Bayes Estimation for the Parameter θ of Burr Type XII Distribution

Haziem M. Hazaimeh
College of Business Administration
Al Falah University
Mailbox 128821, Dubai, UAE

Abstract: - We consider in this article that n independent items having lifetimes $X_1, \dots, X_n$ from Burr type XII distribution with parameter θ. Assuming conjugate prior for θ (Gamma prior) and squared error loss, we computed Bayes estimates of the scale parameter θ using the following sampling planes; complete sample, type I censoring, partial ordering I, type II censoring, and partial ordering II. By using the simulation method, we compare the sampling mentioned above.

Keywords: - Bayes, Estimation, Distribution, Burr Type XII.

I. INTRODUCTION

The Burr system of distribution was introduced by Irving W. Burr (1942). Some applications of the Burr system of distribution are n Reliability and engineering, simulation modeling, and more. These distributions have been used as models in a variety of applied fields such as Business (e.g. Rodriguez and Taniguchi), Economics and Engineering (e.g. McDonald and Richards), and Medicine (Wingo). Burr chose to work with the cumulative distribution function (CDF) $F(x)$ satisfying the differential equation

$$\frac{dy}{dx} = y(1-y)\, g(x,y), \quad y = F(x)$$

that generate the Pearson system.

The Pearson system was originated by Pearson between 1880 and 1900. For every member of Pearson's family, the probability density function $p(x)$ satisfies the differential equation

$$\frac{1}{p}\frac{dy}{dx} = \frac{a+x}{c_0 + c_1 x + c_2 x^2}$$

For Burr families, different choices of $g(x,y)$ generate various solutions of $F(x)$, for example, if $g(x,y) = g(x)$, then

$$F(x) = \frac{1}{1 + EXP(-\int_{-\infty}^{x} g(u)du)}.$$

The solution $F(x)$ of Burr XII is

$$F_X(x) = 1 - (1 + x^c)^{-\theta}, \qquad x > 0.$$

Burr XII was studied by many authors, Shah considered the estimation of the parameters of this distribution by several methods. He compared his results through simulation. Also, he derived the maximum likelihood estimator (MLE) of θ and c. Al-Hussaini gave a characterization of the Burr XII distribution. Wingo derived a maximum likelihood method for fitting the Burr XII distributions to life test data.

Let $X_1, \dots, X_n$ be a random sample from Burr XII distribution with probability density function given by

$$f(x|\theta, c) = \frac{c\theta x^{c-1}}{(1+x^c)^{\theta+1}}.$$

Let $Y_1, \dots, Y_n$ be a future sample from the same population but with outlier observation of the type $\theta_0\theta$ where θ_0 known. Assuming conjugate prior to θ, namely $G\left(\alpha, \frac{1}{\beta}\right)$, and noninformative prior to c, they predicted the minimum $Y_{(1)}$ and the maximum $Y_{(n)}$ of the future samples using squared error loss function.

II. STATEMENT OF THE PROBLEM

Assuming conjugate prior (Gamma prior) and squared error loss function, we computed Bayes estimator of the parameter θ using the following plans:

1. Complete sample, where we observed all lifetimes of the n items.
2. Type I censoring, where we observed the failure times in the interval $(0, t_0]$ such that $Y_1 \le Y_2 \le \cdots \le Y_r$ where t_0 is fixed.
3. Partial ordering I, where we observed the failure times in the interval $(0, t_0]$ such that $Y_1 \le Y_2 \le \cdots \le Y_r$ and the number of failures in the interval $(t_0, t_1]$ where t_0, t_1 are fixed.
4. Type I censoring, where we observe the first r failure $Y_1 \le Y_2 \le \cdots \le Y_r$.
5. Partial ordering II, where we obtain the number of failures observed in the interval $(0, t_0]$. And obtain the number of failures in the interval $(t_0, t_1]$ where t_0, t_1 are fixed.

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III. BAYES ESTIMATION FOR THE PARAMETER θ OF BURR TYPE XII DISTRIBUTION, WHEN c IS KNOWN

Let $X_1, \dots, X_n$ be independent and identically distributed random variables ($iid\ rv's$) from Burr XII distribution with parameters θ and c. The probability density function of X is given by

$$f(x|\theta, c) = \frac{c\theta x^{c-1}}{(1+x^c)^{\theta+1}}, \quad x \geq 0, c > 0, \theta > 0 \qquad (1)$$

where θ and c are positive parameters.

