Relative Strength of Common Fixed Point Theorem of Self-Mappings Satisfying Rational Inequalities in Real Valued Generalized Complete Metric Spaces

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Abstract: - In this paper, we have introduced relative strength of common fixed point theorem of self-mappings satisfying rational type inequalities in real valued complete metric space. The purpose of this paper is to generalized and unify some of previous works.

Key-Words: - Common fixed points, Self-continuous mappings, Complete metric spaces.

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1 Introduction

As you can see for the title of the paper you must The Banach contraction principle is a most powerful tool in solving existence problems in many branch of mathematics (see, e.g. [1]-[7]). The specific extension of this principle were obtained by generalizing the domain of signals or by extending the contractive condition on the signals [8-10]. As a consequence of those generalizations so many metric were introduced namely uniformly convex Banach spaces, cone metric spaces, pseudo metric spaces, B-metric spaces, fuzzy metric spaces, etc. A set of huge work have been done in this direction, for example, the recent works are see, [10-49] these fixed point results are useful in establishing the uniqueness of the solution of non-linear differential and integral equation.

In recent paper author [4] proved some new results on fixed point and common fixed points under the specific condition of continuity of the signals. For this, let R^+ denote the set of non-negative real numbers. Let χ be a family of signals such that $\zeta : \left(R^+\right)^5 \to R^+$ and $\zeta \in \chi$ is upper semi continuous and monotonically increasing in each coordinate variable. Also, we consider a new signal in such a way that M: $R^+ \to R^+$ and $M(z) = \zeta(z, z, p_1 z, p_2 z, z)$

where $3 = p_1 + p_2$.

Lemma ([4]): Let z be a positive real number and $M(z) < I$ if and only if $\lim_{x\to\infty} M^x(z) = 0$.

Proof: It is clear that ζ and M are upper semi continuous.

Let $\lim_{x\to\infty} M^x(z) = F$ where $F \neq 0$.

Thus by hypothesis we may write

$$F = \lim_{x\to\infty} M^{x+1}(z) \leq M \lim_{x\to\infty} M^x(z) = M(F) < F$$

$\therefore\ F < F$ which gives a contradiction. So, our supposition is wrong.

Then we must have $F = 0$

For converse part,

It is clear that ζ and M are monotonically increasing.

We have, $\lim_{x\to\infty} M^x(z) = 0$.

Suppose, if possible that $M(z) > z$ for some $z \in R^+$.

$\Rightarrow M^x(z) > z$ for some $z \in R^+$ and x is natural number.

Thus by hypothesis, $\lim_{x\to\infty} M^x(z) \neq 0$.

Which gives a contradiction.
So our supposition is wrong.

Then we must say that $M(z) \neq z$.

Again, we suppose $M(z) = z$ for some $z \in R^+$

then by hypothesis

$$\lim_{x\to\infty} M^x(z) \to 0$$

Consequently, we many write $M(z) < z\mathrm{M}$ for some $z \in R^+$.

we prove the following theorem which is motivated by the work of the authors [41, 42].

<u>Theorem</u> 1: Let (x,d) be a complete metric space and F, f_1 and g_1 are continuous self-signals of X satisfying the following conditions having

(i) $f_1(x) \cap g_1(x) \supset F(x)$

(ii) for $\zeta \in \chi$, obtaining

For $n, n_1 \in X$,

$$d(F(n_1), F(n_2)) \leq \zeta\{d(f_1(n), g_1(n_1)),$$
$$d(f_1(n), F(n)), d(f_1(n), F(n_1))$$
$$d(g_1(n_1), F(n)), d(g_1(n_1), F(n_1))\}$$

(iii) for all $z > 0$,

$$\zeta(z, z, p_1 z, p_2 z, z) < z,$$

(iv) $(p_1, p_2) \in [(1,2),(2,1)]$

(v) $[F, f_1]$ and $[F, g_2]$ are weakly commuting. Then there exist a point $n_0 \in X$ such that n_0 is a unique common fined point of continuous self-signals F, f_1 and g_1.

