A New Equation of Mean Velocity in Flow, Its Verification and Consequences: H.B. Fischer's Experiment in the Caltech Canal.

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Abstract: - The Chezy-Manning equation has long been the macroscopic description of flow in real channels, and its foundation is the balance of Newtonian mechanical forces. This Article presents the development and verification of a new equation of average velocity in the flow, defined this time in terms of electric forces (Van der Waals), having the same structure as the classical equation. Its application can be very broad, since in addition to having a new and powerful theoretical tool, in engineering practice it allows reducing the degree of uncertainty in the measurement of natural channels. The validity of the Elder equation, which links hydraulics and Dispersion, is analyzed here and an adjustment to its application is proposed, considering the Longitudinal Transport Coefficient as a function of time.

Key-Words: - Hydraulics; Fick, Chezy-Manning; Elder; turbulence, state functions.

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1. Introduction

Environmental and Hydraulic Impact studies use advanced software models that require feeding with precise data that adequately represents the complex reality of natural channels. Although these models have advanced a lot, serious limitations persist in the quality and quantity of the data series, since the theoretical tools have not advanced at that pace, and isolated techniques and concepts are available, which are approximate in their domain. local" application, its general validity is limited and meager, especially in large channels. The foundations of the problem and an alternative model that can solve it are then presented.

2. Problem Formulation.

2.1 The problem of modeling environmental impacts: The basic concepts.

Industrial Society generates an astronomical amount of waste of all types that in a certain percentage end up in natural channels, negatively impacting the quality of the resource. To characterize and control this, multiple efforts have been developed in the physics and chemistry of the evolution of pollutant transport in flows.

A first result was Taylor's one-dimensional mass balance equation in 1954, where ***C*** and ***U*** are average values of the concentration and velocity over the cross section of the flow, ***Ayz***, and ***D*** as the longitudinal dispersion coefficient. [1]:

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} = D\frac{\partial\left(\frac{\partial C}{\partial X}\right)}{\partial X} \quad (1)$$

Although it was accepted that the main cause of the dispersion of the pollutant was the variation of the elemental velocities over ***Ayz***, thanks to the shear effect, the process could be approximated by a Fickian expression (including concentration gradients). Likewise, Taylor found that a value of ***D*** for the case of a long, straight tube, with Radius a, the density of water, ρ, and τo, as the "shear stress" (tangential tension of the edge in the viscous medium).

$$D \approx 10.1 * a * \sqrt{\frac{\tau_o}{\rho}} \quad (2)$$

Based on this analysis, in 1959, J.W. Elder[2], applying velocity variations on the vertical axis, ***Δvz***, in accordance with the logarithmic (turbulent) definition of the velocity distribution in a viscous medium, according to Prandtl. [3][4]. Figure 1.

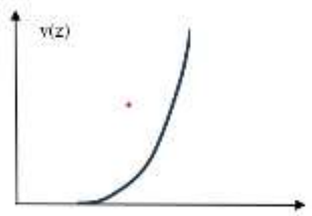

Figura 1.- Perfil no lineal de velocidad

Elder found for an open channel of infinite width, that ***D*** could be expressed as:

$$D \approx 5.93 * z * \sqrt{z * g * S} \qquad (3)$$

Where h is the average depth of the flow, S its slope, and g the acceleration due to gravity.
Although Elder's definition could be applied to natural flows with some approximation, the experimental values in natural channels were dispersed in a ratio of 1 to 15, calling into question even the Fickian concept of mass transport in turbulence. Furthermore, if we consider that the analytical solution of the differential equation (1) is the basic Fick expression, with ***M*** as the mass suddenly poured into the flow, and ***Xo*** the observation distance from the injection point:

$$C(X,t) \approx \frac{M}{Ayz*\sqrt{4\pi*D*t}} e^{-\frac{(Xo-U*t)^2}{4*D*t}} \qquad (4)$$

Despite the rigor of Taylor and Elder's previous arguments, it was verified that not only did the ***D*** values not correspond to those actually observed in natural channels, but that the asymmetric shape of the measured Fickian curve had an "abnormal" bias. Figure 2.

