Numerical Investigation of Magnetohydrodynamic Flow of Reiner–Philippoff Nanofluid with Gyrotactic Microorganism Using Porous Medium

S.K. PRASANNA LAKSHMI[1], S. SREEDHAR[1*], S.V.V RAMA DEVI[2]
[1]Department of Mathematics, GITAM School of Science,
GITAM Deemed University, Visakhapatnam, Andhra Pradesh, INDIA
[1*]Department of Mathematics, GITAM School of Science,
GITAM Deemed University, Visakhapatnam, Andhra Pradesh, INDIA
[2]Department of Mathematics, Raghu Engineering College (A), Visakhapatnam, Andhra Pradesh, INDIA

Abstract: Nanoparticles facilitate the enrichment of heat transmission, which is crucial in many industrial and technical phenomena. The suspension of nanoparticles with microbes is another intriguing study area that is pertinent to biotechnology, health sciences, and medicinal applications. In the dispersion of nanoparticles, the conventional non-Newtonian fluid Reiner-Philippoff flows across a stretching sheet, which is examined in this article using numerical analysis. This study investigates the numerical investigation of Arrhenius reaction, heat radiation, and vicious variation variations on a Reiner-Philippoff nanofluid of MHD flow through a stretched sheet. Thus, for the current nanofluid, nanoparticles and bio-convection are highly crucial. The set of nonlinear differential equations is translated into Ordinary Differential Equations (ODEs) utilizing the requisite translation of similarities. These collected simple ODE are solved using the MATLAB computational tool bvp4c method. The graphical results for the velocity, concentration, motile microorganisms, and temperature profile are defined using the thermophoresis parameter and the Brownian motion respectively. Consider a tube containing gyrotactic microbes and a regular flow of nanofluid which is electrically conducted through a porous stretched sheet surface. This nonlinear differential problem is solved by a hybrid numerical solution method using fourth-order Runge-Kutta with shooting technique. The optimization method also performs well in terms of predicting outcomes accurately. As a result, the research applies the Bayesian Regularization Method (BRM) to improve the accuracy of the prediction results. Physical constraints are plotted against temperature, velocity, concentration, and microorganism profile trends and they are briefly described.

Keywords: Non-Newtonian Fluid, Reiner-Philippoff Nanofluid, Bioconvection, Gyrotactic Microorganism, Thermal Radiation, MHD, Bayesian Regularization, Fourth Order Runge-Kutta.

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1. Introduction

Researchers have focused a lot of emphasis on the study of non-Newtonian fluids because of its numerous demand in science and engineering. Because of the complex interactions between stress and shear rate strain, it is challenging to develop a single relationship that entirely accounts for the behaviour of non-Newtonian fluids [1]. The Reiner-Philippoff fluid exhibits shear-thinning (Pseudo-plastic), shear-thinning (dilatant), and Newtonian behaviour. In the same category as Power-law, Sisko, Powell-Eyring, Prandtl-Eyring and Sutterby are some more fluids that fall under the same category. On a stretching sheet, Gangadhar thought about using heat radiation and micropolar Ferrofluid [3]. Nawaz discovered that the index power-law of the velocity field grows when a Sutterby fluid impinges on an extendible surface, taking into account the impacts of the nanoparticles together with Ohmic effect and viscous dissipation [4]. A slippery extensible surface was used in Jyothi's (2020) investigation into the nanoparticles characterization of gold on Sisko fluid. Eyring-Powell fluid with nanoparticles of aluminium oxide, and found that the thermal boundary thickness increases when the radiation parameter is increased [5]. Non-Newtonian heat transfer has a variety of applications, including paper production, wire coating, heating and flavouring food. The effects of nanoparticles on thermal conductivity phenomena are enhanced. The well-known substance known as a nanofluid is used in a number of technical, commercial, and scientific domains [6]. They have creative thermal applications. The thermophysical characteristics of nanofluids are followed by certain dynamic applications in nuclear heating and cooling devices, solar concerns, magnetic retention, astronomy and safety, automated operation, etc. [7].

Typically, it is assumed that the nanoparticles are tiny metallic particles slighter than 100 nm. The nanofluids, as opposed to the more traditional viscous liquids, make up for increased energy transmission. Choi was the first researcher to pay close attention to the thermal aspect of nanofluids [8]. There are numerous industrial and medical uses for nanofluids, which has drawn the attention of researchers worldwide. Mahanthesh (2019) studied magneto nanofluids with a rotating disc and convective boundary conditions. The Reiner-Philippoff models [9] are the most important models for understanding the characteristics of these fluids. The Reiner-Philippoff model pertains to the pseudoplastic/shear-thinning fluids class. There aren't many study works in the literature that discuss the Reiner-Philippoff fluid boundary layer properties. MHD (Magneto-hydrodynamics) is the research of the motion of the electrically conducting fluid induced by external magnetic forces [10]. The MHD flow problem of a nanofluid in a boundary layer across a porous medium across an exponentially extending sheet was the focus of earlier investigations. It has been demonstrated that unstable viscous nanofluid boundary-layer flow along a vertically extended sheet may transfer mass and heat in the aspect of heat generation, magnetic field, chemical reaction, and thermal radiation [11]. They found that the velocity, concentration profiles, and temperature of the unstable flow varied from those of the corresponding components in the steady-state flow scenario. The energy emission is known as radiation, where the particles or waves through a physical medium, such as space [12].

Smaller-scale physical laws that result in larger-scale events are known as bioconvection. During the bioconvection process, microorganisms with a density greater than water flow higher, resulting in a top-heavy density stratification that commonly destabilises [13]. Many kinds of microbes include those that travel through oxygen (oxytaxis), rotate and spin (gyrotaxis), or move under the influence of gravity (gravitaxis). Interestingly, gyrotactic microorganisms boost the stability of the suspension when added to nanofluid [14]. Magnetic fields and microbes have an impact on the issue of flowing natural boundary layer convection in porous media around a vertical cone saturated by a nanofluid generated by gyrotactic microorganisms. Self-propelled microbe suspensions are the subject of bioconvection. It is crucial to understand how minute, heavier-than-water particles affect the stability suspension of gyrotactic, motile bacteria in a finite depth horizontal fluid layer. The motile bacteria interaction and nanoparticles cause' nanofluid bioconvection, which results from the magnetic field interaction and buoyancy forces [15]. Recently, researchers examined Stefan blow impacts and numerous slides on bioconvection nanofluid buoyancy-driven movement of microorganisms. Computationally, a lot of researchers have written about these remarkable events. A rise in the concentrations of motile microorganisms establishes the efficiency of bio convection. They found that fluid velocity raises as the unsteadiness effect is enhanced while decreasing the viscoelastic term [16]. Consequently, the study discusses the dynamic contribution of Reiner Phillipoff nanofluids' Arrhenius reaction, thermal radiation, and viscosity variation features. To achieve this, the hybrid numerical solution is used to transform the nonlinear differential equation into an ODE. After that, the optimization approach is utilized to successfully forecast accurate results. The MATLAB computational tool bvp4c technique is used to solve these acquired simple ODE.

