On Instability of a Composite Rotating Stellar Atmosphere

PARDEEP KUMAR
Department of Mathematics, ICDEOL, Himachal Pradesh University,
Summer-Hill, Shimla-171005 (HP) INDIA

Abstract: The aim of the present work was to study the thermal-convective instability of a composite rotating stellar atmosphere in the presence of a variable horizontal magnetic field to include, separately, the effects of medium permeability and solute gradient. Following the linearized stability theory, Boussinesq approximation and normal mode analysis, the dispersion is obtained in each case. The criteria for monotonic instability in each case have been obtained which generalize the Defouw's criterion derived for thermal-convective instability in the absence of above effects.

Key-words: Convection, Medium Permeability, Rotation, Solute Gradient, Variable Magnetic Field

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1 Introduction

Defouw [1] has termed 'thermal-convective instability' as the instability in which motions are driven by buoyancy forces of a thermally unstable atmosphere. He has generalized the Schwarzschild criterion for convection to include departures form adiabatic motion and has shown that a thermally unstable atmosphere is also convectively unstable, irrespective of the atmospheric temperature gradient.

Defouw [1] has found that an inviscid stellar atmosphere becomes unstable if

$$D = \frac{1}{C_p}\left(L_T - \rho\alpha L_\rho\right) + \kappa k^2 < 0 \ , \qquad (1)$$

where L is the heat-loss function (the energy lost minus the energy gained per gram per second) and ρ , α , κ , C_p , k , L_T , L_ρ denote, respectively, the density, the coefficient of thermal expansion, the coefficient of thermometric conductivity, the specific heat at constant pressure, the wave number of perturbation, the partial derivatives of L with respect to T , ρ; both evaluated in the equilibrium state. In general, the instability due to inequality (1) may be either oscillatory or monotonic. The effects of a uniform rotation and a uniform magnetic field on thermal-convective instability of a stellar atmosphere have been studied, separately by Defouw [1] and simultaneously by Bhatia [2].

Quite frequently it happens that the plasma is not fully ionized but, instead, may be permeated with neutral atoms. Stromgren [3] has reported that ionized hydrogen is limited to certain rather sharply bounded regions in space surroundings, for example, O-type stars and clusters of such stars, and that the gas outside these regions is essentially non-ionized. As a reasonably simple approximation, the plasma may be idealized as a composite mixture of a hydromagnetic (ionized) component and a neutral component, the two interacting through mutual collisional (frictional) effects. Hans [4] made this simplified approximation and found that these collisions have a stabilizing effect on the Rayleigh-Taylor instability. The thermal hydromagnetic instability of a partially-ionized plasma, for incompressible and compressible cases, has been studied by Sharma [5] and Sharma and Misra [6]. Usually the magnetic field has a stabilizing effect on the instability. However, Kent [7] has studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable. The thermal and convective heat transfer in flat solar collectors have been studied by Amirgaliyev et al. [8]. In stellar interiors and atmospheres, the magnetic field may be variable and may altogether alter the nature of the instability. The Coriolis force also plays an important role on the stability of stellar atmospheres.

A detailed account of thermal convection, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [9]. Veronis [10] has considered the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. In the stellar case, the physics is quite similar to Veronis [10] thermohaline configuration in that helium acts like salt in raising the density and in diffusing more slowly than heat. In thermosolutal-convective instability problem, buoyancy forces can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentrations. The conditions under which convective motions are important in stellar atmospheres are usually far removed from the consideration of a single component fluid and rigid boundaries and, therefore, it is desirable to consider one gas component acted on by solute concentration gradient and free boundaries. Marcu and Ballai [11] have studied the thermosolutal linear stability of a composite two-component plasma in the presence of Coriolis forces, finite Larmor radius, taking into account the collisions between neutral and ionized particles. The thermosolutal instability appears due to a material convection (thermosolutal convection) in a two component fluid with different molecular diffusivities which contribute in an opposing sense to a locally vertical density gradient. Jamwal and Rana [12] have studied the magnetohydrodynamic Veronis's thermohaline convection.