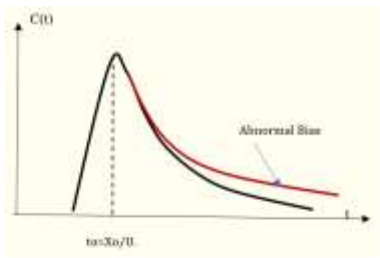

Figure 2.- Sesgo anormal en la respuesta real.

To help solve this incongruence between theory and reality, many hypotheses and procedures were presented, with varying degrees of success (and failure), among them the method of H.B. Fischer [5][6] who considered, unlike Elder, that the primary cause of turbulent diffusion was not the vertical distribution of velocities, ***v(z),*** but the transverse distribution of velocities, ***v(y).***

2.2 Vertical and transversal velocity distributions: Two sides of the same reality.

A first consideration to solve the problem is to ask a simple first question: How different are the turbulent velocity distributions in the vertical axis (Elder) and those in the transverse axis (Fischer)? Or put another way: Is their nature really different?
To resolve these questions, it must be observed that the basic mechanism for generating the "dispersion" itself is the "random separation" of pairs (contiguous) of fluid particles, due to a "fluctuation" (difference) in the velocities in that pair. of very close points, and that produces the separation at different distances of the two particles that were initially "united" (at the same point). That is, how the action of a du generates a ***dX***. [7] Figure 3.

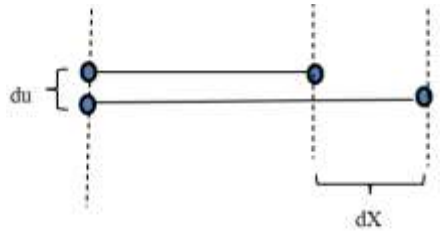

Figure 3.- Random separation of a pair of particles

If we idealize the longitudinal velocity field in the cross section of the flow, we can separate an average velocity, ***<U>,*** and the alterations (pulsations) that are an "indivisible" mixture between wave and turbulent motion, ***du***, [8][9]

$$U(x) \approx < U > +du \qquad (5)$$

If it is considered that "the entire" systematic part of the water movement, and therefore equal, is concentrated in ***<U>,*** then the total variations will be concentrated in du.
Since the part of this definition that generates the separation is du, the question now arises whether this differential will be different if it is placed on the Z axis or the Y axis. Clearly not, because the speed differentials are due only to turbulence, and if this is considered approximately homogeneous and isotropic [10], we have approximately that:

$$du \approx dY \approx dZ \qquad (6)$$

Therefore, the hypothesis of H.B. Fischer, an essential difference in the distribution of increases in turbulent velocity according to the axis, as a cause of the theory-reality discrepancies for the movement of tracers, is not supported as an underlying reason.

2.3 The Longitudinal Diffusion Coefficient as a function of time.

Now, although in the various developments proposed to solve the problem, the value of the Coefficient D itself is a function of certain parameters, as seen in equations (2) and (3), if these do not vary, it is assumed that ***D*** would not vary either. , that is, it is not considered a function of time, since this fact would introduce certain problems of interpretation of the dispersion mechanism, such as the idealization of the "uniform flow" condition [11][12]. An interesting idea that has been presented previously, but has not been developed in detail [13] is that the Coefficient ***D*** should be a function of time.
If you see closely, the temporal nature of the Transport Coefficient is a no small issue from the point of view of Mechanics, it involves the notion of relative motion and its relationship with various inertial observers [14].
A first mobile observer located at the peak of the distribution, moving at speed U, does not compose it (U=0) and describes the Fickian dynamics of a symmetrical tracer plume, as shown in Figure 4.

