Theoretical Contributions Regarding the Modelling of the Superfinishing Process

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Abstract: - The mathematical modeling applied on the machining process by superfinishing includes the study of the forces that appear during the action of the abrasive body on the surface of the parts subjected to processing, as well as the study of the mathematical equations of the trajectories of the abrasive grains within the various working methods used. This study is necessary to be able to obtain a quality of the processed surfaces in accordance with the technical execution documentation. In this way, various movements of the abrasive tool and the processed part can be combined in order to be sure of obtaining the quality of the part's surface finish in the shortest possible time. The study allows obtaining mathematical formulas that facilitate the use of optimal technological parameters for the processing of different shapes of pieces.

Key-Words: - Mathematical modeling, equations of crosshatch pattern, surface finish, optimal parameters.

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1 Introduction

The theoretical foundation of the superfinishing process is still not perfectly elucidated. That is why it was necessary to study some aspects regarding the shape and parameters of the trajectories of the abrasive particles of the cutting tool, the effect of these trajectories on the quality of the surface, the determination of the cutting speed, the acceleration of the particle, etc. Mastering these aspects will allow ensuring a rigorous control of the cutting process and at the same time will facilitate the recommendation of optimal working parameters.

During superfinishing processing, a grain of abrasive material travels a certain trajectory on the surface of the part, which is generally a harmonic oscillatory movement.

The study of the kinematics of a point of the abrasive tool on the surface of the part arose from the need to know the shape and length of the trajectory of a grain during a rotation of the part. This is necessary to determine the parameters of the chipping regime of the length of the curve generated by a particle of the abrasive tool on the surface of the part, to observe which elements intervene in the equation of the trajectory of the movement of an abrasive grain to intervene on them and optimize the chipping regime.

Also, based on the relationships obtained to determine the space traveled by an abrasive grain, the respective velocities and accelerations can be determined by differentiating with respect to time, and it can be observed that during the entire processing these parameters have variable values over a period. This fact leads to a variation of the parameters of the cutting regime and explains the fact that the roughness obtained after processing is not constant over the entire surface of the piece.

In the dynamics of the cutting process, the generation of harmonic oscillatory motion can be achieved by several methods such as: mechanical generation, pneumatic, hydraulic, and electrical generation. The methods of generating the oscillatory movement correctly used in production are mainly the mechanical and pneumatic ones, due to the simplicity of the realization and safety in operation.

2 Development of a theoretical model of the superfinishing process

The superfinishing process is a relatively new processing process compared to other known technological processes, being introduced in the parts manufacturing lines in the period 1930-1935, after this date superfinishing has become more and more widespread due to the advantages that has them in comparison with other smoothing procedures.

From the desire to explain the phenomena that take place in the superfinishing process, the specialists created a theoretical model in which they believe that from the point of view of the physical phenomenon of the chipping process, the superfinishing process with an abrasive bar is similar to that of broaching, each abrasive grain being considered a pin tooth.

At the same time, a first limit of the theoretical model is noted: that the distance between two abrasive grains during superfinishing is much smaller than that between two teeth of the broach, which leads to the difficulty of evacuating the chips. It is considered that the alternative movement of the abrasive bar is the one that allows the evacuation of the chips and avoids clogging of the pores. The authors of the theoretical model explain the small size of the detached chips by the relatively small length of the chipping stroke of the abrasive grain. The theoretical model explains the phenomenon of automatic interruption of the chipping process during superfinishing through the prism of the intervention of the chipping liquid characterized by its viscosity.

The presented theoretical model explains the fine roughness of the superfinished surface by "cutting" the micro asperities that form the rough profile of the part left over from the previous processing operation. It also provides an explanation of the self-interruption of the chipping process. It is considered that the cutting process is interrupted automatically, when following the "recutting" of the tips of the initial geometric micro asperities (the roughness of the semi-finished product), the bearing surface of the processed material increases, the contact pressure decreases, and the cutting fluid forms a bearing film that no longer allows contact between abrasive grains and metal.

Unlike the theoretical models developed for other cutting processes, the theoretical model of the superfinishing process considers the cutting fluid as an active element in the cutting tool-workpiece contact area. According to the model, this liquid - the superfinishing oil, non-emulsifiable oil characterized by viscosity, has the primary role in interrupting the cutting process.

