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Verification of Collatz Conjecture: An algorithmic approach

M.VENKATESULU[1] and C. DEVI PARAMESWARI[2]
[1]Department of Information Technology
[2]Department of Computer Applications
Kalasalingam Academy of Research and Education
Anandnagar, Krishnankoil
626126,Srivilliputtur(Taluk)
Virudhunagar (Dist.),
Tamilnadu, INDIA

Abstract-Lothar Collatz had proposed in 1937 a conjecture in number theory called Collatz conjecture. Till today there is no evidence of proving or disproving the conjecture. In this paper, we propose an algorithmic approach for verification of the Collatz conjecture based on bit representation of integers. The scheme neither encounters any cycles in the so called Collatz sequence and nor the sequence grows indefinitely. Experimental results show that the Collatz sequence starting at the given integer , oscillates for finite number of times, never exceeds 1.7 times (scaling factor) size of the starting integer and finally reaches the value 1. The experimental results show strong evidence that conjecture is correct and paves a way for theoretical proof.

Keywords: Collatz Conjecture, Binary Representation, Collatz Sequence, Expanded size, Predicted size

I. INTRODUCTION

In 1937 Lothar Collatz proposed a conjecture which states that given any positive integer x, the function f(x) defined as x/2 if x is even and 3x+1if x is odd generates a finite sequence called the Collatz sequence which eventually reaches the value 1. Then after it loops between the values 1 □ 4 □2 □1, called a trivial cycle. In other words, the conjecture says that the Collatz sequence does not contain nontrivial cycles and also does not grow indefinitely. In the past and present many authors [1-12] try to establish the validity of the conjecture in affirmatively but till today there is no theoretical proof for the same and there is no counter example to disprove the conjecture as well. In this paper, we propose an algorithmic approach based on binary representation of integers for verification of the Collatz conjecture. The scheme encounters no cycles in the so called Collatz sequence and observes that the Collatz sequence does not grow indefinitely. An experimental results show that the Collatz sequence starting at any given positive integer, oscillates for finite number of times, no element in the sequence ever exceeds 1.7 times (the scaling factor) size of the starting integer and finally reaches the value 1. In other words, for any given positive integer, the elements of Collatz sequence could grow to a size less than 1.7 times the size of the starting number and in finite number of steps, called the stopping time, the sequences eventually reaches the value 1. These results showstrong evidence that conjecture is correct andpaves a way forapossible theoretical proof.

II. VERIFICATION SCHEME

The given positive integer is converted into binary form. Usually, the left most bit represents the most significant bit and the right most bit is the least significant bit. But for our scheme the binary representation is reversed and taken as the input for the verification algorithm. The binary representation of 2x is just ONE bit right shift (note that we have reversed the form) of the binary form of x. The expression 3x + 1 is written x + 2x +1, all taken in binary form with the usual binary addition operations. For an even integer x, x/2 is just ONE bit left shift of the binary expression of x.

A. *Algorithm*

Step 1: Without loss of generality, we may assume that x is an odd integer otherwise keep dividing the integer successively by 2 until we get an odd integer.

Step 2: Convert the given integer x into binary form and reverse the binary representation.

Step 3: Represent the expression 3x + 1 as binary multiplication and addition.

Step 4: Do the division of an even integer by 2 by shifting the resulting binary representation by one place to the right.

Step 5: Repeat steps 3 and 4, appropriately, until step4 results in the value 1.

III. EXPERIMENTAL RESULTS

In this section we present experimental results on verification of Collatz conjecture. The algorithm is implemented using C Programming in a DELL system with Intel®Pentium®3558U @1.70 GHz processor with 4GB RAM. The graphs are drawn using R.

A. *Covert Binary Form*

We have taken RSA-100:
152260502792253336053561837813263742971806811496
13
806886579084945801229632589528976540003506920061
39and converted it into binary form. The decimal form contains100 Decimal digits and its converted binary form has 330.The binary list is reversed (as per our design, leftmost bit should be the least significant bit) and given as an input for step 3 of the algorithm. The experimental result gave 1566 right shifts (even number & division by 2) and 780 loop iterations (odd integer & 3x+1) before the Collatz sequence reaches value 1.

