Application of Analytical Hierarchy Process (AHP) in the selection of a flexible production system

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Abstract: - The Analytic Hierarchy Process (AHP) is a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology. It has application in group decision-making and is used worldwide in a wide variety of decision-making situations. Rather than prescribing a "correct" decision, AHP helps decision-makers choose the decision that best fits their goal and their understanding of the problem. The technique provides a comprehensive and rational framework for structuring a decision problem, representing and quantifying its elements, relating those elements to general objectives, and evaluating alternative solutions. Given each specific situation, making the right decisions is probably one of the most difficult challenges for managers. The obtained results allow the manager to evaluate the employees in an objective way and make an objective decision for their promotion. This tool not only supports and qualifies decisions, but also allows managers to justify their choices as well as simulate possible outcomes.

Key-Words: - Analytic, decision, choice, technique, process.

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1 Introduction

The terms problem, decision, decision-making, decision-making process, decision-making, etc., are not only used in everyday life but also in modern business and professional work, and at the same time, their true meaning is not always known. The identification and analysis of the problem to be solved, the determination of the possible solutions to the problem, and the criteria according to which the possible solutions are evaluated, i.e. the alternatives and the choice of the best possible solution is a process of making a decision - DO (English Decision Making - DM), i.e. decision-making process, and as a result of the decision-making process, the decision emerges. It represents the very choice of the best, from the most possible alternative solutions to the problem. [1]

The analytical hierarchy process has application in group decision-making and is used worldwide in a wide variety of decision-making situations in areas such as government, business, industry, health, shipbuilding, and education. [2]

The AHP method is one of the possible solutions for the construction and application of a multi-criterion decision-making system. It was developed in the 70s of the last century in the USA. During the past decades, it has been the subject of much methodological research and it has been used with success in solving many practical problems. [3-9]

The founder of the AHP method is Thomas Saaty, who worked out the methodological foundations of this concept as a professor at the Wharton Business School in Philadelphia in the early 70s. Wider interest in the method came in the 80s after the publication of the publication The Analytic Hierarchy Process by the renowned publisher MCGRAW -Hill. It seems that the method began to be popularized and spread from the establishment of the foundations of systems theory, as well as from the attempts to develop and provide a formal description of one of the basic characteristics of this system, which L. Bertalanffy already calls "hierarchical order". [3-9]

Saaty [6] describes seven basic pillars of the AHP method, which are the following:

- ratio scales, proportional and normalized ratio scales.
- mutual comparison of pairs.
- sensitivity of the basic right eigenvector.
- clustering and using pivots to scale.
- Synthesis of the created one-dimensional scale of relationships that represent the total result.

• Rank retention and vice versa.
• integrating group reasoning i.e., evaluate.

Using a ratio scale for comparisons from the perspective of the result helps us unify the multidimensionality of the problem into a single dimension.

1.1 Steps in applying the AHP method

When applying the AHP method, five steps are set:
Step 1: Define the problem and the criteria.
Step 2: Define the alternatives.
Step 3: Prioritize the criteria and alternatives using pairwise comparison.
Step 4: Check for consistency between pairwise comparisons.
Step 5: Evaluate the relative weights of the pairwise comparisons and obtain the calculated total priorities for the alternatives. [3-9]

2 Problem Formulation

To show the essence of the AHP model, a concrete example model for the evaluation of different flexible production systems is presented. The first step of AHP consists of developing a hierarchical structure of the estimation problem. In this case, the objective is the selection of the best flexible manufacturing system.