The proposed theoretical model [6] starts from the following specifications:

- the active part of the abrasive grain (which has a diameter Dmax = 9 μm) not embedded in the binder, has dimensions comparable to the maximum height of the micro asperities (about 4 μm) that form the profile of the processed part (Rt= 4-5 μm).
- the chipping liquid present in the processing area can be assimilated to a continuous environment of spherical particles that try to form an adsorption layer on the surface of solid bodies but, due to the dimensions close to those of the microrelief of the part, they fail to penetrate the micro asperities profile.

In the first seconds of the superfinishing process, the abrasive granule "squeezes" the oil drops from the surface of the piece, making direct contact between the granule and the metal ridges, simultaneously achieving the "cutting" of the metal tips and the blunting, breaking, or tearing of the abrasive granule from the binder.

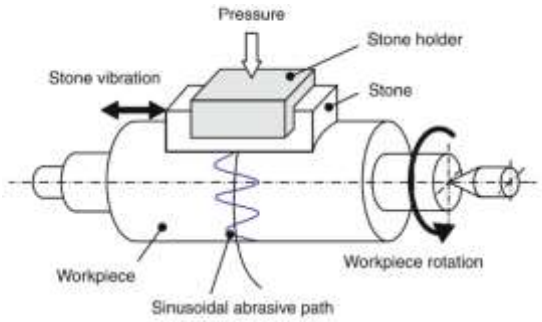

Fig.1
Schematic representation of superfinishing process.

Superfinishing is a chip removing machining process used to refine the surface of a metal component to an extremely fine surface with low roughness.

All the superfinishing grains in contact with the workpiece create a machining pattern by means of overlaying individual sinusoidal lines that cross each other at a particular angle (Fig.1). This generates a specific, defined pattern of grooves and plateaus, which, in turn, results in the advantages of the superfinishing process. The grooves act as channels to aid uniform distribution of the lubricant, while the plateau guarantees a high percentage contact area.

Part of the metal tips cut are fragmented and pulled from the machining area by the cutting liquid, and others remain and penetrate into the "depths" of the valeys. The alternative movement of the tool favors the "stuffing" of the chip pulled into the microrelief of the part.

In the next 10 seconds of machining, the abrasive tool removes the entire layer of material that constituted the initial microrelief and begin to "dig" into the compact material. The traces left by the abrasive grains in the machined part are proportional to the size of the active part (approximately 4 μm) and therefore much smaller than those left by the abrasive grain in the body to be rectified (250-300 μm), so the roughness of the machined surface is also much smaller. The peaks of the superfinished surface are much more frequent and finer, so the bearing surface is larger and the pressure lower, the oil particles are no longer pierced and the superfinishing process is much more difficult (Fig.3).

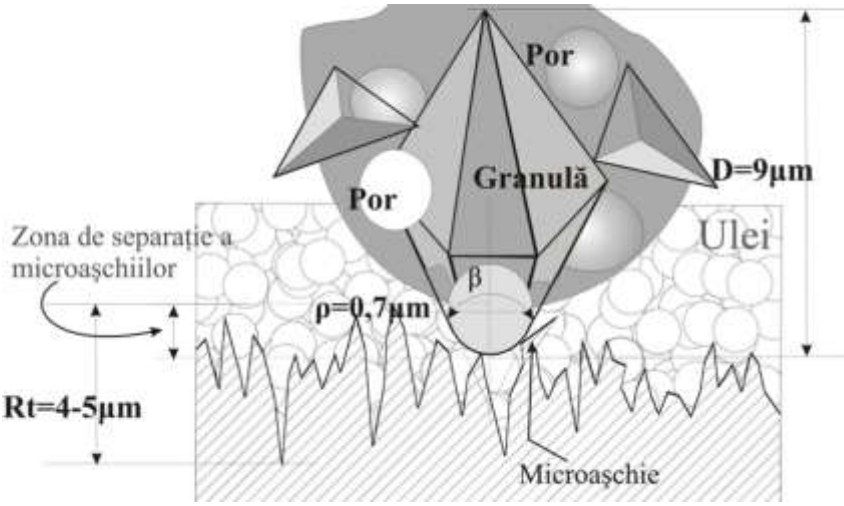

Fig. 2 The first phase of machining.