The distribution function is

$F_x(x) = 1 - (1+x^c)^{-\theta},\ x > 0.$

We like to derive Bayes' estimator of θ using different priors with respect to squared error loss under several sampling plans.

1. The case of the complete sample

The likelihood function is

$$L(\underline{x}|\theta) = \prod_{i=1}^{n} f(x_i, \theta) = \frac{c^n \theta^n (\prod_{i=1}^{n} x_i)^{c-1}}{[\prod_{i=1}^{n}(1+x_i{}^c)]^{\theta+1}} \qquad (2)$$

Assume θ has prior $G(\alpha, \frac{1}{\beta})$ $i.e.$,

$$\pi(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\theta\beta}, \quad \theta, \alpha, \beta > 0.$$

Let

$$H(\underline{x}, \theta) = f(\underline{x}|\theta)\pi(\theta) \qquad (3)$$

then

$$H(\underline{x}, \theta) = \frac{c^n \theta^{n+\alpha-1} \beta^\alpha e^{-\theta\beta} (\prod_{i=1}^{n} x_i)^{c-1}}{\Gamma(\alpha)[\prod_{i=1}^{n}(1+x_i{}^c)]^{\theta+1}} \qquad (4)$$

And the posterior of θ given $\underline{x}$ is,

$$\pi(\theta|\underline{x}) = \frac{H(\underline{x}, \theta)}{\int_0^\infty H(\underline{x}, \theta)\, d\theta} \qquad (5)$$

then equation (5) becomes

$$\pi(\theta|\underline{x}) = \frac{\theta^{n+\alpha-1}}{\Gamma(n+\alpha)} [\beta + \sum \ln(1 + x_i{}^c)]^{n+\alpha} e^{-\theta(\beta + \sum ln(1+x_i{}^c))} \qquad (6)$$

so

$(\theta|\underline{x}) \sim G(n+\alpha, \frac{1}{\beta + \sum ln(1+x_i{}^c)})$.

Under squared error loss, the Bayes estimator of the parameter θ of Burr XII distribution is the mean of the posterior of θ and is given by

$$\hat{\theta}_1 = E(\theta|\underline{x}) = \int_0^\infty \theta\, \pi(\theta|\underline{x}) d\theta = \frac{\alpha+n}{\beta + \sum ln(1+x_i{}^c)} \qquad (7)$$

when $\alpha = 0, \beta = 0$, equation (7) becomes

$$\hat{\theta}_1 = \frac{n}{\sum ln(1+x_i{}^c)}$$

$i.e.$, the same as the MLE.

Estimation of Reliability

Reliability is defined by

$$R = P(X < t) = 1 - P(X \leq t) = 1 - F_X(t) = 1 - [1 - (1+t^c)^{-\theta}] = (1+t^c)^{-\theta}, \quad t > 0 \qquad (8)$$

Under square error loss, the Bayes estimator of reliability is given by

$$\hat{R}_1 = E[(1+t^c)^{-\theta}|\underline{x}] = \int_0^\infty (1+t^c)^{-\theta} \pi(\theta|\underline{x}) d\theta$$

$$= \frac{1}{\Gamma(n+\alpha)} [\beta + \sum \ln(1+x_i{}^c)]^{n+\alpha} \int_0^\infty \theta^{n+\alpha-1} e^{-\theta(\beta + \sum ln(1+x_i{}^c) + ln(1+t^c))} d\theta$$

$$= [\frac{\beta + \sum ln(1+x_i{}^c)}{\beta + \sum ln(1+x_i{}^c) + ln(1+t^c)}]^{n+\alpha} \qquad (9)$$

2. The case of type I censoring

Instead of observing the complete sample $X_1, \dots, X_n$, suppose that we observe the failure items $Y_1 \leq Y_2 \leq \dots \leq Y_r$ in the interval $(0, t_0]$. Then the conditional pdf of the observed sample is given by

$$L(\underline{Y}|\theta, R = r) = \frac{n!}{(n-r)!} \prod_{i=1}^{r} f(y_i)[1 - F(t_0)]^{n-r} = \frac{n!}{(n-r)!} \frac{c^r \theta^r (\prod_{i=1}^{r} y_i)^{c-1}}{(\prod_{i=1}^{r}(1+y_i{}^c))^{\theta+1}} (1 + t_0{}^c)^{-\theta(n-r)}, \qquad (10)$$

$0 \leq Y_1 \leq Y_2 \leq \dots \leq Y_r,\ \theta > 0$, where $\underline{y} = (y_1, \dots, y_r)$.

