Analysis of laminar boundary layer flow along a stretching cylinder in the presence of thermal radiation

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Abstract: - An analysis has been carried out to obtain the effects of thermal radiation on axi-symmetric laminar boundary layer flow of a viscous incompressible fluid along a stretching cylinder. Rosseland approximation has been use to model the radiative heat transfer. Using the similarity transformation, the partial differential equations corresponding to the momentum and heat equations have been transformed to a set of non-linear ordinary differential equations. These equations have been solved numerically using Runge-Kutta Fehlberg method with shooting technique. In the present reported work, the effects of radiation parameter, curvature parameter, Prandtl number and temperature exponent parameter on flow and heat transfer characteristics have been discussed. Variations of these parameters have been graphically presented. The reported results have been found to be in good agreement with the available published work in the literature.

Key-Words: - Laminar flow, boundary layer, stretching cylinder, radiation.

1 Introduction

Flows over cylinder are considered to be two dimensional when the body radius is large as compared to the boundary layer thickness. For lean cylinder, the radius of the cylinder may be of the same order as that of the boundary layer thickness. Therefore, the flow may be considered as axisymmetric instead of two dimensional. In such a case, the governing equations contain the transverse curvature term. It strongly influences the velocity and temperature fields and correspondingly affects the skin friction coefficient and heat transfer rate at the wall. The study of boundary layer flow and heat transfer over a moving stretching surface is of interest in many industrial applications. Their application included polymer extrusion process, where the object enters the fluid for cooling below a certain temperature, hot rolling, paper production, wire drawing, aerodynamic extrusion of plastic sheets etc. The quality of the final product depends on the rate of heat transfer at the stretching surface.

Cebeci [1] investigated the magnitude of the transverse curvature effect for isothermal laminar flows. He has reported that the local skin friction can be altered by an order of magnitude due to an appropriate change in the ratio of boundary layer thickness to cylinder radius. Sakiadis [2] was the first to consider the boundary layer flow on a moving continuous solid surface. Crane [3] extended the work of Sakidas [2] by studying the flow past over a stretching plate. Many authors extend Crane work for stretching sheet in different cases [4-7]. Wang [8] has presented the effect the steady flows in a viscous and incompressible fluid outside of a stretching hollow cylinder in an ambient fluid. Lin and Shih [9, 10] considered the laminar boundary layer and heat transfer along horizontally and vertically moving cylinders with constant velocity. They found that the similarity solutions could not be obtained due to the curvature effect of the cylinder. Ishak and Nazar [11] considered the laminar boundary layer and heat transfer along horizontal moving cylinder with variable velocity. They found that the similarity solutions may be obtained by assuming that the cylinder is stretched with linear velocity in the axial direction. Their study has been considered as the extension of the papers by Grubka and Bobba [12] and Ali [13], from a stretching sheet to a stretching cylinder. Laminar boundary layer flow along stretching surfaces has been discussed by Elbashbeshy et al. [14], Sami [15], Mukhopadhyay [16].

The study of heat transfer in fluid becomes much more interesting due to its outstanding applications. It is used in the heat removal from nuclear plant, underground disposal of radioactive waste material. Its applications are found in wide range of engineering fields such as geophysics, space technology, petroleum engineering. In the context of space technology and in process involving high temperatures, the effects of radiation are of vital importance. Arpaci [17], Thaker et al. [18] investigated combined heat and mass transfer along a vertical moving cylinder with a free stream. Das et al. [19] studied the unsteady flow past a moving vertical plate in the presence of free convection and radiation. Prasad et al. [20] have reported the effects of radiation and mass transfer effects on two-dimensional flow past an impulsively started isothermal vertical plate. The combined radiation and free convection flow over a vertical cylinder has been presented by Yih [21].

In the present study, the effect of thermal radiation on laminar boundary layer flow along a stretching horizontal cylinder has been discussed. The results obtained have been compared with those of Ishak and Nazar [11] and Elbashbeshy et al. [14], as the limiting case of the presented study. Results obtained shows that the flow field is influenced appreciably by the radiation and curvature parameter. Estimation of skin friction and heat transfer coefficient which are very important for the industrial application have been calculated.

