Chimeras-States in the Distributed Systems in the Case of Multi-Valued Solutions. To the Statement of Possible Research Problems

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Abstract: - Nonlinear science has about a hundred years of rapid development. One of the new directions is the study of chimera solutions, i.e. solutions that exhibit significantly different solution behaviour in different regions of space or time. Just a few examples of such behavior have been studied in different systems. However, all examples of chimeras have been single-valued solutions. Recently, however, many nonlinear models of physical and social phenomena have multivalued solutions. It is conceivable that one of the possible directions for further research in chimera science is to allow multivalued solutions to be considered.
The paper is devoted to describing some possible research settings in this area. Some modifications of existing models are also proposed. The interesting class of such objects is the models with strong anticipation (models with advanced arguments). Furthermore, some problems of computational theory and mind with such models are discussed.

Keywords: - Chimeras, distributed systems, multi-valuednes strong anticipation, computation theory

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1 Introduction

Complexity and synchronization phenomena in technical and natural systems have recently become one of the key topics for investigation in philosophy, physics, humanities, and biological sciences. The reason is the globalization of processes, hierarchical structures with many levels, interconnections between elements, existence of many sub-processes with different time and space scales.

Synchronization theory has a history of about two centuries and a more or less well-defined list of topics [1-4]. Usually, the objects of synchronization research are collections of elements with internal dynamics, with a set of boundaries between elements and types of behavior in such systems.

The recent development of theory and practice in the field of synchronization has many different ways. Usually, the nature of a specific field of investigation follows the specific new research problems and new objects. One of the most recent examples is

One of the new, rapidly developing directions in nonlinear science is the study of so-called chimeras as solutions of nonlinear equations. Roughly speaking, these are solutions of distributed systems with coherent and incoherent behavior of solutions in different regions of space. This behavior of solutions was first discovered in computer experiments and described in the paper [5]. The actual name of such solutions, "chimera", appeared a little later in [6]. Since then, the flow of publications on this topic has been steadily increasing, e.g. in Physical Review, Nonlinearity, Chaos, etc., as well as in the proceedings of many conferences. The current state of the art is reflected in the papers of the Dresden conference on chimeras in May 2022 [7].

Much of the research on chimeras has been based on the study of initial equations and various aspects of their behavior. In addition, new sources of chimera solutions have been explored, especially in the context of new applications. Some (far from complete) insights into new applications may be proposed- for example in neuroscience [8-10]. One of the research directions has also been the transition from one-dimensional (1D) to two-dimensional (2D) and then to three-dimensional

(3D) systems [11]. We should also mention the studies on the system of oscillators, taking into account the inertia [12]. Note that for further studies of chimeras in systems with inertia, studies on hydrodynamics with memory can be useful [13]. In this case, the 'bursts' in the flows can be considered as the chimeras.
In all cases of studies of chimeras so far, objects with single-valued solutions (o.d.e., chains of mappings, chains of oscillators, etc.) have been considered. This is while that most models for many different processes have only ONE-valued solutions. However, it is now becoming increasingly clear that one should also consider cases where model solutions for processes have MULTIPLE-valued solutions. Without giving a special survey here, let us mention some areas where the study of multivaluedness is important: mechanical systems with friction, the Everett interpretation in quantum mechanics, underdetermined systems, controlled systems, some models of hydrodynamics, etc. Note also the possibility of multivaluedness in systems with strong anticipation - called hyperincursion [14, 15].
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2. General Distributed Media With Anticipation

Many more models can be proposed for the case of considering continuous media with some kind of anticipation. The simplest variant is when only the nonlinear source $f(u)$ in the equations of such media (where $u(x,t)$ is field value) has anticipatory property, i.e. $f_1 = f(u(x,t), u(x,t+\tau))$ in the simplest case or $f_2 = \int_{-\infty}^{+\infty} f(u(x,\tau), K(x,t-\tau))d\tau$ in more complex case. An example of such a model is the parabolic diffusion equation with such a non-linear source. As an example of such a system, we can propose models with continuous variables of neuronal activation fields (counterparts of the well-known models of Amari or H. Wilson & J. Cowan). Note that much more complex and developed models of such phenomena are the counterparts of the well-known [13] rigorous models of media with non-locality and memory in theoretical physics-
Note for illustration that in the case of discrete-time equations with strong anticipation, they have the form: (D. Dubois [14]) "The definition of a discrete system with strong anticipation: it is a system which calculates the current state at time t as a function of past states, $t-3$, $t-2$, $t-1$, the present state and the state in the future $t+1$, $t+2$, $t+3, \dots$.

$$\begin{aligned} x(t+1) = f(\dots, x(t-2), x(t-1), \\ x(t), x(t+1), x(t+2), \dots) \end{aligned} , \qquad (1)$$

where the variation in future time is computed indirectly from the equation.