Now, the posterior pdf can be calculated as the follows

$$\pi(\theta|\underline{y}) = \frac{L(\underline{y}|\theta)\pi(\theta)}{\int_0^\infty L(\underline{y}|\theta)\pi(\theta) d\theta}$$

$$= \frac{\theta^{r+\alpha-1}e^{-\theta\beta}[\beta+\sum ln(1+y_i{}^c)+(n-r)\,ln(1+t_0{}^c)]^{r+\alpha}}{(1+t_0{}^c)^{\theta(n-r)}(\prod_{i=1}^{n}(1+y_i{}^c))^{\theta}\Gamma(\alpha+r)} \quad (11)$$

Under squared error loss, the Bayes estimator of θ is the posterior mean given by

$$\hat{\theta}_2 = E\left(\theta\middle|\underline{y}\right) = \int_0^\infty \theta\,\pi(\theta|\underline{y})\,d\theta$$

$$= \frac{\alpha+r}{\beta+\sum ln(1+y_i{}^c)+(n-r)\,ln(1+t_0{}^c)} \quad (12)$$

when $\alpha = 0$ and $\beta = 0$, then equation (12) becomes

$$\hat{\theta}_2 = \frac{r}{\sum ln(1+y_i{}^c)+(n-r)\,ln(1+t_0{}^c)}$$

$i.e.$, the same as MLE.

The Bayes estimator of reliability is

$$\hat{R}_2 = E\left[(1+t^c)^{-\theta}\middle|\underline{y}\right] = \int_0^\infty (1+t^c)^{-\theta}\,\pi\left(\theta\middle|\underline{y}\right)d\theta$$

$$= \left[\frac{\beta+\sum ln(1+y_i{}^c)+(n-r)\,ln(1+t_0{}^c)}{\beta+\sum ln(1+y_i{}^c)+(n-r)\,ln(1+t_0{}^c)+ln(1+t^c)}\right]^{r+\alpha} \quad (13)$$

3. The case of partial ordering I

Assume that we observe the lifetime of the units which failed in the interval $(0,t_0]$ and obtain the number of failures in the interval $(t_0,t_1]$. Let $\underline{y} = (y_1,\ldots,y_r)$, the conditional joint probability density function (jpdf) of the observed sample is given by

$$L\left(\underline{y}\middle|\theta\right) = f(r,k)f\left(\underline{y}\middle|R=r,K=k,\theta\right) = \frac{n!}{k!\,(n-r-k)!}[F(t_0)]^r[F(t_1)-F(t_0)]^k[1-F(t_1)]^{n-r-k}\prod_{i=1}^{r}\frac{f(y_i)}{F(t_0)}$$

$$= \frac{n![(1+t_0{}^c)^{-\theta}-(1+t_1{}^c)^{-\theta}]^k[1+t_1{}^c]^{-\theta(n-r-k)}c^r\theta^r(\prod_{i=1}^{r}y_i)^{c-1}}{(n-r-k)!k!\,(\prod_{i=1}^{r}(1+y_i{}^c))^{\theta+1}} \quad (14)$$

Then the pdf of the posterior is

$$\pi\left(\theta\middle|\underline{y}\right) = \frac{L(\underline{y}|\theta)\pi(\theta)}{\int_0^\infty L\left(\underline{y}\middle|\theta\right)\pi(\theta)d\theta} \quad (15)$$

$$= \frac{[(1+t_0{}^c)^{-\theta}-(1+t_1{}^c)^{-\theta}]^k\theta^{r+\alpha-1}e^{-\theta\beta}}{[1+t_1{}^c]^{\theta(n-r-k)}[\prod 1+y_i{}^c]^{\theta}\int_0^\infty \frac{[(1+t_0{}^c)^{-\theta}-(1+t_1{}^c)^{-\theta}]^k\theta^{r+\alpha-1}e^{-\theta\beta}}{([1+t_1{}^c]^{\theta(n-r-k)}[\prod 1+y_i{}^c]^{\theta})^{\theta}}d\theta}$$