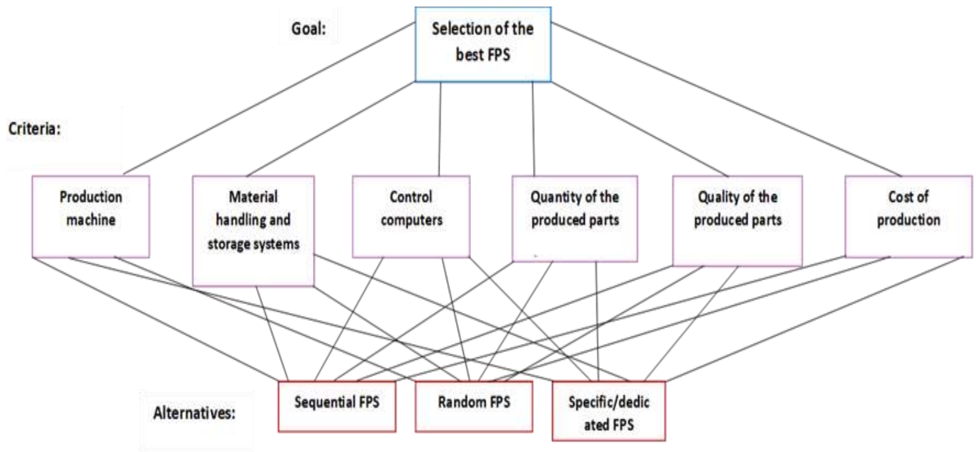

Fig.1 Hierarchical structure of the problem

The criteria are production machine, material handling and storage systems, control computers, quality of produced parts, quantity of produced parts and production cost, and the alternatives are the selection of the best FPS: sequential FPS, random FPS and specific/dedicated FPS.

According to Fig. 1, in this case, the first level, that is, at the top of the goal, is the selection of the most favorable flexible production system.

On the second level, the next level is the criteria, i.e.:

K1 - production machine,
K2 - systems for handling and storing materials,
K3- control computers,
K4- quality of manufactured parts,
K5- quantity of produced parts and
K6- production cost.

Finally, at the third or last level (alternative) of the hierarchy, are the three different types of flexible production systems, i.e., FPS1, FPS2 and FPS3, (hereafter denoted as A1, A2 and A3), which must be evaluated, and compared and chosen the best among them.

3 Problem Solution

Based on the established hierarchical structure and based on the preferences set by the decision maker, an evaluation matrix is formed, i.e., a matrix of comparison pairs (Table 1.).

The next step is to calculate an eigenvector corresponding to the eigenvalues. The values for the eigenvector are given in Table 2.

To obtain the values of the eigenvector, which is also called the priority vector, the procedure is as follows: first, we obtain the elements of the newly formed matrix by dividing the elements of the reciprocal matrix (Table 2) by the sum of the corresponding column, i.e., the value of element K_{11}
is obtained as follows: 1/15=0.066667.

To obtain the final eigenvector, i.e., the weights of the criteria according to which the ranking will be performed, it is necessary to calculate the arithmetic mean for each row of the normalized matrix (Table 2), i.e., the sum of each row is divided by the number of elements:

$$w_1 = 0.066667 + 0.024756 + 0.03876 + 0.1 + 0.25 + 0.090164 = = \frac{0.399335422}{6} = 0.06 \quad (1)$$

	K_1	K_2	K_3	K_4	K_5	K_6
K_1	1	0.33	0.25	0.5	0.5	0.33
K_2	3	1	0.2	0.33	0.5	0.5
K_3	4	5	1	0.5	1	0.5
K_4	2	3	2	1	1	0.33
K_5	2	2	1	1	1	1
K_6	3	2	2	3	1	1
SUM	**15**	**13.33**	**6.45**	**6.33**	**5**	**3.66**

Table 1. Evaluation matrix/comparison of criteria

	K_1	K_2	K_3	K_4	K_5	K_6	ABSOLUTE WEIGHTS
K_1	0.06	0.02	0.03876	0.078989	0.1	0.090164	**0.0665559**
K_2	0.2	0.07	0.031008	0.052133	0.1	0.136612	**0.09912854**
K_3	0.26	0.37	0.155039	0.078989	0.2	0.136612	**0.20206669**
K_4	0.1	0.22	0.310078	0.157978	0.2	0.090164	**0.18610149**
K_5	0.13	0.15	0.155039	0.157978	0.2	0.273224	**0.17826859**
K_6	0.2	0.150	0.310078	0.473934	0.2	0.273224	**0.26787879**

Table 2. Normalized matrix/weight values

The values of the remaining eigenvectors are identical to the previously described procedure. After obtaining the values for the weights of each criterion, we need to perform a consistency check on the comparisons. In doing so, we search for the largest eigenvalue of the corresponding matrix. For this purpose, we find the values of the average matrix, i.e. the matrix of the sum of the weights. We do this by multiplying each row of the initial evaluation, that is: $B_i(a_{ij})(w_i)$.

$$\left[1\frac{1}{3}\frac{1}{4}\frac{1}{2}\frac{1}{2}\frac{1}{3}\right] \text{x} \begin{bmatrix} 0.066555904 \\ 0.099128538 \\ 0.202066694 \\ 0.186101489 \\ 0.178268588 \\ 0.267878787 \end{bmatrix} = 0.420370033 \quad (2)$$

The remaining values (Table 3) from the average matrix are obtained identically $\boldsymbol{B_i}$.