At the same time, the impurities settled in the microstructure’s valeys contribute to the hardening of the surface layer (max.1µm). Thus, the abrasive grains can no longer detach chips and the machining process self-stops after approximately 150-200 seconds.

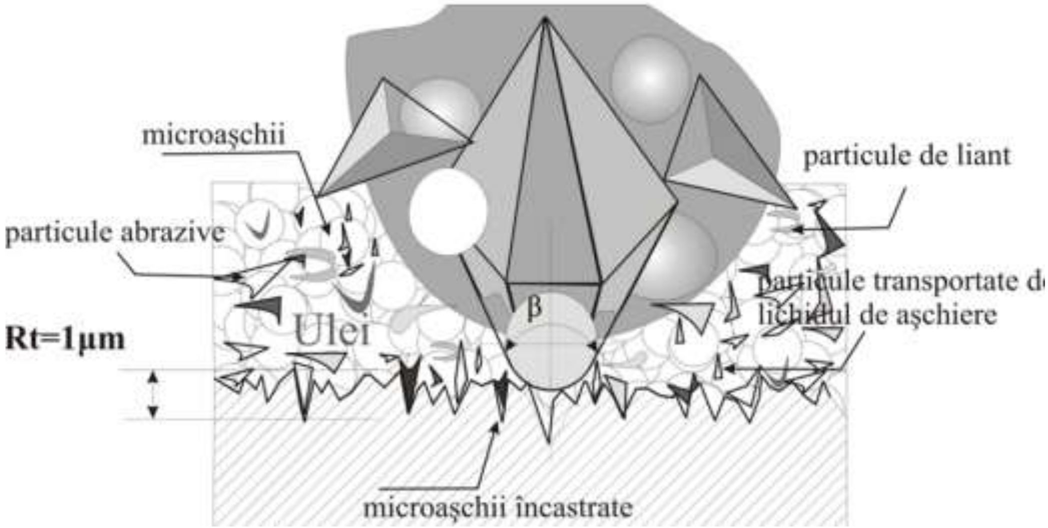

Fig.3 The second phase of machining.

3 Kinematics of a current point of the tool in relation to the part

The kinematics of a current point of the tool in relation to the part will be studied for three processing methods: (a) superfinishing with plunge feed; b) superfinishing with through feed; c) superfinishing with two oscillatory movements.

a) Superfinishing with plunge feed

During the processing of the parts by superfinishing, the abrasive grains describe trajectories of different shapes and lengths on the surface of the part. The length of these trajectories is considered to represent the "degree of coverage" of these surfaces with traces of processing. There is a direct link between the degree of coverage and the quality of the surface of the piece, in the sense that with the increase of the degree of coverage, the roughness of the surface, the bearing coefficient, the state of the surface layer improve.

Fig.4 shows the kinematic diagram of superfinishing with plunge feed.

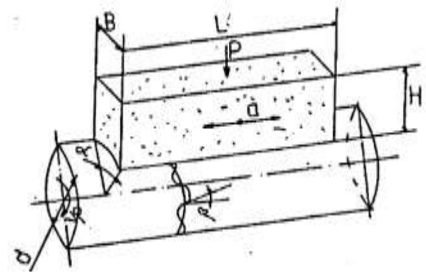

Fig.4 Superfinishing with plunge infeed.

where:

B, L, M = dimensions of the abrasive stone.

P = pressing force of the abrasive stone on the part surface.

a = the amplitude of the oscillation movement.

Vp= peripheral speed of the piece.

α = the angle at the center covered by the abrasive tool.

β = the angle of inclination of the trajectory of the abrasive grain relative to the axis of the part.

An abrasive particle has a periodic sinusoidal movement during processing, whose equation is:

$x(t) = a\sin \omega t$ (1)

where ω is angular velocity of the part being machined.

An Oxyz system related to the part will be considered (Fig.5).

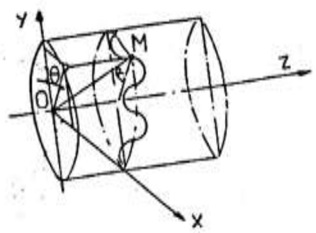

Fig.5 Coordinate system in the case of processing with transverse advance.

