Tsunami Wave Simulation Models Based on Hexagonal Cellular Automata

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Abstract: - The devastating effect of tsunamis on mankind has clearly established that many improvements are needed in disaster management. In this work we propose a new hexagonal cellular automata model based on the transfer of fractional traversed area. The rates of spread of tsunami waves for both homogeneous and non homogeneous oceans under different topological conditions are derived. Graphical representation of rate of spread has been found successfully.

Key-Words: - Tsunami wave, Simulation, Homogeneous, Non-homogeneous, Cellular automata, Discrete time step, Primary wave front, Secondary wave front

1 Introduction

After the repeated attacks of tsunamis around the world, various countries across the globe are installing tsunami warning centres. An integrated socio-technological infrastructure has to be built, with advanced sensors, fast predictive algorithm and reliable communication networks for early warning system. The real-time accurate simulation systems are needed for predicting the spread of tsunamis [1].Tsunamis are known as long gravity waves, and hence their travel time in the ocean depends only on the water depth and gravity and linear approximation. The tsunami model is to be realistic both geometrically and in terms of behaviour. The simulation will enable one to determine where the tsunami is most likely to spread first and how fast. A variety of tsunami simulation models exist [2]. These models focus on the correct propagation thereof. In this research, a model of tsunami wave propagation is developed that aims to be accurate in homogeneous and non- homogeneous cases. In general, the initial and boundary conditions are taken in tsunami propagation. The tidal range was usually neglected during tsunami modelling in shallow areas with strong tidal activity. The friction is important only in shallow water, whereas in the deep ocean the effect is negligible [2].Tsunami travel outward in all direction from the generating area, with the directions of the main energy propagation generally being orthogonal to the direction of the earthquake fracture. Their speed depends on the depth of the water. In the deep and open ocean, they travel at speed of 500 to 1000 km/hr [3][4][5].

Nowadays, assessing hazard conditions related to complex natural phenomena increasingly take advantage of computer-assisted analyses and simulations. To study the tsunami waves, oceans area is divided in to a matrix of identical square cells, with side length L, and it is represented by a cellular automaton with this value, the more clarity of the wave details. Specifically, the efforts to model the growth of the wave spread by means of mathematical models can be classified in to two categories according to the approach: Vector models and cellular automata models. If wave spreading conditions are uniform, a single shape can be used to determine the wave size, the perimeter over time and the area by means of the use of fractals [6]**.**

Simulation is important tools for displaying the spread of the tsunami waves [7].Large, shallow earthquakes also occur along transform faults, but there is only minor vertical motion during the faulting so no tsunamis are generated.

In this work, we propose a new cellular automaton model based on the transfer of fractional traversed area. A simulation tool that can be used to mimic the real time wave spread using hexagonal cellular automata under spatially variable topographic, slope and meteorological conditions. This model introduces factor of propagation from nearest and adjacent cells and includes, in a detailed form, the rate of wave spread. This will add lot of value in the testing and calibration of tsunami computer models and in improving future tsunami early warning systems [8][9].

Working models of tsunami wave, visual representation are obtained using the java programming language. It is used for finding the rate of spread of the tsunami wave under two types of ocean, eight topological and wave conditions.

In the first section of this paper, various existing approaches for the rate of spread of tsunami wave propagation modelling are examined. In section 2, the basic theory of cellular automata and hexagonal cellular automata is presented. In section 3, the new model is proposed. In section 4, homogeneous and non- homogeneous ocean modules are checked and their simulation results are shown. In section 5, Graphic representation of hexagonal cellular automata is presented. Section 6 concludes the paper.

2 Theory of Cellular automata and hexagonal models

In particular, Cellular Automata (CA) is a powerful tool for modelling natural and artificial systems that can be specified in terms of local interactions among their constituent parts. Cellular automata are arrays of 'cells' that interact with their neighbours. These arrays can take on any number of dimensions, starting from a one dimensional string of cells. Each cell has its own state that can be a variable, property or other information. At the beginning of the simulation, cell states are initialized by means of input matrices. Model parameters have also to be assigned in this phase, by taking into consideration their physical/empirical meaning. By simultaneously applying the transition function to all the cells, at discrete steps, states are changed and the evolution of the phenomenon can be simulated.

2.1 Hexagonal Cellular Automata and Its Features

The hexagonal model of a tsunami wave are investigated and the mathematical equation and the physical properties of examining the wave characteristics. The hexagonal model and physical properties of a tsunami wave are involved in this paper for several representative conditions, and a representative real world tsunami is simulated through computer modelling. The graphs, equation and simulation results are represented to understand and create a typical model of a tsunami wave.

In order to fix the values of such essential global parameters, further points must be considered, especially when the phenomenon is complex and involves time and/or space heterogeneity. Our approach differs in its use of discrete space (cells), and discrete time increments (steps): accordingly, continuum limit operations are not required. The Two-dimensional cellular automaton [8**]** does not have accuracy in the shape of the output obtained. Further the rate of spread is high, which decreases the efficiency. Hence the hexagonal cellular automata are used to accurately study the rate of spread and the shape of the output as in real tsunamis. As the state of the cell can be decomposed into substates, the transition function may also be split into local interactions: the “elementary” processes. Different elementary processes may involve different neighbourhoods; the CA neighbourhood is given by the union of all the neighbourhoods associated to each process.