2. Literature Survey

Mohamed E. Nasr *et al* [17] suggested that the magnetohydrodynamics (MHD), the thermal energy, and the mass transport boundary layer flow characteristics are numerically investigated in non-Newtonian Reiner-Philippoff fluid. The species reaction with respect to the activation energy, double diffusions of Cattaneo-Christov, surface convective conditions, and nonlinear radiation are all explored in terms of energy and mass transfer. Using the right similarity variables, the specified governing system of PDEs is drained into a non-linear differential system. Numerical solutions to the determined flow equations have been found. The electrically conductive nanofluid flow of viscosity change is examined by O.A.Famakinwa *et al* [18], in which arrhenius reaction modifications, and thermal radiation across a convectively heated surface. In the built-in bvp4c software package of MATLAB, the shooting method and the 4th order Runge-Kutta formula are used to solve the nonlinear coupled ODE that results from the mathematical model governing the fluid flow. The data point statistical tool is also used to introduce the slope of linear regression.

Taseer Muhammad *et al* [19] explore the properties of the nanofluid named Jeffrey while taking the influence of motile microorganisms and activation energy into consideration. This review has taken into consideration the magnetic field influence, another significant physical factor in the flow analysis. Thermophoresis is relevant for mass transportation processes in operating systems with substantially larger temperature gradients. To convert PDE into ODE conveniently, the necessary similarity transformation is used. To estimate the numerical results of the obtained normal system of flow, the well-known shooting technique is used. To determine the solution, the MATLAB built-in program's bvp4c strategy is used to integrate the governing dimensionless equations. In Muhammad Jebran Khan *et al* [20] study, the numerical study of a model including bioconvection processes and a gyrotactic motile microbe of magnetohydrodynamics is examined. The model employs the change in thickness effect and the thermal conductivity feature. Nanofluid bioconvection is beneficial for bioscience applications such as content detection, drug delivery, micro-enzymes, biosensors, and blood flow. For the linear regression designed for the suggested model, the New Iterative nuMerical (NIM) technique is adopted and used for numerical simulation.

Najiyah SafwaKhashi *et al* [21] explore the effects of MHD and dissipation on viscous in Reiner-Philippoff fluid flow radiative heat transfer through a nonlinearly contracting sheet. The multivariable differential equations partial derivatives are converted into similarity equations of a certain form by using the proper similarity transformations. In MATLAB software, the resulting mathematical model is explained using the bvp4c method. Meanwhile, raising the suction parameter value that has been demonstrated to enhance the efficiency of heat transfer and the skin friction coefficient, the Reiner-Philippoff fluid is greatly impacted by the suction action. The first solution validity is supported by the stability analysis that results from the establishment of the dual solutions.

In this paper, Iskandar Waini *et al* [22] investigate numerically the flow of radiative non-Newtonian fluid across a decreasing sheet while there is an aligned magnetic field. Multivariable differential equations governing partial derivatives are converted into a particular class of similarity equations. The bvp4c method is employed to get the numerical results. The analysis showed that as the suction parameter increased, the skin friction rate and heat transfer increased as well. When the alignment angle and magnetic parameter are taken into account, an identical pattern shows up. However, the performance of heat transmission is harmed by the adjunct to Bingham number, Reiner-Philippoff fluid, and thermal radiation factors. An investigation of stability confirms the correctness of the solution when the dual solutions are formed.

An analysis of the bioconvection phenomenon for Reiner-Philippoff nanofluid-induced Darcy-Forchheimer flow is presented by Yun-XiangLi *et al* [23]. For the flow, the impact of slip through higher relations is broken down. The radiative pattern for the thermally produced flow was examined. Modified Cattaneo Christov expressions have been exploited to scrutinize the evaluation of mass and heat transfer. The dimensionless form is created by transforming the flow equations related to momentum, volumetric friction, and motile microbe density. Using the computing programme MATLAB, transformed dimensionless non-linear equations are tracked using the shooting approach, and the outcomes of significant parameters are depicted via various graphs. The accessible dynamic property of Reiner-Philippoff nanofluid is discussed by Sami UllahKhan *et al* [24] with bioconvection phenomenon applications. The Reiner-Philippoff nanomaterial's radiative analysis is also designed to be performed using the activation energy features and magnetic force impact. To examine the flow, the higher-order relations of slip features are included. To suggest variations from the energy equation, thermal radiation and its nonlinear relationship are used. The flow equations are numerically solved by applying a shooting strategy, which leads to their non-dimensionless form. For the endorsed parameters, a detailed thermal analysis is presented. To analyse the variations in motile density, mass and heat function, numerical data is obtained.

The flow of Eyring-nanofluid Powell's through a porous plate vulnerable to surface suction and heat radiation is said to produce a flow of two-dimensional gyrotactic microorganisms suggested by Naseer M. Khan *et al* [25]. The nonlinear approximation of Rosseland was introduced to incorporate solar radiation parameters into the energy equations, while the Buongiorno model of nanofluid was developed to include the momentum and energy equations. The problem's numerical solution was discovered using the MATLAB "bvp4c" scheme. The analysis is done on how different physical parameters affect the distribution of velocity, temperature, and concentration. Although suction decreases the temperature, it speeds up the heat transmission. A thixotropic fluid flow of a gravity-driven system containing both gyrotactic microorganisms and nanoparticles beside a vertical surface was examined by Olubode Kolade Koriko *et al* [26]. Special instances of controls nanoparticles of passive and active are studied to further explain the transport phenomena. The governing ODE of the gyrotactic microorganisms' momentum, energy, concentration, and density are parameterized and turned into a PDE system. The OHA method is later employed to deduce the series solutions. With regard to the characteristics of mobile microorganisms, the relevant crucial parameters are examined and displayed.