The radius vector OM will be projected on the three coordinate axes, resulting in:

$$\begin{cases} x = R\sin\theta \\ y = R\cos\theta \\ z = 0 \end{cases} \tag{2}$$

where: $\theta = \omega\tau$; (3)

$\omega = \pi n/30$ (4)

The projection on the Oz axis is zero because both the part and the tool have no longitudinal displacements when processing with transverse feed. Substituting (3) in (2) we get:

$$\begin{cases} x = R\sin(\omega\tau) \\ y = R\cos(\omega\tau) \\ z = 0 \end{cases} \tag{5}$$

Differentiating with respect to time, we obtain velocity:

$$\begin{cases} V_x = \frac{dx}{d\tau} = \omega R\cos(\omega\tau) \\ V_y = \frac{dy}{d\tau} = -\omega R\sin(\omega\tau) \\ V_z = 0 \end{cases} \tag{6}$$

To find out the length of the trace of the trajectory of an abrasive grain on the surface of the part, unfold the surface of the part and work in the Ouz coordinate system, using the variable u = Rθ, (Fig.6).

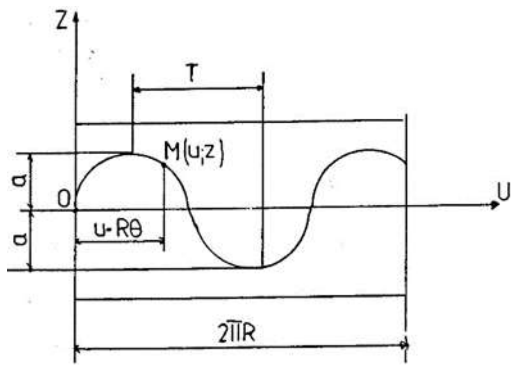

Fig.6 Trajectory of an abrasive grain.

In the new coordinate system using the notation

$$\theta = \frac{u}{R} = k_1' \tag{7}$$

relations (2) will become:

$$\begin{cases} x = Rsin(k_1') \\ y = Rcos(k_1') \\ z = 0 \end{cases} \tag{8}$$

And relations (6) have the form:

$$\begin{cases} V_x = \omega Rcos(k_1') \\ V_y = -\omega Rsin(k_1') \\ V_z = 0 \end{cases} \tag{9}$$

According to Fig.6, the equation of the trajectory of an abrasive grain in the Ouz coordinate system is:

$$z = a\sin(k_1') \tag{10}$$

where a is the amplitude of the motion.

The length of the elementary arc ds will be:

$$ds = du\sqrt{1+\left(\frac{dz}{du}\right)^2} \tag{11}$$

Differentiating eqution (10) with respect to u we get:

$$\frac{dz}{du} = k_1' aRcos(k_1' u) \tag{12}$$

Taking into account relations (11) and (12), the arc of the curve included between the abscissas u_1 and u_2 will have the length:

$$S = \int_{u_1}^{u_2} du\sqrt{1+[k_1' aRcos(k_1' u)]^2} \tag{13}$$

If it is considered that only the arc length of the trajectory of the abrasive grain will be taken into account at one rotation of the part, then the limits of the parameter u will be 0 and 2πR, it will result:

$$S = \int_{0}^{2\pi} du\sqrt{1+[k_1' aRcos(k_1' u)]^2} \tag{14}$$

From relation (14) it can be seen which factors lead to the increase in the length of the trajectory of an abrasive grain on the surface of the part, because the quality obtained after processing is directly proportional to this length. The amplitude of the oscillatory movement also has an important influence, because as the amplitude increases, so does the quality of the processed surface.
The value of the amplitude of the oscillatory movement cannot be too high, because it is limited by technological and functional considerations. The technological considerations are given by the need for a small grain stroke because the space between the grains does not allow the accumulation of a large volume of chips resulting from chipping. Functional and constructive considerations are related to the complexity of realizing a greater amplitude of the work system and to the large moments of inertia that may occur. That is why the optimal amplitude for this method of processing is 1.5 - 3 mm.

b) Superfinishing with through feed
If the length of the processed parts is greater than the length of the abrasive bar, then in addition to the harmonic oscillatory and plunge feed, the tool or part must also perform a longitudinal feed. For example, let's assume that the workpiece has a rotation movement *n* and a through feed with an axial speed v_0 (Fig.7).