We present below experimental results for integers of varying length in binary form with all bits equal to 1. These represent the largest possible integer in the binary form with that size. Numbers of zeros represent division by 2 and loop iteration represents occurrence of an odd integer before the Collatz sequence reaches value 1.

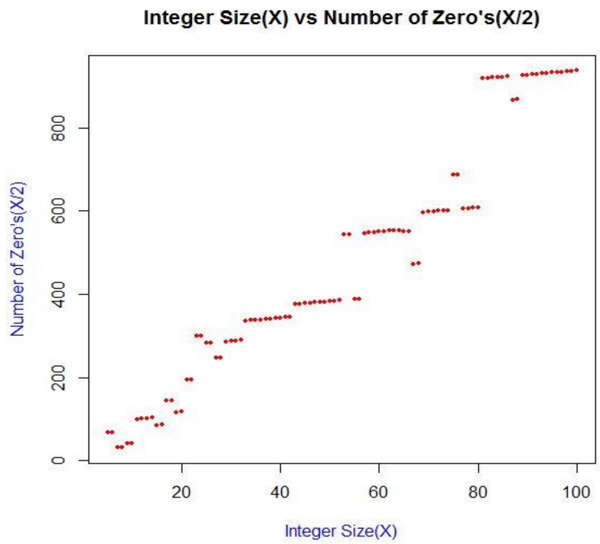


Fig. 1. Integer Size(x) vs Number of Zeros (x/2

In fig.1, integer size(x) indicates the size of the numbers in binary form and x/2 indicates the number of times division by 2 is carried out before the Collatz sequence reaches the value 1.

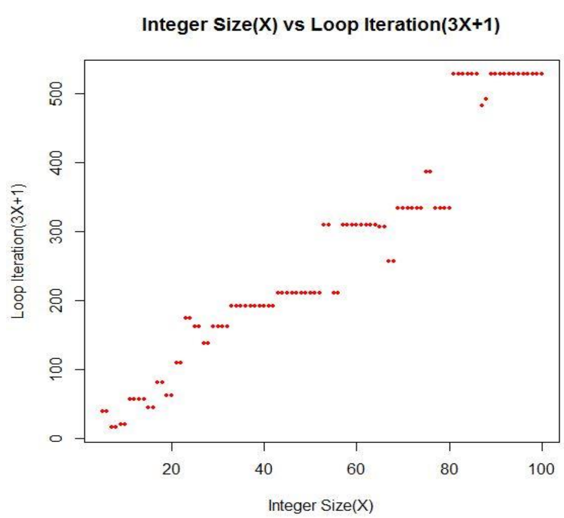


Fig. 2. Integer Size(x) vs Loop Iteration (3x+1)

In fig 2, integer size(x) denotes the size of the numbers in binary form and 3x+1 denotes the number of times the integer becomes odd during the process before the Collatz sequence reaches value 1.

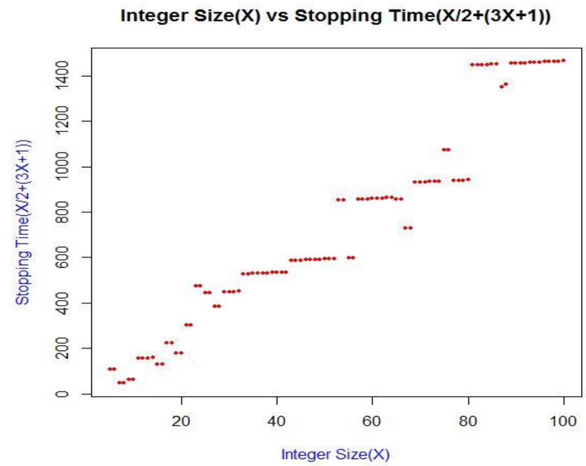


Fig. 3. Integer Size(x) vs Stopping Time(x/2) + (3x+1)

In fig 3, integer size(x) denotes the size of the numbers in binary form and stopping time denotes the length of the Collatz sequence reaches value 1.

