# **Unsteady flow of rheologically complex fluids in cylindrical pipes.**

### NAVRUZOV KURAL, SHARIPOVA SHOKHISTA, ABDIKARIMOV NABIJON Department of Mathematical Engineering, Urgench State University, Urgench, 220100, **UZBEKISTAN**

*Abstract: -* In this article, we have studied the process of transition from the non-stationary state to the stationary state of the mixture formed by adding a Newtonian fluid to a rheologically complex fluid in a cylindrical tube. We used the analytical method. The advantage of this type of fluid flow research is that it allows for a more comprehensive study of fluid motion. Among the obtained results, it is necessary to note new results that are of great importance in revealing the state of the physical phenomenon. The graphs of the results were made using Matlab mathematical software. For a better understanding of the physical properties of some of the obtained results, their graphical representation is shown. These results are effective for further development of unsteady flow of rheologically complex fluids in cylindrical pipes.

*Key-Words: -* Viscoelastic fluid, unsteady flow, transfer function, oscillatory flow, phase

Received: March 21, 2024. Revised: August 16, 2024. Accepted: September 19, 2024. Published: October 17, 2024.<br>  $\int \rho \frac{\partial \theta_x}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r\tau), \ \tau = \alpha_1 \tau_1^0 + \alpha_2 \tau_2^0,$ 

## **1. Introduction**

Channel and channel of Newtonian fluids by scientists devoted to the problems of oscillations and non-stationary flows in pipes it can be noted that many scientific and practical studies have been carried out. Including [1,2, 3; 4;] in research work non-stationary and stationary oscillation of liquids in channels and pipes flows have been sufficiently studied. The first is a viscous liquid characteristic of velocity profiles in oscillatory flow in cylindrical pipes characteristics [5] are presented in the form of experimental results.

Hydrodynamics in transition observed in flat channels. It is also important to determine the changes in characteristics in circular pipes is significant. In this work, the rheologically complex liquid under the influence of a constant pressure gradient in a sufficiently long cylindrical pipe we consider the problem of unsteady flow. In this case, the axis is along the axis of the cylinder we direct, and we define the radius of the pipe by R. The length of the pipe the longitudinal velocity is in the stream because it is sufficiently large compared to the radius, we think that it will not appear. For this case, the cylindrical pipe is rheologically complex system of differential equations defining nonstationary fluid motion is expressed in the following simplified form:

$$
\begin{cases}\n\rho \frac{\partial \mathcal{G}_x}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r\tau), \ \tau = \alpha_1 \tau_1^0 + \alpha_2 \tau_2^0,\n\\
\tau_1^0 = \eta_1^0 \frac{\partial \mathcal{G}_x}{\partial r}, \ \lambda \frac{\partial \tau_2^0}{\partial t} + \tau_2^0 = \eta_2^0 \frac{\partial \mathcal{G}_x}{\partial r}, \ \eta = \alpha_1 \eta_1^0 + \alpha_2 \eta_2^0.\n\end{cases}
$$
\n(1)

To solve this system of equations (1), it is necessary to formulate initial and boundary conditions. for this  $t = 0$  liquid is considered to be at rest[6,7,8] i.e

$$
t=0
$$
  $\partial a$   $\theta_x = 0$ ,  $\frac{\partial p}{\partial x} = 0$  (2)

At positive values of time, motion occurs under the influence of an invariant pressure gradient, the boundary conditions for this case are defined as<br>follows [74,76]:<br> $t \ge 0$   $\omega a$   $r = 0$   $\frac{\partial \theta_x}{\partial r} = 0$ ;  $t \ge 0$   $\omega a$   $r = R$   $\omega a$   $\frac{\partial \omega}{\partial x} = 0$ ,  $\frac{\partial p}{\partial x} \ne 0$ follows [74,76]:

follows [74,76]:  
\n
$$
t \ge 0
$$
 *ea*  $r = 0$  *oa*  $\frac{\partial g_x}{\partial r} = 0$ ;  $t \ge 0$  *ea*  $r = R$  *oa*  $g_x = 0$ ,  $\frac{\partial p}{\partial x} \ne 0$   
\n(3)

The formed system of equations (1) together with initial and boundary conditions (2) and (3) represents the non-stationary movement of a rheologically complex fluid in a cylindrical pipe [9,18].