Now

$$\int_0^\infty \frac{[(1+t_0{}^c)^{-\theta}-(1+t_1{}^c)^{-\theta}]^k\theta^{r+\alpha-1}e^{-\theta\beta}}{([1+t_1{}^c]^{\theta(n-r-k)}[\prod 1+y_i{}^c]^{\theta})^{\theta}}\,d\theta$$

$$= \int_0^\infty [(1+t_0{}^c)^{-\theta}-(1+t_1{}^c)^{-\theta}]^k\theta^{r+\alpha-1}\,e^{-\theta[\beta+(n-r-k)\ln(1+t_1{}^c)+\sum ln(1+y_i{}^c)]}\,d\theta$$

$$= \int_0^\infty e^{r+\alpha-1}\sum_{j=0}^{k}(-1)^j(1+t_1{}^c)^{-j\theta}(1+t_0{}^c)^{-(k-j)\theta}e^{-\theta[\beta+(n-r-k)\,ln(1+t_1{}^c)+\sum ln(1+y_i{}^c)]}d\theta$$

$$= \sum_{j=0}^{k}(-1)^j\int_0^\infty \theta^{r+\alpha-1}e^{-\theta\,A_j}\,d\theta$$

$$= \sum_{j=0}^{k}(-1)^j\frac{\Gamma(r+\alpha)}{A_j{}^{r+\alpha}} \quad (16)$$

where

$$A_j = \beta+(n-r-k)\,ln(1+t_1{}^c)+\sum ln(1+y_i{}^c)+(k-j)\ln(1+t_0{}^c)$$

Therefore, equation (15) becomes

$$\pi\left(\theta\middle|\underline{y}\right) = \frac{[(1+t_0{}^c)^{-\theta}-(1+t_1{}^c)^{-\theta}]^k\theta^{r+\alpha-1}e^{-\theta\beta}}{[1+t_1{}^c]^{\theta(n-r-k)}[\prod(1+y_i{}^c)]^{\theta}\sum_{j=0}^{k}(-1)^j\frac{\Gamma(r+\alpha)}{A_j{}^{r+\alpha}}}$$

Under squared error loss, the Bayes estimate of the parameter θ is the posterior mean given by

$$\hat{\theta}_3 = E\left(\theta\middle|\underline{y}\right) = \int_0^\infty \theta\,\pi\left(\theta\middle|\underline{y}\right)d\theta$$

$$= (r+\alpha)\frac{\sum_{j=0}^{k}\frac{(-1)^j}{A_j{}^{r+\alpha+1}}}{\sum_{j=0}^{k}\frac{(-1)^j}{A_j{}^{r+\alpha}}} \quad (18)$$

and the Bayes estimate of R is

$$\hat{R}_3 = \int_0^\infty [1+t^c]^{-\theta}\pi\left(\theta\middle|\underline{y}\right)\,d\theta \quad (19)$$

$$= \int_0^\infty \frac{[(1+t_0{}^c)^{-\theta}-(1+t_1{}^c)^{-\theta}]^k\theta^{r+\alpha-1}e^{-\theta\beta}}{[1+t^c]^{\theta}[1+t_1{}^c]^{\theta(n-r-k)}[\prod(1+y_i{}^c)]^{\theta}\sum_{j=0}^{k}(-1)^j\frac{\Gamma(r\ldots}{A_j{}^{\ldots}}}$$

$$= \frac{1}{\Gamma(r+\alpha)\sum_{j=0}^{k}\frac{(-1)^j}{A_j{}^{r+\alpha}}}\int_0^\infty \frac{\sum_{j=0}^{k}(-1)^j(1+t_1{}^c)^{-j\theta}(1+t_0{}^c)^{-(k\ldots}}{[1+t^c]^{\theta}[1+t_1{}^c]^{\theta(n-r-k)}[\prod\ldots}$$

$$= \frac{1}{\Gamma(r+\alpha)\sum_{j=0}^{k}\frac{(-1)^j}{A_j{}^{r+\alpha}}}\int_0^\infty \sum_{j=0}^{k}(-1)^j\theta^{r+\alpha-1}e^{-\theta B_j}\,d\theta$$

$$= \frac{\sum_{j=0}^{k}\frac{(-1)^j}{B_j{}^{r+\alpha}}}{\sum_{j=0}^{k}\frac{(-1)^j}{A_j{}^{r+\alpha}}} \quad (20)$$

where

$$A_j = \beta+(n-r-k)\,ln(1+t_1{}^c)+\sum ln(1+y_i{}^c)+(k-j)\ln(1+t_0{}^c)$$

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$$B_j = \beta + (n+j-r-k)\, ln(1+{t_1}^c) + (k-j)\, ln(1+{t_0}^c) + \sum ln(1+{y_i}^c) + \ln(1+t^c)$$