2 Problem Formulation

A steady, axi-symmetric boundary layer flow of a viscous and incompressible fluid along a continuously stretching horizontal cylinder of radius a as shown in Fig.1 has been considered.

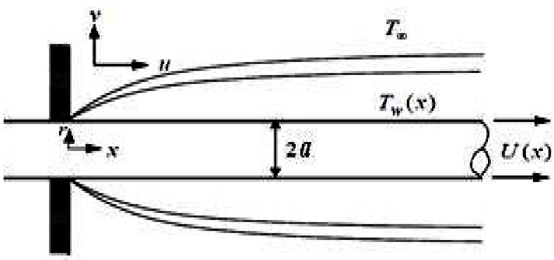

Fig. 1 Physical model of the problem

It is assumed that stretched surface has the velocity $U_w(x) = U_0(x/l)$ and $T_w(x) = T_\infty + T_0(x/l)^n$ is the prescribed surface temperature, where U_0, T_0 are the reference velocity and temperature respectively, T_∞ is the ambient temperature, l is the characteristic length and n is the temperature exponent.

The continuity, momentum and energy equations governing such type of flow are written as:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) \quad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) - \frac{1}{\rho c_p}\frac{1}{r}\frac{\partial}{\partial y}(rq_r) \quad (3)$$

where u and v are the components of velocity in x and r directions respectively, $\nu = \mu / \rho$ is the kinematic viscosity, ρ is the fluid density, μ is the coefficient of fluid viscosity, T be the fluid temperature, α is the thermal diffusivity and q_r is the radiation heat flux.

Here q_r is approximated by Rosseland approximation, which gives: $q_r = -\frac{4\sigma_s}{3k}\frac{\partial T^4}{\partial y}$ (4)

where k is mean absorption coefficient, σ_s is Stefan-Boltzmann constant. It is assumed that the temperature difference within the flow is so small that T^4 can be expressed as a linear function of T_∞. This can be obtained by expanding T^4 in a Taylor series about T_∞ and neglecting the higher order terms. Thus we get, $T^4 = 4T_\infty^3 T - 3T_\infty^4$.

Therefore, using above equation in (4), change in radiative flux with respect to y has been obtained as

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma_s T_\infty^3}{3k}\frac{\partial^2 T^4}{\partial y^2} \quad (5)$$

subject to the boundary conditions,

$u = U_w(x), v = 0, T = T_w(x)$ at $r = a$ and

$u \to 0, T \to T_\infty$ as $r \to \infty$ (6)

The equation of continuity is satisfied if we choose the stream function $\psi(x,r)$ such that $u = \frac{1}{r}\frac{\partial \psi}{\partial r}$ and $v = -\frac{1}{r}\frac{\partial \psi}{\partial x}$. Introducing the similarity variables as $\eta = \frac{r^2 - a^2}{2a}\left(\frac{U}{\nu x}\right)^{1/2}$, $\psi = (U\nu x)^{1/2} af(\eta)$ and

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (7)$$

The momentum and the energy equations can be transformed into the corresponding ordinary differential equations as

$$(1 + 2\gamma\eta) f''' + 2\gamma f'' + ff' - f'^2 = 0 \quad (8)$$

$$(1 + 2\gamma\eta)(3R + 4)\theta'' + 2\gamma\theta' + \Pr(f\theta' - nf'\theta) = 0 \quad (9)$$

subject to the boundary conditions

$f(0) = 0$, $f'(0) = 1$, $\theta(0) = 1$ and

$f'(\infty) \to 0$, $\theta(\infty) \to 0$ (10)

where the primes denotes the differentiation with respect to η, $\gamma = \frac{1}{a}\left(\frac{\nu l}{U_0}\right)^{1/2}$ is the curvature parameter, $R = \frac{kK}{4\sigma_s T_\infty^3}$ is the radiation parameter, where K is the thermal conductivity and $\Pr = \nu / a$ is the Prandtl number. The physical quantities of interest here are the skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as:

$C_f = 2(R_e)^{-1/2}\bar{x}f''(0)$ and $Nu_x = -(R_e)^{-1/2}\bar{x}\theta'(0)$ where $\bar{x} = x/l$ and $R_e = lU_0/\nu$.