### **2. Problem Formulation**

To solve this system of differential equations through the formulated initial and boundary conditions,

Laplace-Carson substitution in terms of time variable was used. By performing several steps, the inhomogeneous Bessel equation is derived to find the longitudinal velocity distribution in the image.

$$
\overline{g}_x(r) = \frac{1}{\rho} \frac{1}{s} \left( -\frac{dp}{dx} \right) \left( 1 - \frac{J_0 \left( i \sqrt{\frac{\rho s}{\overline{\eta}(s)}} r \right)}{J_0 \left( i \sqrt{\frac{\rho s}{\overline{\eta}(s)}} R \right)} \right) \tag{4}
$$

As a result of solving the Bessel equation, the formula for finding the longitudinal velocity distribution in the image is determined. Laplace-Carson integral

Carson integral  
\n
$$
\mathcal{G}_x(r,t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} e^{st} \frac{1}{\rho s} \left( -\frac{dp}{dx} \right) \left( 1 - \frac{J_0 \left( i \sqrt{\frac{\rho s}{\overline{\eta}(s)}} r \right)}{J_0 \left( i \sqrt{\frac{\rho s}{\overline{\eta}(s)}} R \right)} \right) \frac{ds}{s}
$$
(5)

using the exact solution of the problem was found based on the formula of longitudinal speed distribution.

### **3. Problem Solution**

To calculate the integral with complex variable (5), we use the fact that this integral is the Laplace integral, and taking into account that the function under the integral is a meromorphic function, we determine its special points.

These special points are:

special points are:  
\n
$$
s = 0
$$
,  $s = -v \frac{\overline{S}_{1,n}}{R^2}$ ,  $s = -v \frac{\overline{S}_{2,n}}{R^2}$ .

the solution of the equation is determined in  
\nthe following form:  
\n
$$
g_x(r,t) = \frac{1}{4\eta} \left( -\frac{\partial p}{\partial x} \right) R^2 \left( 1 - \frac{r^2}{R^2} - 8 \sum_{n=1}^{\infty} \sum_{k=1}^2 \frac{J_0 \left( \beta_n \frac{r}{R} \right) e^{-\frac{v}{R^2} x_{nt}}}{J_1(\beta_n) \beta_n^3 \frac{1 - 2De\overline{s}_{kn} + De^2 \overline{s}_{kn}^2 X}{(1 - De\overline{s}_{kn})^2}} \right)
$$
\n
$$
(6)
$$

The ratio of the non-stationary velocity on the axis of the pipe to the stationary velocity on the

axis of the pipe is determined by the following formula:

remula:  
\n
$$
\frac{\mathcal{G}_x(0,t)}{\mathcal{G}_x(0)} = 1 - 8 \sum_{n=1}^{\infty} \sum_{k=1}^{2} \frac{e^{-\frac{V}{R^2} \bar{x}_{kn}t}}{J_1(\beta_n) \beta_n^3 \frac{1 - 2De\bar{x}_{kn} + De^2 \bar{x}_{kn}^2 X}{(1 - De\bar{x}_{kn})^2}}
$$
\nHere  
\n
$$
\mathcal{G}_x(0) = \frac{1}{4\eta} \left( -\frac{\partial p}{\partial x} \right) R^2.
$$
\n(7)

### **3.1 Subsection**

In the analysis of the solution, the ratio of the maximum longitudinal velocity value of the nonstationary state of the rheological complex fluid flow to the maximum longitudinal velocity value in the stationary flow (at different relaxations of the fluid subject to the Maxwell model and at different concentrations of the Newtonian fluid) as a function of time changes using a graph described. At a small value of the Debor number, the process of transition of a rheological complex fluid flow from a nonstationary state to a stationary state is slightly different from a Newtonian fluid flow transition process, and in this case, the transition process of a rheological complex fluid flow from a non-stationary state to a stationary state can be considered as a Newtonian fluid transition process shown to be possible. At large values of the Debor number, the transition process of a rheologically complex fluid flow from a non-stationary state to a stationary state is shown in Fig. 1. As a result of such sudden changes, the fluid consumption at the initial values of the time compared to the steady-state value  $De = 1$ it was determined to be four times larger. Changes in the unsteady motion of a mixture resulting from the addition of a Newtonian fluid to a rheologically complex fluid have been shown to be more flowstabilizing than turbulent changes in the rheologically complex fluid motion, and by increasing the concentration of a Newtonian fluid in the mixture, the turbulent flow Change management capabilities are created.