In recent years, the investigations of flow of fluids through porous media have become an important topic due to the recovery of crude oil from the pores of reservoir rocks. The study of the onset of convection in a porous medium has attracted considerable interest because if its natural occurrence and of its intrinsic importance in many industrial problems, particularly in petroleum exploration, chemical, and nuclear industries. The derivation of the basic equations of a layer of fluid heated from below in porous medium, using the Boussinesq approximation, has been given by Joseph [13]. The study of a layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering disciplines. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous medium. When a fluid permeates an isotropic and homogeneous porous medium, the gross effect is represented by Darcy's law. A great number of applications in geophysics may be found in the books by Phillips [14], Ingham and Pop [15], and Nield and Bejan [16]. Generally, it is accepted that comets consist of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice versa. The physical properties of comets, meteorites, and interplanetary dust strongly suggest importance of porosity in astrophysical context (see McDonnel [17]). Purkayastha and Choudhury [18] have studied the Hall current and thermal radiation effect on MHD convection flow of an elastico-viscous fluid in a rotating porous channel. The porosity is important in several geophysical situations. Effects of permeability on double diffusive MHD mixed convective flow past an inclined porous plate have been studied by Uddin et al. [19]. Kumar and Singh [20] have considered the thermal convection of a plasma in porous medium to include simultaneously the effect of rotation and the finiteness of the ion Larmor radius (FLR) in the presence of a vertical magnetic field.

Keeping such astrophysical and geophysical situations in mind, thermal-convective instability of a composite rotating stellar atmosphere in the presence of a variable horizontal magnetic field is considered in the present paper to include, separately, the effects of medium permeability and solute gradient.

2 Presence of Porous Medium

2A: Formulation of the Problem and Perturbation Equations

Consider an infinite horizontal composite layer consisting of a finitely conducting hydromagnetic fluid of density ρ and a neutral gas of density ρ_d , which is in a state of uniform rotation $\vec{\Omega}(0,0,\Omega)$, acted on by a variable horizontal magnetic field $\vec{H}(H_0(z),0,0)$ and gravity force $\vec{g}(0,0,-g)$ through a porous medium of porosity ε and medium permeability k_1. This layer is heated such that a steady temperature gradient $\beta(=dT/dz)$ is maintained. Regard the model under consideration so that both the ionized fluid and the neutral gas behave like continuum fluids and that the effects on the neutral component resulting from the presence of gravity and pressure are neglected. The magnetic field interacts with the ionized component only.

Let $\delta\rho, \delta p, \vec{q}(u,v,w)$ and $\vec{h}(h_x, h_y, h_z)$ denote the perturbations in density ρ, pressure p, filter velocity, and magnetic field $\vec{H}$, respectively; $g, \nu, \eta, \overrightarrow{q_d}$, and ν_c denote, respectively, the gravitational acceleration, the kinematic viscosity, the resistivity, the velocity of neutral gas, and the collisional frequency between the two components of the composite medium. Then the linearized perturbation equations governing the motion of the mixture of the hydromagnetic fluid and a neutral gas are

$$\frac{\partial\vec{q}}{\partial t} = -\frac{\varepsilon}{\rho}\nabla\delta p - \frac{\nu\varepsilon}{k_1}\vec{q} + \frac{\varepsilon}{4\pi\rho}\left[\left(\nabla\times\vec{h}\right)\times\vec{H} + \left(\nabla\times\vec{H}\right)\times\vec{h}\right] + \vec{g}\varepsilon\frac{\delta\rho}{\rho} + 2\left(\vec{q}\times\vec{\Omega}\right) + \frac{\rho_d\nu_c}{\rho}\left(\overrightarrow{q_d}-\vec{q}\right) , \quad (2)$$

$$\nabla.\vec{q} = 0 , \quad (3)$$

$$\nabla.\vec{h} = 0 , \quad (4)$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t} = \nabla\times\left(\vec{q}\times\vec{H}\right) + \varepsilon\eta\nabla^2\vec{h} , \quad (5)$$

$$\frac{\partial\overrightarrow{q_d}}{\partial t} = -\nu_c\left(\overrightarrow{q_d}-\vec{q}\right) . \quad (6)$$

The first law of thermodynamics can be written as

$$C_v\frac{dT}{dt} = -L + \frac{K}{\rho}\nabla^2 T + \frac{p}{\rho^2}\frac{d\rho}{dt} , \quad (7)$$

where K, C_v, T, and t denote, respectively, the thermal conductivity, the specific heat at constant volume, the temperature, and the time. d/dt is the convective derivative.