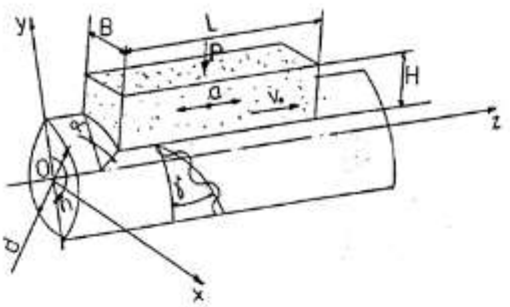

Fig.7 Machining with through feed.

The abrasive bar has a harmonic oscillatory movement with an oscillating stroke and is pressed on the surface of the workpiece with a force P. In this case, the elements of the technological system are related to the Cartesian coordinate axes Oxyz, where we determined the equation of the trajectory of a certain point M of the abrasive grain (Fig. 8).

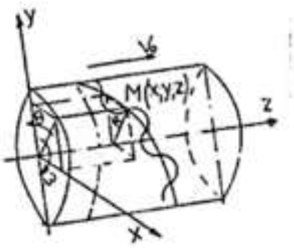

Fig.8 The trajectory of an abrasive grain in the Oxyz coordinate system.

Thus, from Fig.8, the radius vector OM of a certain point M of the abrasive tool is broken down into components on the 3 coordinate axes given by the relations:

$$\begin{cases} x = Rsin\theta \\ y = Rcos\theta \\ z = v_0\tau \end{cases} \tag{15}$$

where $\theta = \omega R$ (16)

Using (16), z will become:

$$z = v_0\tau = v_0\frac{\theta}{\omega} = k\theta$$

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where, $k = \frac{v_0}{\omega}$

The system of relations (15) becomes:

$$\begin{cases} x = Rsin\theta \\ y = Rcos\theta \\ z = k\theta \end{cases} \quad (16)$$

Differentiating with respect to time, we obtain velocity:

$$\begin{cases} V_x = \omega Rcos\theta \\ V_y = -\omega Rsin\theta \\ V_z = v_0 \end{cases} \quad (17)$$

Unfolding the surface of the part in Fig.7, in the Ouz coordinate system, one can observe the shape of the trajectory of an abrasive grain during one rotation of the part.

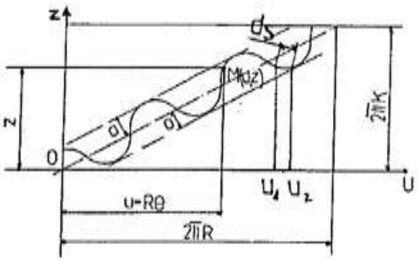

Fig.9 The trajectory of an abrasive grain in the Ouz coordinate system.

Using relations: $\theta = \frac{u}{R}, \frac{1}{R} = k_1$

The system of relations (16) becomes:

$$\begin{cases} x = Rsin(k_1 u) \\ y = Rcos(k_1 u) \\ z = k\theta \end{cases} \quad (18)$$

The Cartesian components of the velocity of point M will be:

$$\begin{cases} V_x = \omega Rcos(k_1 u) \\ V_y = -\omega Rsin(k_1 u) \\ V_z = v_0 \end{cases} \quad (19)$$

In the Ouz coordinate system, point M has the coordinates:

$$z = \text{asin}\,(k_1' u) + v_0 \tau \quad (20)$$

$$\frac{dz}{du} = arcos(k_1' u) + \frac{v_0}{\omega} \quad (21)$$

The length of the elementary arc ds is calculated with the relation:

$$ds = du\sqrt{1 + (\frac{dz}{du})^2} \quad (22)$$

Taking into account (21) and (22), the arc of the curve described between the abscissas u_1 and u_2 has the length:

$$S = \int_{u_1}^{u_2} du \sqrt{1 + [aRcos(k_1' u) + \frac{v_0}{\omega}]^2} \quad (23)$$