3. Research Problem Definition and Motivation

Due to its numerous industrial and technological applications, the study of heat transfer and boundary layer flow over a stretching surface had significant success over the decades. Recently, several industrial processes have benefited greatly from the research of Nanofluids and convective heat transfer. The nanofluid's thermal conductivity has been the subject of several theoretical and experimental studies by many scientists. Over the past ten years, nanofluids have gained importance due to their wide range of applications, particularly in the problem of boundary layer flow. Nanofluids are useful for improving the thermo-physical characteristics of the governing fluid, such as thermal diffusivity, convection, and conductivity. The word "bioconvection" refers to the additional mobility of swimming microorganisms caused by the macroscopic movement of the fluid on account of the spatial variation of density over a region. A bio-convective stream is created when the self-driven motile microorganisms improve the base fluid in a certain direction. Smaller density microorganisms float, which is the basis of the bioconvection process. The non-uniform instability structure is typically too responsible for the microorganisms swimming in the top region.

Due to biological germs swimming on the higher surface, the upper section becomes unstable which enlightens the increased stratification density. The interplay between nanoparticles and bioconvection aspects is crucial for microfluidic applications since swimming microbes cannot travel in nanoparticles' uniform motion. Additionally, the interplay of buoyancy forces, nanomaterials, and microorganisms result in bio convection of nanoparticles, which is related to pattern formation and stratification density. The nanoparticle suspension stability is frequently observed to be greatly increased when gyrotactic bacteria are present. Numerous important physical processes are regularly modelled using nonlinear differential equations in the relevant field of science and technology. These equations are frequently difficult or impossible to solve analytically. However, analytical approximation techniques for obtaining reasonably correct results have grown significantly in importance recently. This study then describes how bioconvection can be used in Reiner-Philippoff nanofluid flow while also dealing with thermal radiation and viscosity variation. The thermal applications of the Reiner-Philippoff nanofluid are addressed via nonlinear radiation relations. With intriguing applications, the physical perspective of parameters is graphically operated out. Additionally, the numerical calculations are exposed to show how mass, motile density, and heat rate change in relation to flow parameters.

4. Research Proposed Methodology

The objective of the ongoing research is to create a thin Reiner-Philippoff nanofluid MHD flow over a stretching sheet using thermal radiation and heat transfer. The physically demonstrated geometry yields boundary-layer equations. Thermophoresis effects and Brownian motion are also encircled with different physical parameters. With the aid of new variables, a similar solution is attained, which causes a complex model to be reduced to a straightforward coupled ODE. The simplified system is solved using an analytical approach. For converting the nonlinear differential equation into regular equations, Runge Kutta's fourth-order and shooting method are suggested. The outcomes are plotted, tabulated, and systematically discussed using a number of different physical characteristics. The optimization approach is also used to increase the precision of the numerical solution.

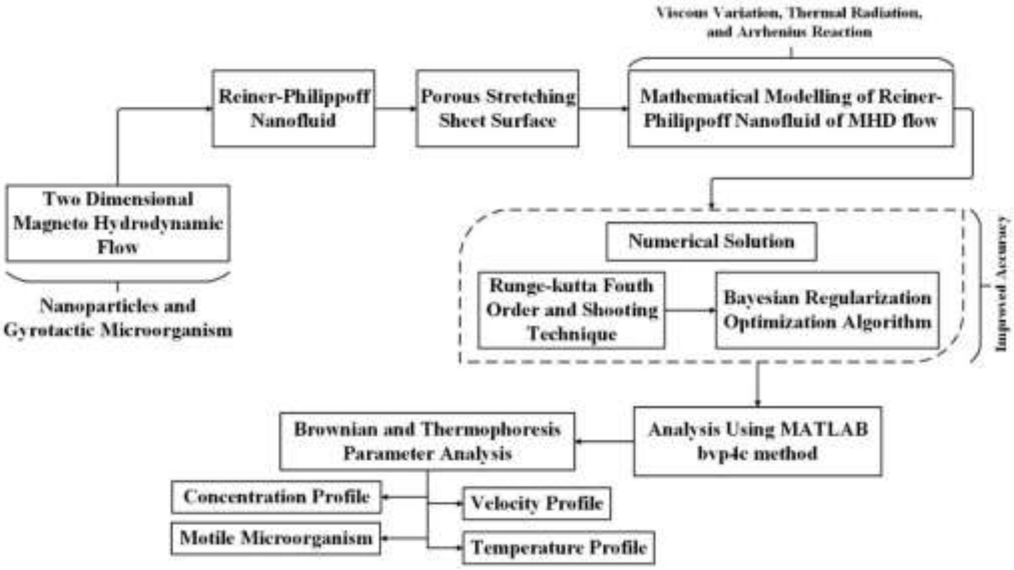

Figure 1: Proposed Work Flow Diagram

The flow diagram for the suggested task is shown in Figure 1. This article examines the numerical investigation of changes in Arrhenius reaction, thermal radiation, and viscous variation on a Reiner-Philippoff nanofluid of MHD flow using the stretching sheet. Nanoparticles and bio-convection are therefore quite important for the current nanofluid. The required translation of similarities is used to convert the set of nonlinear PDE into ODE for this purpose, this also mentioned the parameter analysis of Brownian and thermophoresis. The numerical solution method of the Runge-Kutta method of fourth order with shooting technique is used to solve this nonlinear differential problem. The article introduced the BR optimization Algorithm to increase the suggested technique's accuracy.

The MATLAB computational tool bvp4c technique is used to solve these acquired simple ODEs. By using the thermophoresis parameter and the Brownian motion parameter, respectively, the graphical results are defined for the velocity, motile microorganisms, concentration and temperature profile. The study considers the thermal radiation effect on a continuous Reiner-Philippoff nanofluid flow that is two-dimensional and transports nanoparticles and gyrotactic microorganisms through a vertical porous medium of the stretched sheet. Both the horizontal y-axis and the vertical x-axis are subjected to a fluctuating magnetic field. Assume that the fluid is being subjected to a strong magnetic field at a particular rate, and the surface is stretched in the x-direction. The Buongiorno model allows for the observation of the Brownian and thermophoresis distinctiveness.