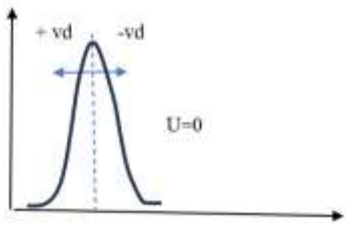

Figure 4.- Symmetrical kinematic composition of the plume.

A second fixed observer located on the bank of the channel does compose the velocity U, such that U>0, and describes the Fickian dynamics of an asymmetric tracer plume, as shown in Figure 5

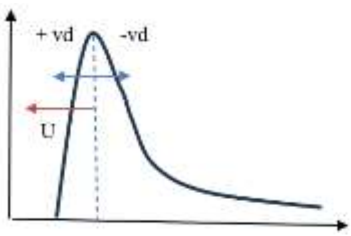

Figure 5.- Asymmetric kinematic composition of the plume.

In this case, with the average velocity ***U***, two other opposite velocities are composed: ***+vd*** and ***-vd,*** associated with the movement of diffusion, expanding to both sides of the central point of the mass injection. This composition leads to the initial segment of the Fickian curve being much steeper than the subsequent segment, generating the bias that is observed experimentally.
The net velocity, composed kinematically as a function of time, can then be described:

$$U(t) = < U > \mp vd \tag{7}$$

Since the motion of the tracer is described as a Brownian one, it can be written in a general way as [15]:

$$vd \approx \frac{\Delta}{\tau} \approx \frac{\sqrt{2D\tau}}{\tau} \approx \sqrt{\frac{2D}{\tau}} \tag{8}$$

Here, the "characteristic time", τ, can be expressed as a function of the general time, t, in the following way, incorporating the Feigenbaum Number, δ, in such a way that the self-similar, chaotic nature of the phenomenon is manifested. [16]

$$\tau \approx \frac{t}{\delta} \approx \frac{t}{4.669} \approx 0.2142 * t \tag{9}$$

And you can define a State Function, ***Φ(U,E,t),*** that complies with the following:

$$\oint d\Phi = 0 \tag{10}$$

And

$$\Phi \approx \frac{vd}{U} \approx \frac{\sqrt{\frac{2D}{\tau}}}{U} \tag{11}$$

From here then, the average velocity is:

$$U \approx \frac{1}{\Phi}\sqrt{\frac{2D}{\tau}} \tag{12}$$

This equation has the same mathematical structure as the classical Chezy-Manning relationship, for the mean velocity of uniform flow, using mechanical quantities. [17]:

$$U \approx \frac{R^{\frac{2}{3}}}{n}\sqrt{S} \tag{13}$$

In equation (13) the source magnitude of the movement is gravitation acting through the slope, S≈ ***dy/dx***, while in equation (12) they are the electric forces (Van der Waals) acting through the state function , ***Φ***.

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This function reflects the exchange of energy between the mass of the tracer and its liquid environment, basically due to the expenditure of Enthalpy of formation, which is expelled as heat to the outside. As this process depends on the square root of the Concentration, $\sqrt{C}$, it can be shown that the expression of $\boldsymbol{\Phi}$ depends on this factor.[18] Figure 6.

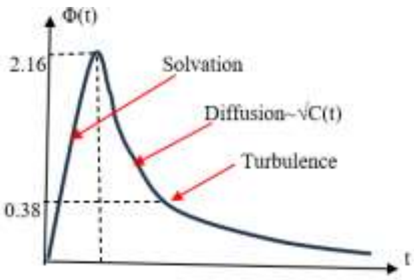

Figure 6.- State function Φ(t).

