply, this method the parameters must be independent of each another and must be of same order. In addition , these parameters must describe different physical and fluid properties such as the material and dynamic properties and must be small so that the higher powers and products can be neglected.

2. Mathematical Formulation:

An oscillatory unsteady flow of a visco-elastic incompressible fluid through highly porous medium which is bounded between two infinite parallel plates has been considered. Here w_0 is a constant injection velocity which is applied at the lower stationary plate and the same constant suction velocity w_0 is applied at the upper plate which is oscillating in its own plane with a non-zero constant mean velocity U_0.A co-ordinate system is taken with origin at the lower stationary plate lying in the $x^{'}y^{'}$-plane with $x^{'}$ axis along the plate in the upward direction and $y^{'}$ axis normal to the plate. The $z^{'}$ axis is taken normal to the plane of the plates, which is the axis of the rotation about which the entire system is rotating with constant angular velocity $\Omega^{'}$. Since the plates are of infinite in length, all physical quantities except the pressure depends only $z^{'}$ and $t^{'}$. We consider the velocity components $u^{'}, v^{'}, w^{'}$ in the $x^{'}, y^{'}, z^{'}$ directions respectively and the flow in the rotating system is governed by the following equations:

Equation of continuity:

$$\frac{\partial w^{'}}{\partial z^{'}} = 0 \tag{1}$$

Momentum equations:

$$\frac{\partial u^{'}}{\partial t^{'}} + w_0 \frac{\partial u^{'}}{\partial z^{'}} = -\frac{1}{\rho}\frac{\partial p^{'}}{\partial x^{'}} + \nu \frac{\partial^2 u^{'}}{\partial z^{'2}} - k_0 \left[\frac{\partial^3 u^{'}}{\partial t^{'}\partial z^{'2}} + w_0 \frac{\partial^3 u^{'}}{\partial z^{'3}}\right] + 2\Omega^{'} v^{'} - \nu \frac{u^{'}}{K^{'}} \tag{2}$$

$$\frac{\partial v^{'}}{\partial t^{'}} + w_0 \frac{\partial v^{'}}{\partial z^{'}} = -\frac{1}{\rho}\frac{\partial p^{'}}{\partial y^{'}} + \nu \frac{\partial^2 v^{'}}{\partial z^{'2}} - k_0 \left[\frac{\partial^3 v^{'}}{\partial t^{'}\partial z^{'2}} + w_0 \frac{\partial^3 v^{'}}{\partial z^{'3}}\right] - 2\Omega^{'} u^{'} - \nu \frac{v^{'}}{K^{'}} \tag{3}$$

where $\nu = \frac{\eta_0}{\rho}$

The boundary conditions for the problem are

$$\left.\begin{aligned} & u^{'} = v^{'} = 0\,,\ w^{'} = w_0 \quad at \quad z^{'} = 0 \\ & u^{'} = U^{'}(t) = u_0(1 + \varepsilon cos\omega^{'} t^{'})\,, \\ & v^{'} = 0\,, w^{'} = w_0 \quad at \quad z^{'} = d \end{aligned}\right\} \tag{4}$$

where $\omega^{'}$ is the frequency of oscillation and ε is very small positive number.

Eliminating the pressure gradient , under the usual boundary layer approximation, we get,

$$\frac{\partial U^{'}}{\partial t^{'}} = -\frac{1}{\rho}\frac{\partial p^{'}}{\partial x^{'}} - \nu \frac{U^{'}}{K^{'}} \tag{5}$$

$$\frac{\partial u^{'}}{\partial t^{'}} + w_0 \frac{\partial u^{'}}{\partial z^{'}} = \nu \frac{\partial^2 u^{'}}{\partial z^{'2}} + \frac{\partial U^{'}}{\partial t^{'}} + 2\Omega^{'} v^{'} - \frac{\nu}{K^{'}}(u^{'} - U^{'}) - k_0 \left[\frac{\partial^3 u^{'}}{\partial t^{'}\partial z^{'2}} + w_0 \frac{\partial^3 u^{'}}{\partial z^{'3}}\right] \tag{6}$$

$$\frac{\partial v'}{\partial t'} + w_0 \frac{\partial v'}{\partial z'} = \nu \frac{\partial^2 v'}{\partial z'^2} - 2\Omega'(u' - U') - \nu \frac{v'}{K'} - k_0 \left[\frac{\partial^3 v'}{\partial t' \partial z'^2} + w_0 \frac{\partial^3 v'}{\partial z'^3}\right] \quad (7)$$

We introduce the following non-dimensional quantities ,

$$\left.\begin{aligned} &\eta = \frac{z'}{d}, t = \omega' t', u = \frac{u'}{U_0}, v = \frac{v'}{U_0} \\ &, \Omega = \Omega' \frac{d^2}{\nu}, \omega = \omega' \frac{d^2}{\nu}, s = \frac{w_0 d}{\nu}, \\ &K = \frac{K'}{d^2}, U = \frac{U'}{U_0}, \end{aligned}\right\} \quad (8)$$

where Ω is the rotation of parameter, ω is the frequency parameter, s is the injection/suction parameter and K is the permeability parameter.

