Sensorless Adaptive Observers for Linearly Parameterized Class of Induction Machine

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Abstract: In this paper a new adaptive flux, speed, load torque and rotor resistance observer is proposed for induction machine drive system. The structure of the proposed observer is simple and it is able to give rise to different observers among which adaptive high gain and adaptive sliding mode. We adopt a mild change of coordinates that allows to easily design of adaptive observer. Computer simulation results verify the validity of the proposed estimation algorithm.

Key–Words: Adaptive observer, high gain observer, sliding mode observer, induction machine, sensorless speed.

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1 Introduction

On line joint estimation of states and parameters in state space systems is of practical importance for adaptive control and for fault detection and isolation. The algorithms designed for this purpose are called adaptive observers. Some early works on this subject can be found in [1, 2, 3, 4] .These results are essentially for linear time invariant (LTI) systems, though some of them have been proposed for nonlinear systems which can be linearized by coordinate change and by output injection. Recently, adaptive observers for multi input multi output (MIMO) linear time varying (LTV) systems have been developed [5, 6] .Some results on truly nonlinear systems have also been reported [7, 8, 5].

The adaptive observer for MIMO LTV systems proposed in [6] is conceptually simple and computationally efficient. Its global exponential convergence for joint state parameter estimation has been proved in the noise free case. When the considered system is noise corrupted, the convergence in the mean has been established, but not the consistency. In other words, under appropriate assumptions, the means of the estimation errors tend to zero, but not the errors themselves. Up to our knowledge, this was the first reported result on the convergence of adaptive observer for noise

In this paper, we propose an adaptive observers for a class of uniformly observable nonlinear systems. This observers, based on techniques of High gain and Sliding mode, is applied to jointly estimate states (rotor flux, rotating speed and torque load) and unknown constant parameters (rotor resistance).

Firstly, the convergence of the proposed observer is guaranteed under a well-defined persistent excitation condition. Secondly, the structure of the proposed observer is simple and it is able to give rise to different observers among which adaptive high gain like observers [9, 10, 11] and adaptive sliding mode like observers [12, 13, 14].

This paper is organized as follows. The next section introduces the dynamic model of an induction machine. In Section 3, the observer design is detailed. The equations of the proposed adaptive observer are given and a full convergence analysis is made. Besides, different expressions of the observer design function are specified and it is shown that they give rise to different observers. A simulation of IM is given in Section 4 in order to illustrate the theory.

2 Description of Induction Machine

Assuming linear magnetic circuits, the dynamics of a balanced non-saturated induction machine in a fixed reference frame attached to the stator are given in the form follows :

$$\left\{\begin{array}{lcl} \dot{i}_s & = & -\gamma i_s + KA(\Omega,\alpha_r)\psi_r + \frac{1}{\sigma L_s}u \\ \dot{\psi}_r & = & M\alpha_r i_s - A(\Omega,\alpha_r)\psi_r \\ \dot{\Omega} & = & \frac{p}{J}\frac{M}{L_r} i_s^T \mathcal{J}_2 \psi_r - \frac{1}{J}T_L \\ \dot{T}_L & = & 0 \end{array}\right. \tag{1}$$

The states variables accessible to measurement are the stator currents $i_s = \begin{bmatrix} i_{s\alpha} & i_{s\beta} \end{bmatrix}^T$ but to in no case rotor flux $\psi_r = \begin{bmatrix} \psi_{r\alpha} & \psi_{r\beta} \end{bmatrix}^T$ and possibly rotating speed. The source of energy $u = \begin{bmatrix} u_{s\alpha} & u_{s\beta} \end{bmatrix}^T$. The model parameters are : rotor moment of inertia J, rotor and stator winding's resistances R_r, R_s, mutual inductance M. To simplify notations we use the parameterization : $K = \dfrac{M}{\sigma L_s L_r}$, $\sigma = 1 - \dfrac{M^2}{L_s L_r}$, $\alpha_r = \dfrac{R_r}{L_r}$, $\alpha_s = \dfrac{R_s}{L_s}$,

$\gamma = \dfrac{\alpha_s}{\sigma} + \dfrac{\alpha_r M^2}{\sigma L_s L_r} = \dfrac{\alpha_s}{\sigma} + \alpha_r MK$, $A(\Omega, \alpha_r) = \alpha_r \mathcal{I}_2 - p\Omega \mathcal{J}_2$, $\mathcal{I}_2$ is the 2- dimensional identity matrix and $\mathcal{J}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is a skew - symmetric matrix.

We need to transform system (1) to the triangular form. One will introduce the change of variable according to:

$$\left\{ \begin{array}{l} z_1 = i_s \\ z_2 = -M\alpha_r i_s + (\alpha_r \mathcal{I}_2 - p\Omega \mathcal{J}_2)\, \psi_r \\ z_3 = \begin{bmatrix} \Omega & T_l \end{bmatrix}^T = \begin{bmatrix} z_{31} & z_{32} \end{bmatrix}^T \end{array} \right. \tag{2}$$

Using this transformation and a time derivative of these states, we can rewrite from model (1), a following model :

$$\left\{ \begin{array}{lcl} \dot{z}_1 & = & -\frac{\alpha_s}{\sigma} z_1 + K z_2 + \frac{1}{\sigma L_s} u_s \\ \dot{z}_2 & = & -pd\omega \mathcal{J}_2 \psi_r - (M\dot{z}_1 + z_2)\, \alpha_r \\ & & +p z_{31} \mathcal{J}_2 z_2 + \frac{p}{J} z_{32} \mathcal{J}_2 \psi_r \\ \dot{z}_3 & = & \begin{bmatrix} d\omega - \frac{1}{J} T_L & 0 \end{bmatrix}^T \\ y & = & z_1 \end{array} \right. \tag{3}$$

With $d\omega = \dfrac{p}{J} \dfrac{M}{L_r} z_1^T \mathcal{J}_2 \psi_r$.