3 Numerical Solution

The set of non-linear coupled differential equations (8) and (9) subject to boundary conditions (10) constitute a two-point boundary value problem. In order to solve these equations numerically, we follow most efficient numerical shooting technique with Runge-Kutta Fehlberg integration scheme. In this method, it is more important to choose the appropriate finite values of $\eta \to \infty$. The solution process is repeated with another large values of η_∞ until two successive values of $f''(0)$ and $\theta'(0)$ same upto the desired significant values. The last values of η_∞ is chosen as appropriate values of the limit $\eta \to \infty$ for that particular set of parameters. The ordinary differential equations (8) and (9) were first converted into a set of five first-order simultaneous equations. To solve this system we require five initial conditions but we have only three initial conditions $f(0)$ and $f'(0)$ on $f(\eta)$ and one initial condition $\theta(0)$ on $\theta(\eta)$. Still there are two initial conditions $f''(0)$ and $\theta'(0)$ are required, which are not prescribed. However the values of $f'(\eta)$ and $\theta(\eta)$ are known at $\eta \to \infty$. Shooting technique has been employed to find the unknown initial values utilizing these two ending boundary conditions. After finding the required boundary conditions, the problem has been solved numerically using Runge-Kutta Fehlberg integration scheme.

4 Result and Discussion

In the absence of an analytical solution of a problem, a numerical solution is indeed an obvious and natural choice. Thus, the governing boundary layer and thermal boundary layer equations (8) and (9) with boundary conditions (10) are solved using Runge-Kutta Fehlberg method with shooting technique. The results are given to carry out a parametric study showing influence of several non-dimensional parameters, namely Prandtl number $\Pr$, curvature parameter γ, temperature exponent n, radiation parameter R. For the validation of numerical method used, we have compared our results with Ishak and Nazar [11] and Elbashbeshy et al. [14] for $\gamma = 0$ (stretching flat plate) and large values of R with different values of $\Pr$ and n and they are found to be in a good agreement as shown in Table 1.

Table 1 Comparison of $-\theta'(0)$ for various values of $\Pr$ and n when $\gamma = 0$

Pr	n	Elbashbeshy et al. [14]	Ishak and Nazar [11]	Present Result
1	0	0.5820	0.5820	0.5822
	1	1.0000	1.0000	1.0005
	2	1.3333	1.3333	1.3322
10	0	2.3080	2.3080	2.3062
	1	3.7207	3.7207	3.7180
	2	4.7969	4.7969	4.7935

Fig. 2 represents the variation of velocity profile for different values of curvature parameter. In all cases, the velocity vanishes at some large distance from the sheet. The velocity increases with increasing values of γ. The velocity gradient at the surface is larger for small values of γ, which produces larger skin friction coefficient.

Effects of curvature parameter on the dimensionless temperature in presence of radiation parameter are presented in Fig. 3. The temperature is found to be increase with the increasing curvature parameter γ. The value of local Nusselt number $-\theta'(0)$, is increased by increasing curvature parameter γ which means that the heat transfer rate at the surface are larger for a cylinder compared to that for the flat plate.

From the Table 2, the value of skin friction coefficient, $f''(0)$ is negative for all values of the different parameters. Physically, the negative values of $f''(0)$ means the surface exerts a drag force on the fluid and the stretching cylinder will induce the flow. The value of $f''(0)$ is decreased by increasing curvature parameter γ. Hence, in order to minimize the skin friction values, the curvature parameter γ

is decreases. This has been found to be useful in many industrial and engineering applications.