Using (8) into the equations (6) and (7) , we have

$$\omega \frac{\partial u}{\partial t} + s \frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2} + \omega \frac{\partial U}{\partial t} + 2\Omega v - \frac{1}{K}(u - U) - \frac{k_0}{d^2}\left[s \frac{\partial^3 u}{\partial \eta^3} + \omega \frac{\partial^3 u}{\partial \eta^2 \partial t}\right] \quad (9)$$

$$\omega \frac{\partial v}{\partial t} + s \frac{\partial v}{\partial \eta} = \frac{\partial^2 v}{\partial \eta^2} + \omega \frac{\partial U}{\partial t} - 2\Omega(\mathrm{u} - \mathrm{U}) - \frac{v}{K} - \frac{k_0}{d^2}\left[s \frac{\partial^3 v}{\partial \eta^3} + \omega \frac{\partial^3 v}{\partial \eta^2 \partial t}\right] \quad (10)$$

subject to the boundary conditions

$$\left.\begin{aligned} &u = v = 0 \quad at \ \eta = 0 \\ &u = U(t) = 1 + \varepsilon cost, \\ &\quad v = 0, \ at \ \eta = 1 \end{aligned}\right\} \quad (11)$$

Equation (9) and (10) can now be combined into a single equation , by introducing the complex function $q = u + iv$ as

$$\omega \frac{\partial q}{\partial t} + s \frac{\partial q}{\partial \eta} = \frac{\partial^2 q}{\partial \eta^2} + \omega \frac{\partial U}{\partial t} - 2\Omega i(q - U) - \frac{1}{K}(q - U) - k\left[s \frac{\partial^3 q}{\partial \eta^3} + \omega \frac{\partial^3 q}{\partial \eta^2 \partial t}\right] \quad (12)$$

where $k = \frac{k_0}{d^2}$

The relevent boundary conditions are:

$$\left.\begin{aligned} &q = 0 \quad at \ \eta = 0 \\ &q = U(t) = 1 + \frac{\varepsilon}{2}\left(e^{it} + e^{-it}\right) \\ &\quad at \ \eta = 1 \end{aligned}\right\} \quad (13)$$

3 .Method of Solution:

In order to solve the equation (12) subject to the boundary conditions (13) , we look for a solution of the form

$$q(\eta, t) = q_0(\eta) + \frac{\varepsilon}{2}\{q_1(\eta)e^{it} + q_2(\eta)e^{-it}\} \quad (14)$$

Substituting (14) into the equation (12), and comparing the harmonic and non-harmonic terms, we get

$$ksq_0''' - q_0'' + sq_0' + \left(l^2 + \frac{1}{K}\right)q_0 = \left(l^2 + \frac{1}{K}\right) \quad (15)$$

$$ksq_1''' + (k\omega i - 1)q_1'' + sq_1' + \left(m^2 + \frac{1}{K}\right)q_1 = \left(m^2 + \frac{1}{K}\right) \quad (16)$$

$$ksq_2''' - (k\omega i + 1)q_2'' + sq_2' + \left(n^2 + \frac{1}{K}\right)q_2 = \left(n^2 + \frac{1}{K}\right) \quad (17)$$

where $l^2 = 2\Omega i$, $m^2 = i(2\Omega + \omega)$,

$n^2 = i(2\Omega - \omega)$

Here prime denotes differentiation with respect to η.

The transformed boundary conditions are:

$$\left.\begin{aligned} &q_0 = q_1 = q_2 = 0 \quad at \ \eta = 0 \\ &q_0 = q_1 = q_2 = 1 \quad at \ \eta = 1 \end{aligned}\right\} \quad (18)$$

To solve the equations (15) to (17) under boundary conditions (18) , we consider the transformation as

$$q_0(\eta) = q_{00}(\eta) + kq_{01}(\eta) + o(k^2) \quad (19)$$

$$q_1(\eta) = q_{10}(\eta) + kq_{11}(\eta) + 0(k^2) \quad (20)$$

$$q_2(\eta) = q_{20}(\eta) + kq_{21}(\eta) + 0(k^2) \quad (21)$$

where $k \ll 1$ for small shear rate.