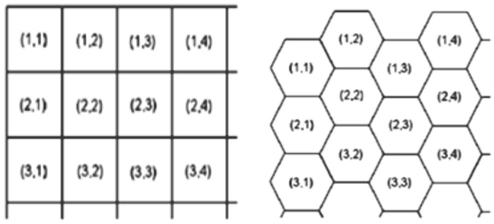
Let us focus on simplicity of a single CA cell of the two-dimensional space: it is considered limited to the universe of its neighbourhood, which consists of m cells (the central cell and its adjacent cells). Indexes are utilized to indicate the central cell O and the adjacent ones (1, 2 . . . m − 1), respectively. The states considered are 0 if the cell is not traversed or partially traversed and 1 if the cell is fully traversed [8].Two-dimensional cellular automata (CA) are discrete dynamical systems formed by a finite number of identical objects called cells, which are arranged uniformly in a two dimensional space. They are endowed with a state that changes at every discrete step of time according to a deterministic rule. More precisely, a CA can be defined as a 4-tuplet $U = (R, N, Q, k)$ Where R is the cellular space formed by a two-dimensional array of q x b cells: $\{(x, y), 1 \leq x \leq q, 1 \leq y \leq b\}$, such that each of which can assume a state. In the bi-dimensional case, the cells are usually represented as identical hexagonal areas (Fig.1 (a)), but in this work, the cells will be represented by means of regular hexagonal areas (Fig. 1(b)), making a tessellation of the plane. This new representation allows us to obtain a more realistic simulation like 2D-CA **[8].**The state of each cell is an element of a finite or infinite state set, N. Moreover, the state of the cell (x, y) at time t is denoted by $N_{xy}^{(t)}$.

The set of indices of the CA is the ordered finite subset $Q \subset \mathbb{S} \times \mathbb{S}, |Q| = c,$ such that for every cell(x, y)its Neighbourhood $Q_{(x,y)}$is the ordered set of c cells given by

$$Q_{(x,y)} = \left\{ \begin{matrix} (x + \lambda_1, y + \delta_1), \dots\dots\dots\dots, \\ (x + \lambda_c, y + \delta_c) : (\lambda_r, \delta_r) \in Q \end{matrix} \right\} \qquad (1)$$

Depending on the process to be modelled, one can choose an appropriate neighbourhood. In this work, the neighbourhood of a cell *O=(x, y)* is given by the set

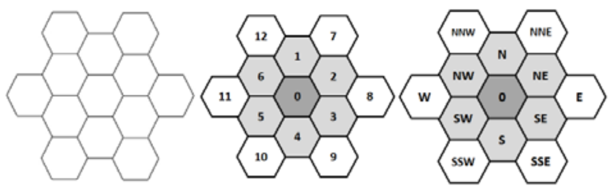
Q_O={O,N,NE,SE,S,SW,NW,NNE,E,SSE,SSW,W,NNW} as it is shown in Fig.1(a) and 1(b).



1(a) 1(b)

Fig.1 (a): square cellular space
Fig.1 (b): Hexagonal cellular space

We can also distinguish two types of neighbour cells of O, depending on whether the neighbour of the cell O is an adjacent cell or not as seen in Fig.2.

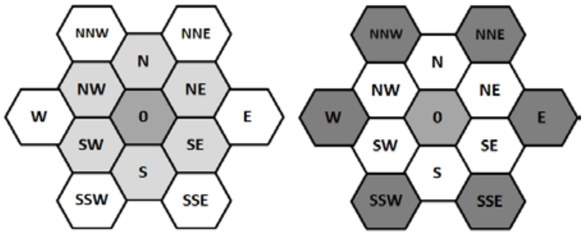


2(a) 2(b) 2(c)

Fig.2 (a): Hexagonal model
Fig.2 (b) and Fig.2(c): tessellation of the plane.
The near neighbour cells are the set

Q_n= {N, NE, SE, S, SW, NW}, whereas the distant neighbour cells are given by the set

Q_d= {NNE, E, SSE, SSW, W, NNW}. Furthermore, each distant neighbour cell has two near neighbour cells associated, those with common sides:



3(a) 3(b)

Fig. 3: Hexagonal model neighbour cells
3 (a).Associated near neighbour cells
3 (b). Distant neighbour cell

In this way, if $(\lambda, \delta) \in Q_d$ then the associated neighbour cells of $(x + \lambda, y + \delta)$ are denoted by $(x + \lambda^+, y + \delta^+)$and $(x + \lambda^-, y + \delta^-)$.

Moreover, in this case, the set defining the neighbourhood, Q depends on *O=(x,y)* to be considered.

In this sense, if y is odd, then

$$Q^{odd} = \left\{ \begin{matrix} (0,0), (-1,0), (0,1), (1,1), (1,0), (1,-1), (0,-1), \\ (-1,1), (0,2), (2,1), (2,-1), (0,-2), (-1,-1) \end{matrix} \right\}$$

Consequently, the neighbourhood of the cell (x,y), with y odd, is (see Fig. 4(b)):

$$Q_{(x,y)}^{odd} = \{(x,y), (x-1,y), (x,y+1), (x+1,y+1), (x-1,y+1), (x,y+2), (x+2,y+1), (x+2,y-1), (x,y-2), (x-1,y-1)\} \qquad (2)$$

