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Contact Forces of Spur Gearing

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Abstract : - In this paper, on the basis of the bending produced by the normal force which is applied in contact point of teeth flanks, the contact force of external spur gearing was determined. For the calculus methodology presented in this paper, a computer program has been developed.

Key-Words: - contact forces, spur gearing, tooth pair of gearing.

1 Introduction

As has been known, during the meshing, the contact point moves from the tooth tip to its root and from the conjugated gear tooth root to its tip (from the initial point of contact to the final point of contact). The normal force is mobile on the tooth flank, it changes continuously the position respect to the tooth fixing zone. Calculating the contact forces in conditions as near as possible by the reality, the gears dimension designing can be developed more exactly.

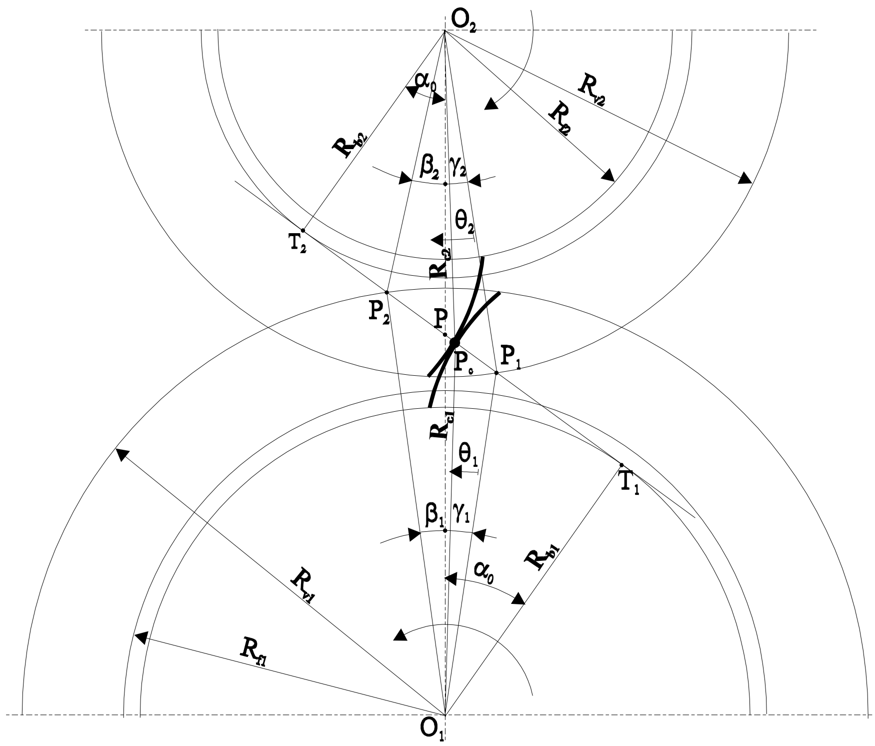

Fig.1 Geometric parameters at the contact point

2 Bases of design

In this paper are presented the following bases of design:

a) The involute tooth is considered to be a beam which is fixed at one end in the body of gear.

b) The gear tooth is a non-uniform cantilever beam, as shown in figure 2.

c) It is taken into consideration only the bending produced by the normal force.

d) Gears are rigid bodies except for their teeth.

e) The load is uniformly distributed along the tooth width.

In figure 1 is presented an external spur gearing, where:

P_c-contact point

T_1T_2-line of contact

P_1P_2-length of path of contact

$\theta_1 \in [0, \gamma_1 + \beta_1]$

$\theta_2 \in [0, \gamma_2 + \beta_2]$ - angles corresponding to length of path of contact.