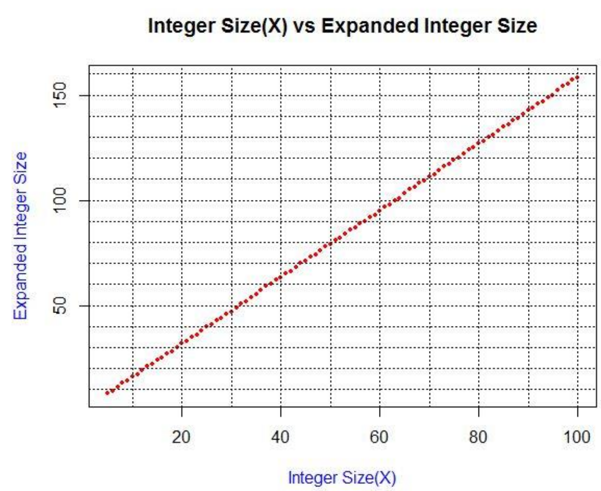
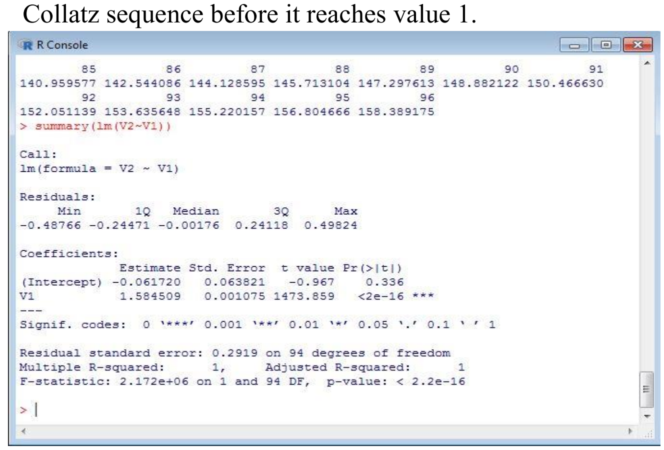


Fig. 4. Integer Size(x) vs Expanded Integer Size

In fig 4, integer size(x) denotes the size in binary form and expanded integer size denotes the maximum size in binary form of any element of the Collatz sequence before it reaches value 1.



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R Console
       85         86         87         88         89         90         91
140.959577 142.544086 144.128595 145.713104 147.297613 148.882122 150.466630
       92         93         94         95         96
152.051139 153.635648 155.220157 156.804666 158.389175
> summary(lm(V2~V1))

Call:
lm(formula = V2 ~ V1)

Residuals:
     Min       1Q   Median       3Q      Max
-0.48766 -0.24471 -0.00176  0.24118  0.49824

Coefficients:
             Estimate Std. Error  t value Pr(>|t|)
(Intercept) -0.061720   0.063821   -0.967    0.336
V1           1.584509   0.001075 1473.859   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2919 on 94 degrees of freedom
Multiple R-squared:      1,     Adjusted R-squared:      1
F-statistic: 2.172e+06 on 1 and 94 DF,  p-value: < 2.2e-16

> |
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Fig. 5. Regression Analysis

Here fig 5 , we have considered integers with bit size from 5 bits to 100 bits and the maximum size of any element in the Collatz sequence before it reaches value 1.The graph is drawn by applying Linear regression model in R. We got R-squared value is 1 which means that model is totally perfect.

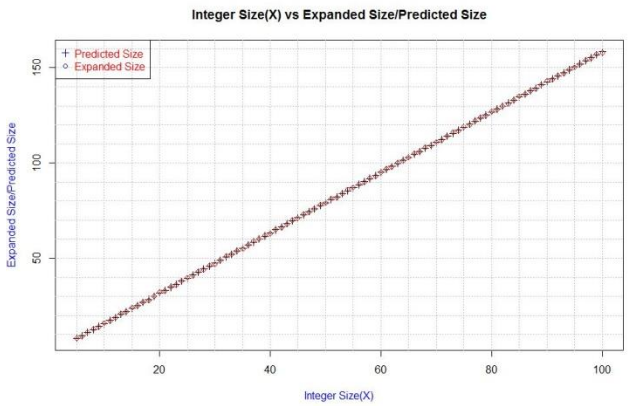

Fig. 6. Integer Size(x) vs. Expanded/Predicted Size

Fig.6 shows that expanded size and Predicted size of any element of the Collatz sequence are approximately the same.