One can show that this transformation puts system (1) under the following form:

$$\left\{ \begin{array}{l} \dot{z} = F(z) z + G(z,u) + H(z)\rho, \\ \\ y = Cz = z_1. \end{array} \right. \tag{4}$$

where

$$F(z) = \begin{bmatrix} 0_{2,2} & K\mathcal{I}_2 & 0_{2,2} \\ 0_{2,2} & 0_{2,2} & \begin{bmatrix} p\mathcal{J}_2 z_2 \\ \frac{p}{J} \mathcal{J}_2 \psi_r \end{bmatrix}^T \\ 0_{2,2} & 0_{2,2} & 0_{2,2} \end{bmatrix},$$

$$G(z,u) = \begin{bmatrix} -\frac{\alpha_s}{\sigma} z_1 + \frac{1}{\sigma L_s} u_s \\ -pd\omega \mathcal{J}_2 \psi_r \\ \begin{bmatrix} d\omega - \frac{1}{J} T_L & 0 \end{bmatrix}^T \end{bmatrix},$$

$$H(z) = \begin{bmatrix} 0_{2,1} \\ -(M\dot{z}_1 + z_2)\, \frac{1}{L_r} \\ 0_{2,1} \end{bmatrix} \text{ and } \rho = R_r.$$

3 Observers Design

3.1 Observers synthesis

As in the works related to the high gain observers synthesis [10, 11] and adaptive observers [15], one pose the hypothesis :

$\mathcal{H}1$ **:** The functions $F(z)$ is globally LIPSCHITZ with respect to z

$\mathcal{H}2$ **:** The functions $G(z,u)$ is globally LIPSCHITZ with respect to z uniformly in u.

$\mathcal{H}3$ **:** The functions $H(z)$ is globally LIPSCHITZ with respect to z

Before giving our candidate observers, one introduces the following notations.

1) Λ block diagonal matrix and Λ^{-1} is his left inverse:

$$\Lambda = diag\left(\mathcal{I}_2,\ K\mathcal{I}_2,\ K \begin{bmatrix} p\mathcal{J}_2 z_2 \\ \frac{p}{J} \mathcal{J}_2 \psi_r \end{bmatrix}^T \right)$$

2) Let Δ_θ is a block diagonal matrix defined by:

$$\Delta_\theta = diag\left(\mathcal{I}_2, \frac{1}{\theta} \mathcal{I}_2, \frac{1}{\theta^2} \mathcal{I}_2 \right);\ \theta > 0$$

is a real number.

Easy computations allow us to check the following identities:

- $\theta \mathcal{A} = \Delta_\theta \mathcal{A} \Delta_\theta^{-1} = \Delta_\theta \Lambda F(\hat{z}) \Lambda^{-1} \Delta_\theta^{-1}$
- $C\Delta_\theta^{-1} = C$
- $S_\theta^{-1} C^T = \theta \Delta_\theta^{-1} S^{-1} C^T$
- $\bar{z} = \Delta_\theta \Lambda \tilde{z}$

where $\mathcal{A}$ is matrix:

$$\mathcal{A} = \begin{bmatrix} 0_{2,2} & \mathcal{I}_2 & 0_{2,2} \\ 0_{2,2} & 0_{2,2} & \mathcal{I}_2 \\ 0_{2,2} & 0_{2,2} & 0_{2,2} \end{bmatrix}$$

3) Let $S = S_{\theta=1}$ is a definite positive solution of the ALGEBRAIC LYAPUNOV EQUATION:

$$S + \mathcal{A}^T S + S\mathcal{A} - C^T C = 0. \tag{5}$$

Note that (5) is independent of the system and the solution can be expressed analytically. For a straightforward computation, its stationary solution is given by: $S_{(n,p)} = (-1)^{n+p} C_{n+p-2}^{n-1}$ where $C_n^p = \frac{n!}{p!(n-p)!}$ for $n \geq 1$ and $p \leq 3$; and then we can explicitly determinate the correction gain of (3) as follows:

$$\theta \Lambda^{-1} \Delta_\theta^{-1} S^{-1} C^T = \begin{bmatrix} 3\theta \mathcal{I}_2 \\ \frac{3\theta^2}{K} \mathcal{I}_2 \\ \frac{\theta^3}{K} \left(\begin{bmatrix} p\mathcal{J}_2 z_2 \\ \frac{p}{J} \mathcal{J}_2 \psi_r \end{bmatrix}^T \right)^{-1} \end{bmatrix}. \tag{6}$$

4) $\forall \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$, set $\bar{\xi} = \Delta_\theta \Lambda \xi$ and let $\Upsilon(\xi) = \begin{bmatrix} \Upsilon_1(\xi_1) \\ \Upsilon_2(\xi_2) \end{bmatrix}$ be a vector of smooth functions satisfying:

$$\forall \xi \in \mathbb{R}^2 :\ \bar{\xi}^T \Upsilon(\xi) \geq \frac{1}{2} \xi^T C^T C \xi. \tag{7}$$

$$\exists \kappa > 0;\ \forall \xi \in \mathbb{R}^2 :\ \|\Upsilon(\xi)\| \leq \kappa \|\xi\|. \tag{8}$$

The system

$$\left\{\begin{array}{rcl}\dot{\hat{z}} & = & F(\hat{z})\hat{z}+G(\hat{z},u)+H(\hat{z})\hat{\rho}+\Lambda^{-1}\Gamma\dot{\hat{\rho}} \\ & & -\theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}C^T\Upsilon(\tilde{z}), \\ \dot{\hat{\rho}} & = & -\theta P\Gamma^T C^T\Upsilon(\tilde{z}), \\ \dot{\Gamma} & = & \theta\mathcal{A}\Gamma-\theta S^{-1}C^TC\Gamma+\Delta_\theta H(\hat{z}), \\ \dot{P} & = & -\theta P\Gamma^TC^TC\Gamma P+\theta P.\end{array}\right. \tag{9}$$

With $\Gamma(0)=0$ and $P(0)=P^T(0)>0$ means that the initial condition of the Ordinary Differential Equation (ODE) governing P is chosen Symmetric Positive Definite (SPD).

is an adaptive observer for system (4) with an exponential error convergence for relatively high values of θ.