	K_1	K_2	K_3	K_4	K_5	K_6	SUM OF WEIGHTS
K_1	0.066556	0.032712	0.050517	0.093051	0.089134	0.0884	**0.420370033**
K_2	0.199668	0.099129	0.040413	0.061413	0.089134	0.133939	**0.623696767**
K_3	0.266224	0.495643	0.202067	0.093051	0.178269	0.133939	**1.369191726**
K_4	0.133112	0.297386	0.404133	0.186101	0.178269	0.0884	**1.287400887**
K_5	0.133112	0.198257	0.202067	0.186101	0.178269	0.267879	**1.165684442**
K_6	0.199668	0.198257	0.404133	0.558304	0.178269	0.267879	**1.806510017**

Table 3. Average matrix B_i matrix of the sum of the weights

The next step is the determination of own values. We do this by dividing the values from the average matrix by the weights from the normalized matrix: $\boldsymbol{B_i w}$.

$$\lambda = \frac{0.420370033}{0.0665559} = 6.3 \tag{3}$$

Table 4. Eigenvalues λ

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	SUM	λ_{MAX}
6.3	6.2	6.7	6.9	6.5	6.7	39.6	**6.597**

Next, we find the largest eigenvalue as follows, sum all the obtained eigenvalues and divide by the number of elements λmax.

After finding the largest eigenvalue, the next step is to find the consistency index. We do that through the following formula: **CI**

$$CI = \frac{\lambda_{max} - n}{n-1} = \frac{6.597-6}{6-1} = 0.119 \tag{4}$$

The next and the last step is finding the consistency ratio: **CR**

$$CR = \frac{CI}{CI(r)} = \frac{0.1195}{1.25} = 0.095 < 10\% \tag{5}$$

From this it follows that the ratio of consistency is acceptable and that our estimates are accepted.

After all the necessary calculations have been performed, the final ranking (Figure 2.) can be displayed for the importance of the criteria according to which the selection will be made.

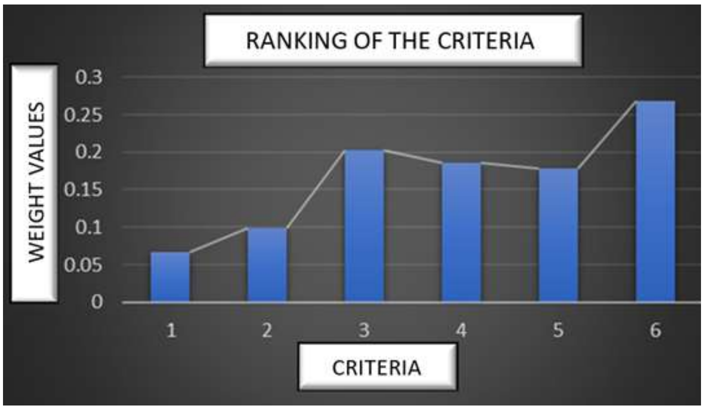

Fig.2. Ranking criteria according to importance

In the following, only the results of the research are given. The method of their calculation is identical to the one that was previously explained in detail.

Table 5. Average matrix B_i matrix of the sum of the weights according to K_1 - production machine

	A_1	A_2	A_3	SUM OF WEIGHTS
A_1	0.411111	0.522222	0.327778	**1.261111111**
A_2	0.205556	0.261111	0.327778	**0.794444444**
A_3	0.411111	0.261111	0.327778	**1**

According to the production machine criterion, the best ranked is FPS_1.

Table 6. Average matrix B_i matrix of the sum of the weights according to K_2 - the criterion systems for handling and storing materials

	A_1	A_2	A_3	SUM OF WEIGHTS
A_1	0.328296	2.397525	0.04115	**0.922323586**
A_2	0.045961	0.342504	2.962807	**1.117090778**
A_3	2.626365	0.037675	0.329201	**0.997747009**

According to the material handling and storage systems criterion, the best ranked is FPS_2.

Table 7. Average matrix B_i matrix of the sum of the weights according to K_3 - control computers

	A_1	A_2	A_3	SUM OF WEIGHTS
A_1	0.106014	0.085796	0.126799	**0.318609891**
A_2	0.318043	0.259989	0.209219	**0.787250374**
A_3	0.530071	0.779966	0.633997	**1.944034375**

According to the controller computer's criterion, FPS_3 is the best-ranked.