Substituting (19) to (21) into equations (15) to (17) , and comparing the like powers of k , neglecting higher powers of k we get

$$\left.\begin{aligned} q_{00}'' - sq_{00}' - \left(l^2 + \frac{1}{K}\right) q_{00} &= -\left(l^2 + \frac{1}{K}\right) \\ sq_{00}''' - q_{01}'' + sq_{01}' + \left(l^2 + \frac{1}{K}\right) q_{01} &= 0 \end{aligned}\right\} \quad (22)$$

with boundary conditions :

$$\left.\begin{aligned} q_{00} = q_{01} = 0 \quad at \ \eta = 0 \\ q_{00} = 1 , q_{01} = 0 , at \ \eta = 1 \end{aligned}\right\} \quad (23)$$

and

$$\left.\begin{aligned} q_{10}'' - sq_{10}' - \left(m^2 + \frac{1}{K}\right) q_{10} &= -\left(m^2 + \frac{1}{K}\right) \\ sq_{10}''' + i\omega q_{10}'' - q_{11}'' + sq_{11}' + \left(m^2 + \frac{1}{K}\right) q_{11} &= 0 \end{aligned}\right\} \quad (24)$$

with boundary conditions :

$$\left.\begin{aligned} q_{10} = 0 , q_{11} = 0 \ at \ \eta = 0 \\ q_{10} = 1 , q_{11} = 0 \ at \ \eta = 1 \end{aligned}\right\} \quad (25)$$

Also

$$\left.\begin{aligned} q_{20}'' - sq_{20}' - \left(n^2 + \frac{1}{K}\right) q_{20} &= -\left(n^2 + \frac{1}{K}\right) \\ sq_{20}''' - i\omega q_{20}'' - q_{21}'' + sq_{21}' + \left(n^2 + \frac{1}{K}\right) q_{21} &= 0 \end{aligned}\right\} \quad (26)$$

with boundary conditions

$$\left.\begin{aligned} q_{20} = 0 , q_{21} = 0 \quad at \ \eta = 0 \\ q_{20} = 1 , q_{21} = 0 , at \ \eta = 1 \end{aligned}\right\} \quad (27)$$

Solving the equations (22) , (24) , (26) with boundary conditions (23) , (25) , (27) respectively and substituting these values in (19) to (21) we get the solutions as

$$q_0 = 1 - \left[\frac{e^{\alpha_1 + \alpha_2 \eta} - e^{\alpha_2 + \alpha_1 \eta}}{e^{\alpha_1} - e^{\alpha_2}}\right] + k\left[A_4 e^{\beta_1 + \alpha_1 \eta} - A_4 e^{\beta_1 + \alpha_2 \eta} + A_2 \eta e^{\alpha_1 + \alpha_2 \eta} - A_1 \eta e^{\alpha_2 + \alpha_1 \eta}\right] \quad (28)$$

$$q_1 = 1 - \left[\frac{e^{\alpha_3 + \alpha_4 \eta} - e^{\alpha_4 + \alpha_3 \eta}}{e^{\alpha_3} - e^{\alpha_4}}\right] + k\left[A_{15}\left(e^{\beta_2 + \alpha_3 \eta} - e^{\beta_2 + \alpha_4 \eta}\right) + A_9 \eta e^{\alpha_3 + \alpha_4 \eta} - A_{10} \eta e^{\alpha_4 + \alpha_3 \eta} + i\left\{A_{16}\left(e^{\beta_2 + \alpha_4 \eta} - e^{\beta_2 + \alpha_3 \eta}\right) + A_{11} \eta e^{\alpha_3 + \alpha_4 \eta} - A_{12} \eta e^{\alpha_4 + \alpha_3 \eta}\right\}\right] \quad (29)$$

$$q_2 = 1 - \left[\frac{e^{\alpha_5 + \alpha_6 \eta} - e^{\alpha_6 + \alpha_5 \eta}}{e^{\alpha_5} - e^{\alpha_6}}\right] + k\left[A_{27}\left(e^{\beta_3 + \alpha_5 \eta} - e^{\beta_3 + \alpha_6 \eta}\right) + A_{21} \eta e^{\alpha_5 + \alpha_6 \eta} - A_{22} \eta e^{\alpha_6 + \alpha_5 \eta} + i\left\{A_{28}\left(e^{\beta_3 + \alpha_5 \eta} - e^{\beta_3 + \alpha_6 \eta}\right) - A_{23} \eta e^{\alpha_5 + \alpha_6 \eta} + A_{24} \eta e^{\alpha_6 + \alpha_5 \eta}\right\}\right] \quad (30)$$

where the constants are obtained but not given here due to sake of brevity.