Following Defouw [1], the linearized perturbation form of equation (7) is

$$\frac{\partial\theta}{\partial t} + \frac{1}{C_p}\left(L_T - \rho\alpha L_\rho\right)\theta - \kappa\nabla^2\theta = -\frac{1}{\varepsilon}\left(\beta + \frac{g}{C_p}\right)w , \quad (8)$$

where θ is the perturbation in temperature T. We have employed the Boussinesq approximation modified modified so as to apply to thin layers of compressible fluids (cf. Spiegel and Veronis [21]) and used the Boussinesq equation of state

$$\rho = -\rho\alpha\theta ,$$

where we consider the case in which both boundaries are free and the medium adjoining the fluid is non-conducting. The case of two free boundaries is the most appropriate for stellar atmospheres (Spiegel [22]).

The boundary conditions for the problem are (cf. Chandrasekhar [9]; Lapwood [23]):

$$w = \frac{\partial^2 w}{\partial z^2} = \theta = \frac{\partial\zeta}{\partial z} = \xi = 0 , \quad (9)$$

and h_x, h_y, h_z are continuous with an external vacuum field, where ζ and ξ denote the z-components of vorticity and current density, respectively.

2B: The Dispersion Relation

Analyzing in terms of normal modes, we seek solutions whose dependence on space- and time-coordinates is of the form

$$\sin k_z z\exp\left(ik_x x + ik_y y + nt\right) , \quad (10)$$

where n is the growth rate and $k_z = s\pi/d$, s being any integer and d is the thickness of the layer and $k = \left(k_x^2 + k_y^2 + k_z^2\right)^{1/2}$ is the wave number of the perturbation.

If we eliminate $\overrightarrow{q_d}$ between equations (2) and (6), Equations (8) and (2)-(6), using expression (10), we find that

$$\left(n' + \frac{\nu\varepsilon}{k_1}\right)\nabla^2 w = g\alpha\varepsilon\nabla_1^2\theta - 2\vec{\Omega}\frac{\partial\zeta}{\partial z} + \frac{\varepsilon}{4\pi\rho}\nabla_1^2\left\{H_0\left(\frac{\partial h_z}{\partial x} - \frac{\partial h_x}{\partial z}\right) - h_x\frac{\partial H_0}{\partial z}\right\} - \frac{\varepsilon}{4\pi\rho}\frac{\partial}{\partial z}\left\{\frac{\partial}{\partial x}\left(h_z\frac{\partial H_0}{\partial z}\right) + H_0\frac{\partial}{\partial y}\left(\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}\right)\right\}, \quad (11)$$

$$\left(n' + \frac{\nu\varepsilon}{k_1}\right)\zeta = 2\vec{\Omega}\frac{\partial w}{\partial z} + \frac{H_0\varepsilon}{4\pi\rho}\frac{\partial\xi}{\partial x} - \frac{\varepsilon}{4\pi\rho}\frac{\partial}{\partial y}\left(h_z\frac{\partial H_0}{\partial z}\right), \quad (12)$$

$$(\eta - \eta\nabla^2)\vec{h} = \frac{1}{\varepsilon}\left\{\hat{\imath}\left(H_0\frac{\partial u}{\partial x} - w\frac{\partial H_0}{\partial z}\right) + \hat{\jmath}H_0\frac{\partial v}{\partial x} + \hat{k}H_0\frac{\partial w}{\partial x}\right\}, \quad (13)$$

$$(n + D)\theta = -\frac{1}{\varepsilon}\left(\beta + \frac{g}{C_p}\right)w, \quad (14)$$

where

$$n' = n\left(1 + \frac{\alpha_0\nu_c}{n + \nu_c}\right), \quad \alpha_0 = \frac{\rho_d}{\rho}, \quad \nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \text{ and } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

If we eliminate $\zeta, \xi, \theta, h_x, h_y$ and h_z from equations (11) – (14) and using (10), we obtain the dispersion relation

$$n^7 + A_6n^6 + A_5n^5 + A_4n^4 + A_3n^3 + A_2n^2 + A_1n + A_0 = 0, \quad (15)$$

where

$$A_6 = D + 2\nu_v\overline{1 + \alpha_0} + 2\left(\frac{\nu\varepsilon}{k_1} + \eta k^2\right),$$

$$A_0 = \nu_c\left(k_x^2V_A^2 + \frac{\nu\varepsilon}{k_1}\eta k^2\right)\left[\eta k^2\nu_c\left\{\frac{\nu\varepsilon}{k_1}D + \Gamma\left(\beta + \frac{g}{C_p}\right)\right\} + k_x^2V_A^2D\nu_c\left(1 + 2\frac{k_z^2}{k^2}\right)\right] + 4\nu_c^2\eta^2k^2k_z^2\Omega^2D, \quad (16)$$

and

$$\Gamma = g\alpha\left(\frac{k_x^2 + k_y^2}{k^2}\right), \quad V_A^2 = \frac{H^2}{4\pi\rho},$$

A_1 to A_5 having a large number of terms and being not needed in the discussion on instability, have not been written here.