4. <u>Case of type II censoring</u>

Observe the first r failures $Y_1 \le Y_2 \le \cdots \le Y_r$. The posterior distribution is

$$\pi\left(\theta \middle| \underline{y}\right) = \frac{f(\underline{y}|\theta)\pi(\theta)}{\int_0^\infty f\left(\underline{y}\middle|\theta\right)\pi(\theta)d\theta} \qquad (21)$$

where

$$\pi(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\,\theta^{\alpha-1}\,e^{-\theta\beta}, \quad \theta,\alpha,\beta > 0$$

$$f\left(\underline{y}\middle|\theta\right) = \frac{n!}{(n-r)!}\prod_{i=1}^{r} f(y_i)\ [1 - F(y_r)]^{n-r} \qquad (22)$$

$$= \frac{n!}{(n-r)!}\,\frac{c^r\theta^r(\prod_{i=1}^r y_i)^{c-1}(1+{y_r}^c)^{-\theta(n-r)}}{(\prod_{i=1}^r(1+{y_i}^c))^{\theta+1}}, \quad 0 \le Y_1 \le Y_2 \le \cdots \le Y_r$$

therefore,

$$\pi\left(\theta\middle|\underline{y}\right) = \frac{\theta^{r+\alpha-1}(1-{y_r}^c)^{-\theta(n-r)}e^{-\theta\beta}[\beta+(n-r)\,ln(1+{y_r}^c)+\sum ln(1+{y_i}^c)]^{r+\alpha}}{\Gamma(r+\alpha)[\prod(1+{y_i}^c)]^\theta}$$

Under squared error loss, the Bayes estimator of the parameter θ is the posterior mean and is given by

$$\hat{\theta}_4 = E\left(\theta\middle|\underline{y}\right) = \int_0^\infty \theta\,\pi\left(\theta\middle|\underline{y}\right)d\theta$$

$$= \frac{r+\alpha}{\beta+(n-r)\,ln(1+{y_r}^c)+\sum ln(1+{y_i}^c)} \qquad (24)$$

And the Bayes estimator of reliability is

$$\hat{R}_4 = \int_0^\infty [1+t^c]^{-\theta}\pi\left(\theta\middle|\underline{y}\right)\,d\theta$$

$$= \left[\frac{\beta+(n-r)\,ln(1+{y_r}^c)+\sum ln(1+{y_i}^c)}{\beta+(n-r)\,ln(1+{y_r}^c)+\sum ln(1+{y_i}^c)+ln(1+t^c)}\right]^{r+\alpha}$$

5. <u>The case of partial ordering II</u>

Assume that we obtain k_1 be the number of failures observed in the interval $(0,t_0]$ and obtain k_2 be the number of failures observed in the interval $(t_0,t_1]mi.e.,$

$k_1 =$ number of observed in $(0,t_0]$

$k_2 =$ number of observed in $(t_0,t_1]$

$k_3 = n - k_1 - k_2.$

Let

$p_1 = P(x \le t_0) = F_x(t_0) = 1-(1+{t_0}^c)^{-\theta}$

$p_2 = P(t_0 < x \le t_1) = F_x(t_1) - F_x(t_0) = (1+{t_0}^c)^{-\theta} - (1+{t_1}^c)^{-\theta}$

$p_3 = P(x > t_1) = 1 - P(x \le t_1) = 1 - F_x(t_1) = (1+{t_1}^c)^{-\theta}$

The likelihood function is given by

$$L(k_1,k_1|\theta) = \frac{n!}{k_1!\,k_2!\,k_3!}[1-(1+{t_0}^c)^{-\theta}]^{k_1}[(1+{t_0}^c)^{-\theta}-(1+{t_1}^c)^{-\theta}]^{k_2}[1+{t_1}^c]^{-\theta k_3} \quad (26)$$