Proof: Let m_0 be any point of X. Then, by lemma 1, we choose n_{2k+1} and n_{2k+2} in X such that $f_1(n_1) = F(n_0)$, $f_1(n_3) = F(n_2)$, $f_1(n_5) = F(n_4)$,

$f_1(n_7) = F(n_5)$, $f_1(n_9) = F(n_7)$,

$,\cdots, f_1(n_{2k+1}) = F(n_{2k})$

And $g_1(n_2) = F(n_1), g_1(n_4) = F(n_3)$,

$g_1(n_6) = F(n_5), g_1(n_8) = F(n_7), \cdots,$

$g_1(n_{2k+2}) = F(n_{2k+1})$.

Let $d_0 = d(F(n_0), F(n_1))$,

$d_1 = d(F(n_1), F(n_2))$,

$d_2 = d(F(n_2), F(n_3))$,

$d_3 = d(F(n_3), F(n_4))$,

$d_k = d(F(n_k), F(n_{k+1}))$.

We have to show that $d_{2k} \le d_{2k-1} \forall k$.

Suppose, if possible that $d_{2k} > d_{2k-1}$ for some k

Now,

$d_{2k} = d(F(n_{2k}), F(n_{2k+1}))$

$d(F(n_{2k+1}), F(n_{2k}))$

$\le \zeta\{d(f_1(n_{2k+1}), g_1(n_{2k})),$

$d(f_1(n_{2k+1}), F(n_{2k+1})), d(f_1(n_{2k+1})),$

$F(n_{2k}), d(g_1(n_{2k}), F(n_{2k+1})),$

$d(g_1(n_{2k}), F(n_{2k}))\} = \zeta\{d(F(n_{2k}), F(n_{2k-1})),$

$d(F(n_{2k}), F(n_{2k+1})),$

$d(F(n_{2k}), F(n_{2k})), d(F(n_{2k-1})),$

$F(n_{2k+1}), d(F(n_{2k-1}), F(n_{2k}))\}$

$\le \zeta(d_{2k-1}, d_{2k}, 0, d_{2k-1} + d_{2k}, d_{2k-1})$

$\le \zeta(d_{2k}, d_{2k}, d_{2k}, 2d_{2k}, d_{2k})$

$< d_{2k}$

i.e., $d_{2k} < d_{2k}$ gives a contradiction.

Thus $d_{2k} \le d_{2k-1} \forall k$.

Similarly, we can show that

$d_{2k+1} \le d_{2k}$ for $k = 0,1,2,...$

$\therefore \{d_k\}$ is monotonically decreasing sequence.

Then $d_1 = d(F(n_1), F(n_2))$

$\le \zeta\{d(f_1(n_1), g_1(n_2)), d(F_1(n_1), F(n_1))$

$, d(f_1(n_1), F(n_2)$
$d(g_1(n_2), F(n_1)), d(g_1(n_2), F(n_2))\}$

$= \varsigma\{d(F(n_0), F(n_1)), d(F(n_0), F(n_1)) d(F(n_0), F(n_2))$
$, d(F(n_1), F(n_1)), d(F(n_1), F(n_2))\}$

$\le \zeta(d_0, d_0, d_0 + d_1, 0, d_1)$
$\le \zeta(d_0, d_0, 2d_0, d_0, d_0)$
$= M(d_0)$,

similarly, we can show that

$d_2 \le M^2(d_0)$

$d_3 \le M^3(d_0)$

$d_4 \le M^4(d_0)$

$d_k \le M^k(d_0)$.

If $d_0 > 0$ then by lemma 1, $\lim_{k\to\infty} d_{k=0}$.

If for $d_0 = 0$, $\lim_{k\to\infty} d_{k=0}$.

Thus $d_{k=0}$ for each k .

We next show that $\{F(n_k)\}$ is a Cauchy sequence.

It is sufficient show that $\{F(n_{2k})\}$ is a Cauchy sequence.