The proof of the stability is given in the next subsection. However, before detailing this proof, one shall give some comments and facts which will be used throughout the proof.

Please notice that the time derivative of $\dot{\hat{z}}$ given in (9) can be written as follows:

$$\begin{array}{rcl}\dot{\hat{z}} & = & F(\hat{z})\hat{z}+G(\hat{z},u)+H(\hat{z})\hat{\rho}+\Lambda^{-1}\Gamma\dot{\hat{\rho}} \\ & & -\theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}C^T\Upsilon(\tilde{z}).\end{array} \tag{10}$$

The equation of $\dot{\hat{z}}$ consists in a copy of the model (4) with a correcting term. The correcting term is composed by two terms. The first one, $-\theta\Delta_\theta^{-1}S^{-1}C^T\Upsilon(\tilde{z})$ is rather classical and is met in classical high gain state observers [16]. The second term, $\Delta_\theta^{-1}\Gamma\dot{\hat{\rho}}$ is similar to the expression used for updating the unknown parameters, i.e. the term used in the expression of $\dot{\hat{\rho}}$.

3.2 Convergence analysis

Set $\tilde{z}=\hat{z}-z$ and $\tilde{\rho}=\hat{\rho}-\rho$ From (9) and (10), one has

$$\begin{array}{rcl}\dot{\tilde{z}} & = & F(\hat{z})\tilde{z}+(F(\hat{z})-F(z))\,z+G(\hat{z},u) \\ & & -G(z,u)+H(\hat{z})\tilde{\rho}+(H(\hat{z})-H(z))\,\rho \\ & & -\theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}C^T\Upsilon(\tilde{z})+\Lambda^{-1}\Delta_\theta^{-1}\Gamma\dot{\hat{\rho}}.\end{array} \tag{11}$$

Set $\bar{z}=\Delta_\theta\Lambda\tilde{z}$. Using the notations 2 from subsection §.3.1, one obtains:

$$\begin{array}{rcl}\dot{\bar{z}} & = & \Delta_\theta\Lambda\dot{\tilde{z}}+\Delta_\theta\dot{\Lambda}\tilde{z}, \\ & = & \theta\mathcal{A}\bar{z}+\Delta_\theta\Lambda\,(F(\hat{z})-F(z))\,z+\Delta_\theta\Lambda H(\hat{z})\tilde{\rho} \\ & & +\Delta_\theta\Lambda\,(G(\hat{z},u)-G(z,u)) \\ & & +\Delta_\theta\Lambda\,(H(\hat{z})-H(z))\,\rho \\ & & -\theta S^{-1}C^T\Upsilon(\tilde{z})+\Gamma\dot{\hat{\rho}}+\Delta_\theta\dot{\Lambda}\tilde{z}.\end{array} \tag{12}$$

Now, as in [17], set: $\bar{z}=\eta+\Gamma\tilde{\rho}$. For writing convenience and as long as there is no ambiguity, the time variable t shall be omitted in the sequel. Using the fact that Γ is governed by the ODE given in (9), one can show that:

$$\begin{array}{rcl}\dot{\eta} & = & \theta\mathcal{A}\eta-\theta S^{-1}C^T\Upsilon(\tilde{z})+\theta S^{-1}C^TC\Gamma\tilde{\rho} \\ & & +\Delta_\theta\Lambda\,(F(\hat{z})-F(z))\,z \\ & & +\Delta_\theta\Lambda\,(G(\hat{z},u)-G(z,u)) \\ & & +\Delta_\theta\Lambda\,(H(\hat{z})-H(z))\,\rho+\Delta_\theta\dot{\Lambda}\Lambda^{-1}\Delta_\theta^{-1}\bar{z}\end{array} \tag{13}$$

Set $V_1=\eta^TS\eta,\ \ V_2=\tilde{\rho}^TP^{-1}\tilde{\rho}^T$ where P is given in (9) and let $V=V_1+V_2$ be a LYAPUNOV CANDIDATE FUNCTION. Using (5), one gets:

$$\begin{array}{rcl}\dot{V}_1 & = & -\theta\eta^TS\eta+\theta\eta^TC^TC\eta+\theta S^{-1}C^TC\Gamma\tilde{\rho} \\ & & +2\eta^TS\Delta_\theta\Lambda\,(F(\hat{z})-F(z))\,z \\ & & +2\eta^TS\Delta_\theta\Lambda\,(G(\hat{z},u)-G(z,u)) \\ & & +2\eta^TS\Delta_\theta\Lambda\,(H(\hat{z})-H(z))\,\rho \\ & & +2\eta^TS\Delta_\theta\dot{\Lambda}\Lambda^{-1}\Delta_\theta^{-1}\bar{z}.\end{array} \tag{14}$$

It is obvious that

$$\|\bar{z}\|\leq\|\eta\|+\|\Gamma\|\,\|\tilde{\rho}\|\,. \tag{15}$$

By the Mean Value Theorem, one gets:

$$\begin{array}{rl}\Delta_\theta\,(G(\hat{z},u)-G(z,u))=\Delta_\theta\dfrac{\partial G(\xi,u)}{\partial z}\tilde{z}, & \\ =\Delta_\theta\dfrac{\partial G(\xi,u)}{\partial z}\Delta_\theta^{-1}\Lambda^{-1}\bar{z}. & \end{array} \tag{16}$$