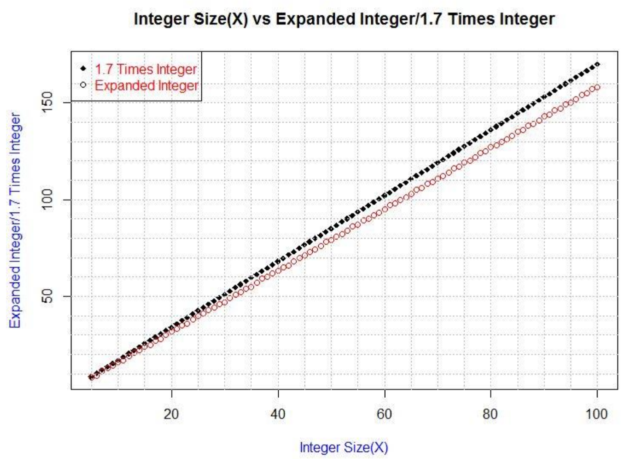

Fig. 7. Integer Size(x) vs.Expanded Integer/1.7 Times Integer

Fig.7 shows that the maximum expanded size of the integer is always less than 1.7 times size of the integer.

We also performed the experiments for integers with binary representation from 100 to 3000 bits size.

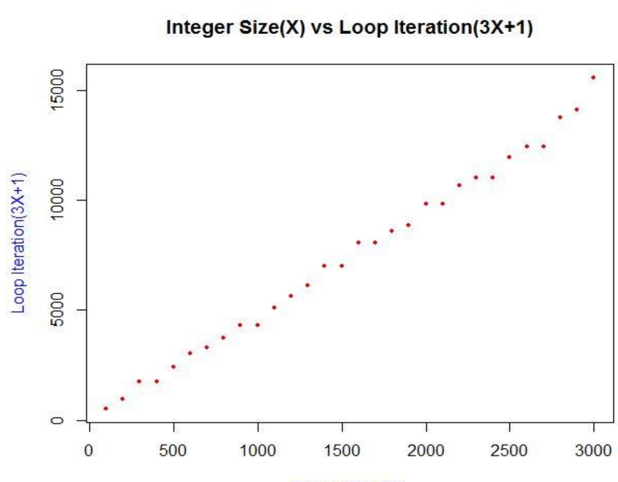

Fig. 8. Integer Size(x) vs Loop Iteration (3x+1)

In fig 8, integer size(x) denotes the size of the numbers in binary form and 3x+1 denotes the number of times the integer becomes odd during the process before the Collatz sequence reaches value 1.

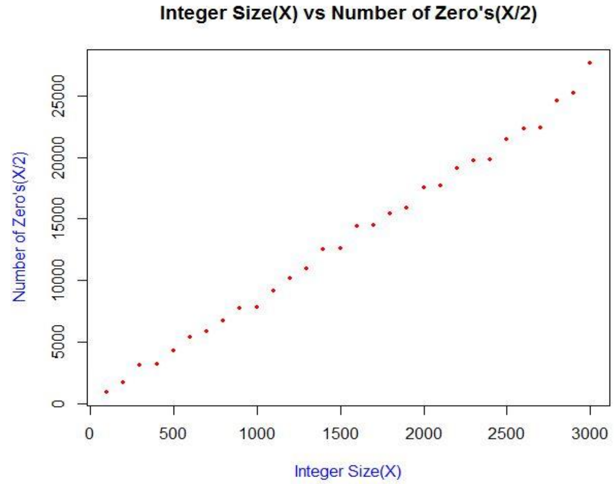

Fig. 9. Integer Size(x) vs Number of Zeros (x/2

In fig.9, integer size(x) indicates the size of the numbers in binary form and x/2 indicates the number of times division by 2 is carried out before the Collatz sequence reaches the value 1.

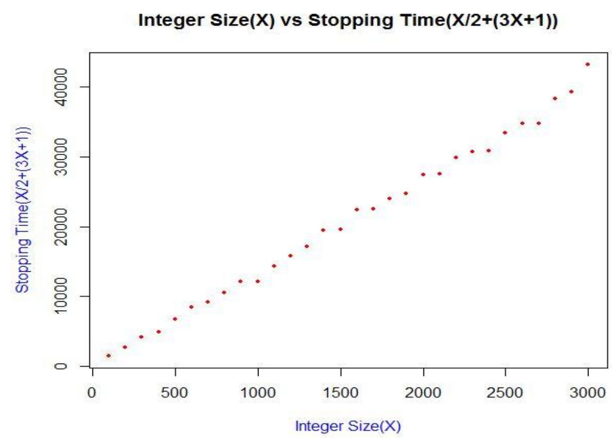

Fig. 10. Integer Size(x) vs Stopping Time(x/2) + (3x+1)

In fig 10, integer size(x) denotes the size of the numbers in binary form and stopping time denotes the length of the Collatz sequence reaches value 1.