Now , for the resultant velocity and the shear stress of the steady and unsteady flow , we write

$$u_0(\eta) + iv_0(\eta) = q_0(\eta) \quad (31)$$

and $u_1(\eta) + iv_1(\eta) =$

$$q_1(\eta) e^{it} + q_2(\eta) e^{-it} \quad (32)$$

The solution (31) corresponds to the steady part which gives u_0 as the primary and v_0 as the secondary velocity components. The amplitude and phase difference due to these primary and secondary velocities for the steady flow are given by

$$\left.\begin{aligned} R_0 &= \sqrt{u_0^2 + v_0^2} \\ \theta_0 &= \tan^{-1}\left(\frac{v_0}{u_0}\right) \end{aligned}\right\} \quad (33)$$

The amplitude and the phase difference of the shear stress at the stationary plate $(\eta = 0)$ for the steady flow can be obtained as

$$\tau_{0r} = \sqrt{\tau_{0x}^2 + \tau_{0y}^2} \qquad \theta_{0r} = \tan^{-1}\left(\frac{\tau_{0y}}{\tau_{0x}}\right) \quad (34)$$

where $\tau_{0x} + i\tau_{0y} = \left(\frac{\partial q}{\partial \eta}\right)_{\eta=0}$

$$\tau_{0x} = \left[\frac{\partial u_0}{\partial \eta} - k\left(\omega \frac{\partial^2 u_0}{\partial \eta \partial t} + s\frac{\partial^2 u_0}{\partial \eta^2}\right)\right]_{\eta=0}$$

$$\tau_{0y} = \left[\frac{\partial v_0}{\partial \eta} - k\left(\omega \frac{\partial^2 v_0}{\partial \eta \partial t} + s\frac{\partial^2 v_0}{\partial \eta^2}\right)\right]_{\eta=0}$$

Here , τ_{0x} and τ_{0y} are , respectively , the shear stresses at the stationary plate due to the primary and secondary velocity components.

The solutions of (20) and (21), together give the unsteady part of the flow. The unsteady primary and secondary velocity components can be obtained as

$$u_1 = (q_1 + q_2)cost$$

$$v_1 = (q_1 - q_2)sint$$

The resultant velocity and phase difference for the unsteady flow can be obtained as

$$\left.\begin{aligned} R_1 &= \sqrt{u_1^2 + v_1^2} \\ \theta_{1r} &= \tan^{-1}\left(\frac{v_1}{u_1}\right) \end{aligned}\right\} \qquad (35)$$

The amplitude and phase difference of shearing stress for the unsteady part of the flow, at the stationary plate $\eta = 0$ can be obtained as

$$\tau_{1x} + i\tau_{1y} = \left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0} + i\left(\frac{\partial v_1}{\partial \eta}\right)_{\eta=0}$$

which gives

$$\tau_{1r} = \sqrt{\tau_{1x}^2 + \tau_{1y}^2} \ , \ \theta_{1r} = \tan^{-1}\left(\frac{\tau_{1y}}{\tau_{1x}}\right)$$

3. Results and Discussion:

For the purpose of discussing the effects of various physical parameters on the flow behaviours, numerical calculations have been carried out for different values of these parameters viz. Rotation parameter Ω , the frequency parameters ω, the injection /suction parameters S, the permeability parameters K. Emphasis has been given on visco-elastic parameter which is exhibited through the non-dimensional parameter k. The non-zero values of k characterize the visco-elastic fluid and $k = 0$ represents the Newtonian fluid. The fluid velocity and the shearing stress at the stationary plate have been analyzed graphically for various values of flow parameters involved in the solution.

The resultant fluid velocity R_0 (for steady part) and R_1 (for unsteady part) against η have been depicted in the figures 1 to 4 and 5 to 9 respectively. In steady part (figures 1 to 4), it is observed that the resultant fluid velocity, accelerates with the growth of visco-elasticity in compared to simple Newtonian Newtonian fluid with the variation of physical parameters

K , s and Ω. Again, for unsteady part (figures 5 to 9), the resultant fluid velocity R_1 reveals an enhancement for both Newtonian and visco-elastic fluids with the variation of physical parameters S , Ω and ω in the fluid flow region. The pattern is same in all the figures.