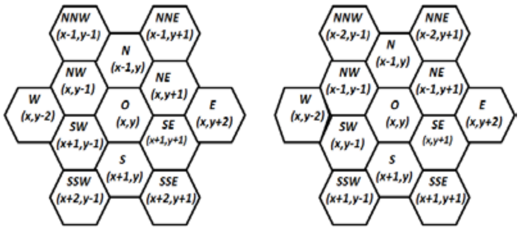
On the other hand, if y is even, then

$$Q^{even} = \left\{ \begin{matrix} (0,0), (-1,0), (-1,1), (0,1), (1,0), (0,-1), (-1,-1), \\ (-2,1), (0,2), (1,1), (1,-1), (0,-2), (-2,-1) \end{matrix} \right\}$$

and, as a consequence, the neighbourhood of the cell (x,y), with y even, is (see Fig. 4(a)):

$$Q_{(x,y)}^{even} = \{(x,y), (x-1,y), (x-1,y+1), (x,y+1), (x+1,y), (x,y-1), (x-1,y-1), (x-2,y+1), (x,y+2), (x+1,y+1), (x+1,y-1), (x,y-2), (x-2,y-1)\} \qquad (3)$$

As these labels do not affect to the calculus below, thereafter we will work with a generic neighbour Q for the sake of simplicity as seen in Fig.4.



4(a) 4(b)

Fig.4 (a): Even neighbour cells
Fig.4 (b): Odd neighbour cells

As was mentioned above, the CA evolves deterministically in discrete time steps, changing the states of the cells according to a local transition function$f: S^{13} \rightarrow S$. The updated state of the cell *(a,b)* depends on the thirteen variables of the local transition function, which are the previous states of the cells constituting its neighbourhood, that is

$$N_{xy}^{(t+1)} = k\left(N_{x+\lambda_1,y+\delta_1}^{(t)}, \ldots, N_{x+\lambda_{13},y+\delta_{13}}^{(t)}\right) \quad (4)$$

Moreover, the matrix

$R^{(t)} = \left(N_{xy}^{(t)}\right), 1 \leq x \leq q, 1 \leq y \leq b,$ is called the configuration at time t of the CA, and B(0)is the initial configuration of the CA. As the number of cells of the CA is finite, boundary conditions must be considered in order to assure the well-defined dynamics of the CA. One can state several boundary conditions but in this work, we will consider null boundary conditions, that is

If $(x, y) \notin \{(i, j), 1 \leq i \leq q, 1 \leq j \leq b\} \Rightarrow N_{xy}^{(t)}=0$

A very important type of CA is linear CA, whose local transition function is as follows:

$$N_{xy}^{(t+1)} = f\left(\sum_{(\lambda,\delta)\in Q} \mu_{\lambda\beta}\, N_{x+\lambda,y+\delta}^{(t)}\right), \mu_{\lambda\delta} \in \mathbb{Z} \qquad (5)$$

Where $f: \mathbb{Z} \rightarrow \mathbb{R}$ is a suitable discretization function.

3 The Cellular Automata Based Model for Tsunami Wave Spreading

In this section the model for predicting tsunami wave spreading based on two-dimensional linear cellular automata with hexagonal cellular space is proposed.

3.1 The New Model

Here ocean area can be interpreted as the hexagonal cellular space by simply dividing it into a two-dimensional array of identical hexagonal regions of side length L **.**Obviously, each one of these region stands for a cell of the CA.

The state of a cell *(x,y)* at a time *t*, is defined as follows:

$N_{xy}^{(t)}$= traversed area of *(x,y)* at a time *t* / total area of (x,*y*) where as a simple calculus shows, the total area of the hexagonal cell (x,*y*) is $3\sqrt{3}L^2/2$.

If $N_{xy}^{(t)}$= 0, then the cell (x,*y*) is said to be untraversed at time *t*; if $0 < N_{xy}^{(t)} < 1$, then the cell (x,*y*) is partially traversed out at time *t*, and finally if $N_{xy}^{(t)}$= 1, the cell is said to be completely traversed out at time *t*. Observed that the values $N_{xy}^{(t)}$may else be greater than 1. In this case, the state of the cell (x,*y*) at time *t* is taken to be equal to 1.

The dynamics of such automata basically supposes that the state of a cell (x,*y*) at time *t* +1 linearly depends on the states of its neighbour cells at time *t*; specifically one has

$$N_{xy}^{(t+1)} = f\ (N_{xy}^{(t)} + \sum_{(\lambda,\delta)\in Q_n} \mu_{\lambda\delta}^{(x,y)} N_{x+\lambda,y+\delta}^{(t)}$$

$$+ \sum_{(\lambda,\delta)\in Q_d} \mu_{\lambda\delta}^{(x,y)} N_{x+\lambda,y+\delta}^{(t)}\ ) \qquad (6)$$

where $\mu_{\lambda\delta}^{(x,y)} \in \mathbb{Z}$ are parameters involving some physical magnitudes of the cells, and the discretization function f is given by

f: [0,1]→ N

$a \mapsto f(a) = \frac{[10_a]}{10},\ where\ [c]$ stands for the closest integer to c.

As it is mentioned above, each cell, (x,y), represents a small hexagonal area of the ocean. Then, it is endowed with three parameters: the rate of wave spread$RA_{(x,y)}$, the wave speed$WA_{(x,y))}$, and the depth $DE_{(x,y)}$of the cell.

Consequently, the expression of the parameter $\mu_{\lambda\delta}^{(x,y)}$ is as follows:

$$\mu_{\lambda\delta}^{(x,y)} = \omega a_{\lambda\delta}^{(x,y)}.de_{\lambda\delta}^{(x,y)}.ra_{\lambda\delta}^{(x,y)} \qquad (7)$$

where $WA_{(a,b)}$ stands for the wave influence of the neighbor cell $x+\lambda, y+\delta$)on (x,y), such that

$WA_{(a,b)} = \{\omega a_{\lambda\delta}^{(x,y)}, (\lambda,\delta) \in Q\}$;$de_{\lambda\delta}^{(x,y)}$ represents the height influence and, as is shown below,

It is a function of $DE_{(x,y)} - DE_{(x-\lambda,y+\delta)}$ where $DE_{(x,y)}$)is the height in the central point of the hexagonal area which is represented by the cell(x,y).