Actually, what the $\boldsymbol{\Phi(t)}$ curve represents is a phase transition, in which the solid tracer becomes a gas, which occurs at the point $\boldsymbol{\Phi \approx 0.38}$, at which almost all of the mass of the tracer is lost. their mutual interactions, and the description from there is properly of water in turbulence. This point is very important because the practical description of this state does not require the use of non-linear differential expressions.[19]

2.1 Data obtained from the tracer curve.

When a mass M of tracer is injected abruptly, a tracer concentration distribution is generated that corresponds to the solution of the Taylor equation (1), and is the Fick equation (4), only if we do not have consider that the Coefficient of Dispersion is a function of time, D(t), its development will not appropriately replicate the experimental data, in particular it will present incorrect bias.
The way to correct this problem is to solve for ***D(t)*** from equation (12) and replace it in (4), as shown below, where to is the time measured after passing the peak of the distribution at the point of measurement:

$$D \approx \frac{\Phi^2 * U^2 * 0.214 * to}{2} \tag{14}$$

And then:

$$C(X,t) \approx \frac{M}{Q * \Phi * t * 1.16} e^{-\frac{(to-t)^2}{2 * 0.214 * (\Phi * t)^2}} \tag{15}$$

In this expression ***Q≈U*Ayz***, is the flow rate (in l/s) and t is the general time. The validity of this equation is verified when the experimental curve agrees with the model, and the correct dispersive data are those that allow this coincidence: basically, the $\boldsymbol{\Phi(to)}$ data, the flow rate and the average velocity, U(to), having the ***Xo*** data and the Mass, ***M*** in μg if a fluorescent tracer is used, such as Rhodamine WT or Fluorescein. These data are obtained from the special measurement equipment, FLUVIA F-1[20], but can also be calculated from the tracer curves of cases that have not been measured with said equipment. Figure 7.

Figure 7.- Tracer measurement equipment.

2.3 Calculation of roughness and its verification with Elder's equation.

The existence of a definition of the mean flow velocity that is based on Newton forces, the Chezy-Manning equation (13), and also on Coulomb forces (12), allows the calculation and verification of hydraulic data (sometimes difficult to obtain) from Dispersion data (easier to achieve). In this way, a data such as the Roughness of a flow, ***n***, can be stated by equating (12) and (13) as follows:

$$n \approx \frac{\Phi * R^{\frac{2}{3}} * \sqrt{S}}{\sqrt{\frac{2D}{\tau}}} \tag{16}$$

3 Analysis of an experimental case.

To apply the methodology explained here, an experiment carried out by H.B. Fischer in the Caltech calibrated channel, in 1966. Figure7. [21][22]

Figure 7.- Caltech Calibrated Channel.

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Likewise, the two consecutive results of injecting a saline solution are shown, at ***Xo1=14.05 m*** and ***Xo2=25.06 m.*** [23] Figure 8

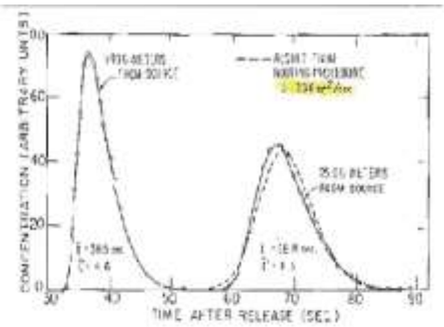

Figure 8.- Salt tracer curves at two distances.

3.1 Channel data.

This experiment was carried out in the 40.0 m calibrated channel of the M. Keck Laboratory of hydraulics of CALTECH, especially the experiment called “Series 2700”, with adjustable slope and bed material. The geometric data of the channel are as follows, Table 1:

Table 1: Channel data

Specifications	Characteristics	Metrics.
Shape	Rectangular	Ayz≈0.140 m2
Distance Xo1	linear	Xo1=14.06 m
Distance Xo2	linear	Xo2=26.06 m
Bottom	Smooth	Wide=1.09 m
Sides	Smooth	Depth=0.128 m
Slope	Mechanical adj.	S≈0.0002 min.
Hydraulic radius	Mean value of flow	R≈0.104 m

3.2 Datos de la Advección y dispersion y datos de la modelación con la ecuación (4).

Los datos del transporte advectivo y dispersivo se muestran en el siguiente Table 2.