Figure 1. Dimensionless time-dependent change of the ratio of the maximum longitudinal velocity of a rheologically complex fluid in an unsteady flow to the maximum longitudinal velocity in a stationary flow (for a fluid subject to the Maxwell model  $De = 1$  at different concentrations of a Newtonian fluid)

### **4. Conclusion**

In this work, it was shown that the changes in the unsteady motion of the mixture resulting from the addition of a Newtonian fluid to the rheologically complex fluids have a more stabilizing property than the turbulent changes in the motion of the rheologically complex fluids, and by increasing the concentration of the Newtonian fluid in the mixture, g in the flow It was concluded that opportunities for effective change management will be created.

*References:*

- [1]. [Lei Tang](https://pubs.rsc.org/en/results?searchtext=Author%3ALei%20Tang) et al. Ion current rectification properties of non-Newtonian fluids in conical nanochannels // Physical Chemistry Chemical Physics. 2024. – Т. 2895.<https://doi.org/10.1039/D3CP05184F>
- [2]. Navruzov K. et al. Mathematical modeling of hydrodynamic resistance in an oscillatory flow of a viscoelastic fluid VT− 92 and two-fluid model to the problem of a subsonic axisymmetric jet //E3S

Web of Conferences. – EDP Sciences, 2023. – Т. 401. – С. 02010**.** 

- [3]. Navruzov K. et al. Study of Oscillatory Flows of a Viscoelastic Fluid in a Flat Channel Based on the Generalized Maxwell Model //Russian Mathematics. – 2023. – T. 67. – №. 8. – C. 27-35.
- [4]. Navruzov K. et al. Tangential Shear Stress in an Oscillatory Flow of a Viscoelastic Fluid in a Flat Channel //International Conference on Next Generation Wired/Wireless Networking. – Cham: Springer Nature Switzerland, 2022. – С. 1-14.
- [5]. Navruzov K., Sharipova S. B. Tangential shear stress in oscillatory flow of a viscoelastic incompressible fluid in a plane channel //Fluid Dynamics. – 2023. – Т. 58. – №. 3. – С. 360-370.
- [6]. Xingyu Chen et.al. Streaming potential of viscoelastic fluids with the pressure-dependent viscosity in nanochannel// Physics of Fuids-2024.- T35.<https://doi.org/10.1063/5.0197157>
- [7]. G.T. Ayubov1, G.I. Mamatisaev, F.A. Usanov, Sh.I. Askarhodjaev, and B. Urinov, (2023). Seismic resistance evaluation of multi-story buildings using the modern LIRA-SAPR SP// E3S Web of Conferences 402, 07019, https://doi.org/10.1051/e3sconf/202340207019
- [8]. Casanellas L., Ortin J. Laminar oscillatory flow of Maxwell and Oldroyd-B fluids// J. Non-Newtonian Fluid. Mechanics. 166. 2011. P. 1315-1326.
- [9]. Ding Z., Jian Y. Electrokinetic oscillatory flow and energy microchannelis: a linear analysis // J. Fluid. Mech. 2021. vol. 919. A20. P.1-31.
- [10]. A. Sh. Begjanov. et al. Pulsating flow of stationary elastic-viscous fluids in flat-wall channel// In E3S Web of Conferences Vol. 401, p. 01030 (2023).
- [11]. Ibrokhimov A. et al. Numerical study of particle motion in a two-dimensional channel with complex geometry //BIO Web of Conferences. – EDP Sciences, 2024. – Т. 84. – С. 05037.
- [12]. Ibrokhimov A. et al. Mathematical modeling of particle movement in laminar flow in a pipe //BIO Web of Conferences. – EDP Sciences, 2024. – Т. 84. – С. 02026.
- [13]. Ibrokhimov A. et al. Comparison of the results of applying the turbulence model VT− 92 and twofluid model to the problem of a subsonic axisymmetric jet //E3S Web of Conferences. – EDP Sciences, 2023. – Т. 452. – С. 02026.
- [14]. Usarov Makhamatali. et al. On solution of the problem of bending and vibrations of thick plates on the basis of the bimoment theory//Cite as: AIP Conference Proceedings 2637, 030016 (2022); <https://doi.org/10.1063/5.0118598>

#### **Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)**

Kuralbay Navzruzov and Shokhista Sharipova conceived of the presented idea. Kuralbay Navzruzov developed the theory and performed the computations. Shokhista Sharipova verified the analytical methods. Both authors discussed the results and contributed to the final manuscript. Both authors contributed to the article and approved the submitted version.

### **Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### **Conflict of Interest**

The author declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

### **Data availability**

quiries can be directed to the corresponding author/s.

### **Creative Commons Attribution License 4.0 (Attribution 4.0 International, CC BY 4.0)**

This article is published under the terms of the Creative Commons Attribution License 4.0

https://creativecommons.org/licenses/by/4.0/deed.en \_US