4.1 Mathematical Modelling

With velocity components u that are rejected along with the sheet, the surface is stretched in the guidance of flow, whereas v is taken in the normal flow direction. The magnetic force often affects the flow direction. In the flow domain, the nanofluid's temperature (T), concentration (C), and motile density (N) are all regarded as being uniform. Utilizing specific relations for the analysis of thermally radiative flow modifies the energy equation.

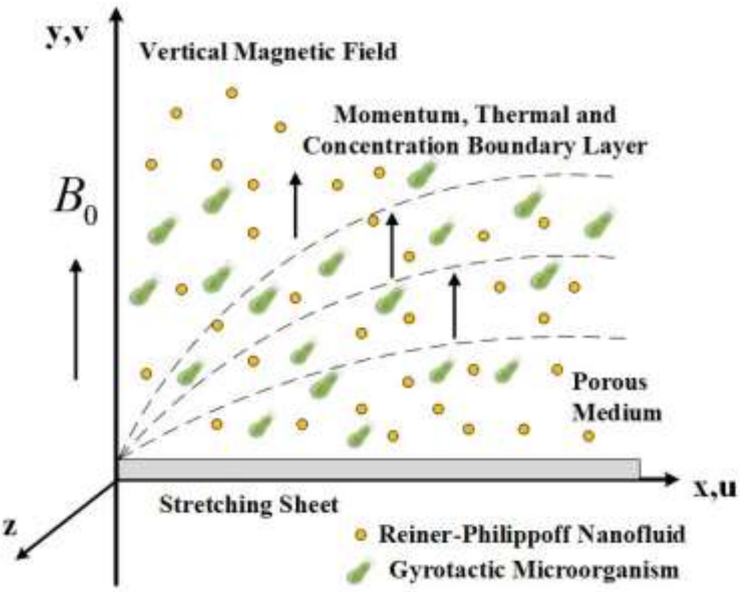

Figure 2: Geometrical Configurations

The geometrical arrangement of a two-dimensional nanofluid flow above the stretching sheet is exposed in Figure 2. Between these two values of viscosity μ, the non-Newtonian character in Reiner-Philippoff fluid is presented. The Reiner-Philippoff fluid model's deformation rate and shear stress relationships are defined as

$$\frac{\partial u}{\partial y}=\frac{\tau}{\mu_\infty+\dfrac{\mu_0-\mu_\infty}{1+\left(\dfrac{\tau}{\tau_s}\right)}} \tag{1}$$

with shear stress τ and deformation rate $\frac{\partial u}{\partial y}$. The boundary layer assumptions form the foundation of the equation system for the current task.

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_f}\frac{\partial \tau}{\partial y}-\frac{\sigma_c B_0^2}{\rho_f}+\frac{g_1}{\rho_f}\begin{bmatrix}(1-C_f)\rho_f\beta^*(T-T_\infty)-\\(\rho_p-\rho_f)(C-C_\infty)-(N-N_\infty)\gamma^*(\rho_m-\rho_f)\end{bmatrix} \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\Lambda_f\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^*}{3(\rho c)_f\kappa^*}\frac{\partial}{\partial y}\left(T^3\frac{\partial T}{\partial y}\right)+\frac{(\rho c_p)_p}{(\rho c_p)_f}\left[D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-Kr^2(C-C_\infty)\left(\frac{T}{T_\infty}\right)^n\exp\left(\frac{-E_a}{k_1 T}\right) \tag{5}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}+\frac{\hat{b}\hat{w}}{(C_w-C_\infty)}\left[\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right)\right]=D_m\frac{\partial^2 N}{\partial y^2} \tag{6}$$

The nonlinear thermal relations for the radiative phenomenon are incorporated into the energy equation using the Roseland approximations. Additionally, the activation energy is represented by the final term in Equation (6), which is obtained by applying the Arrhenius relations.

The boundary conditions are

$$\left.\begin{aligned}&u=u_w(x)=ax^{1/3}+u_{sl},\ \ u_{sl}=A\frac{\partial u}{\partial y}+B\frac{\partial^2 u}{\partial y^2},\ at\ \ v=0\\&-k\frac{\partial T}{\partial y}=h_f(T_f-T),\ \ D_B\frac{\partial C}{\partial y}+\frac{DT}{T_\infty}\frac{\partial T}{\partial y},N=N_w,\ at\ \ y=0\end{aligned}\right\} \tag{7}$$

$$u\to 0, T\to T_\infty, C\to C_\infty, N\to N_\infty \text{ as } y\to\infty \tag{8}$$

Physical quantities such as fluid density ρ_f, kinematic viscosity v, Stefan Boltzmann constant σ^*, nanoparticles heat capacity $(\rho c_p)_p$, applied Boltzmann constant κ^*, thermal diffusivity Λ_m, swimming cells speed $\hat{w}$, electrical conductivity σ_c, volume suspension coefficient $\Omega*$, diffusion constant D_B, chemotaxis constant $\hat{b}$, mean absorption coefficient k^*, microorganisms diffusion constant D_m, magnetic field strength B_0, gravity is represented as g, heat capacity $(\rho c_p)_f$, motile microorganisms is denoted as ρ_m and nanofluid density as ρ_p. For the current flow model, the following non-dimensional set of expressions are anticipated:

$$\left.\begin{aligned}&\eta=\sqrt{\frac{a}{v}}\frac{y}{x^{1/3}},\ \psi=\sqrt{avx^{\frac{2}{3}}},\ \ \tau=\sqrt{a^3vg(\eta)},\ \ \theta(\eta)=\frac{T-T_w}{T_w-T_\infty}\\&\phi(\eta)=\frac{C-C_w}{C_\infty},\ \chi(\eta)=\frac{N-N_w}{N_w-N_\infty}\end{aligned}\right\} \tag{9}$$

$$g = f''\frac{g^2 + A\gamma^2}{g^2 + \gamma^2} \tag{10}$$

The equation of momentum is articulated as follows

$$g' = \frac{1}{3}f'^2 - \frac{2}{3}f\,f'' - M\,f' + Gr(\theta - Nr\phi - Rb\chi) = 0 \tag{11}$$

$$\begin{aligned}&\left[1 + \frac{4}{3}Rd\{1 + (\theta_w - 1)\theta\}^3\right]\theta'' + 4Rd(\theta_w - 1)[1 + (\theta_w - 1)\theta]^2\theta'^2 + \\ &\Pr\left[f\theta' + Nb\theta'\phi' + N_t(\theta')^2\right] = 0\end{aligned} \tag{12}$$

$$\phi'' + Le\Pr f\phi' + \frac{N_t}{N_b}\theta'' - \Pr Le\sigma(1 + \delta\theta)^n \exp\left(\frac{-E}{1 + \delta\theta}\right)\phi = 0 \tag{13}$$