Suppose, if possible suppose that $\{F(n_{2k})\}$ is not a Cauchy sequence. Then for $\in > 0$, such that for even number $2k, k = 0,1,2,...$, there exists even integer $2k(a)$ and $2\ell(a), 2a \le 2k(a) < 2\ell(a)$ such that

$$d\left(F\left(n_{2\ell(a)}\right), F\left(n_{2\ell(a)}\right)\right) > \in \quad (1)$$

Let, for each integer $2a, 2\ell(a)$ be the least integer exceeding $2k(a)$ satisfying equation (1).

Thus $d\left(F\left(n_{2k(a)}\right), F\left(n_{2\ell(a)-2}\right)\right) \ge \in \quad (2)$

and $d\left(F\left(n_{2k(a)}\right), F\left(n_{2\ell(a)}\right)\right) > \in \quad (3)$

Then for each integer $2a$,

$$\{< d\left(F\left(n_{2k(a)}, F\left(n_{2\ell(a)}\right) < d\left(F\left(n_{2k(a)}\right), F\left(n_{2\ell(a)-2}\right)\right)\right)\right)$$

$$+ \; d\left(F\left(n_{2\ell(a)-2}, F\left(n_{2\ell(a)-1}\right)\right)\right) +$$

$$d\left(F\left(n_{2\ell(a)-1}\right), F\left(n_{2\ell(a)}\right)\right)$$

then by equation (2) and equation (3) and $d_{k\to 0}$,

we obtain

$$d\left(F\left(n_{2k(a)}\right), F\left(n_{2\ell(a)}\right)\right) \to \in \quad (4)$$

as $a \to \infty$.

Then by triangular inequality

$$\left| d(F(n_{2k(a)}), F(n_{2l(a)-1})) - d(F(n_{2k(a)}), F(n_{2l(a)}\right|$$
$$\le d_{2l(a)-1}$$

and

$$\left| d(F(n_{2l(a)+1}), F(n_{2l(a)-1})) - d(F(n_{2l(a)}), F(n_{2l(a)}\right|$$
$$\le d_{2l(a)-1} + d_{2(k)a.}$$

By equation (4), as $a \to \infty$

$$d\left(F\left(n_{2k(a)}\right), F\left(n_{2\ell(a)-1}\right)\right) \to \in \quad (5)$$

and

$$d\left(F\left(n_{2k(a)+1}\right), F\left(n_{2\ell(a)-1}\right)\right) \to \in \quad (6)$$

Again,

$$d\left(F\left(n_{2k(a)}\right), F\left(n_{2\ell(a)}\right)\right) \le$$

$$\le d\left(F\left(n_{2k(a)}\right), F\left(n_{2k(a)+1}\right)\right) + d\left(F\left(n_{2k(a)+1}\right), F\left(n_{2\ell(a)}\right)\right)$$
$$\le d_{2k(a)} + \zeta\, d\left(F\left(n_{2k(a)}\right) F\left(n_{2\ell(a)-1}\right)\right), d_{2k(a)},$$
$$d\left(F\left(n_{2k(a)}\right), F\left(n_{2\ell(a)}\right)\right),$$
$$d\left(F\left(n_{2\ell(a)-1}\right), F\left(n_{2k(a)+1}\right)\right), d_{2\ell(a)-1}$$

By equation (5),

$\lim_{k\to\infty} d_k = 0$, and upper semi-continuity of ζ , we obtain

$$\in \le \zeta(\in, 0, \in, \in, 0)$$
$$\le M(\in)$$
$$< \in$$

i.e. $\in < \in$

which gives a contradiction.

Therefore $\{F(n_k)\}$ is a Cauchy sequence and using completeness property of X, there is a point $c \in X$ such that $F(n_k) \to c$.

It is clear that $\{f_1(n_{2k+1})\}$ and $\{g_1(n_{2k})\}$ are subsequences of $\{F(n_k)\}$ and hence $\{f_1(n_{2k})\} \to c$ and $\{g_1(n_{2k+1})\} \to c$.