Table 2: Tracer experiment two points data

Specifications	Characteristics	Metrics.
Mass of saline solution (NaCl).	Ionic tracer to be measured with conductivimeter	M≈40500 mgr
Time of observation at Xo1	Measured in peak of distribution	to1≈ 38.5 s
Time of observation at Xo2	Measured in peak of distribution	to2≈ 68.8 s
Mean Velocity	Measured in peak of distribution	U≈0.372 m/s
Shear Velocity	-----	U*≈0.0159 m/s
Discharge	-----	Q≈ 50.8 L/s
State function at to1	-----	Φ1≈0.137
State function at to2	-----	Φ2≈0.130
Longitudinal Diffusion Coefficient at to1	As time function	D1≈0.0106 m2/s
Longitudinal Diffusion Coefficient at to2	As time function	D2≈0.0169 m2/s

The modeling of the two tracer curves using the “Modified Fick” equation (4) is shown below, superimposed on the experimental data, showing that the simulation is quite good, representing the asymmetries of the real curves. Figure 9.

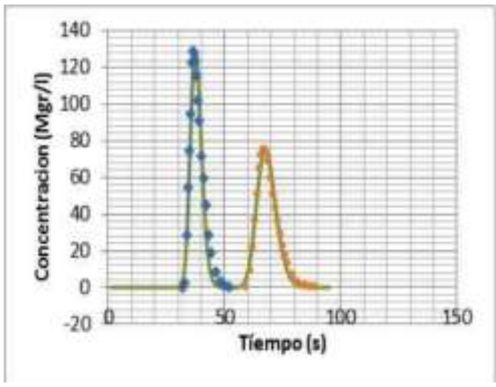

Figure 9.- Modeling of saline tracer curves at two distances.

3.3 Calculation of the Manning Roughness of the channel from the new equation.

The great advantage is that this experiment with a salt tracer is profusely documented, although the value of the channel roughness is not in the list of values, so it is interesting to calculate it with (16) and then review its probable value in a Table of roughness.

Considering the initial data from both the channel and the tracer experiment, its value is solved according to equation (16). For the two measurement points, like this:

$$n1 \approx \frac{\Phi 1 * R^{\frac{2}{3}} * \sqrt{S}}{\sqrt{\frac{2D1}{\tau 1}}} \approx \frac{0.137 * 0.104^{\frac{2}{3}} * \sqrt{0.0002}}{\sqrt{\frac{2*0.0106}{0.214*38.5}}} \approx 0.0085 \qquad (17)$$

And:

$$n2 \approx \frac{\Phi 2 * R^{\frac{2}{3}} * \sqrt{S}}{\sqrt{\frac{2D2}{\tau 2}}} \approx \frac{0.130 * 0.104^{\frac{2}{3}} * \sqrt{0.0002}}{\sqrt{\frac{2*0.0169}{0.214*68.8}}} \approx 0.0085 \qquad (18)$$

The two values are coincident with the figure of ***n≈0.0085***, a value that is the reference figure for artificial laboratory channels, in the text of V.T. Chow [24].

3.4 Verification of the Slope, S, with Elder's equation.

Considering that Elder's equation is valid for all types of velocity distribution (vertical or transversal), it will be applied to this experiment for which the maximum depth, ***h≈0.128 m***, and the shear velocity, ***U*≈0.0159 m/s***, are known, therefore, for the two distances the Slope is solved:

$$S \approx \frac{D^2}{35.2*h^3*g} \tag{19}$$

Then:

$$S1 \approx \frac{D1^2}{35.2*h^3*9.83} \approx \frac{0.0106^2}{345.7*0.128^3} \approx 0.00016 \tag{20}$$

And:

$$S2 \approx \frac{D2^2}{35.2*h^3*9.83} \approx \frac{0.0169^2}{345.7*0.128^3} \approx 0.0004 \tag{21}$$

The average value of these values is ***<S>≈0.00028***, that is, with a percentage error of 40% but within the same order of magnitude. This error probably corresponds to the fact that, as Elder's formula for "***D***" is defined, this parameter does not depend on time, but as has been shown here, this dependence does exist, therefore the behavior of the Elder relation actually it should be expressed graphically as in Figure 9.