According to the Concentration Equation,

$$\chi'' + Lbf\chi' - Pe[\phi''(\chi + \delta_1) + \chi'\phi'] = 0$$

$$\left.\begin{aligned}&f(0) = 0,\ f'(0) = 1 + \Omega_1 f''(0) + \Omega_2 f'''(0), \theta'(0) = -Bi(1 - \theta(0))\\ &\phi'(0) + \frac{N_t}{N_b}\theta'(0) = 0, \chi(0) = 1\\ &f'(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0, \chi(\infty) \to 0\end{aligned}\right\} \tag{14}$$

Brownian constant $N_b = (\rho c)_p D_B\left(\frac{(C_w - C_\infty)}{(\rho c)_f}\right)v$, bioconvection Rayleigh number $Rb = \frac{\gamma^*(\rho_m - \rho_f)(N_w - N_\infty)}{\beta^*(1 - C_f)T_\infty}$, Philippoff fluid parameter $A = \frac{\mu_0}{\mu_\infty}$, microorganism difference parameter $\delta_1 = \frac{N_\infty}{N_w - N_\infty}$, buoyancy ratio parameter $Nr = \frac{(C_w - C_\infty)(\rho_P - \rho_f)}{\beta^*(1 - C_f)T_\infty}$, Bingham number $\gamma = \frac{\tau_s}{\sqrt{a^3 v}}$, Prandtl number $\Pr = \frac{v}{\Lambda_f}$, Peclet number $Pe = \frac{\hat{b}\hat{w}}{D_m}$, thermophoresis constant $N_t = \frac{((\rho c)_p D_T(T_w - T_\infty))}{(\rho c)_f T_\infty v}$, activation energy $E = \frac{E_a}{k_1 T}$, chemical reaction parameter $\sigma = \frac{kr^2}{a}$, temperature difference parameter $\delta = \frac{T_w - T_\infty}{T_\infty}$, bioconvection Lewis number $Le = v/D_m$ are some of the other parameters that are used.

The dimensionless version of the decreased Sherwood number, Nusselt number, and motile density number is given as follows:

$$\left.\begin{aligned}&Nu(\mathrm{Re}_x)^{-0.5} = -\left(1 + \frac{4}{3}Rd\left(1 + (\theta_w - 1)\theta(0)\right)^3\right)\theta'(0)\\ &Nn(\mathrm{Re}_x)^{-0.5} = -\chi'(0)\end{aligned}\right\} \tag{15}$$

$$Sh(Re_x)^{-0.5} = -\phi'(0)$$

4.2 Numerical Solution of Governing Equation

The shooting approach and the fourth-order Runge-Kutta integration scheme are used to solve the two-point boundary value problem created by the nonlinear coupled differential Equation (7) and the boundary conditions (Equation 8), which is then transformed into an initial value problem. The governing equations are initially converted into ODEs by using an appropriate similarity transformation. After that, this system of ODEs is numerically solved using the Runge-Kutta-Fehlberg technique of fourth order. The unbounded domain $[0,\infty)$ has been changed for numerical solutions by $[0,\eta_{\max}]$ where $\eta_{\max}$ is a real number selected hence that there are no appreciable fluctuations in the solution for $\eta > \eta_{\max}$. The fact that $\eta_{\max} = 7$ guarantees the anticipated level of convergence for all the numerical results described in this paper is remarkable. The momentum equations (11) and (12) will be solved collectively using the shooting method, with the temperature and concentration equations solution using f as a known function.

The momentum equations (11) and (12) are transformed into the first-order ODEs, f is represented as y_1, f' and y_3 is denoted by y_2, g, and the missing initial condition is stated as

$$\left.\begin{aligned} y_1' &= y_2 \qquad y_1(0)=0 \\ y_2' &= \frac{y_3\left(y_3^2+\gamma^2\right)}{\left(y_3^2\right)},\ y_2(0)=1 \\ y_3' &= \frac{1}{3}y_2^2 - \frac{2}{3}y_1 y_2',\ y_3(0)=s \end{aligned}\right\} \tag{16}$$

The Runge Kutta fourth-order (RK4) algorithm has been used to numerically manage the aforementioned system (16). Additionally, Newton's technique is used to update the missing initial conditions until the criteria are satisfied.

$$max\{|y_2(\eta_{max}) - 0|\} < \epsilon \tag{17}$$

In this case, the symbol ϵ denotes the positive number with the value $\epsilon = 10^{-6}$ and $\eta_{max} = 7$.

The temperature equation (13) is transformed into a system, which uses differential equations of the first order (16) with known functions f to represent θ by u_1 and θ' is represented as u_2. With the initial conditions, it is feasible to achieve the system of ODEs in (18).

$$\left.\begin{aligned} u_1' &= u_2 \qquad ,u_1(0)=1 \\ u_2' &= \frac{\left[1+4Rd(\theta_w-1)(1+(\theta_w-1)u_1)^2u_2^2+(2/3)Prfu_2+(A^*exp^{-\eta}+B^*u_1)\right]}{((1+u_1)+(4/3)Rd(1+(\theta_w-1)u_1)^3)} \\ &\qquad ,u_2(0)=1 \end{aligned}\right\} \tag{18}$$

To produce θ and θ', equation (18) is treated in the same manner as equation (16).

By designating ϕ as z_1 and ϕ' as z_2, and using f as a known function, the concentration equation (14) is converted into the first-order ODEs. The system of equations that results is

$$\left. z_1' = z_2 z_1(0) = 1,\quad z_2' = \frac{\left(z_2^2+(2/3)fz_2\right)}{1+z_1},\ \ z_2(0)=u \right\} \tag{19}$$

Furthermore, the optimization process maintains the precision of numerical solutions. The following is a description of the solution process that was used based on ANN's back propagated with BR.