It is supposed that this height is the same in every point of such cell. Finally, $ra_{\lambda\delta}^{(x,y)}$is a parameter which stands for the influence of the different rates of tsunami wave spread.

3.2 The Size of Discrete Time Step

Since cellular automata evolve in discrete time steps, it is a basic point to decide what is the size of such step of time, $\tilde{t}$. In the proposed model, this step is equal to the time needed for a one and only near neighbour cell to be traversed.

The rate of tsunami wave spread of the cell *(x,y)*, $RA_{(x,y)}$,determines the time needed for this cell to be completely traversed out and depends on the physical composition of the cell **[9]**. Note that if the cell *(x,y)* stands for an untraversed hexagonal area, then $RA_{(x,y)}$ =0 and $N_{xy}^{(t)}$ = 0 for every t.

The importance of this parameter lies in the setting-up of the size of the time step,$\tilde{t}$. Suppose that the ocean area modeled is homogeneous, i.e. the value of the rate of wave spread is the same for all cells:$\{RA_{(x,y)}, 1 \le x \le q, 1 \le y \le b\}$

Then, it is easy to check that if the only traversed cell at time t in the neighborhood of *O* is for example N, then the time needed for *O* to be completely traversed out is $\tilde{t} = \sqrt{3}\frac{L}{RA}$

Consequently, if all cells in the neighborhood of $O = (x,y)$ are untraversed at time t except only one adjacent cell, which is completely traversed out, then at time t + 1, the cell (x,y) is completely traversed: $N_{xy}^{(t+1)}$=1

However, since almost all real oceans are non-homogeneous, the step size is taken to be the time needed for the cells with the larger rate spread to be completely traversed out,that is

$$\tilde{t} = \sqrt{3}\frac{L}{RA} \quad (8)$$

where RA=max$\{RA_{(x,y)}, 1 \le x \le q, 1 \le y \le b\}$

It is followed that if the only completely traversed out neighbor cell at time t, is a distant neighbor cell of *O = (x,y), say NNE,then* $N_{xy}^{(t+1)}$ =γ<1. That value is calculated as follows. In a step time $\tilde{t}$, the near neighbor cells of the cell NNE, N and NE, and a little portion (a circular sector) of the cell O will be traversed out. Specifically, as the distance covered by the tsunami wave spread in t with speed R is√3L then the radius of the circular sector of *O* traversed out is $\sqrt{3}L - L$,as seen in Fig.5.

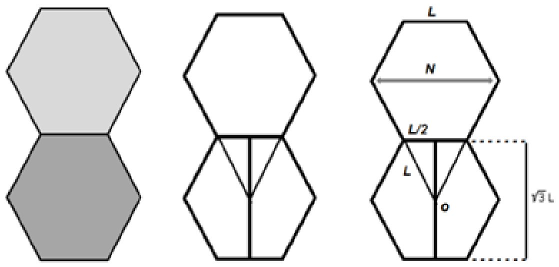


Fig. 5: Determining the size of the discrete time step

As a consequence, the traversed out area of the cell O will be

$\frac{\pi(\sqrt{3}-1)^2 L^2 \frac{2\pi}{3}}{2\pi} = \frac{4-2\sqrt{3}}{3}\pi L^2$ So, if all neighbor cells of (x,y) are un traversed at time t, except a distant neighbour which is fully traversed out, then

$$N_{xy}^{(t+1)} = \gamma = \frac{\frac{4-2\sqrt{3}}{3}\pi L^2}{\frac{3\sqrt{3}}{2}L^2} = \frac{8\sqrt{3}-12}{27}\pi \approx 0.21 \quad (9)$$

3.3 The Influence of Tsunami Wave

Another factor to be incorporated to the model is the wave speed and direction, due to their important influence to the tsunami wave spreading [10].

As was stated above, the effect of the wave on the cell O is given by the set

$$WA_{(x,y)} = \{\omega a_{\lambda\delta}^{(x,y)}, (\lambda,\delta) \in Q\} \quad (10)$$

where $\omega a_{\lambda\delta}^{(x,y)}$>0, in such a way that if no wave is traversed on $O = (x,y)$then $\omega a_{\lambda\delta}^{(x,y)} = 1$ for every $(\lambda,\delta) \in V$; if the wave is traversed from North to South, then the coefficients $\omega a_{NW}^{O}, \omega a_{NNW}^{O}, \omega a_{N}^{O},, \omega a_{NNE}^{O}$, ωa_{NE}^{O} must be larger than the rest of coefficients, and so on. The value of such coefficients stand for the magnitude of the wave as seen in Fig.6.