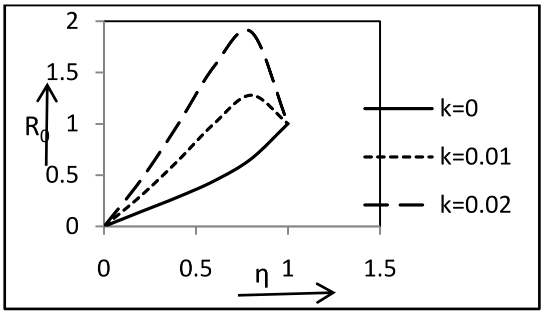

Figure 1: Resultant velocity R_0 against η for K=2 , S=3 , Ω=2

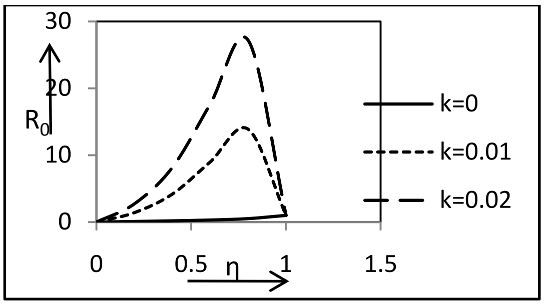

Figure 2: Resultant velocity R_0 against η for K=2 , S=5 , Ω=2

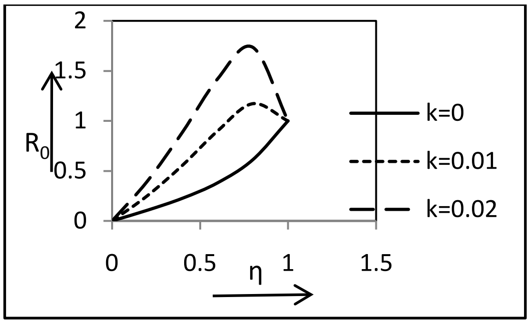

Figure 3: Resultant velocity R_0 against η for K=2 , S=3 , Ω=1

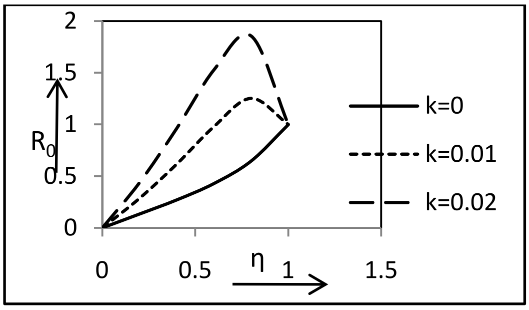

Figure 4: Resultant velocity R_0 against η for K=4 , S=3 , Ω=2

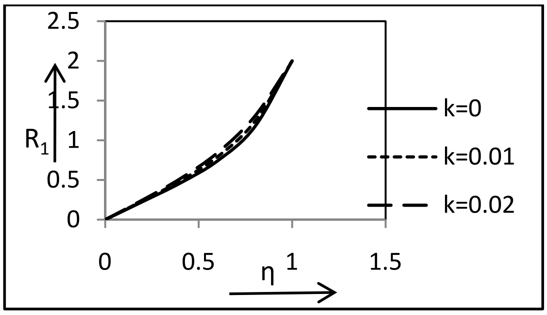

Figure5: Resultant velocity R_1 against η for K=2 , S=3 , Ω=2 , ω=1

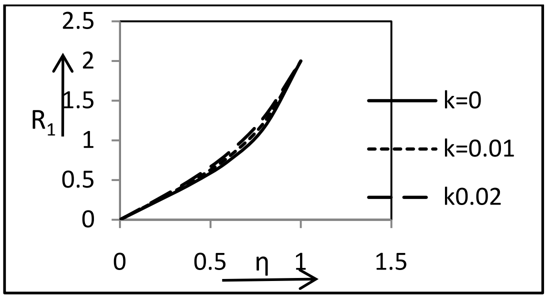

Figure 6: Resultant velocity R_1 against η for K=2 , S=5 , Ω=2 , ω=1

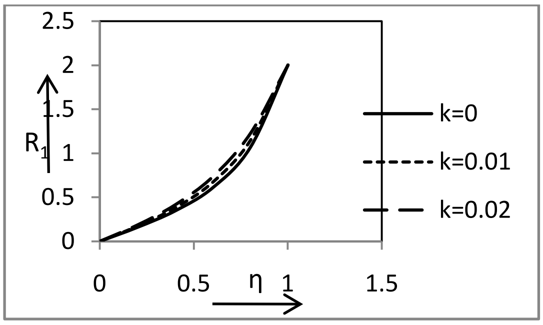

Figure 7: Resultant velocity R_1 against η for K=2 , S=3 , Ω=1 , ω=1

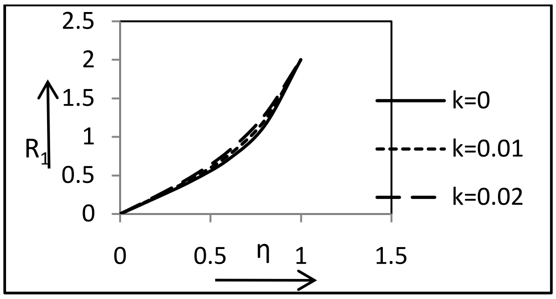

Figure 8: Resultant velocity R_1 against η for K=4 , S=3 , Ω=2 , ω=1