As a result, for $\theta\geq1$ and from (16), one obtains

$$\begin{array}{l}\|\Lambda\|\,\|\Delta_\theta\,(G(\hat{z},u)-G(z,u))\| \\ \leq\|\Lambda\|\left\|\Delta_\theta\dfrac{\partial G(\xi,u)}{\partial z}\Delta_\theta^{-1}\right\|\left\|\Lambda^{-1}\right\|\|\bar{z}\|\,, \\ \leq\sup\{\Lambda\}\left\|\Delta_\theta\dfrac{\partial G(\xi,u)}{\partial z}\Delta_\theta^{-1}\right\|\sup\left\{\Lambda^{-1}\right\}\|\bar{z}\|\,, \\ \leq c_1\,\|\bar{z}\|\,.\end{array} \tag{17}$$

where c_1 is a constant which does not depend on θ for $\theta\geq1$. Using (17) and (15) , one obtains:

$$\|\Lambda\|\,\|\Delta_\theta\,(G(\hat{z},u)-G(z,u))\|\leq c_1\,\|\eta\|+c_2\,\|\tilde{\rho}\|\,. \tag{18}$$

where $c_2=c_1\sup\{\Gamma\}$. Therefore, one has:

$$\begin{array}{l}2\eta^TS\Delta_\theta\Lambda\,(G(\hat{z})-G(z)) \\ \leq2\left\|\eta^T\right\|\|S\|\,\|\Lambda\|\,\|\Delta_\theta\,(G(\hat{z})-G(z))\|\,, \\ \leq c_1^1\,\|\eta\|^2+c_1^2\,\|\eta\|\,\|\tilde{\rho}\|\,, \\ \leq\dfrac{c_1^1}{\lambda_{\min}(S)}V_1+\dfrac{c_1^2}{\sqrt{\lambda_{\min}(S)\lambda_{\min}(P)}}\sqrt{V_1}\sqrt{V_2}, \\ \leq c_{11}V_1+c_{12}\sqrt{V_1}\sqrt{V_2}.\end{array} \tag{19}$$

where $c_1^1=2c_1\,\|S\|$, $c_1^2=2c_2\,\|S\|$, $c_{11}=\frac{c_1^1}{\lambda_{\min}(S)}$ and $c_{12}=\frac{c_1^2}{\sqrt{\lambda_{\min}(S)\lambda_{\min}(P)}}$ are positive constants which do not depend on $\theta\geq1$, $\lambda_{\min}(S)$ and $\lambda_{\min}(P)$ denoting the smallest eigenvalue of the matrix S and P.

In ways similar, we will have:

$$\|\Lambda\| \, \|\Delta_\theta \left(F(\hat{z}) - F(z)\right) z\| \leq c_3 \|\eta\| + c_4 \|\tilde{\rho}\|. \quad (20)$$

where $c_4 = c_3 \sup \{\Gamma\}$. Therefore, one has:

$$\begin{aligned} & 2\eta^T S \Delta_\theta \Lambda \left(F(\hat{z}) - F(z)\right) z \\ & \leq 2 \left\|\eta^T\right\| \|S\| \, \|\Lambda\| \, \|\Delta_\theta \left(F(\hat{z}) - F(z)\right) z\|, \\ & \leq c_2^1 \|\eta\|^2 + c_2^2 \|\eta\| \, \|\tilde{\rho}\|, \\ & \leq \frac{c_2^1}{\lambda_{\min}(S)} V_1 + \frac{c_2^2}{\sqrt{\lambda_{\min}(S)\lambda_{\min}(P)}} \sqrt{V_1}\sqrt{V_2}, \\ & \leq c_{21} V_1 + c_{22} \sqrt{V_1}\sqrt{V_2}. \end{aligned} \quad (21)$$

where $c_2^1 = 2c_3 \|S\|$, $c_2^2 = 2c_4 \|S\|$, $c_{21} = \frac{c_2^1}{\lambda_{\min}(S)}$ and $c_{22} = \frac{c_2^2}{\sqrt{\lambda_{\min}(S)\lambda_{\min}(P)}}$ are positive constants which do not depend on $\theta \geq 1$.

Since each column of the matrix H assumes a triangular structure and since ρ is bounded, the arguments developed above are still be valid for bounding $2\eta^T S \Delta_\theta \Lambda \left(H(\hat{z}) - H(z)\right) \rho$ and indeed by proceeding in a similar way as above, one obtains:

$$2\eta^T S \Delta_\theta \Lambda \left(H(\hat{z}) - H(z)\right) \rho \leq c_{31} V_1 + c_{32} \sqrt{V_1}\sqrt{V_2}. \quad (22)$$

where c_{31} and c_{32} are positive constants (depending on the bounds of ρ which do not depend on $\theta \geq 1$.

And

$$2\eta^T S \Delta_\theta \dot{\Lambda} \Lambda^{-1} \Delta_\theta^{-1} \bar{z} \leq c_{41} V_1 + c_{42} \sqrt{V_1}\sqrt{V_2}. \quad (23)$$

Using (19), (21), (22) and (23), inequality (14) can be written as follows:

$$\begin{aligned} \dot{V}_1 &= -\theta V_1 + \theta \eta^T C^T C \eta + 2\theta \eta^T C^T C \Gamma \tilde{\rho} \\ & \quad -2\theta \eta^T C^T \Upsilon(\tilde{z}) + k_1 V_1 + k_2 \sqrt{V_1}\sqrt{V_2}. \end{aligned} \quad (24)$$

with $k_1 = c_{11} + c_{21} + c_{31} + c_{41}$ and $k_2 = c_{12} + c_{22} + c_{32} + c_{42}$.