Calculating the length of the trajectory curve of the abrasive grain at a single rotation of the part, then the integration limits will be *0* and *2π*, and relation (23) becomes:

$$S = \int_{0}^{2\pi} du \sqrt{1 + [aRcos(k_1' u) + \frac{v_0}{\omega}]^2} \quad (24)$$

From relation (24) it can be seen that the length of the curve traced by an abrasive grain on the surface of the part will be greater if the amplitude of the oscillatory movement and the axial speed v_0 impressed on the tool or the part are increased. The amplitude of the oscillation cannot be affected too much, because a value higher than 3 mm would lead to a worsening of the evacuation of the chips and therefore the machining capacity of the abrasive grains. You can increase the advance speed v_0, or you can intervene on the angular speed ω of the part, which must not have values that lead to cutting speeds higher than 30-35 m/min.
The greater the length of the curve described by the abrasive grain, the higher the surface finish of the machined surface.

c) Superfinishing with two oscillatory movements
To increase the intensity of the machning process and implicitly the productivity of the process, it is recommended that the abrasive tool perform two types of oscillatory movements with different amplitudes and frequencies. Thus, for example, an oscillatory movement produced pneumatically with a smaller amplitude (1-3 mm) and a high frequency,1500-2000 cpm (cycles per minute) and another mechanically generated oscillatory movement with a higher amplitude, 5-20 mm, with a lower frequency, 60-150 cpm. The composition of the two oscillatory movements increases the degree of coverage of the machined surface with cross hatch patterns of abrasive grains, resulting in a better surface finish in a shorter time.

We will study the case where the oscillatory motion of the abrasive tool is composed of an oscillatory movement produced by a mechanical generator and another oscillatory movement produced pneumatically. The mechanical generation of oscillations is produced by a crank-slider plane mechanism as seen in Fig.10.

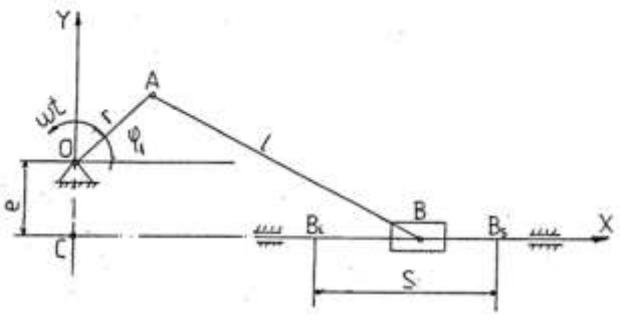

Fig.10 Generation oscillations by a crank-slider mechanism.

For the mechanism in Fig.10, the explicit form of the displacement motion has the next form:

$$s = rcos\varphi_1 + l\sqrt{1-(\lambda sin\varphi_1 + k)^2} \quad (25)$$

where: $\lambda = \frac{r}{e}; k = \frac{e}{l}$
The principle kinematic diagram of the mechanism with two oscillatory movements is shown in Fig.11.

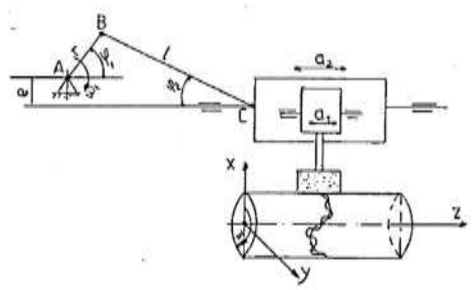

Fig.11 Kinematic diagram of the mechanism with two oscillatory movements.

From Fig.11 it can be seen that the path left by an abrasive grain on the surface of the part is actually a composition of two harmonic oscillatory movements. The part has a rotational movement with angular velocity ω, and the abrasive tool is under the influence of two oscillatory movements, one with the amplitude a1 mechanically generated by the mechanism in Fig.10, and the other with the amplitude a2 generated pneumatically by a pneumatic vibration generator. In order to increase the machining capacity, a feed movement with axial speed v0 is also applied to the part.
In order to obtain the equation of the displacement of an abrasive grain on the surface of the part, everything is related to an Oxyz coordinate system (Fig.12).

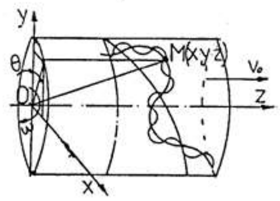

Fig.12 The path left on part surface in the case of two oscillating movements.