Then the posterior of θ is given by

$$\pi\left(\theta\middle|\underline{x}\right) = \frac{H(\underline{x},\theta)}{\int_0^\infty H(\underline{x},\theta)\,d\theta}$$

$$= \frac{\sum_{j=0}^{k_1}\sum_{l=0}^{k_2}(-1)^{j+l}\theta^{\alpha-1}e^{-\theta A_{jl}}}{\Gamma(\alpha)\sum_{j=0}^{k_1}\sum_{l=0}^{k_2}\frac{(-1)^{j+l}}{{A_{jl}}^\alpha}} \qquad (27)$$

where

$$A_{jl} = \beta + (l+k_3)\ln(1+{t_1}^c) + (j+k_2-l)\,ln(1+{t_0}^c)$$

Under squared error loss, the Bayes estimator of the parameter $\theta$of Burr XII distribution is the mean of the posterior of θ is given by

$$\hat{\theta}_5 = E\left(\theta\middle|\underline{y}\right) = \int_0^\infty \theta\,\pi\left(\theta\middle|\underline{y}\right)d\theta$$

$$= \alpha\,\frac{\sum_{j=0}^{k_1}\sum_{l=0}^{k_2}\frac{(-1)^{j+l}}{{A_{jl}}^{\alpha+1}}}{\sum_{j=0}^{k_1}\sum_{l=0}^{k_2}\frac{(-1)^{j+l}}{{A_{jl}}^\alpha}} \qquad (28)$$

And the Bayes estimator of reliability is

$$\hat{R}_5 = \int_0^\infty [1+t^c]^{-\theta}\pi\left(\theta\middle|\underline{y}\right)\,d\theta$$

$$= \frac{\sum_{j=0}^{k_1}\sum_{l=0}^{k_2}\frac{(-1)^{j+l}}{[A_{jl}+\ln(1+t^c)]^\alpha}}{\sum_{j=0}^{k_1}\sum_{l=0}^{k_2}\frac{(-1)^{j+l}}{{A_{jl}}^\alpha}} \qquad (29)$$

IV. CONCLUSION

To compare between the five different sampling plans by MSE we take

Case (1): Complete sample.

Case (2): Type I censoring.

Case (3): Partial ordering I.

Case (4): Type I censoring.

Case (1): Partial ordering II.

where c known and θ unknown.

To compare the results obtained in each sampling plan, we take (1)$\alpha = 2,\ \beta = 3$ (2) $\alpha = 2,\ \beta = 6$ and generate a value of θ from Gamma distribution and then generate random sample of size $10, 20, 40$ from Burr XII when $c = 2$ and generated $\theta = 0.6414, 0.3417$. We repeated the simulation 1000 times. For each sampling plan we computed Bayes estimator $\hat{\theta}\ of\ \theta,\ \hat{R}\ of\ R,$ and MSE for each. Stopping $t_0 = 2, t_1 = 5, and\ t = 3$ for reliability.

The following notation were used

$\overline{\hat{\theta}_i}$ ≡means of Bayes estimator of θ in the i −th sampling plan. $i.e.$,

$$\overline{\hat{\theta}_i} = \frac{1}{1000}\sum_{j=1}^{1000} \hat{\theta}_{ij}$$

and

$$MSE = \frac{1}{1000}\sum_{j=1}^{1000} \left(\hat{\theta}_{ij} - \theta_{(given)}\right)^2$$

$\overline{\hat{R}_i}$ ≡mean of Bayes estimator of R in the i −th sampling plan. $i.e.$,

$$\overline{\hat{R}_i} = \frac{1}{1000}\sum_{j=1}^{1000} \hat{R}_{ij}$$

and

$$MSE = \frac{1}{1000}\sum_{j=1}^{1000} \left(\hat{R}_{ij} - R_{(given)}\right)^2$$

The results are given in tables (1) − (4). We write $case\ (i) < case(j)$ to mean MSE of the estimator in $case\ (i)$ < MSE of the estimator in $case(j)$. Prior to $\theta\ is\ G\left(\alpha, \frac{1}{\beta}\right)$.