4. 3 Bayesian Regularization Optimization Algorithm

It is decided that the solution requires a description of the neural network-based approach, including the layer structure, hidden neurons, topology of the networks, and arbitrary selection of an input and target data set for training, testing, and validation samples. The suggested approach based on ANN back propagated with BR, or ANN-BR is executed using the MATLAB software's "nftool," which takes advantage of the neural networks environment. The solution process includes a description of a significant dataset and an execution process for the suggested ANN-BR. Figure 3 displays the solution methodology's overall workflow.

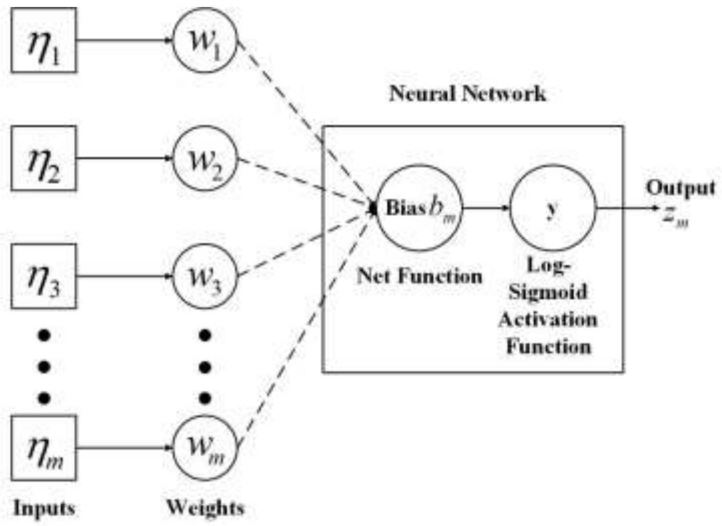

Figure 3: BR Network Mathematical Representation

The data point's total number for the ANN-BR model is 1001, which was discovered between 0 and 1 by utilising the Runge-Kutta method in Mathematica's "NDSolve" function with 0.001 as the step size. To get the best convergence, these data are randomly dispersed for f, f', θ, and ϕ into sets for training, testing, and validation. The reference datasets for the temperature profile $\theta(\eta)$, $\theta'(\eta)$ and velocity profile $f(\eta)$, $f'(\eta)$ respectively. Additionally, for 61 inputs, the concentration profile is denoted as $\phi(\eta)$ and $\phi'(\eta)$, along with the motile density, profile is represented by $\chi(\eta)$ and $\chi'(\eta)$ are generated. As 85% of the data in this problem are used for training, 10% are used for testing, and 5% are used to validate the suggested ANN-BR using a neural network. Additionally, the neurons amount is mutable; 20 neurons gives the computational findings of high accuracy.

5. Analysis of Results and Discussion

This section provides results of a systematic analysis obtained from numerical computations on Equations (11–14) using the MATLAB software's bvp4c module. The analysis includes the effects of several physical parameters that come into play in the suggested model, with the calculations' results shown in graphical and tabular form. Using the default settings of the dimensionless parameter, effects of viscosity variation analysis, Arrhenius reaction, and thermal radiation flow under study are displayed as tables and figures. The effects of several selected factors have been assessed and shown for velocity profiles $f'(\eta)$, temperature profiles $\theta(\eta)$, nanoparticle concentration profiles $\phi(\eta)$, and motile microorganism density profiles is denoted as $w(\eta)$. The governing parameters have the theoretical values of $M = Nr = Rb = 0.5$, $N_b = N_t = \delta_t = \delta_n = K_1$ and $K_2 = 0.1$, $Sh = Pe = 1.0$, $\Pr = Le = 2.0$. The number of local Nusselt is demonstrated in Table 1 to be a decreasing function of $\Pr, Nb$ and Nt for the nanoparticles.

Table 1: Sh_x, Nu_x and Nn_x for Various M and P_0

M	P_0	Nu_x	Sh_x	Nn_x
2	0	2.5231	-0.2700	2.8226

M	P_0	Nu_x	Sh_x	Nn_x
3	1	3.4493	-0.1697	3.5544
4	10	4.2423	-0.0506	4.2734
5	100	5.0333	0.0800	4.9863

The current results also indicate that higher convergence of the bioconvection parameter can be attained if the porosity of the medium rises. Due to the huge size of holes ($0 < P_0 < 100$), which prevent them from ingesting microorganisms, there is no discernible variation in the concentration Nn_x of motile bacteria (Table 1). Because the porosity parameter is not included in the concentration and heat equation, it has little impact on the rates of mass and heat transport. Additionally, only a significant shear stress shift is observed, due to the inclusion of P_0 in the momentum equation. The thermal radiation presence has no influence on bioconvection across porous surfaces, and the action of heat radiation results in a growth in the motile microorganisms' density Nn_x. The obtained results depict that the Nn_x density of local motile microbes is only stable at high porosity.

Table 2: Numerical Analysis of $-\phi'(0)$ Flow Parameter

Pr	λ	Nr	Nc	Rd	Nb	Nt	$-\phi'(0)$
0.1 0.7 1.1	0.4	0.4	0.2	0.6	0.4	0.4	0.63547 0.67878 0.71657
0.2	0.3 0.7 1.1						0.59786 0.53653 0.49775
	1.0	0.3 0.5 0.8					0.58855 0.57435 0.55534
		0.2	0.3 0.5 0.9				0.55764 0.52764 0.50134
				0.3 0.9 1.5			0.60744 0.63765 0.64748
2.0	1.0				0.1 0.5 0.9		0.68741 0.71745 0.7326
						0.1 0.7 1.3	0.62953 0.59664 0.53743

Due to this $\Pr$, Nb and Nt a rising shift in heat transmission is observed, while Nr and Rd attain lower numerical data. Table 2's numerical data illustrated the variation in the $-\phi'(0)$assertion, the transfer rate of mass slows with increasing values of Nb, Nr and Nc.

5.1 Brownian Motion and Effect of Thermophoresis Parameter Analysis

The fluid's velocity is decreased as a consequence of the overall effect. It has been demonstrated that lowering the boundary layer solute results in a Brownian motion parameter increase. As a result, nanoparticles in the inactive fluid began to move away from the surfaces as the boundary layer warmed up. The effect of thermophoresis and the Brownian motion on the velocity, concentration profiles, motile microorganisms, and temperature is depicted in Figures (4–8). A fluid's fast-moving molecules collide with suspended particles, causing Brownian motion, which is the random, "indecisive," movement of the particles.