Let us now derive the time derivative of V_2. One has:

$$\begin{aligned} \dot{V}_2 &= 2\tilde{\rho}^T P^{-1} \dot{\tilde{\rho}} - \tilde{\rho}^T P^{-1} \dot{P} P^{-1} \tilde{\rho}, \\ &= -\theta V_2 - 2\theta \tilde{\rho}^T \Gamma^T C^T \Upsilon(\Delta_\theta^{-1} \bar{z}) + \theta \tilde{\rho}^T \Gamma^T C^T C \Gamma \tilde{\rho}, \\ &= -\theta V_2 - 2\theta \tilde{\rho}^T \Gamma^T C^T \Upsilon(\bar{z}) + \theta \tilde{\rho}^T \Gamma^T C^T C \Gamma \tilde{\rho}. \end{aligned} \quad (25)$$

Hence, using (24) and (25), one obtains

$$\begin{aligned} \dot{V} &= V_1 + V_2, \\ &\leq -(\theta - k_1) V_1 - \theta V_2 + k_2 \sqrt{V_1}\sqrt{V_2} \\ & \quad +\theta \eta^T C^T C \eta \\ & \quad +2\theta \eta^T C^T C \Gamma \tilde{\rho} - 2\theta \eta^T C^T C \Upsilon(\tilde{z}) \\ & \quad -2\theta \tilde{\rho}^T \Gamma^T C^T \Upsilon(\bar{z}) + \theta \tilde{\rho}^T \Gamma^T C^T C \Gamma \tilde{\rho}, \\ &\leq -(\theta - k_1) V_1 - \theta V_2 + k_2 \sqrt{V_1}\sqrt{V_2} \\ & \quad +\theta \left(\bar{z}^T C^T C \bar{z} - 2(C\bar{z})^T \Upsilon(\bar{z})\right), \\ &\leq -(\theta - k_1) V_1 + k_2 \sqrt{V_1}\sqrt{V_2} - \theta V_2. \end{aligned} \quad (26)$$

The last inequality is obtained according to the inequality (7). Now, set $V_1^* = (\theta - k_1) V_1$, $V_2^* = \theta V_2$ and $V = V_1 + V_2$.Please notice that $V^* = (\theta - k_1) V$.

Inequality (26) yields to

$$\begin{aligned} \dot{V} &\leq -V^* + \frac{k_2}{2\sqrt{\theta(\theta - k_1)}} V^*, \\ &\leq (\theta - k_1) \left(1 - \frac{k_2}{2\sqrt{\theta(\theta - k_1)}}\right) V. \end{aligned} \quad (27)$$

Now, it suffices to choose θ such that $\left(1 - \frac{k_2}{2\sqrt{\theta(\theta - k_1)}}\right) > 0$. This ends the proof.

3.3 Adaptive high gain observers

Consider the following expression of $\Upsilon(\tilde{\xi})$:

$$\Upsilon_{HG}(\tilde{z}) = C^T \tilde{z}_1 = C^T C \tilde{z} \quad (28)$$

Replacing $\Upsilon(\tilde{z})$ by expression (28) in (10) gives rise to a high gain observer :

$$\begin{aligned} \dot{\hat{z}} &= F(\hat{z})\hat{z} + G(\hat{z}, u) + H(\hat{z})\hat{\rho} + \Lambda^{-1} \Delta_\theta^{-1} \Gamma \dot{\hat{\rho}} \\ & \quad -\theta \Lambda^{-1} \Delta_\theta^{-1} S^{-1} C^T (\hat{z}_1 - z_1), \quad (29) \\ \dot{\hat{\rho}} &= -\theta P \Gamma^T C^T (\hat{z}_1 - z_1), \\ \dot{\Gamma} &= \theta \mathcal{A} \Gamma - \theta S^{-1} C^T C \Gamma + \Delta_\theta H(\hat{z}), \\ \dot{P} &= -\theta P \Gamma^T C^T C \Gamma P + \theta P. \end{aligned}$$

Referring to (2), the rotor flux is governed by the following equations:

$$\hat{\psi}_r = \left(\frac{\hat{R}_r}{L_r} \mathcal{I}_2 - p \hat{\Omega} \mathcal{J}_2\right)^{-1} \left(\hat{z}_2 + M \frac{R_r}{L_r} \hat{z}_1\right). \quad (30)$$

3.4 Adaptive sliding mode like observers

At first glance, the following vector seems to be a potential candidate for the expression of $\Upsilon(\tilde{x})$:

$$\Upsilon_{\text{sign}}(\tilde{z}) = C^T \quad (31)$$

$$\text{sign}(\tilde{z}_1) = C^T C \text{sign}(\tilde{z}). \quad (32)$$

where sign is the usual signe function with $\text{sign}(\tilde{z}_1) = \begin{bmatrix} \text{sign}(\tilde{z}_{11}) \\ \text{sign}(\tilde{z}_{12}) \end{bmatrix}$; then:

$$\begin{aligned} \dot{\hat{z}} &= F(\hat{z})\hat{z} + G(\hat{z}, u) + H(\hat{z})\hat{\rho} + \Lambda^{-1} \Delta_\theta^{-1} \Gamma \dot{\hat{\rho}} \\ & \quad -\theta \Lambda^{-1} \Delta_\theta^{-1} S^{-1} C^T \text{sign}(\hat{z}_1 - z_1), \quad (33) \\ \dot{\hat{\rho}} &= -\theta P \Gamma^T C^T \text{sign}(\hat{z}_1 - z_1), \\ \dot{\Gamma} &= \theta \mathcal{A} \Gamma - \theta S^{-1} C^T C \Gamma + \Delta_\theta H(\hat{z}), \\ \dot{P} &= -\theta P \Gamma^T C^T C \Gamma P + \theta P. \end{aligned}$$

Indeed, condition (7) is trivially satisfied by (31). Similarly, for bounded input bounded output systems. However, expression (31) cannot be used due the discontinuity of sign function(see.[18]). Indeed, such discontinuity makes the stability problem not well posed since the LYAPUNOV method used throughout the proof is not valid. In order to overcome these difficulties, one shall use continuous functions which have similar properties that those of the signfunction. This approach is widely used when implementing sliding mode observers. Indeed, consider the following function:

Tanh function:

$$\Upsilon_{\tanh}(\tilde{x}) = C^T \tanh(\tilde{z}_1) = C^T C \tanh(\tilde{z}). \quad (34)$$

where $\tanh$ denotes the hyperbolic tangent function; then:

$$\begin{aligned} \dot{\hat{z}} &= F(\hat{z})\hat{z} + G(\hat{z}, u) + H(\hat{z})\hat{\rho} + \Lambda^{-1}\Delta_\theta^{-1}\Gamma\dot{\hat{\rho}} \\ &\quad -\theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}C^T \tanh(\tilde{z}_1), \quad (35) \\ \dot{\hat{\rho}} &= -\theta P\Gamma^T C^T \tanh(\tilde{z}_1), \\ \dot{\Gamma} &= \theta\mathcal{A}\Gamma - \theta S^{-1}C^T C\Gamma + \Delta_\theta H(\hat{z}), \\ \dot{P} &= -\theta P\Gamma^T C^T C\Gamma P + \theta P. \end{aligned}$$

Arctan function:

$$\Upsilon_{\arctan}(\tilde{x}) = C^T \arctan(\tilde{z}_1) = C^T C \arctan(\tilde{z}). \quad (36)$$

Similarly to the hyperbolic tangent function, one can easily check that the inverse tangent function:

$$\begin{aligned} \dot{\hat{z}} &= F(\hat{z})\hat{z} + G(\hat{z}, u) + H(\hat{z})\hat{\rho} + \Lambda^{-1}\Delta_\theta^{-1}\Gamma\dot{\hat{\rho}} \\ &\quad -\theta\Lambda^{-1}\Delta_\theta^{-1}S^{-1}C^T \arctan(\tilde{z}_1), \quad (37) \\ \dot{\hat{\rho}} &= -\theta P\Gamma^T C^T \arctan(\tilde{z}_1), \\ \dot{\Gamma} &= \theta\mathcal{A}\Gamma - \theta S^{-1}C^T C\Gamma + \Delta_\theta H(\hat{z}), \\ \dot{P} &= -\theta P\Gamma^T C^T C\Gamma P + \theta P. \end{aligned}$$

4 Simulation of Sensorless Observers

To examine practical usefulness, the proposed observer has been simulated for a three-phase 1.5kw induction machine(see [19]), whose parameters are depicted in Table1.

Table 1: Induction machine parameters used in simulations.

Notations	value	unit
p	2	
f	50	Hz
L_s	0.464	H
L_r	0.464	H
M	0.4417	H
R_s	5.717	Ω
R_r	3	Ω
J	0.0049	$Kg.m^2$

In order to evaluate the observer behaviour in the realistic situation, the measurements of i_s issued from the model simulation have been corrupted by noise measurements with a zero mean value. The torque lead takes the step value.

High gain observer :

The adjustment parameter of the observer (29) is to chosen $\theta = 0.1$. The dynamic behaviour of the error of rotor flux is depicted in Figure 1 graph (a); when graph (b) shows the gaussian errors density and empirical errors histogram of rotor flux error. The means of error flux equal 0.0067 with small variance 9.1199×10^{-4} . The pace of speed error is given by the figure 2 graph (a) and the gaussian errors density and empirical errors histogram of rotor speed error are presented in graph (b)where means of error rotating speed equal -0.3873 and variance equal 28.0212; the curve of load torque is illustrated on figure 3 graph (a).In graph (b) appear gaussian errors density and empirical errors histogram of load torque error where means of error load torque equal 1.4429×10^{-4} and variance equal 1.1506×10^{-10}; the curve of stator resistance is illustrated on figure 4 graph (a).In graph (b) appear gaussian errors density and empirical errors histogram of stator resistance error where means equal -0.0034 and variance equal 1.3214×10^{-7}.

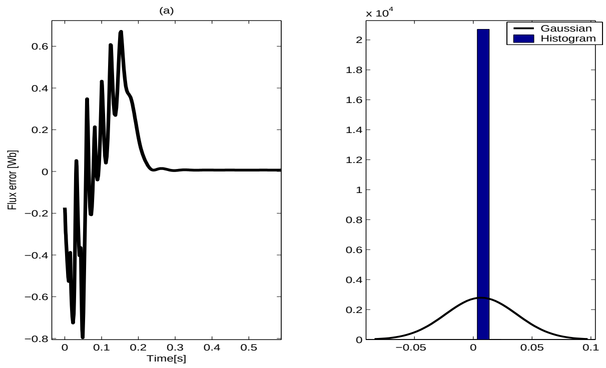

Figure 1: (a) Flux error. (b) Gaussian and histogram of error flux.

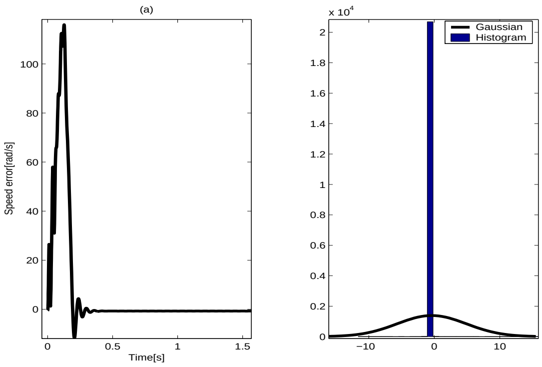

Figure 2: (a) Speed error. (b) Gaussian and histogram of error speed.

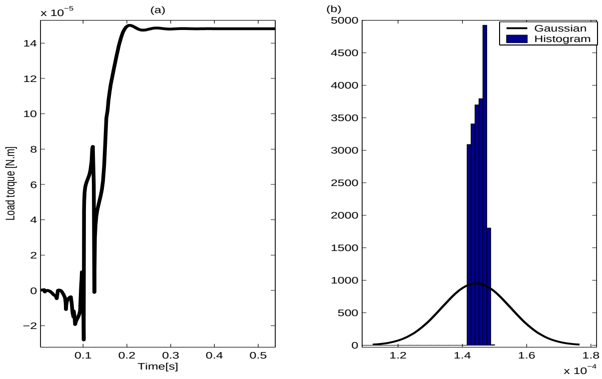

Figure 3: (a) Load torque error. (b) Gaussian and histogram of error load torque.