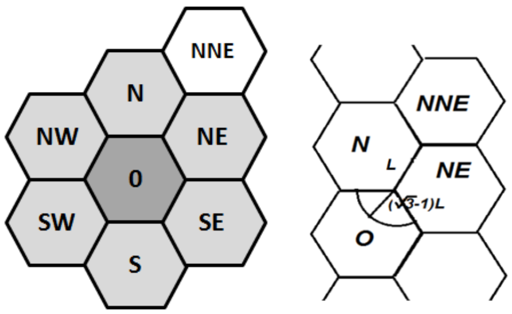


Fig. 6: The calculus of γ

3.4 The Influence of Topography

The height differences between various points in a ocean also affects to the wave spreading. As is well-known, the waves show a higher rate of spread when they descend i, whereas waves show a smaller rate of spread when they ascend.The height influence of a near neighbour cell $(x+\lambda, y+\delta)$, on a cell $O=(x,y)$ is given by $de_{\lambda\delta}^{(x,y)}$,which depends on the difference of height between each pair of cells considered, that is,

$$de_{\lambda\delta}^{(x,y)} = \emptyset(HA_{(x,y)} - HA_{(x+\lambda,y+\delta)}) \quad (11)$$

The function $=\emptyset(a)$, where x stands for the height difference, must be determined according to the characteristic of the tsunami, and, also, it has to satisfy the following conditions:

If $a>0$, then $\emptyset(a)<1$

If $a=0$, then $\emptyset(a)=1$

If $a<0$, then $\emptyset(a)>1$

Note that the first condition establishes that if $DE < DE_{(x+\lambda,y+\delta)}$, the tsunami wave increases its rate of spread; the second condition states that when $DE_{(x,y)} = DE_{(x+\lambda,y+\delta)}$, the topography does not affect to the tsunami wave spread; and the third condition establishes that

If $DE_{(x,y)} > DE_{(x+\lambda,y+\delta)}$ the tsunami wave restrains its spreading.

Moreover, the height influence of a distant neighbour cell is affected by the influence of its associated near neighbour cells. For example, if the distant neighbour cell of O is NNE, then:

$$de_{NNE}^{O} = \frac{1}{4}\begin{bmatrix}\emptyset(DE_O - DE_N) + \emptyset(DE_N - DE_{NNE}) + \\ \emptyset(DE_O - DE_{NE}) + \emptyset(DE_{NE} - DE_{NNE})\end{bmatrix} \quad (12)$$

and so on.

Note that if the wave motion is horizontal (all cells have the same height), then $\emptyset(a) \leq 1$ and consequently $de_{\lambda\delta}^{(x,y)} = 1$ for every cell (x,y) and any $\lambda, \delta) \in Q$

3.5 The Rate of Wave Spread

Let us consider a non-homogeneous ocean and set R the maximum rate of wave spread. Let $O=(x,y)$ be a cell with $RA_O \leq R$A. The purpose of this section is to determine the value of the parameter, $q_{\lambda\delta}^{(x,y)}$, which stands for the influence of the different rates of wave spread of the neighbor cells. If all neighbor cells are untraversed at time t except only one near neighbor cell of O, say for example N, then after a time step, $\tilde{t}$ the space traversed by the wave spread is given by :

$$q = RA_0\tilde{t} = \sqrt{3\frac{RA_O}{RA}L} \quad (13)$$

and consequently, there are two cases to be considered: When $q \leq L$, and when q > L.

If $q \leq L$,

then $\sqrt{3}\frac{RA_O}{RA}L \leq L$, and $\frac{RA_O}{RA} \leq \frac{\sqrt{3}}{3} \approx 0.57735$

As a consequence, the traversed out area of the cell O after a time step ~t is given by

$$L_q + 2\frac{q^2\frac{\pi}{6}}{2} = (\sqrt{3} + \frac{\pi}{2}\ \frac{RA_O}{RA})\ \frac{RA_O}{RA}L^2$$

Consequently,

$$q_N^O = \frac{\left(\sqrt{3}+\frac{\pi}{2}\ \frac{RA_O}{RA}\right)\frac{RA_O}{RA}L^2}{\frac{3}{2}\sqrt{3}L^2}$$

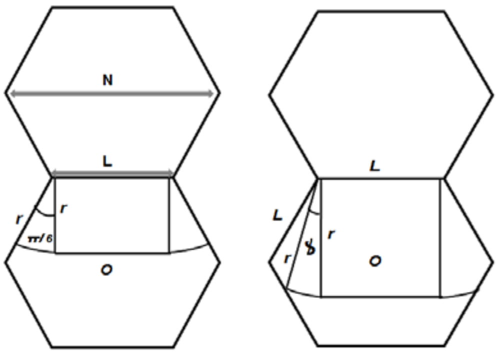
$$= \frac{2\sqrt{3}}{9}\left(\sqrt{3}+\frac{\pi}{2}\ \frac{RA_O}{RA}\right)\frac{RA_O}{RA} \quad (14)$$

If $q>L$, then $L < \sqrt{3}\ \frac{RA_O}{RA}L$, and

$$0.57735 \approx \frac{\sqrt{3}}{3} < \frac{RA_O}{RA} \leq 1$$

As a simple calculus shows, the traversed out area of the cell $O=(x,y)$ after a time step $\tilde{t}$ is given by

$$\left[1+\sin(\frac{\pi}{6}-\lambda)+\sqrt{3}\lambda\frac{RA_O}{RA}\right]\sqrt{3}\ \frac{RA_O}{RA}L^2$$



7(a) 7(b)