Table 8. Average matrix B_i matrix of the sum of the weights according to K_4 - quantity of produced parts

	A_1	A_2	A_3	SUM OF WEIGHTS
A_1	0.334032	0.262603	0.422284	**1.018919388**
A_2	0.668064	0.525207	0.422284	**1.615554665**
A_3	0.110231	0.173318	0.140761	**0.424310118**

According to the quantity of produced parts criterion, FPS_2 ranks best.

Table 9. Average matrix B_i matrix of the sum of the weights according to K_5 - quality of manufactured parts

	A_1	A_2	A_3	SUM OF WEIGHTS
A_1	0.724174	0.5775453	0.966598	**2.268317325**
A_2	0.101384	0.0825065	0.063795	**0.247686292**
A_3	0.144835	0.2475194	0.19332	**0.585673814**

According to the criterion of the quality of the manufactured parts, FPS_1 is ranked best.

Table 10. Average matrix B_i matrix of the sum of the weights according to K_6 - cost of the production

	A_1	A_2	A_3	SUM OF WEIGHTS
A_1	0.106014	0.085796	0.126799	**0.318609891**
A_2	0.318043	0.259989	0.209219	**0.787250374**
A_3	0.530071	0.779966	0.633997	**1.944034375**

According to the cost of production criterion, FPS_3 ranks best.

After receiving the weights for the importance of each flexible production system in relation to each criterion, they are summed up and the final ranking of each of them is obtained.

The subtotal of each flexible production system offered is given in Table 11.

	K_1	K_2	K_3	K_4	K_5	K_6	INTERSUM OF EACH FPS
A_1	0.411	0.328	0.106	0.334	0.724	0.106	**0.301**
A_2	0.261	0.342	0.259	0.525	0.082	0.259	**0.286**
A_3	0.327	0.329	0.633	0.140	0.193	0.633	**0.413**

Table 11. Subtotal of FPS

The final ranking, and thus the result of the research conducted to rank the most favorable flexible production system that would be used by manufacturing enterprises, is given in Table 12 and Figure 3.

Table 12. Final rank of the best FPS

A_3 -DEDICATED FPS	**0.413**
A_1 -SEQUENTIAL FPS	**0.301**
A_2 -RANDOM FPS	**0.286**

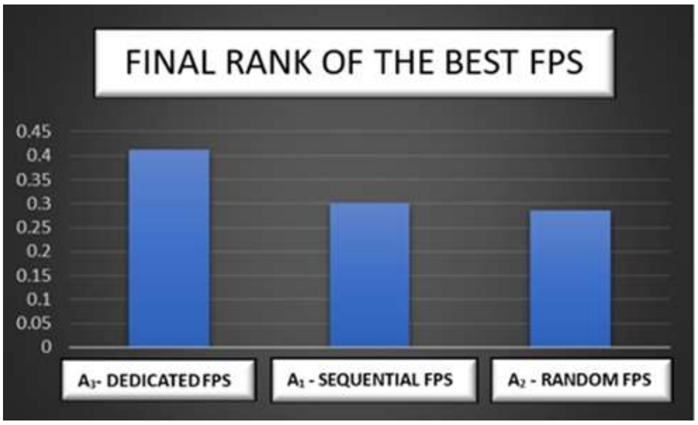

Fig. 3. Final ranking of the best flexible manufacturing system

After the detailed analysis and assessment in this case, the most favorable flexible production system is the third one, that is, a dedicated or specific flexible production system.

4 Conclusion

Production at FPS is largely automated, reducing overall labor costs. Systems typically consist of three main functions: a central control computer, production machinery, and material handling systems that allow the system to remain operational. These systems have a great impact on the future of production, making them an indispensable tool for many companies in the future. Using robots, computer numerical control machines and other automation technologies, FPS can significantly improve production efficiency and reduce labor costs. After detailed analysis and evaluation, the third FPS3, i.e., the dedicated or specific flexible production system, was obtained as the most favorable flexible production system.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Sara Srebrenkoska, Aleksandra Apostolova organized and executed the experiment and the optimization.
- Misko Dzidrov carried out the conceptualization,
- data curation, formal analysis and methodology.
- Dejan Krstev was responsible for the statistics and the visualization.

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