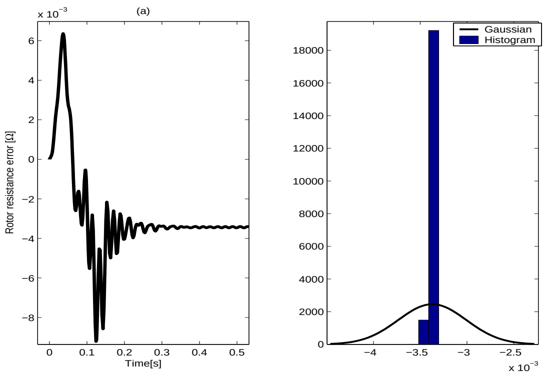

Figure 4: (a) stator resistance error. (b) Gaussian and histogram of error rotor resistance.

Sliding mode observer with $\tanh$ **:**

Estimation results of the proposed algorithm (35) with $\theta = 0.1$ is reported in figure 5, 6 and 7. The behaviour of the error of rotor flux is depicted in figure 5 graph (a); when graph (b) shows the gaussian errors density and empirical errors histogram of rotor flux error. The means of error flux equal 0.0065 with very small variance 9.9232×10^{-4} this is almost surety. The pace of speed error is given by the figure 6 graph (a) and the gaussian errors density and empirical errors histogram of rotor speed error are presented in graph (b)where means of error rotating speed equal -0.3850 and variance equal 29.3175; the curve of load torque is illustrated on figure 7 graph (a).In graph (b) appear gaussian errors density and empirical errors histogram of load torque error where means of error load torque equal 1.1747×10^{-5} and variance equal 4.5253×10^{-12}; the curve of stator resistance is illustrated on figure 8 graph (a).In graph (b) appear gaussian errors density and empirical errors histogram of stator resistance error where means equal -2.7527×10^{-4} and variance equal 1.0119×10^{-8}.

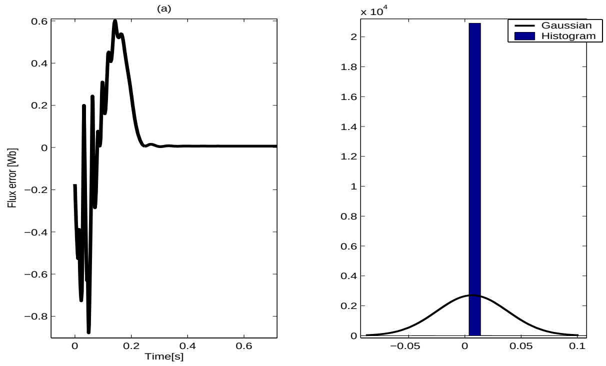

Figure 5: (a) Flux error. (b) Gaussian and histogram of error flux.

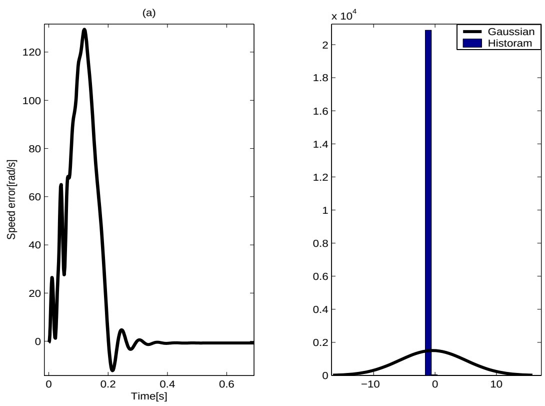

Figure 6: (a) Speed error. (b) Gaussian and histogram of error speed.

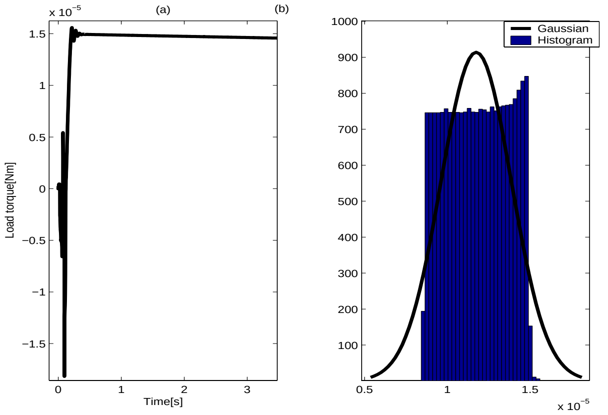

Figure 7: (a) Load torque error. (b) Gaussian and histogram of error load torque.

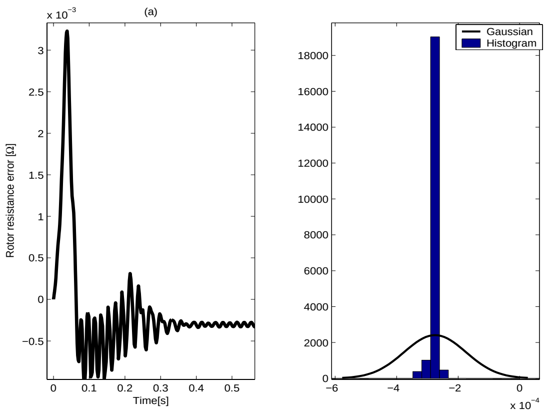

Figure 8: (a) stator resistance error. (b) Gaussian and histogram of error rotor resistance.

Sliding mode observer with $\arctan$ **:**

Under the same conditions with the function $\tanh$. One simulates for the function $\arctan$. The figure 9, 10 and 11 illustrates the pace of error flux, error speed and error load torque in respectively. The behaviour of the error of rotor flux is depicted in figure 9 graph (a); when graph (b) shows the gaussian errors density and empirical errors histogram of rotor flux error. The means of error flux equal 0.0065 with small variance 1×10^{-3} this is almost surety. The pace of speed error is given by the figure 10 graph (a) and the gaussian errors density and empirical errors histogram of rotor speed error are presented in graph (b)where means of error rotating speed equal -0.3747 and variance equal 30.5824; the curve of load torque is illustrated on figure 11 graph (a).In graph (b) appear gaussian errors density and empirical errors histogram of load torque error where means of error load torque equal 2.4285×10^{-5} and variance equal 7.2229×10^{-12}; the curve of stator resistance is illustrated on figure 12 graph (a).In graph (b) appear gaussian errors density and empirical errors histogram of stator resistance error where means equal -6.6832×10^{-4} and variance equal 1.6982×10^{-8}.