Fig.7(a):The rate of spread (near cell) $r \leq L$
Fig. 7(b): The rate of spread (near cell) $r > L$

where,

$$\lambda = \frac{\pi}{6} - arccos\left(\frac{\sqrt{3}}{4}\frac{RA}{RA_O} + \frac{\sqrt{3}}{12}\sqrt{12 - 3\frac{RA^2}{RA_O^2}}\right),$$

$$0 \leq \lambda < \frac{\pi}{6} \qquad (15)$$

As a consequence:

$$q_N^O = \frac{\left[1+\sin(\frac{\pi}{6}-\lambda)+\sqrt{3}\lambda\frac{RA_O}{RA}\right]\sqrt{3}\,\frac{RA_O}{RA}L^2}{\frac{3}{2}\sqrt{3}L^2}$$

$$= \frac{2}{3}\left[1 + \sin(\frac{\pi}{6}-\lambda)\right]\frac{RA_O}{RA} + \frac{2\sqrt{3}}{3}\lambda\frac{RA_O^2}{RA^2} \qquad (16)$$

Note that if the ocean is homogeneous, then $R_{ab} = R$ for every cell (x,y), and

$$\lambda = \frac{\pi}{6} - arccos(\frac{\sqrt{3}}{2}) = 0 \qquad (17)$$

Consequently $q_N^O = 1$ as was expected.

On the other hand, if all neighbour cells are untraversed at time t, except only one distant neighbour cell, say for example NNE, then after a time step $\tilde{t}$, the wave spread traverses the border between the near neighbor cells N and NE, and affects the main cell O.

The border line between N and NE is traversed in $\tilde{t}_0 = L/max\{RA_N, RANE\}$ and consequently, the space traversed along the cell O is:

$$RA_O(\tilde{t} - \tilde{t}_O) = \left(\frac{\sqrt{3}}{RA} - \frac{1}{\max\{RA_N, RA_{NNE}\}}\right) RA_O L$$

As a consequence, the traversed out area of O after a time step $\tilde{t}$ is:

$$\frac{\pi\left(\frac{\sqrt{3}}{RA} - \frac{1}{\max\{RA_N, RA_{NNE}\}}\right)^2 RA_O{}^2 L^2 \frac{2\pi}{3}}{\frac{3}{2}\sqrt{3}L^2}$$

$$= \frac{\pi}{3}\left(\frac{\sqrt{3}}{RA} - \frac{1}{\max\{RA_N, RA_{NNE}\}}\right)^2 RA_O{}^2 L^2 \qquad (18)$$

Obviously, the state of the cell O is

$$= \frac{\frac{\pi}{3}\left(\frac{\sqrt{3}}{RA} - \frac{1}{\max\{RA_N, RA_{NNE}\}}\right)^2 RA_O{}^2 L^2}{\frac{3}{2}\sqrt{3}L^2}$$

$$q_{NNE}^O = \frac{2\pi\sqrt{3}}{27}\left(\sqrt{3}\frac{RA_O}{RA} - \frac{RA_O}{\max\{RA_N, RA_{NNE}\}}\right)^2$$

Note that if the ocean is homogeneous, then

$$q_{NNE}^O = \frac{2\pi\sqrt{3}}{27}(\sqrt{3} - 1)^2 = \frac{8\sqrt{3}-12}{27} = \gamma \qquad (19)$$

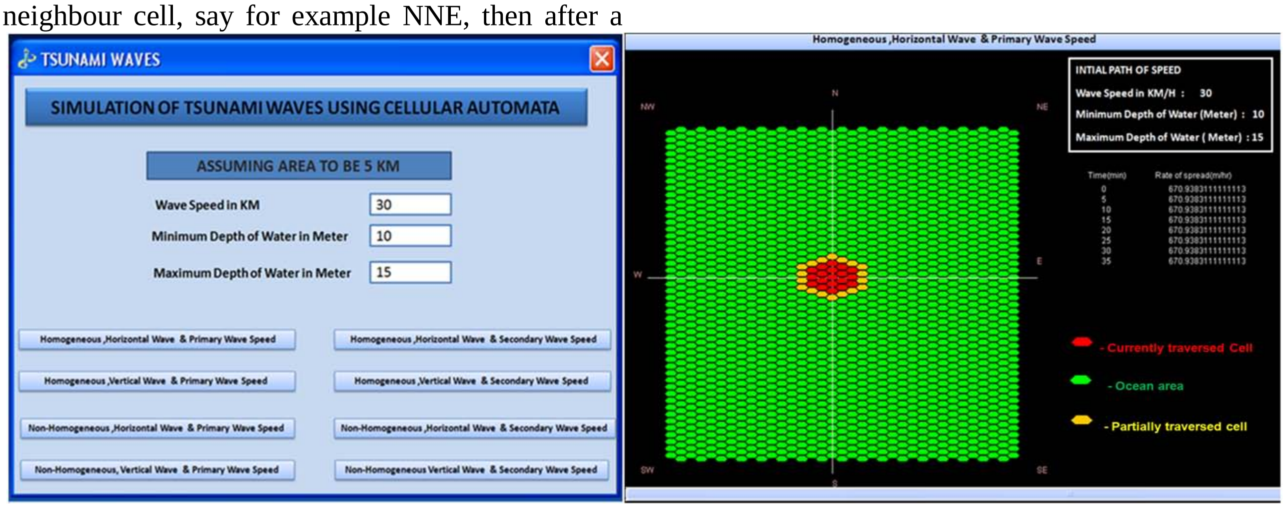


Fig.8: Hexagonal cellular automata simulation results for Tsunami wave propagation with different topographic conditions

4 Simulation results of Testing of modules

4.1 Homogeneous Ocean Rate of Spread Calculation

► Near Cells:

If $q \leq L$, then,

$$q_N^O = \frac{2\sqrt{3}}{9}\left(\sqrt{3} + \frac{\pi}{2}\ \frac{RA_O}{RA}\right)\frac{RA_O}{RA}$$

If $q > L$,then,

$$\left[1 + \sin(\frac{\pi}{6} - \lambda) + \sqrt{3}\lambda\frac{RA_O}{RA}\right]\sqrt{3}\ \frac{RA_O}{RA}L^2;$$

where, λ =0.