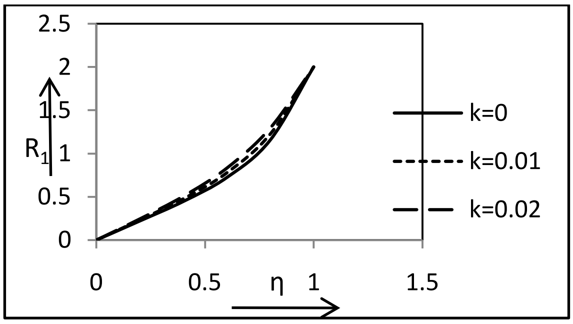

Figure 9: Resultant velocity R_1 against η for K=2 , S=3 , Ω=2 , ω=3

The amplitude τ_{or} of the shearing stress at the stationary plate (η_0) for the steady flow has been presented against the permeability parameter K (figure10), injection/suction parameter S (figure 12) and rotation parameter Ω (figure 14) respectively. In all the cases , it is observed that the amplitude τ_{0r} of the shearing stress enhances in the fluid flow region with the growth of visco-elasticity in comparison with Newtonian fluid with combination of other flow problems but for the amplitude τ_{1r} of the shearing stress at the stationary plate $(\eta = 0)$ for the unsteady flow against the parameters K (permeability parameter), S (suction parameter), Ω (rotation parameter) and ω (frequency parameter) the reverse pattern has been observed (figures 11 , 13, 15, 17).