2C: Discussion

Theorem 1: A criteria for thermal-convective instability of a composite stellar atmosphere in the presence of a variable horizontal magnetic field, rotation and collisional effects through porous medium to be unstable if

$$D < 0 \quad and \quad \left|\frac{\nu\varepsilon}{k_1}D\right| > \Gamma\left(\beta + \frac{g}{C_p}\right).$$

Proof: Taking the dispersion relation (15), when

$$D < 0 \quad and \quad \left|\frac{\nu\varepsilon}{k_1}D\right| > \Gamma\left(\beta + \frac{g}{C_p}\right), \quad (17)$$

the constant term A_0 in equation (15) is negative. Equation (15), therefore, involves at least one change of sign and, hence, contains one positive real root. The occurrence of positive root implies monotonic instability. We thus obtain the criteria for thermal-convective instability of a composite stellar atmosphere in the presence of a variable horizontal magnetic field, rotation and collisional effects through porous medium to be unstable if

$$D < 0 \quad and \quad \left|\frac{\nu\varepsilon}{k_1}D\right| > \Gamma\left(\beta + \frac{g}{C_p}\right).$$

Hence the result.

3 Presence of Solute Gradient

3A: Formulation of the Problem and Dispersion Relation

Here we consider the same configuration as in previous section except that the medium is non-porous and that the system is acted on by a stable solute gradient $\beta'(=|dC/dz|)$. The linearized perturbation equations governing the motion of the mixture of the hydromagnetic fluid and a neutral gas are

$$\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho}\nabla\delta p + \nu\nabla^2\vec{q} + \frac{1}{4\pi\rho}\left[(\nabla\times\vec{H})\times\vec{h} + (\nabla\times\vec{h})\times\vec{H}\right] + \vec{g}\frac{\delta\rho}{\rho} + 2(\vec{q}\times\vec{\Omega}) + \frac{\rho_d\nu_c}{\rho}(\vec{q_d}-\vec{q}), \quad (18)$$

$$\frac{\partial\gamma}{\partial t} = \beta' w + \kappa'\nabla^2\gamma \ , \quad (19)$$

$$\frac{\partial\vec{h}}{\partial t} = \nabla\times(\vec{q}\times\vec{H}) + \eta\nabla^2\vec{h}, \quad (20)$$

$$\nabla.\vec{q} = 0, \qquad \nabla.\vec{h} = 0 \ , \quad (21)$$

$$\frac{\partial\vec{q_d}}{\partial t} = -\nu_c(\vec{q_d}-\vec{q}) \ , \quad (22)$$

$$\frac{\partial\theta}{\partial t} + \frac{1}{C_p}(L_T - \rho\alpha L_\rho)\theta - \kappa\nabla^2\theta = -\left(\beta + \frac{g}{C_p}\right)w \ , \quad (23)$$

together with the Boussinesq equation of state $\delta\rho = -\rho(\alpha\theta - \alpha'\gamma)$.

Here again we consider the case of two free boundaries and the medium adjoining the fluid to be non-conducting. Eliminating $\vec{q_d}$ between equations (18) and (22), equations (18) – (23), using expression (10), give

$$(n'-\nu\nabla^2)\nabla^2 w = g\nabla_1^2(\alpha\theta - \alpha'\gamma) - 2\Omega\frac{\partial\zeta}{\partial z} + \frac{1}{4\pi\rho}\nabla_1^2\left\{H_0\left(\frac{\partial h_z}{\partial x} - \frac{\partial h_x}{\partial z}\right) - h_x\frac{\partial H_0}{\partial z}\right\} - \frac{1}{4\pi\rho}\frac{\partial}{\partial z}\left\{\frac{\partial}{\partial x}\left(h_z\frac{\partial H_0}{\partial z}\right) + H_0\frac{\partial}{\partial y}\left(\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}\right)\right\}, \quad (24)$$