► Distant Cells:

$$q_{NNE}^O = \frac{2\pi\sqrt{3}}{27}(\sqrt{3} - 1)^2 = \frac{8\sqrt{3} - 12}{27} = \gamma$$

4.2 Homogeneous Ocean, horizontal wave motion and Primary Wave Speed

In the case of horizontal wave motion, the spread is even and with primary wave speed, the spread is even in all directions as shown in the Fig.9.

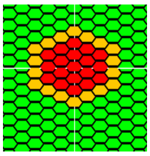


Fig.9 Homogeneous Ocean, horizontal wave motion, primary wave speed and direction

4.3 Homogeneous Ocean, horizontal wave motion and Secondary Wave Direction

In this case, from the central cell, the neighboring cells are partially traversed, and then they are completely traversed, in the specified direction. The spread is even and with secondary wave direction, the spread is more in the specified direction as shown in Fig.10.

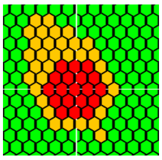


Fig.10 Homogeneous Ocean, horizontal wave motion, secondary wave speed and direction in North-West

4.4 Homogeneous Ocean, Vertical wave motion and Primary Wave Speed

In this case of vertical wave motion, the spread is very fast in the trans-ocean tsunami direction, moderate on the sides if the slope and very slow down the slope. With primary wave speed, the spread is even as shown in the Fig.11.

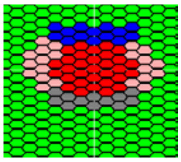


Fig.11 Homogeneous Ocean, Vertical wave motion, primary wave speed

4.5 Homogeneous Ocean, Vertical wave motion and Secondary Wave Speed

In this case, from the central cell, the neighboring cells are partially traversed, and then they are completely traversed, in the specified direction. In vertical wave motion, the spread is very fast in the trans-ocean tsunami direction, moderate in the sides of the slope and very slow down the slope. With secondary wave directions, the spread is more on the specified direction; the spread is shown in the Fig.12.

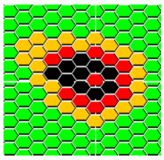


Fig.12 Homogenous Ocean, Vertical wave motion, Secondary wave speed in North-West

4.6 Non-Homogeneous Ocean, Rate of Spread Calculation

► Near Cells:

If $q \leq L$, then,

$$q_N^O = \frac{2\sqrt{3}}{9}\left(\sqrt{3} + \frac{\pi}{2}\ \frac{RA_O}{RA}\right)\frac{RA_O}{RA}$$

If $q > L$, then,

$$q_N^O = \frac{2}{3}[1 + \sin(\frac{\pi}{6} - \lambda)]\frac{RA_O}{RA} + \frac{2\sqrt{3}}{3}\lambda\frac{RA_O^2}{RA^2};$$

where,

$$\lambda = \frac{\pi}{6} - \arccos\left(\frac{\sqrt{3}}{4}\frac{RA}{RA_{(x,y)}} + \frac{\sqrt{3}}{12}\sqrt{12 - 3\frac{RA^2}{RA_{(x,y)}^2}}\right),$$

$0 \leq \lambda \leq \pi/6$

► Distant Cells:

$$q^{O}_{NNE} = \frac{\pi}{3}\left(\frac{\sqrt{3}}{RA} - \frac{1}{\max\{RA_{N,}RA_{NNE}\}}\right)^2 {RA_O}^2 L^2$$

4.7 Non-Homogeneous Ocean, horizontal wave motion and Primary Wave Speed

In the case of horizontal wave motion, the spread is even and with primary wave speed, the spread is even in all directions; which is shown in the Fig.13.

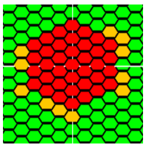


Fig.13 Non-homogenous Ocean, horizontal wave motion with Primary wave

4.8 Non -Homogeneous Ocean, horizontal wave motion and Secondary Wave Direction

In this case of horizontal wave motion, the spread is even and with secondary wave direction, the spread is more in the specified direction, which is shown in the Fig.14.

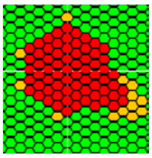


Fig.14 Non-homogenous Ocean, horizontal wave motion, Secondary wave speed in South-West

4.9 Non-Homogeneous Ocean, Vertical wave motion and Primary Wave Speed

In the case of vertical wave motion, the spread is very fast in the trans-ocean tsunami direction, moderate in the sides of the slope and very slow down the slope. With primary wave speed, the spread is even; which is shown in the Fig.15.

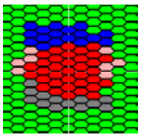


Fig.15 Non-homogenous Ocean, Vertical wave motion, Primary wave speed

4.10 Non-Homogeneous Tsunami, Vertical wave motion and Secondary Wave Direction

In the case of vertical wave motion, the spread is very fast in the trans-ocean tsunami direction, moderate in the sides of the slope and very slow down the slope. With secondary wave direction, the spread is more in the specified direction; which is shown in the Fig.16.