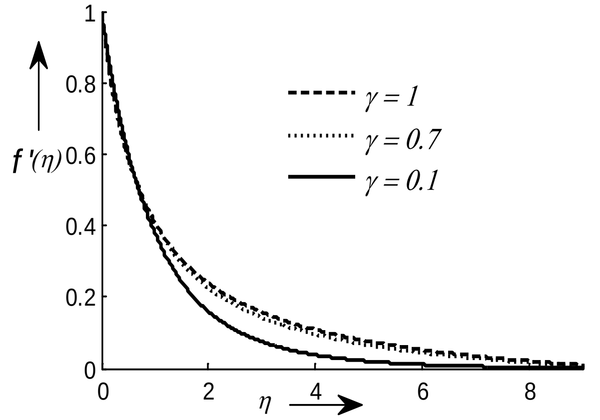

Fig. 2 The velocity profile $f'(\eta)$ for various values of γ at $\Pr = 7,\ R = 10,\ n = 0.5$

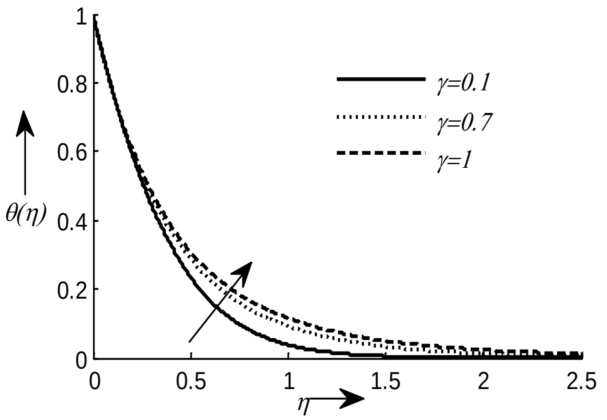

Fig. 3 The temperature profile $\theta(\eta)$ for various values of γ at $\Pr = 7,\ R = 10,\ n = 0.5$

Table 2 The values of $f''(0)$ and $-\theta'(0)$ for various values of γ when $\Pr = 7,\ R = 10,\ n = 0.5$

γ	$f''(0)$	$-\theta'(0)$
0.1	-1.036977	2.386325
0.3	-1.111138	2.429591
0.7	-1.257011	2.513587
1	-1.363865	2.574948

Fig. 4 represents the temperature profiles distribution for the variations of radiation parameters R in presence of curvature parameter γ. The temperature is found to be decreased as we increased in the radiation parameter R. The radiation parameter R defines the relative contribution of conduction heat transfer to thermal radiation transfer. The value of $f''(0)$ is negative and Nusselt number increases with increasing R as shown in Table 3

Table 3 The values of $f''(0)$ and $-\theta'(0)$ for various values of R when $\Pr = 7,\ \gamma = 0.5,\ n = 0.5$

R	$f''(0)$	$-\theta'(0)$
50	-1.18458	2.65902
10	-1.18458	2.47192
5	-1.18458	2.27578
2	-1.18458	1.84960

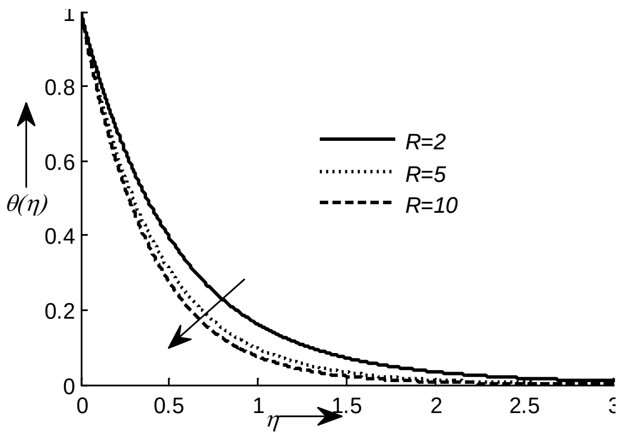

Fig. 4 The temperature profile $\theta(\eta)$ for various values of R when $\Pr = 7,\ \gamma = 0.5,\ n = 0.5$

Table 4 The values of $f''(0)$ and $-\theta'(0)$ for various values of Pr when $R = 10,\ \gamma = 0.1,\ n = 0.5$

Pr	$f''(0)$	$-\theta'(0)$
0.7	-1.03697	0.60566
7	-1.03697	2.38932
10	-1.03697	2.89517
20	-1.03697	4.18392
100	-1.03697	9.61814

Fig. 5 shows the temperature profiles distribution for the variation of Prandtl number Pr in presence of curvature parameter γ and radiation parameter R. The temperature is found to be decreased as we increased in the Prandtl number Pr. An increase in Prandtl number reduces the thermal boundary layer thickness. Prandtl number signifies the ratio of momentum diffusity to thermal diffusity. Fluids with lower Prandtl number will possesses higher thermal conductivity and thicker thermal boundary layer structure so that heat can diffuse from the wall, faster than the higher Prandtl number fluids. Hence, Prandtl number can be used to increase the rate of cooling to conducting flows.