$$(n'-\nu\nabla^2)\zeta = 2\Omega\frac{\partial w}{\partial z} + \frac{H_0}{4\pi\rho}\frac{\partial\xi}{\partial z} - \frac{1}{4\pi\rho}\frac{\partial}{\partial y}\left(h_z\frac{\partial H_0}{\partial z}\right), \quad (25)$$

$$(n-\eta\nabla^2)\vec{h} = \hat{\imath}\left(H_0\frac{\partial u}{\partial x} - w\frac{\partial H_0}{\partial z}\right) + \hat{\jmath}H_0\frac{\partial v}{\partial x} + \hat{k}H_0\frac{\partial w}{\partial x}, \quad (26)$$

$$(n+D)\theta = -\left(\beta + \frac{g}{C_p}\right)w \ , \quad (27)$$

$$(n-\kappa'\nabla^2)\gamma = \beta' w \ . \quad (28)$$

If we eliminate $h_x, h_y, h_z, \zeta, \xi, \theta$, and γ from equations (24) – (28) and using (10), we obtain the dispersion relation

$$n^8 + B_7 n^7 + B_6 n^6 + B_5 n^5 + B_4 n^4 + B_3 n^3 + B_2 n^2 + B_1 n + B_0 = 0, \quad (29)$$

where

$$B_7 = D + k^2(\kappa' + 2\overline{\nu+\eta}) + 2\nu_c\overline{1+\alpha_0} \ ,$$

$$B_0 = \nu_c(k_x^2V_A^2 + \nu\eta k^4)\left[\eta k^2\nu_c\left\{(\nu\kappa' k^4 + \Gamma'\beta')D + \kappa' k^2\Gamma\left(\beta + \frac{g}{C_p}\right)\right\} + \nu_c\kappa' k^2 k_x^2 V_A^2\left(1 + 2\frac{k_z^2}{k^2}\right)D\right] + 4\nu_c^2\kappa' k^4\eta^2 k_z^2\Omega^2 D \ , \quad (30)$$

and

$$\Gamma' = \frac{g\alpha'(k_x^2 + k_y^2)}{k^2} \ .$$

Coefficients B_1 to B_6 having a large number of terms and being not needed in the discussion on stability, have not been written here.

3B: Discussion

Theorem 2: A criteria for thermosolutal-convective instability of a composite stellar atmosphere in the presence of rotation, variable horizontal magnetic field, stable solute gradient and collisional effects to be unstable if

$$D < 0 \; and \; |(\nu\kappa' k^4 + \Gamma'\beta')D| > \kappa' k^2 \Gamma \left(\beta + \frac{g}{C_p}\right).$$

Proof: Taking the dispersion relation (29), when

$$D < 0 \; and \; |(\nu\kappa' k^4 + \Gamma'\beta')D| > \kappa' k^2 \Gamma \left(\beta + \frac{g}{C_p}\right) \;, \quad (31)$$

the constant term in equation (29) is negative. Equation (29), therefore, involves one change of sign and, hence, contains one positive real root, meaning thereby monotonic instability. We have, therefore, obtained the criteria for thermosolutal-convective instability of a composite stellar atmosphere in the presence of rotation, variable horizontal magnetic field, stable solute gradient and collisional effects to be unstable if

$$D < 0 \; and \; |(\nu\kappa' k^4 + \Gamma'\beta')D| > \kappa' k^2 \Gamma \left(\beta + \frac{g}{C_p}\right) .$$

Hence the result.

4 Conclusions

The convective stability of a star has customarily been determined by Schwarzschild criterion and one of the fundamental assumptions used in deriving this criterion is that the motion is adiabatic. The Schwarzschild criterion in the interior of a star, where the photon mean free path is small, the assumption that the motion is adiabatic is justified. The departure from adiabatic motion may be significant in the outer layers of a stellar atmosphere, where the effective heat transfer is no longer prevented by opacity. The Schwarzschild criterion for convection has been generalized to include departures from adiabatic motion by Defouw [1].

The stellar chromospheres, coronae, and the interstellar medium may exhibit thermal-convective instability. For such astrophysical situations the Coriolis force, the variable magnetic field, medium permeability, solute gradient, and collisional effects, play an important role. The thermal-convective instability of a composite rotating stellar atmosphere in the presence of a variable horizontal magnetic field is considered to include, separately, the effects of medium permeability and solute gradient. The criteria for monotonic instability in each case have been obtained which generalize the Defouw's criterion derived for thermal-convective instability in the absence of above mentioned effects.

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