Consequently,

$\{f_1 g_1(n_{2k})\} \to f_1(c)$ and $\{g_1 f_1(n_{2k+1})\} \to g_1(c)$ as f_1 and g_1 are continuous signals.

Again, Let $d(f_1 g_1(n_{2k}), g_1 f_1(n_{2k+1})) =$

$$d(f_1 F(n_{2k-1}), g_1 F(n_{2k}))$$

$$\leq d(f_1 F(n_{2k-1}), Ff_1(n_{2k-1})) +$$

$$d(Ff_1(n_{2k-1}), Fg_1(n_{2k})) + d(Fg_1(n_{2k}), g_1 F(n_{2k}))$$

using given conditions (i) and (ii),

$$d(f_1 g_1(n_{2k}), g_1 f_1(n_{2k+1})) \leq$$

$$d(f_1(n_{2k-1}), F(n_{2k-1})) +$$

$$d(Ff_1(n_{2k-1}), Fg_1(n_{2k})) +$$

$$d(F(n_{2k}), g_1(n_{2k}))$$

$$\leq d(f_1(n_{2k-1}), F(n_{2k-1})) + \zeta\{d(f_1^2(n_{2k-1}), g_1^2(n_{2k})),$$
$$d(f_1^2(n_{2k-1}), Ff_1(n_{2k-1})), d(f_1^2(n_{2k-1}), Fg_1(n_{2k}))$$
$$d(g_1^2(n_{2k}), Ff_1(n_{2k-1})),$$
$$d(g_1^2(n_{2k}), Fg_1(n_{2k}))\} + d(F(n_{2k}), g_1(n_{2k}))$$

$$\leq d(f_1(n_{2k-1}), F(n_{2k-1})) +$$
$$\zeta\{d(f_1^2(n_{2k-1}), g_1^2(n_{2k})),$$
$$d(f_1^2(n_{2k-1}), f_1 F(n_{2k-1})) +$$
$$d(f_1(n_{2k-1}), F(n_{2k-1})),$$
$$d(f_1^2(n_{2k-1}), g_1 F(n_{2k})) +$$
$$d(g_1(n_{2k}), F(n_{2k})), d(g_1^2(n_{2k}), f_1 F(n_{2k-1}))$$
$$d(f(n_{2k-1}), F(n_{2k-1})), d(g_1^2(n_{2k}), g_1 F(n_{2k}))$$
$$+ d(g_1(n_{2k}), F(n_{2k}))\} + d(F(n_{2k}), g_1(n_{2k})).$$

Let $d(f_1(c), g_1(c)) > 0$.

Then by using condition (iii),

$$d(f_1(c), g_1(c)) \leq \zeta\{d(f_1(c), g_1(c)), 0,$$

$$d(f_1(c), g_1(c)), d(g_1(c), f_1(c)), 0\}$$

$$\leq M(d(f_1(z), g_1(z)))$$

$$< d(f_1(c), g_1(c))$$

i.e. $d(f_1(c), g_1(c)) < d(f_1(c), g_1(c))$

which gives a contradiction

Thus, $f_1(c) = g_1(c)$.

We next show that $F(c) = f_1(c)$.

Let $d(f_1 F(n_{2k+1}), F(c)) \leq$

$$d(f_1 F(n_{2k+1}), Ff_1(n_{2k+1})) +$$
$$d(Ff_1(n_{2k+1}), F(c))$$

Again, by condition (iii),

$$d(f_1 F(n_{2k+1}), F(c)) \leq$$

$$d(f_1(n_{2k+1}), F(n_{2k+1})) +$$

$$\zeta\{d(f_1^2(n_{2k+1}), g_1(c)),$$

$d\left(f_1^2\left(n_{2k+1}\right), f_1 F\left(n_{2k+1}\right)\right)+$

$d\left(f_1\left(n_{2k+1}\right), F\left(n_{2k+1}\right)\right),$

$d\left(f_1^2\left(n_{2k+1}\right), F(c)\right),$

$d\left(g_1(c), F\left(n_{2k+1}\right)\right)+d\left(f_1\left(n_{2k+1}\right), F\left(n_{2k+1}\right)\right),$