The following elements are known:

-normal module of tooth-m_n, [mm];

-helix angle- $\beta_d = 0^\circ$

-number of gear 1-z_1

-number of gear 2-z_2

-basic rack:

- pressure angle- $\alpha_0 = 20°$
- whole depth- $2.25 \cdot m_n$, [mm];

3 Equation System of Forces

First of all, on the basis of the known data and figure 1, the following parameters will be determined:

-base radius of gear:

$$R_{b_j} = \frac{m_n \cdot z_j}{2} \cos(\alpha_0); (where\, j = 1,2) \tag{1}$$

-root radius of gear:

$$R_{f_j} = m_n \cdot \left(\frac{z_j}{2} - 1{,}25 \right) \tag{2}$$

-tip radius of gear:

$$R_{v_j} = m_n \cdot \left(\frac{z_j}{2} + 1 \right) \tag{3}$$

-gear center distance:

$$O_1O_2 = A = \frac{m_n \cdot (z_1 + z_2)}{2} \tag{4}$$

-length line of contact:

$$T_1T_2 = \left(R_{b_1} + R_{b_2}\right) \cdot \tan(\alpha_0) = \frac{m_n}{2}(z_1 + z_2)\sin(\alpha_0) \tag{5}$$

3.1 The Calculation of Angles γ_j, β_j.

In $\Delta O_1T_1P_2$

$$T_1P_2 = \sqrt{O_1P_2^2 - O_1T_1^2}$$

$$\beta_1 = \arctan \frac{T_1P_2}{R_{b_1}} - \alpha_0 \tag{6}$$

In $\Delta O_2T_2P_1$

$$\gamma_2 = \arctan \frac{T_2P_1}{R_{b_2}} - \alpha_0$$

and the following relations may be used:

$$PP_1 = T_2P_1 - T_2P$$
$$PP_2 = T_1P_2 - T_1P$$
$$P_1P_2 = PP_1 + PP_2$$
$$P_1T_1 = T_1T_2 - T_2P_1$$
$$\gamma_1 = \alpha_0 - \arctan \frac{P_1T_1}{R_{b_1}}$$
$$\beta_2 = \alpha_0 - \arctan \frac{P_2T_2}{R_{b_2}} \tag{7}$$

3.2 The Calculation of The Radius of The Contact Point P_c

The angle θ_1 takes values in $\theta_1 \in [0, \gamma_1 + \beta_1]$. For a known value of θ_1, the radius R_{c1} is given by:

$$R_{c_1} = \frac{R_{b_1}}{\cos(\alpha_0 - \gamma_1 + \theta_1)} \tag{8}$$
$$P_1P_C = \sqrt{O_1P_1^2 + R_{c_1}^2 - 2 \cdot O_1P_1 \cdot R_{c_1} \cdot \cos(\theta_1)}$$
$$P_c\widehat{P_1}O_1 = \arccos \frac{P_1P_c^2 + O_1P_1^2 - R_{c_1}^2}{2 \cdot P_1P_c \cdot O_1P_1}$$
$$P_c\widehat{P_1}O_2 = \pi - \gamma_1 - \gamma_2 - P_cP_1O_1$$

The angle θ_2 takes values in $[0, \gamma_2 + \beta_2]$. For a known value of θ_2 the radius R_{c2} is given by:

$$R_{c_2} = \sqrt{R_{v_2}^2 + P_1P_c^2 - 2 \cdot R_{v_2} \cdot P_1P_c \cdot \cos(P_c\widehat{P_1}O_2)} \tag{9}$$

By means of R_{c1} and R_{c2}, the teeth depth h_1 and h_2 in the contact point P_c can be determined:

$$h_j = R_{c_j} - R_{f_j} \tag{10}$$

Without of the teeth pair with the contact in P_c, another teeth pairs there are in contact too. The contact points there are on the line P_1P_2 at the distance P_b from point P_c. For each teeth pair which there are in contact it is possible to calculate the depth h_{ji} down to contact point. In the contact point P_i, a contact force F_i appears between the flanks. Under the influence of contact force F_i, the teeth deform with the common dimension:

$$\delta_i = \delta_{1_i} + \delta_{2_i} \tag{11}$$

If the deflection is given by:

$$\delta_{j_i} = F_i \cdot f_{(h_{ji}, z_j)}; j = 1 or\ 2 \tag{12}$$

than, the equations system of forces will be:

$$\begin{cases} \delta_1 = \delta_2 = \cdots = \delta_i \\ \sum F_i \cdot R_{b1} = M_1 \end{cases} \tag{13}$$