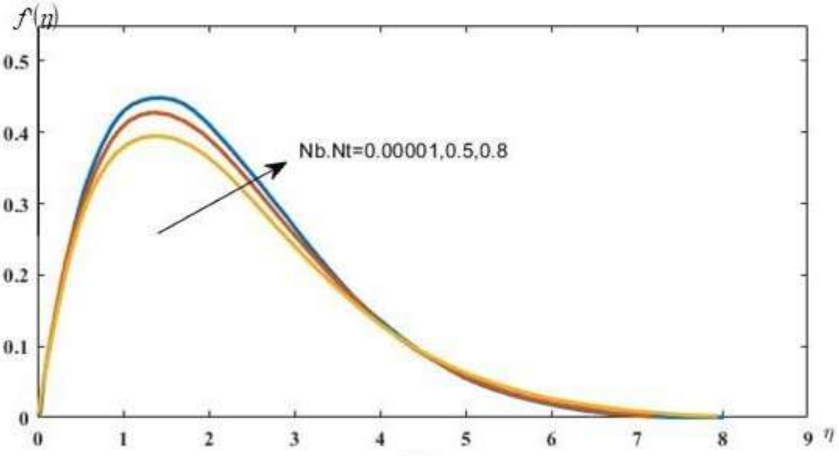

Figure 4: Velocity Profile for Brownian and Thermophoresis

Figure 4 illustrates how parameter Brownian N_b and thermophoresis N_t affect Reiner Philippoff fluid velocity function $f'(\eta)$. Due to the interaction of a growing opposing force, the Lorentz force, the flow speed curve decreases as M values rise. The estimated parameters of brownian motion with the thermophoresis effect on the velocity profiles which is dimensionless for $\Pr = 1$, $Le = 1$, $M = N = 0.5$. It is seen that while the velocity profile $f'(\eta)$ is reduced for the parameter of buoyancy ratio N_r, it is boosted for the bioconvection parameter η.

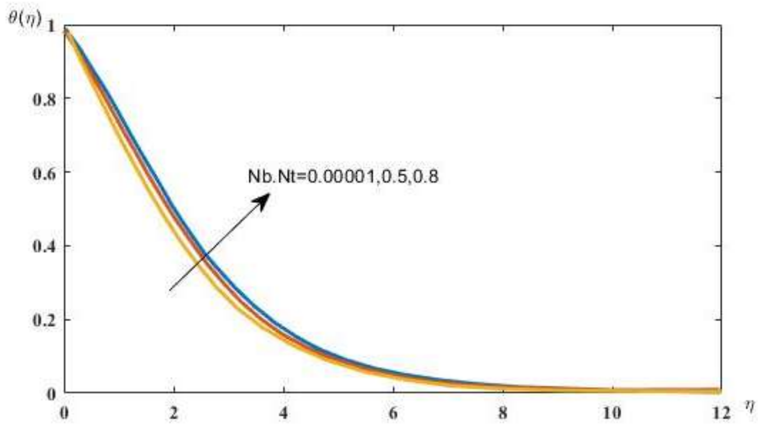

Figure 5: Temperature Profile for Brownian and Thermophoresis

The temperature distribution under the N_t and N_b is shown in Figure 5. Generally speaking, a particle collision results from the particles' erratic velocity. Effects of these parameters on the dimensional fewer temperature profiles for $\Pr = 1$, $Le = 1$, $M = N = 0.5$. The temperature is lowered by means of the Prandtl number. The dimensionless quantity of the Prandtl number correlates with a fluid's viscosity and thermal conductivity. Both the temperature and the width of the boundary layer tend to decrease due to this event. Larger values lead to a rise in temperature, which in turn causes an increase in surface temperature. Similar to this, the temperature profile $\theta(\eta)$ rises as the λ values in the stretching situation increase.

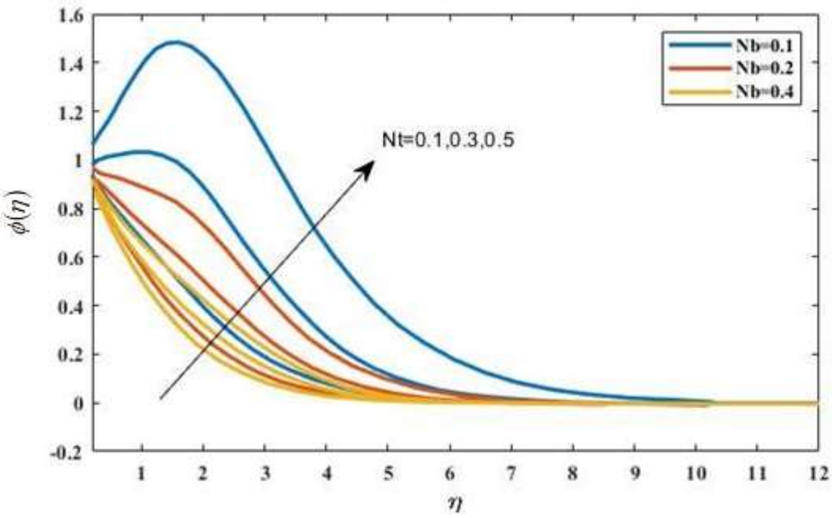

Figure 6: Concentration Profile for Thermophoresis and Brownian motion

The parameter of chemical reaction importance with N_b, N_t on the fluids' concentration Kr, was shown in Figure 6. The improvement in these parameters of the chemical reaction is responsible for a drop in the concentration profile. Meanwhile, viscous fluids ($M = 1$) experience a greater drop. Figure 6 clearly shows that, in the absence of Kr, the concentration distribution grows as increases in the value of N_t. According to Table 3, increasing the viscous variation parameter boosts skin friction coefficients by 0.2786 and heat transmission by 0.02703, but decreases mass transfer by -0.0569 and motile microbial density by -0.0321.

Table 3: Variation in $f''(0), -\theta'(0), -\phi'(0)$ and $-\chi'(0)$ with M when $\eta_{\max} = 7$ and $Kr = 1$

M	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$-\chi'(0)$
1	0.6254	0.0025	1.2345	1.1423
2	0.7893	−0.0489	1.6485	1.3645
3	1.1256	−0.0421	1.5875	1.2678

5.2 Motile Microorganism Profile

The bioconvection is produced by the unstable density stratification. According to recent research, the flow's viscous drag and gravitational torques influence how the microorganisms move (gyrotaxis). The gyrotactic microbe with the nanoparticles was used in the study to determine the bio convective fluid density. The motile microbe density is shown in the following figures, together with thermophoresis and the Brownian motion parameter.