$d\left(g_1(c), F(c)\right)\}.$

$\therefore \quad d\left(f_1(c), F(c)\right) \leq \zeta\{d\left(f_1(c), g_1(c)\right),$

$d\left(f_1(c), f_1(c)\right), d\left(f_1(c), F(c)\right),$

$d\left(g_1(c), f_1(c)\right), d\left(g_1(c), F(c)\right)\}$

$\therefore \quad d\left(f_1(c), F(c)\right)=$

$\zeta\{0,0, d\left(f_1(c), F(c)\right), 0,$

$d\left(f_1(c), F(c)\right)\}$

$\leq M\left(d\left(f_1(c), F(c)\right)\right)$

$<d\left(f_1(c), F(c)\right)$

$\therefore \quad d\left(f_1(c), F(c)\right)<d\left(f_1(c), F(c)\right)$

which gives a contradiction.

Thus $f_1(c)=F(c)$

$\therefore \quad f_1(c)=F(c)=g_1(c).$

Also $d(F(c), c) \leq \zeta\{d\left(f_1(c), c\right),$

$0, d\left(f_1(c), c\right), d(c, F(c)), 0\}$

$\leq M \quad d(F(c), c))$

which also gives a contradiction.

Consequently, $F(c)=c=f_1(c)=g_1(c)$.

Thus c is only common fixed point of the continuous self-signals F, f_1 and g_1.

<u>Example (1):</u>

Let $X=[0,1]$ be a complete metric space.

We consider $F(n)=\frac{n}{n+2}$, $f_1(n)=\frac{n}{2}$,

and $g_1(n)=\frac{3n}{4} \quad \forall n \in X$.

Also, Let $\zeta\left(z_1, z_2, z_3, z_4, z_5\right)=$

$$\frac{1}{5}\left(z_1+z_2+z_3+z_4+z_5\right)$$

Thus,

$$F(n)=\left[0, \frac{1}{3}\right] \subset\left[0, \frac{1}{2}\right] \cap \left[0, \frac{3}{4}\right]=f_1(n) \cap g_1(n).$$

Here, $F g_1(n)=F\left(g_1(n)\right)$

$$=F\left(\frac{3n}{4}\right)=\frac{\frac{3n}{4}}{\frac{3n}{4}+2}$$

$$=\frac{3n}{3n+8},$$

$$g_1 F(n)=g_1\left(\frac{n}{n+2}\right)$$

$$=\frac{3}{4}\left(\frac{n}{n+2}\right)$$

$$=\frac{3n}{4n+8}$$

for $n \in X$,

$$d\left(Fg_1(n), g_1F(n)\right) = \left|\frac{3n}{8+3n} - \frac{3n}{8+4n}\right|$$

$$= \frac{3n^2}{(8+3n)(8+4n)}$$

$$\le \frac{3n^2}{8+4n} \le \frac{3n^2}{8+4n} + \frac{2n}{8+4n}$$

$$= d\left(F(n), g_1(n)\right).$$

Also, for $n, n \in [0,1]$, it is easy to verify the condition (ii) of our theorem.

Thus we must say that 0 is only a common fixed point of continuous self-mapping F, f_1 and g_1.

Example (2):

Let $X = R$ and define F, f_1 and $g_1 : x \to X$ by

$$f(n) = \begin{cases} 2^{-1} & for \quad n > 1 \\ n(n+1)^{-1} & for \ 0 < n \le 1, \\ 0 & for \quad n \le 0 \end{cases}$$

$$f_1(n) = \begin{cases} 1 & for \quad n > 1 \\ n & for \ 0 < n \le 1, \\ 0 & for \quad n \le 0 \end{cases}$$

and $g_1(n) = \begin{cases} n \ for \ n > 0 \\ 0 \ for \ n \le 0 \end{cases}$.