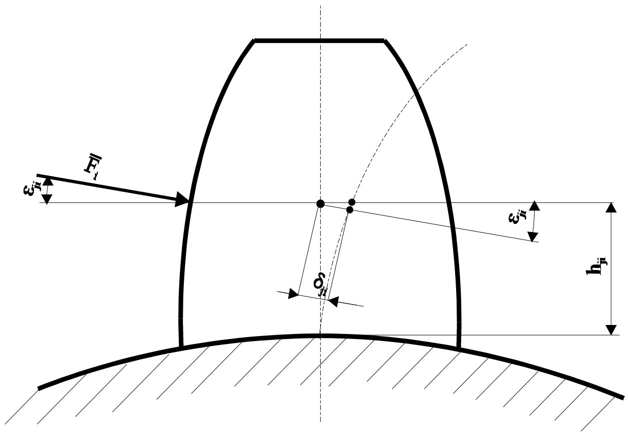

Fig.2
Tooth deflection in the direction of the applied F_i

The system (13) has "n" equations, the unknowns being: F_1, F_2, ...F_n

In assumption that the beam has a variable cross-sectional area, in accordance with involute profile (figure 2), the deflection δ_{ji} is calculated with the following formula:

$$\delta_{ji} = \frac{F_i \cdot \cos^2(\varepsilon_{ji})}{k_{ji}} \tag{14}$$

where: k_{ji} [N/mm] is the elasticity constant of tooth deformed due to contact force F_i applied to depth h_{ji};

$\frac{1}{k_{ji}}$ is the compliance of tooth "j" due to contact force F_i.

Own studies [4],[5] have established that, in this case, the compliance expression is given by:

$$\frac{1}{k_{ji}} = 5.362 \cdot 10^{-10} \cdot h_{ji}^{3.13633} \cdot z_j^{-1,03885} \cdot B \tag{15}$$

where:
z-teeth number of the gear "j".
B-tooth face width, [mm].

ε_{ji} is the angle between the force F_i and the normal line to the symmetry axis of tooth "j" from the "i" teeth pair which is in contact.

By means of the computer program, the equations system (15) was solved. In table 1 are presented the numerical values of the contact forces and bending deflections, having the following parameters:
m_n=10 mm; B=10 mm; α_0=20^0; z_1=40; z_2=60; F=20000N; M_1=3758,77Nm; γ_1=7,385^0; β_1=6,499^0;

Numerical Results

Table 1

Nr crt	θ_1 [^{o}]	Number of contact teeth pairs (i)	F_i [N]	δ_{ji}		δ_i [mm]
				δ_{1i}	δ_{2i}	
1	0,93	2	5770,42	0,000167	0,009189	0,009356
			14229,58	0,007688	0,001668	
2	1,85	2	5711,42	0,000246	0.010853	0.011099
			14288,58	0,009890	0,001209	
3	2,78	2	5762,85	0,000361	0,013005	0,013366
			14237,15	0,012522	0,000844	
4	3,7	2	5883,02	0,000527	0,015697	0,016223
			14116,98	0,015657	0,000566	
5	4,63	2	6042,89	0,000761	0,018985	0,019745
			13957,11	0,019383	0,000362	

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Table 1 (continuation)

6	5,55	2	6223,15	0,001085	0,022934	0,024019
			13776,85	0,023801	0,000219	
7	6,48	1	20000	0,004770	0,086160	0,090930
8	7,4	1	20000	0,006442	0,100396	0,106839

To be repeat to θ_1=13,884

4 Conclusion

The analytical modeling presented in this paper permits to establish the numerical values of the contact forces for external spur gearing. The real tooth profile and the geometry elements which are specific for these gear pairs permit to compute the contact forces and the deflection of a teeth pair with a high accuracy. As part of further work on this research program, the contact or Hertzian deflection will be taken in consideration.

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