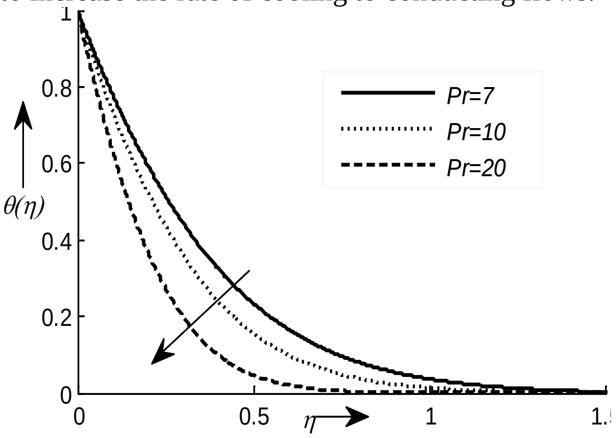

Fig.5 The temperature profile $\theta(\eta)$ for various values of Pr when $R=10,\ \gamma=0.1,\ \ n=0.5$

Table 5 The values of $f''(0)$ and $-\theta'(0)$ for various values of n when $R=10,\ \ \gamma=0.1,\ \ \Pr=7$

n	$f''(0)$	$-\theta'(0)$
0.1	-1.03697	1.91397
0.7	-1.03697	2.59888
2	-1.03697	3.74149
3	-1.03697	4.44659

Fig. 6 represents the temperature distribution profiles for the variations of temperature exponent n in the presence of curvature parameter γ and radiation parameter R. The dimensionless temperature was found to decrease as we increased in the temperature exponent was increased.

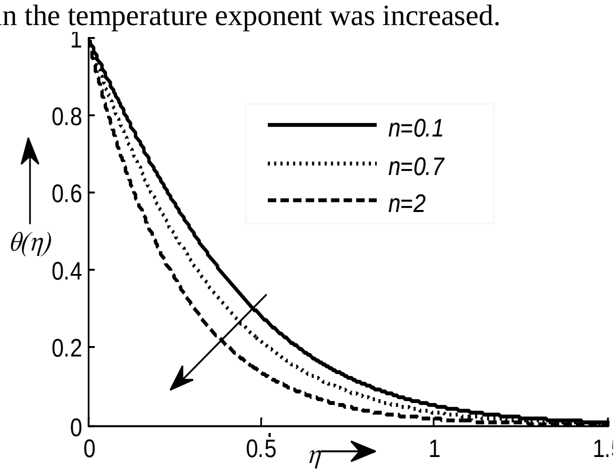

Fig.6 The temperature profile $\theta(\eta)$ for various values of n when $R=10,\ \ \gamma=0.1,\ \ \Pr=7$

5 Conclusion

In the present study, numerical solutions have been reported for steady, axi-symmetric boundary layer flow of a viscous and incompressible fluid along a continuously stretching horizontal cylinder in presence of thermal radiation. With the help of similarity transformations, the governing partial differential equations for momentum and thermal have been reduced to coupled non-linear ordinary differential equations which were then solved numerically using Runge-Kutta Fehlberg method with shooting technique. The effects of Prandtl number, radiation parameter R, curvature parameter γ, temperature exponent n have been studied. The following results have been arrived at in the present study:

- The local Nusselt number increase results in the increase of Prandtl number Pr, radiation parameter R, curvature parameter γ, temperature exponent n.
- The skin friction coefficient at the surface increases with increasing curvature parameter γ.
- The velocity increases with an increase in the value of curvature parameter γ.
- The temperature increases with increase in the value of curvature parameter γ and decreases with increase in the values of radiation parameter R, Prandtl number Pr and temperature exponent n.

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