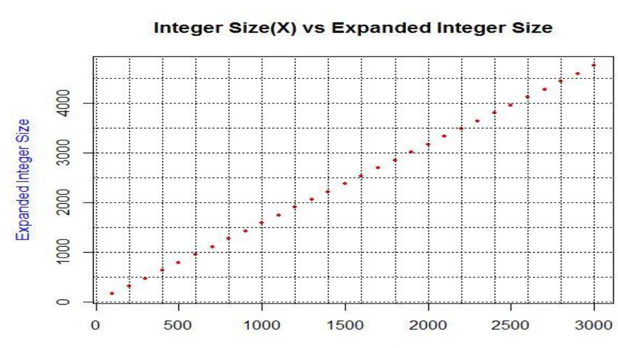

Fig. 11. Integer Size(x) vs Expanded Integer Size

In fig 11, integer size(x) denotes the size in binary form and expanded integer size denotes the maximum size in binary form of any element of the Collatz sequence before it reaches value 1.

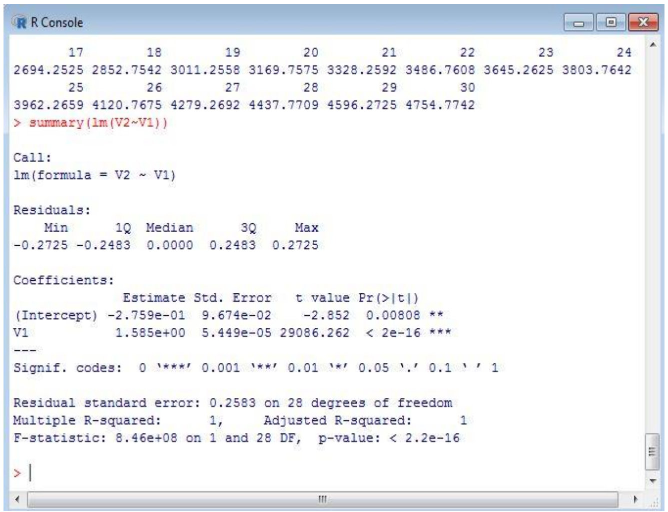

Fig. 12. Regression Analysis

Here fig 12 , we have considered integers with bit size from 100 bits to 3000 bits and the maximum size of any element of any element in the Collatz sequence before it reaches value 1.The graph is drawn by applying Linear regression model in R. We got R-squared value (a statistical measure for model validation) is equal to 1 which means that model is totally perfect.

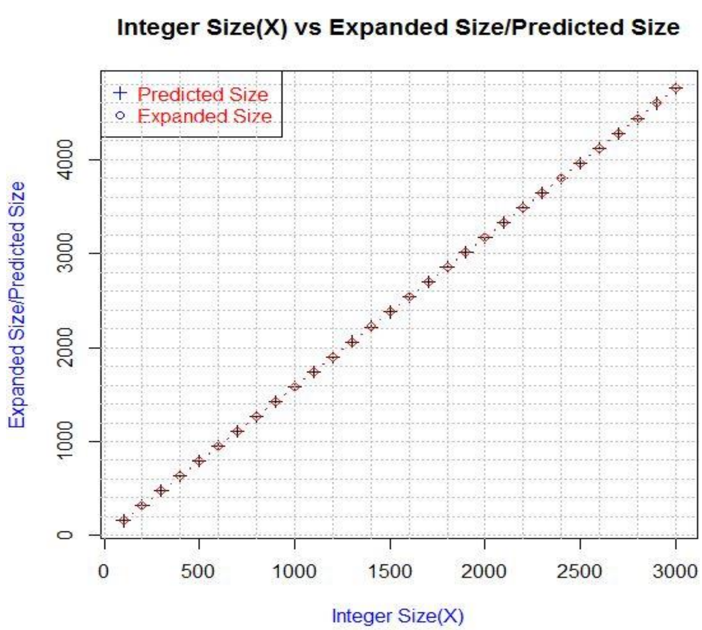

Fig. 13. Integer Size(x) vs. Expanded/Predicted Size

Fig.13 shows that expanded size and Predicted size of any element of the Collatz sequence are approximately the same.