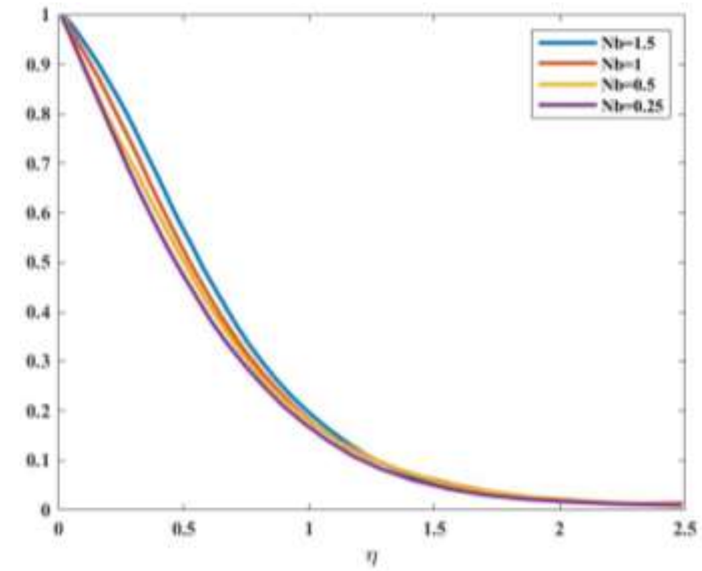

Figure 7: Motile Microorganism Density Parameter Analysis of Brownian motion

For numerous estimates of the bioconvection Lewis and buoyancy number, Figure 7 shows the distribution of the motile microorganisms. The concentration profiles are typically depressed by the expanding values of Le and Br . An increase in the bioconvection Peclet number and motile microorganism's parameter χ values is accompanied by a declaration in the distribution of motile microbes N_b .

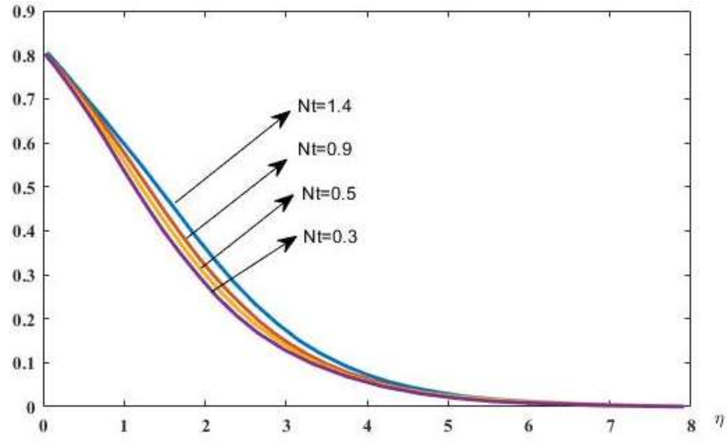

Figure 8: Motile Microorganism Density of Thermophoresis Parameter Analysis

Figure 8 depicts the thermophoresis parameter analysis to show the motile microorganism density. Inversely proportional to Dn (microorganism diffusivity) and directly proportional to one another are the Peclet number and cell swimming speed. As a result, increasing Peclet number values decrease the density profile of motile microbes and increase the flux of wall motile bacteria. The motile microorganisms' density increased due to their diverse variety in Pe and N_t .

Table 4: Comparison of Various Emerging Parameters Values on Nusselt Number

Pr	Rb	θ_w	Nu_x		
			Newtonian Fluid $(\lambda=1)$	**Dilant Fluid** $(\lambda<1)$	**Reiner Philippoff Fluid** $(\lambda>1)$
0.5			1.170370	0.849563	1.271570
1.0			0.985130	0.718474	1.271570
1.5			0.727497	0.411036	1.117839
2.0	1.0		1.167123	0.605998	0.986096
	1.5		1.337873	0.813748	1.648060
	2.0		1.467557	0.909926	1.322524
		0.5	0.865201	1.001301	1.506275
		1.2	1.167123	0.635582	1.465753
		1.5	1.368815	0.925486	1.635462

The obtained values of the local Nu_x for innumerable values of flow issues in the current parameters are revealed in Table 4. For Reiner Philippoff fluid, Newtonian fluid, and dilatant fluid acknowledged that scale back is necessary. The value of Nu_x increases in the pseudoplastic, Newtonian and dilatant fluid is also suggested for rising Rb, Pr and θ_w values. In addition, compared to Newtonian fluid and dilatant fluid, Nu_x is efficiently improved in Reiner Philippoff fluid.

6. Research Conclusion

Nanofluids have been widely used in energy technologies and have already demonstrated significant promise in the thermal amplification of numerous manufacturing industries. The present approach aims to characterise the features of the Reiner Philippoff nanofluid MHD flow while considering the impact of thermal radiation, viscosity change, and the Arrhenius reaction over a porous stretched sheet. Brownian motion along with the effect of thermophoresis features show the extraordinary properties of nanofluid. Thermophoresis is relevant for mass transportation processes in operating systems with substantially larger temperature gradients. To convert PDE into ODE conveniently, the necessary similarity transformation is exploited. It is expected that the process makes use of MATLAB. It is possible to estimate the effects of velocity, concentration, motile microorganism density, and temperature profiles on a number of physical parameters, including the Brownian motion, magnetic and Prandtl number, the thermophoresis parameter, the Lewis number, and the Peclet number.

- The controlling dimensionless equations are integrated using the MATLAB built-in program’s bvp4c technique to attain the solution.
- The shooting method of Runge Kutta's fourth order is used to do numerical computations for the group of equations expressed in coupled and nonlinear forms.
- While Bingham number and permeability of porous space show a decreasing change in velocity, R-P fluid parameter increases fluid velocity.
- The motile microorganisms' density increased due to their varied variety of Pe and N_t
- In the absence of Kr, the concentration distribution is increased by the rising value of N_t.

- The Lewis and Prandtl number, along with the concentration relaxation parameter reveal fading results, although the nanofluids concentration is increased by the thermophoresis constants and activation energy.

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