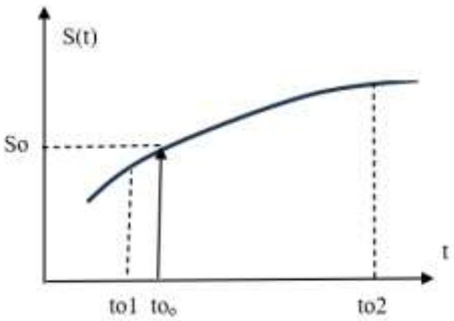

Figure 9. Time dependence of the Elder equation.

This means that Elder's equation must necessarily be interpreted as a function of time, and that there will be an "optimal observation time", to_o. This time can be established with an "Estimation Function", as expressed below.

$$F \approx \Phi^2 * \tau * \left(\frac{C^2}{2}\right) * \left(\frac{h}{R}\right) * \sqrt{\frac{S}{h*g}} \tag{22}$$

Here for simplicity we have written "C" is the Chezy constant, which is:

$$C \approx \frac{R^{\frac{1}{6}}}{n} \approx 82.0 \tag{23}$$

This estimate is exactly ***F=5.93***, for the optimal time, ***t=to₀***. To determine this optimal time, interpolations of values of the time-dependent factor $\boldsymbol{\Phi^2 * \tau}$ are made (since the other values are not considered temporal functions with known values) and we have that ***F≈5.95*** (with an error of 0.4%) with ***Φ ≈0.132*** , ***to_o≈ 46.4 s.*** and ***D(to_o)≈ 0.0121 m2/s.*** for ***S≈0.0002***, in the Caltech channel experiment.

3.5 Methodology to establish approximate values of hydraulics based on the average velocity equations and the Elder equation.

With the tracer, Dispersion and Advection data are obtained, which can be approximately extrapolated from the development of the State Function. In this way, values can be proposed at a relatively large distance from ***Φ(t).*** Using an average value of the depth, h, and the average width, which are measured by bathymetry or by detailed observation, the slope of the channel, ***S***, is obtained. , the Cross Section, ***Ayz***, and from there we can solve for the Manning roughness, n. Figure 10.

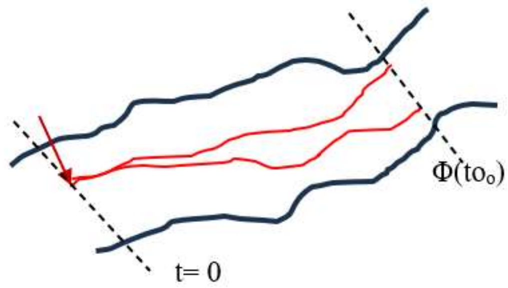

Figure 10. Approximate measurement of "far" values of hydraulics and geomorphology by Φ(t).

This extrapolation is possible as long as the channel is in "dynamic equilibrium", when the production of entropy is minimal and the differences between the values have minimum variance.[25].

4. Conclusions

1.- With a new equation for the average velocity of the flow in a non-uniform regime, it is possible to introduce a Longitudinal Transport Coefficient, ***D(t),*** as a function of time, which eliminates the errors that are introduced in the formation of the tracer plume model. allowing the experimental bias to be appropriately reproduced.

2.- This temporal dependence of the longitudinal transport coefficient arises from the existence of a State Function that describes the evolution of the tracer in turbulence. This function describes the phase transition of the tracer in the flow, and its statistical coupling with it.