Table (1)

$\alpha = 2, \beta = 3, c = 2, \theta = 0.6414, t_0 = 2, and\ t_1 = 5$										
n	$\overline{\hat{\theta}_1}$	MS	$\overline{\hat{\theta}_2}$	MS	$\overline{\hat{\theta}_3}$	MS	$\overline{\hat{\theta}_4}$	MS	$\overline{\hat{\theta}_5}$	MS
10	0.6792	0.0491	0.6373	0.0172	0.6079	0.0554	0.6970	0.0612	0.6237	0.0421
20	0.6521	0.0135	0.6141	0.0452	0.5938	0.0521	0.6503	0.0116	0.6124	0.0382
30	0.6458	0.0091	0.5918	0.0515	0.5911	0.0491	0.6521	0.0085	0.6015	0.0214

$n = 10: case\ 2 < case\ 5 < case\ 1 < case\ 3 < case\ 4$
$n = 20: case\ 1 < case\ 4 < case\ 5 < case\ 2 < case\ 3$
$n = 30: case\ 4 < case\ 1 < case\ 5 < case\ 3 < case\ 2$

Table (2)

$\alpha = 2, \beta = 3, c = 2, \theta = 0.6414, t = 3, R = 0.2283, t_0 = 2, and\ t_1 = 5$										
n	$\overline{\hat{R}_1}$	MS	$\overline{\hat{R}_2}$	MS	$\overline{\hat{R}_3}$	MS	$\overline{\hat{R}_4}$	MS	$\overline{\hat{R}_5}$	MS
10	0.2035	0.0622	0.2223	0.0479	0.2537	0.0903	0.2510	0.0882	0.2168	0.0515
20	0.2138	0.0418	0.2615	0.0603	0.1983	0.0537	0.2632	0.0925	0.2001	0.0525
30	0.2135	0.0401	0.2661	0.0831	0.2235	0.0452	0.2385	0.0511	0.2458	0.0724

$n = 10: case\ 2 < case\ 5 < case\ 1 < case\ 4 < case\ 3$
$n = 20: case\ 1 < case\ 5 < case\ 3 < case\ 2 < case\ 4$
$n = 30: case\ 1 < case\ 3 < case\ 4 < case\ 5 < case\ 2$

Table (3)

$\alpha = 2, \beta = 6, c = 2, \theta = 0.3417, t_0 = 2, and\ t_1 = 5$										
n	$\overline{\hat{\theta}_1}$	MS	$\overline{\hat{\theta}_2}$	MS	$\overline{\hat{\theta}_3}$	MS	$\overline{\hat{\theta}_4}$	MS	$\overline{\hat{\theta}_5}$	MS
10	0.3405	0.0011	0.3465	0.0019	0.3638	0.0125	0.3536	0.0052	0.3622	0.0091
20	0.3563	0.0050	0.3498	0.0037	0.3215	0.0273	0.3512	0.0041	0.3300	0.0180
30	0.32	0.04	0.35	0.00	0.31	0.05	0.34	0.00	0.32	0.04

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	01	39	25	45	88	32	10	10	47	18

$n = 10: case\ 1 < case\ 2 < case\ 4 < case\ 3 < case\ 5$
$n = 20: case\ 2 < case\ 4 < case\ 1 < case\ 5 < case\ 3$
$n = 30: case\ 4 < case\ 2 < case\ 5 < case\ 1 < case\ 3$

Table (4)

$\alpha = 2, \beta = 6, c = 2, \theta = 0.3417, t = 3, R = 0.4553, t_0 = 2, and\ t_1 = 5$										
n	$\overline{\hat{R}_1}$	MS	$\overline{\hat{R}_2}$	MS	$\overline{\hat{R}_3}$	MS	$\overline{\hat{R}_4}$	MS	$\overline{\hat{R}_5}$	MS
10	0.4105	0.0312	0.4889	0.0080	0.4605	0.0013	0.4164	0.0103	0.4823	0.0065
20	0.4219	0.0294	0.4862	0.0072	0.4497	0.0017	0.4134	0.0209	0.4709	0.0051
30	0.4255	0.0235	0.4743	0.0075	0.4364	0.0071	0.4001	0.0471	0.4681	0.0041

$n = 10: case\ 3 < case\ 5 < case\ 2 < case\ 4 < case\ 1$
$n = 20: case\ 3 < case\ 5 < case\ 2 < case\ 1 < case\ 4$
$n = 30: case\ 5 < case\ 3 < case\ 2 < case\ 1 < case\ 4$

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