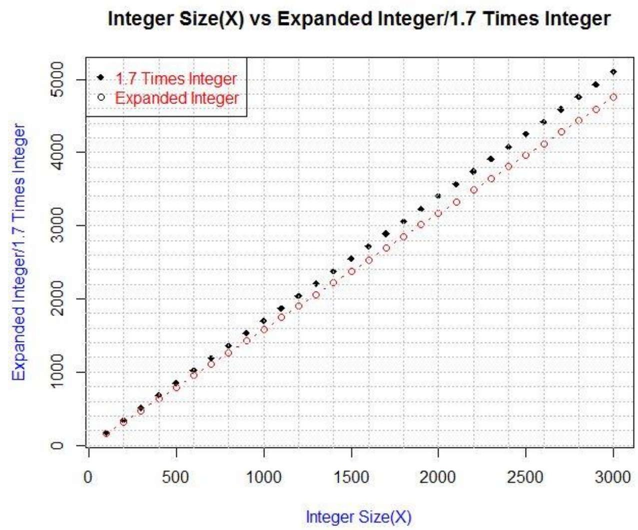

Fig. 14. Integer Size(x) vs. Expanded Integer/1.7 Times Integer

Fig.14 shows that the maximum expanded size of the integer is always less than 1.7 times size of the integer.

Table 1 below shows the experimental results on verification of Collatz Conjecture wherein we have introduced some zeros in the binary representation. The zeros are introduced in a randomized manner. The second column denotes the number of zeros in the binary representation of the given integer size, x/2 and 3x+1. Fourth column denotes the number of times division by 2 happens and fifth column denotes the number of times the odd integer is encountered during the Collatz sequence generation before it reaches value 1.The third column denotes the maximum size of any element of the Collatz sequence.

TABLE I

Integer Size	Number of Zero's	Expanded r Size	x/2	3x+1
100	1	155	937	528
100	2	116	588	309
100	3	112	429	210
100	4	112	708	386
100	5	114	431	210
200	1	245	1102	569
200	2	244	1316	704
200	3	244	1140	593
200	4	244	1316	704
200	5	244	1102	569
300	1	312	1443	721
300	2	312	1454	728
300	3	312	2020	1085
300	4	312	1454	728
300	5	315	2004	1075
400	1	547	3212	1774
400	2	547	3212	1774
400	3	545	2811	1521
400	4	545	3212	1774
400	5	545	2765	1492
500	1	672	3648	1986
500	2	676	4030	2227
500	3	676	3277	1752

500	4	676	3648	1986
500	5	677	3277	1752
1000	1	1246	6370	3388
1000	2	1247	6266	7892
1000	3	1246	6434	7998
1000	4	1246	6266	7892
1000	5	1247	6266	7892
1500	1	2051	10982	10552
1500	2	2049	10982	10552
1500	3	2047	11399	10815
1500	4	2047	10982	10552
1500	5	2046	10981	10552
2000	1	2639	14839	12670
2000	2	2639	14270	12311
2000	3	2639	13709	11957
2000	4	2639	13159	11610
2000	5	2639	13159	11610
2500	1	3506	18192	14470
2500	2	3505	18417	14612
2500	3	3504	18417	14612
2500	4	3504	18417	14612
2500	5	3504	18875	14901
3000	1	4379	23239	17339
3000	2	4378	24298	18007
3000	3	4378	23672	17612
3000	4	4378	23198	17313
3000	5	4379	23198	17313

IV. CONCLUSION

We have presented a new algorithmic approach for Collatz conjecture verification based on binary representation, multiplication, addition and division by 2, all done in binary domain. We observe that there are NO cycles in Collatz sequence and no element of the sequence exceeds in size 1.7 times the size of the given (starting) number. We have verified the results with all entries in binary form equal to 1, being the largest possible integer with that size. Due to limit in available computing resources, we verified the conjecture up to binary string of size 3000. Given, enough computing resources, our scheme can verify the conjecture for any given integer however big it may be.

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