3.- Historical objections to the validity of the Elder equation, which defines ***D(t)*** are not founded, since turbulence can be approximated as an isotropic phenomenon, and the nature of the vertical distribution of velocities is not distinguished in principle. of that of Lateral Distribution.

4.- Using the Chezy-Manning (13), Van der Waals (12), and Elder (3) equations with adjustments, it is possible to propose a methodology for measuring parameters in natural channels, minimizing uncertainties.

5.- This methodology is verified in the experiment carried out by H.B: Fischer in the Caltech calibrated channel in 1966, which was documented in great detail.

References

[1] Fischer H.B. Dispersion predictions in natural Streams. *Journal of Sanitary Engineering*. October (1968).
[2] Elder J:W: The dispersion of market fluid in turbulent shear flow. *Journal of fluid mechanics*. 5. Part 4. May (1959)
[3] French. R. *Hydraulics of open channel*. McGraw-Hill, (1985)
[4] Nekrasov B. *Hidraulica*. Editorial Mir, Moscú. (1968).
[5] Fischer H.B. *PhD Thesis*. (1966)
[6] Fischer H.B. The mechanics of dispersion in natural streams. *Journal of Hydraulics Division*. HY 6. November (1967).
[7]Constaín A. Dispersión Random Walk, irreversibilidad y velocidad en flujo no uniforme. *Revista Guillermo de Okham*. Cali, (2005).
[8]Simonenko S.V. *Non-equilibrium statistical thermodynamics of turbulence*. Nauka, Moscu, 2006.
[9] Meyer R.E. *Introduction to mathematical fluid dynamics*. Dover, New York. (1971)
[10] Simonenko S.V. Ibid (1971)
[11] Holley E.R. Unified view of diffusion and dispersion in natural streams. *Journal of the Hydraulic division*. ASCE, March (1969).
[12] French R. Ibid (1985).
[13]Cushman Roisin B. Beyond eddy diffusivity: An alternative model for turbulent dispersion. *Environ. Fluid Mech.* (2008)
[14] Frish S. & Timoreva A. *Curso de física general*. Tomo 1. Editorial Mir, Moscú, (1969).
[15] Einstein A. *Investigations on the theory of the Brownian movement*. Dover. New York.(1956).
[16] Constaín A., Peña G. & Peña C. Función de estado de evolución de trazadores, Φ(U,E,t), aplicada a una función de potencia que describe las etapas de la turbulencia. *Revista Ingeniería Civil*, CEDEX, Madrid. (2022).
[17] Vennard J. *Elementos de la mecánica de fluidos*. C.E.C.S.A. México.(1965).
[18] Constaín A. The Svedberg number, 1.54, as the basis of a State function describing the evolution of turbulence and dispersion. Chapter *Intech Open book*, London. To be published next.(2024).
[19] Anderson P.W. More is different. *Science*,177(4047). (1972).
[20] Constaín A., Lemos R. & Carvajal A. Tecnología IMHE: Nuevos desarrollos de la hidráulica. *Revista Ingeniería Civil*, CEDEX, Madrid. No. 129. (2003).
[21] Constaín A. Aplicación de una ecuación de velocidad media en régimen no uniforme : Análisis detallado del transporte en el Canal Caltech usando Excel. *Revista Ingeniería Civil*, CEDEX, No. 170. (2013).
[22] Constaín A. Revalidación de la ecuación de Elder para la medición precisa de Coeficientes de dispersión en flujos naturales. *Revista DYNA,* Medellín, No.81. (2014).
[23] Constaín A. & Corredor J. Ecuación de Elder: Una nueva visión de la geomorfología de cauces naturales en Estudios de calidad de aguas. *Revista ACODAL; No.235.* (2014):
[24] Chow V.T. *Hidráulica de canales* abiertos. McGraw Hill, New York. (2004).
[25] Langbein W. & Leopold L. River meander-Theory of minimum variance. Env.Sci. (1966).

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