Let $\zeta : \left(R^+\right)^5 \to R^+$ by

$$\zeta(z_1, z_2, z_3, z_4, z_5) =$$

$$\begin{cases} z_1(1+z_1)^{-1} & for \ 0 \le z_1 \le 1 \\ 2^{-1}z & for \quad z_1 > 1 \end{cases}.$$

Then f_1, g_1 and ζ are monotonically increasing and continuous signals.

Here, $A(n) = \left[0, \frac{1}{2}\right] \cap [0,1] \cap [0, \infty]$

$$= f_1(n) \cap g_1(n).$$

It can be noticed that

$$M(z) = \zeta(z, z, p, z, p, z, z) < z$$

and $p_1 + p_2 = 3$. For this,

case (i) : If $n \le 0, n_1 \le 0$ then

$$d\left(F(n), F(n_1)\right) = 0 = M\left(d\left(f_1(n), g_1(n_1)\right)\right)$$

case (ii) : If $n \le 0, 0 < n_1 \le 1$,

then $d\left(F(n), F(n_1)\right) = \frac{n_1}{1+n} = M(n_1)$

$$= M\left(d\left(f_1(n), g_1(n_1)\right)\right)$$

case (iii) : If $n \le 0, n_1 > 1$

then $d\left(F(n), F(n_1)\right) = \frac{1}{2} < \frac{n_1}{2}$

$$= M\left(d\left(f_1(n), g_1(n_1)\right)\right)$$

case (iv): If $0 < n \le 1, 0 < n_1 \le 1$,

$$d\left(F(n), F(n_1)\right) = \left|\frac{n}{n+1} - \frac{n_1}{n_1+1}\right|$$

$$= \frac{|n_1 - n|}{(n+1)(n_1+1)} \le \frac{|n_1 - n|}{1 + |n_1 - n|}$$

$$= M\left(|n - n_1|\right)$$

$$= M\left(d\left(f_1(n), g_1(n_1)\right)\right)$$

$\because |n - n_1| <|$

case (v) : If $0 < n \le 1$,

$$F(n) = \frac{n}{n+1}, F(n_1) = \frac{1}{2},$$

$$f_1(n) = n, g_1(n_1) = n_1,$$

$$n+1 < 2 \Rightarrow \frac{1}{n+1} > \frac{1}{2}$$

$$\Rightarrow \frac{n}{n+1} > \frac{n}{2}$$

$$\Rightarrow -\frac{n}{n+1} < \frac{-n}{2}$$

$$\Rightarrow \frac{1}{2} - \frac{n}{n+1} < \frac{1}{2} - \frac{n}{2} < \frac{-n}{2} = \frac{n}{2} \text{ where } n_1 > 1$$

then for $n_1 - n > 1$, we deduce that

$$d\left(F(n), F(n_1)\right) = \frac{1}{2} - \frac{n}{n+1} < \frac{n_1 - n}{2} = M\left(d\left(f_1(n), g_1(n_1)\right)\right)$$

similarly, $\frac{n_1 - n}{1 + n_1 - n} \ge \frac{n_1 - n}{2}$

and $d\left(F(n), F(n_1)\right) =$

$$\frac{1}{2} - \frac{3}{n+1} < \frac{n_1 - n}{2} \le \frac{n_1 - n}{1 + n_1 - n} =$$

$M\left(d\left(f_1(n), g_1(n_1)\right)\right)$ where $n_1 \le n+1$

case (vi) : If $n > 1$, $n_1 > 1$

$$d\left(F(n), F(n_1)\right) = 0$$

$$< \begin{cases} \frac{n_1 - 1}{2} = M\left(d\left(f_1(n), g_1(n_1)\right)\right) \text{ if } n_1 \ge 2 \\ \frac{n_1 - 1}{n_1} = \frac{n_1 - 1}{1 + (n_1 - 1)} = M\left(d\left(f_1(n), g_1(n_1)\right)\right) \text{ if } 1 < n_1 < 2 \end{cases}$$

Therefore all assumptions of theorems are satisfied.

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