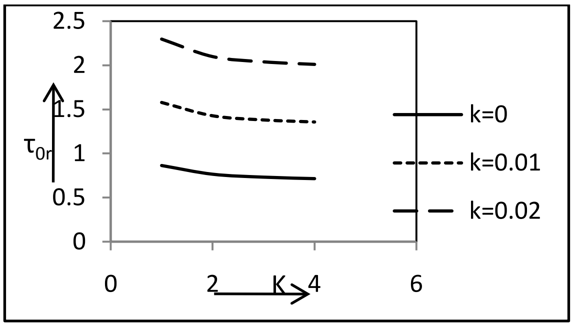

Figure 10: Shearing stress τ_{0r} against K for S=3 , Ω=2 , ω=1

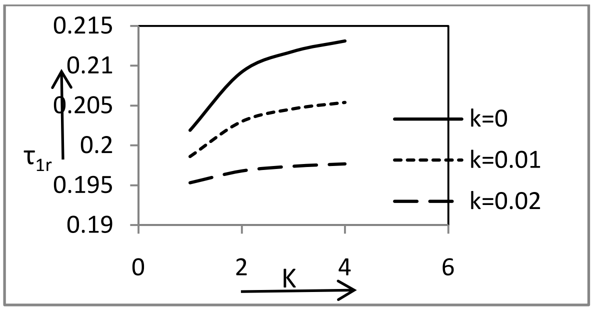

Figure 11: Shearing stress τ_{1r} against K for S=3 , Ω=2, ω=1

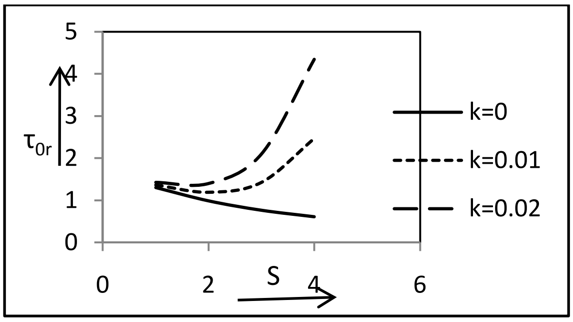

Figure 12: Shearing stress τ_{0r} against S for K=2 , Ω=2 , ω=1

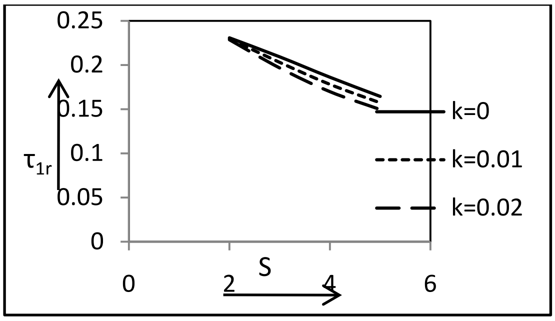

Figure 13: Shearing stress τ_{1r} against S for K=2 , Ω=2, ω=1

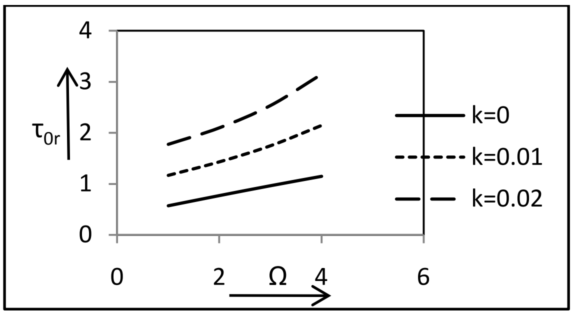

Figure 14: Shearing stress τ_{0r} against Ω for K=2 , S=3, ω=1

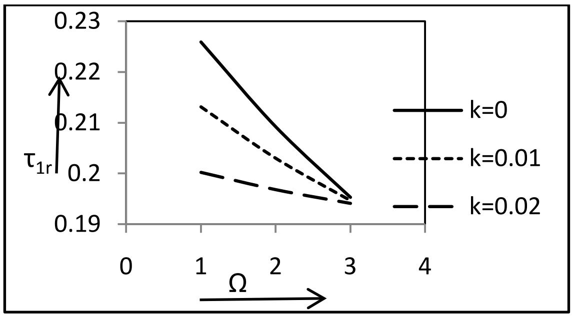

Figure 15: Shearing stress τ_{1r} against Ω for K=2 , S=3, ω=1

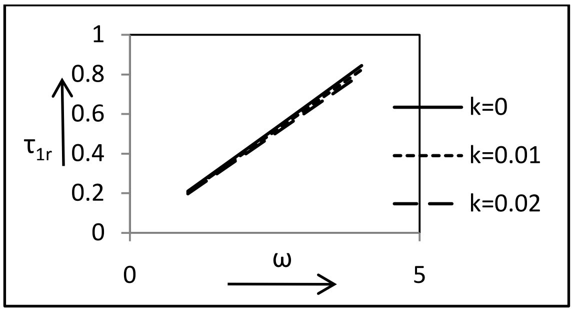

Figure 16: Shearing stress τ_{1r} against ω for K=2 , S=3, Ω=2

Again, the phase difference θ_{0r} (for steady flow) and θ_{1r} (for unsteady flow) of the shearing stress against different physical parameter have been depicted in figures 17 to 23. For steady flow, the phase difference θ_{0r} shows an decelerating trend with the growth of visco-elasticity as well as the increase of permeability parameter K (figure 17), injection/suction parameter S, (figure 19) and the rotation parameter Ω (figure 21) respectively and the same patterns are observed for the unsteady flow (figures 18, 20, 22, 23). It may be remarked that the effects of visco-elastic parameter in combination of other flow parameters play a significant role in this observation.