In Fig.12, the radius vector OM of a point of the tool is broken down into the following coordinates on the three coordinate axes:

$$\begin{cases} x = Rsin\theta \\ y = Rcos\theta \\ z = v_0\tau + s \end{cases} \quad (26)$$

where s is given by (25).

Considering that:

$\varphi_1 = \omega_1\tau\,, \theta = \omega\tau$

$\tau = \frac{\theta}{\omega}$

So, $\varphi_1 = \omega_1\frac{\theta}{\omega} = k_1\theta$ (26)

where $k_1 = \frac{\omega_1}{\omega}$

and $v_0\tau = v_0\ \frac{\theta}{\omega} = k_2\theta$ (27)

where $k_2=\frac{v_0}{\omega}$

Using (26) and (27) system (26) becomes:

$$\begin{cases} x = Rsin\theta \\ y = Rcos\theta \\ z = k_2\theta + rcos(k_1\theta) + l\sqrt{1-(\lambda\sin(k_1\theta)+k)^2} \end{cases} \quad (28)$$

If the cylindrical surface of the part is unfolded (Fig.13), in the Ouz coordinate system, we have:

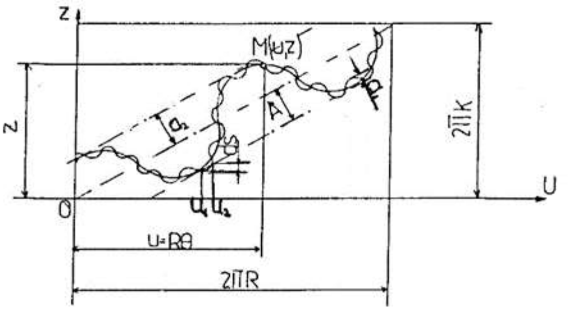

Fig.13 Path left by an abrasive grain in Ouz coordinate system.

$u = R\theta;\ \theta = \frac{u}{R} = k'u$

where $k' = \frac{1}{R}$

But $k_2\theta = k_2\frac{u}{R} = k_3 u$

where $k_3 = \frac{k_2}{R}$

and $k_1\theta = k_1\frac{u}{R} = k_4 u$

where $k_4 = \frac{k_1}{R}$

The system (28) becomes:

$$\begin{cases} x = Rsin(k'u) \\ y = Rcos(k'u) \\ z = k_3u + rcos(k_4u) + l\sqrt{1-(\lambda\sin(k_4u)+k)^2} \end{cases} \quad (29)$$

The length of the elementary arc is:

$$ds = du\sqrt{1+(\frac{dz}{du})^2} \quad (30)$$

Using (29) will have:

$$\frac{dz}{du} = k_3 - k_4rsin(k_4u) - l\frac{[\lambda\sin(k_4u)+k]k_4\lambda cos(k_4u)}{\sqrt{1-(\lambda sin(k_4u)+k)^2}} \quad (31)$$

Taking into account relations (30) and (31), the arc of the path included between the abscissas u_1 and u_2 will have the length:

$$S = \int_{u_1}^{u_2} du\sqrt{1+\left\{k_3 - k_4rsin(k_4u) - l\frac{[\lambda sin(k_4u)+k]k_4\lambda cos(k_4u)}{\sqrt{1-(\lambda sin(k_4u)+k)^2}}\right.}$$
(32)

To find out the length of the trajectory of an abrasive grain at one rotation of the part, *0* and *2πR* are taken as integration limits.

5 Conclusion

From relation (32) we can draw the conclusion regarding the factors that influence the length *S* of the trajectory of an abrasive grain on the surface of the part. Thus, it can be seen that to increase the length *S* of the trajectory of the abrasive grain, the angular speed ω_1 of the radius *r* and the length l of the mechanical oscillation generation mechanism must be increased. Also, to increase the length *S*, the amplitudes a_1 and a_2 of the two oscillatory movements can be increased.
Also from relation (32) it can be seen that in order to have as long as possible the length of the curve *S*, we must reduce the factor *k*, and therefore reduce the eccentricity *e* of the origin of the rotation movement from the mechanical generator of oscillations with respect to the axis of translation of the tool which support abrasives stones.

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