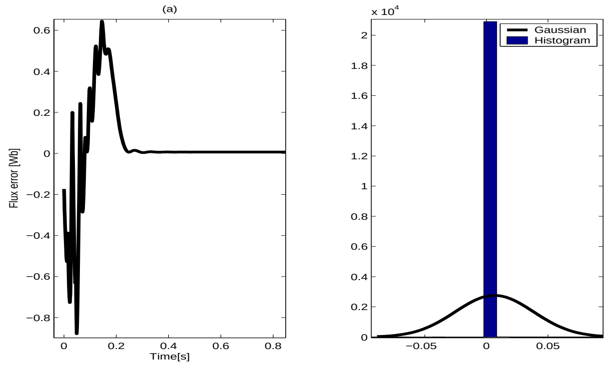

Figure 9: (a) Flux error. (b) Gaussian and histogram of error flux.

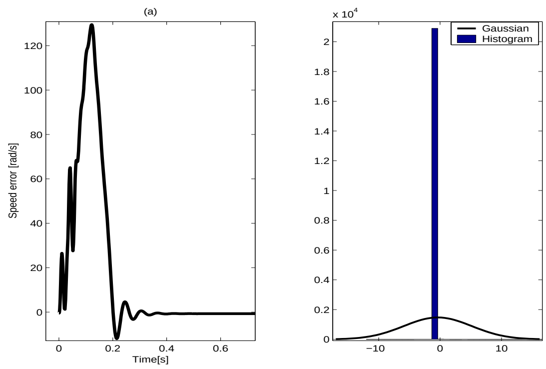

Figure 10: (a) Speed error. (b) Gaussian and histogram of error speed.

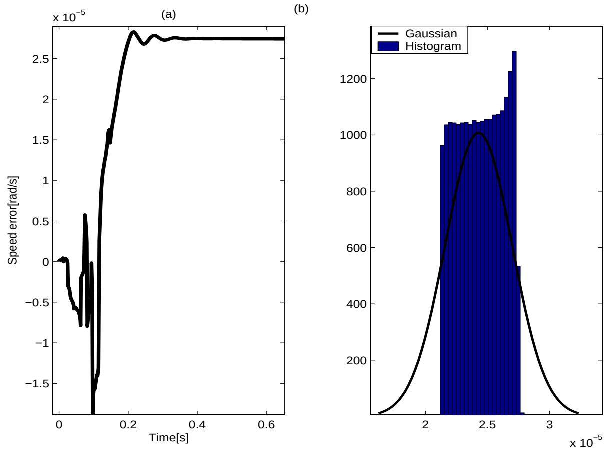

Figure 11: (a) Load torque error. (b) Gaussian and histogram of error load torque.

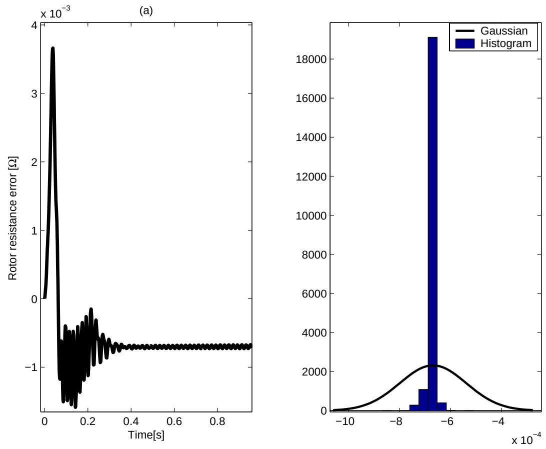

Figure 12: (a) stator resistance error. (b) Gaussian and histogram of error rotor resistance.

Table 2, 3, 4 and 5 are collection of the preceding results, we notice the errors mean of observation with the high gain observer being very near to 0 with the very small variance (almost surely), then the sliding mode observer with the function $\tanh$ is the good in our case.

Table 2: Means and variances of Flux error.

		Flux error
H.G	Mean	67×10^{-4}
	Var	9.1199×10^{-4}
$\tanh$	Mean	65×10^{-4}
	Var	9.9232×10^{-4}
$\arctan$	Mean	65×10^{-4}
	Var	10×10^{-4}

Table 3: Means and variances of speed error.

		Speed error
H.G	Mean	-0.3875
	Var	28.0212
$\tanh$	Mean	-0.3850
	Var	29.3175
$\arctan$	Mean	-0.3747
	Var	30.5824

Table 4: Means and variances of load torque error.

		Load Torque error
H.G	Mean	1.4429×10^{-4}
	Var	1.1506×10^{-10}
$\tanh$	Mean	1.1747×10^{-5}
	Var	4.5253×10^{-12}
$\arctan$	Mean	2.4285×10^{-5}
	Var	7.2229×10^{-12}

Table 5: Means and variances of rotor resistance error.

		Rotor resistance error
H.G	Mean	-34×10^{-4}
	Var	1.3214×10^{-7}
$\tanh$	Mean	-2.7527×10^{-4}
	Var	1.0119×10^{-8}
$\arctan$	Mean	-6.6832×10^{-4}
	Var	1.6982×10^{-8}

5 Conclusions

In this paper, adaptive high gain and alternative form for a adaptive sliding mode observers are presented. they is observer makes possible to observe, rotor flux, rotor speed, load torque and rotor resistance. An observer with adaptive high gain and three others with adaptive sliding mode which the functions sign, $\tanh$ and $\arctan$. Observer whose sign gives chattering. Adaptive sliding mode observer with function $\tanh$ is good for the observation of rotor flux, rotating speed, load torque and rotor resistance.

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