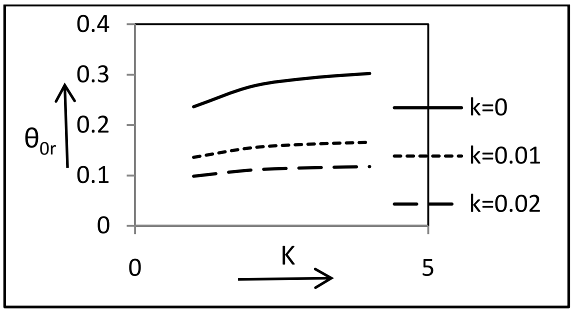

Figure 17: Phase difference θ_{0r} against K for S=3 , Ω=2 , ω=1

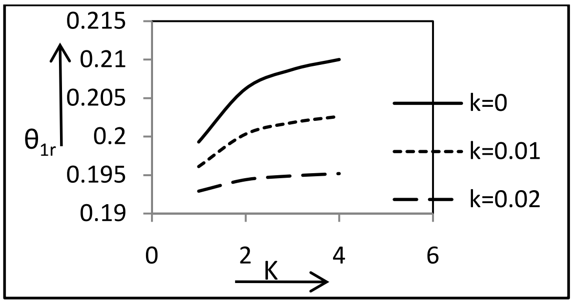

Figure 18: Phase difference θ_{1r} against K for S=3, Ω=2 , ω=1

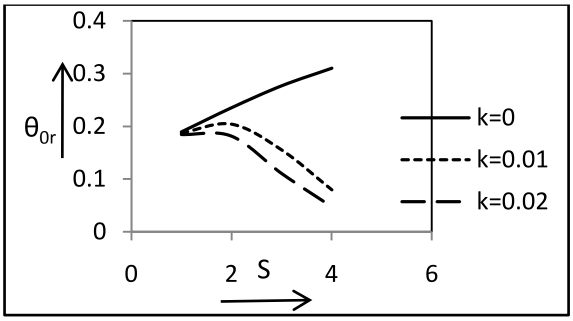

Figure 19: Phase difference θ_{0r} against S for K=2 , Ω=2 , ω=1

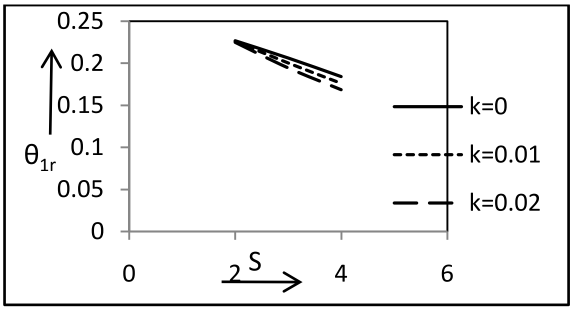

Figure 20: Phase difference θ_{1r} against S for K=2, Ω=2 , ω=1

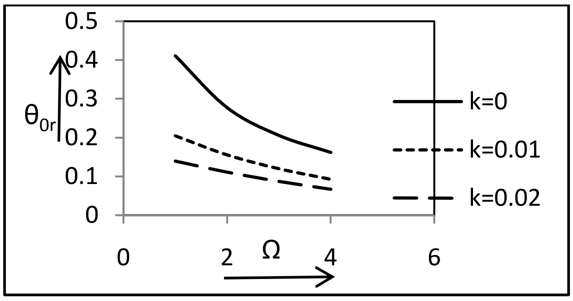

Figure 21: Phase difference θ_{0r} against 'Ω for K=2 , S=2 , ω=1

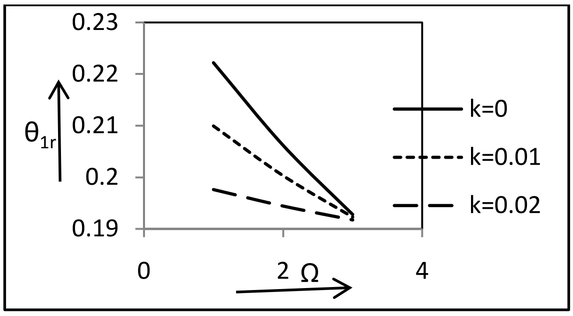

Figure 22: Phase difference θ_{1r} against Ω for K=2, S=3 , ω=1

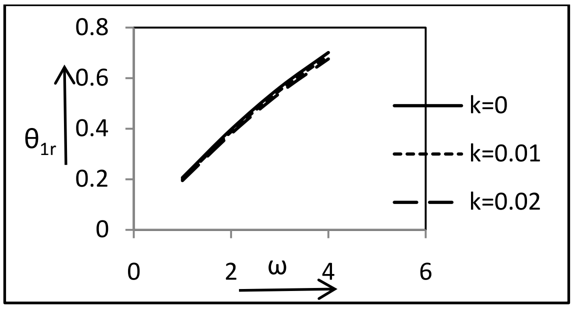

Figure 23: Phase difference θ_{1r} against ω for K=2, S=3 , Ω=2

4.Conclusion:

A theoretical analysis has been performed to study the influence of visco-elasticity on an oscillatory unsteady flow of a visco-elastic fluid through a highly porous medium which is bounded between two infinite parallel plates in presence of constant suction/injection. The investigation gives the following conclusion.

The growth of visco-elasticity accelerates the resultant velocity in both steady and unsteady parts of the fluid flow region.The amplitude of the shearing stress at the stationary plate depicts a rising trend with the increasing values of the visco-elastic parameter for steady part of the flow. The enhancement of visco-elastic parameter decelerates the amplitude of the shearing stress at the stationary plate for unsteady part of the flow.The phase difference of the shearing stress reveals a diminishing trend when the visco-elastic parameter value increases in both steady and unsteady parts of the flow.

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