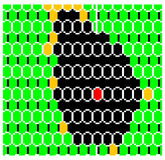


Fig.16 Non-homogenous Ocean, Vertical wave motion, Secondary wave speed in North-West

5 Graphic representation of rate of spread in Hexagonal cellular automata

5.1 Rate of Spread (Homogeneous Oceans)

Here the rate of spread of hexagonal cellular automata for homogeneous ocean **[9]** with horizontal wave motion and primary wave. It is found that the rate of spread is constant for both the cases, as there are same types of waves throughout the ocean.

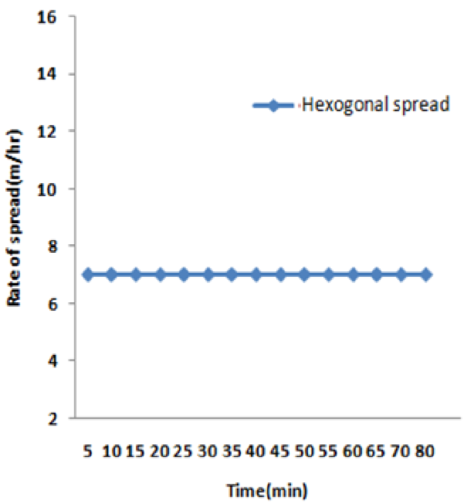


Fig.17 The rate of spread by hexagonal CA in homogeneous Ocean.

5.2 Number of Cells Traversed (Homogeneous Oceans)

Here the number of cells traversed in hexagonal cellular automata for homogeneous ocean with horizontal wave motion and primary wave.

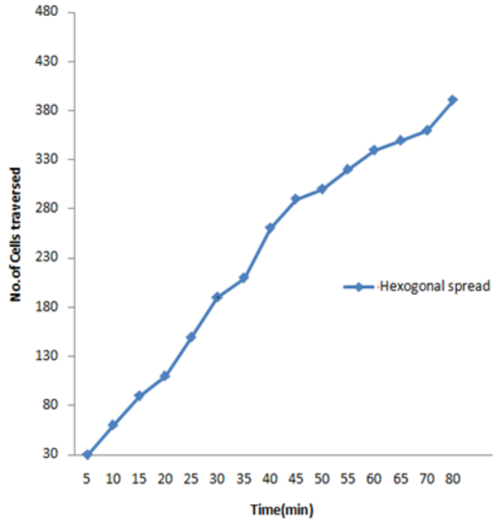


Fig.18 Number of cells traversed by hexagonal CA non- homogeneous Ocean.

5.3 Rate of Spread (Non-Homogeneous Oceans)

Here the rate of spread of hexagonal cellular automata for non-homogeneous oceans with horizontal wave motion and primary wave. The rate of spread is uneven in our results, which is more compatible with the rate of spread in non homogeneous oceans.

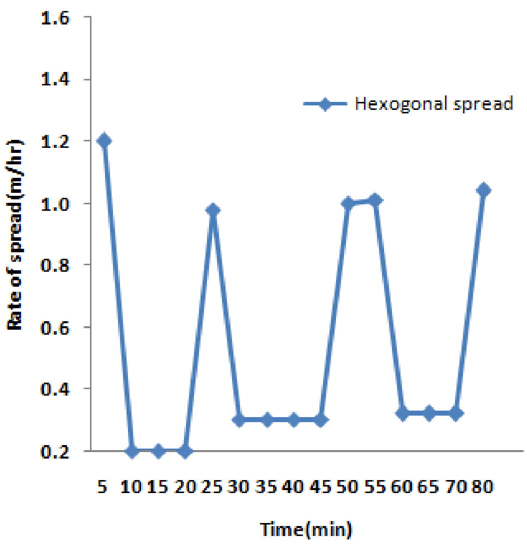


Fig.19 The rate of spread by hexagonal CA in non-homogeneous Ocean.

5.4 Number of Cells Traversed (Non-Homogeneous Oceans)

Here the number of cells traversed in hexagonal cellular automata for non homogeneous oceans with horizontal wave motion and primary wave. In our result, the area of the cells is more and the number of cells traversed is less, which is more accurate.

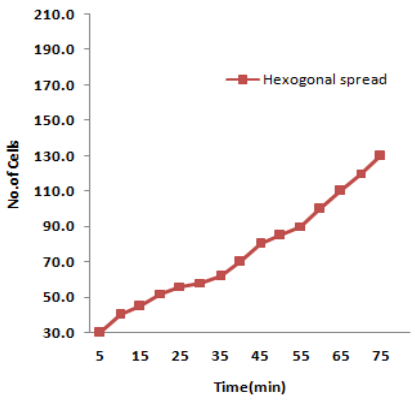


Fig.20 Number of cells traversed by hexagonal cellular automata in non-homogeneous Ocean.

6 Conclusion

We proposed a hexagonal cellular automata model of tsunami wave propagation for independent geographical areas. Basically we have proposed a circular spreading of the wave front, when it comes from nearest and adjacent cells. The hexagonal cellular automata model determines the dynamic of the wave front in both homogeneous and non-homogeneous ocean with eight cases depending on the wave based topography conditions to determine the proposed models. From these results it is evident the hexagonal models are a suitable approach to tsunami wave modelling .This method of modelling tsunami wave is also easily extensible and useful if one wants to experiment with different tsunami wave models. The research also explained how the tsunami and environment are visualized and a number of alternative visualization options are tested. Shallow water wave equation may be used to make the model more accurate.

However, some changes in the notation of the state of the cell can be studied. In this case, a similar Cellular Automata model will be designed in which the states of the cell will be defined by means of transfer energy, instead of transfer of fractional traversed area.

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