3. Towards 'Chimera' States In Anticipatory Systems

Another promising topic for investigation in the field of synchronization is the analog of 'chimera' states in the case of anticipating systems. Recall that 'chimera' states are highly inhomogeneous transitive solutions when different types of behavior coexist in space [5-12].
The 'chimera' states are the solutions in chains of elements or in distributed media that have different behavior in different domains of space. For example, such systems may have coexisting

domains of chaotic and 'smooth' behavior at different locations in space. One of the discrete systems with putative 'chimeras' is the following [5-12]

$$z_i^{t+1} = f(z_i^t) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} [f(z_j^t) - f(z_i^t)] \quad (2)$$

The next example is the chain of oscillators:

$$\frac{d\psi_k(t)}{dt} = \omega - \frac{1}{2R} \sum_{j=k-R}^{k+R} \sin[\psi_k(t) - \psi_j(t) + \alpha] \quad (3)$$

The next general examples with 'chimera' states are the following: It is accepted that the main source for the existence of 'chimera' states is the non-locality in the equations. Note that the nonlocality described in [5] essentially extends the cases with the presumed origin of 'chimeras'. The possibilities of 'chimeras' in systems with anticipation are new [15]. As examples, we should mention the possibilities of 'multivalued chimeras', the coexistence of different 'chimeras' on different branches of multivalued solutions, a coexistence of 'chimeras', and 'smooth' behavior in different branches of the solution. As one of the presumed basic systems with anticipation for the study of 'chimeras', we can propose the next one:

$$z_i^{t+1} = (1-\alpha)[f(z_i^t) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} [f(z_j^t) - f(z_i^t)]] + \alpha[f(z_i^{t+1}) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} [f(z_j^{t+1}) - f(z_i^{t+1})]] \quad (4)$$

The next example is the chains of anticipatory oscillators:

$$\frac{d\psi_k(t)}{dt} = \omega - \frac{(1-\alpha)}{2R} \sum_{j=k-R}^{k+R} \sin[\psi_k(t) - \psi_j(t) + \alpha] + \frac{\alpha}{2R} \sum_{j=k-R}^{k+R} \sin[\psi_k(t+\tau) - \psi_j(t+\tau) + \alpha] \quad (5)$$

Note that more general examples with 'chimera' states can also be proposed.

4. Possible Problems With Chimeras In The Multivalued Case

As mentioned above, an example of systems with multivalued solutions are systems with strong anticipation. Therefore, here we will illustrate some of the possible problem formulations in the field of chimera research. Such systems have been studied previously for models of cellular automata [16], neural networks [17], and discrete dynamical systems [18]. The figure below shows a schematic case of multivalued solutions in the one-dimensional case in space. On the horizontal axis the discrete time moments are plotted (for discrete dynamical systems). On the vertical axis, the states of considered system X at discrete time steps are conditionally plotted. For example, for a system of N-connected one-dimensional elements, a point corresponds to an N-dimensional vector. For cellular automata, a values corresponds to the so-called configuration of the automaton at given moment of time (configuration is the representation of states of all cells of cellular automata).

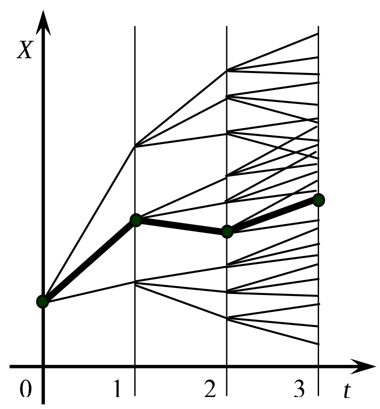

FIGURE 1: Plurality of system states during evolution

The thin lines correspond to the branches of the multi-valued solution. The thick line corresponds to the selection of a single-valued solution. For systems described by differential equations, the point can correspond to an infinite dimensional object.

Because of the possible multivaluedness of the solutions, we can propose the new problem formulations in the field of chimera research.

1. Computational studies of models with multivalued solutions, where the appearance of one-dimensional analog can be expected. It should be assumed that such studies are the first necessary steps in the study of chimeras in the multivalued case (let us call them multivalued chimeras). Even in this case, many difficult problems arise, even for one-dimensional systems, and, for example, in the visualization of computational results.

Theoretically, there is the problem of a strict definition of multivalued chimeras. This also raises the problem of introducing indices that can be used to check multivalued solutions for the presence of chimeras. The inverse problem may also be of interest, i.e. finding the models and initial conditions that can generate chimeras with specific deactivation properties.

3. The next set of problems arises in the study of the peculiarities of multi-valued solutions, namely the existence of different branches of the solution. One of the first interesting problems is whether different chimeras can occur on different branches of a multivalued solution. For example, this leads to the question of can the chimeras exist on some branches and not on others.

4. The possible multivaluedness of chimeras leads to unexpected problems at the interface of nonlinear science and computational theory. Namely, can chimeras be used in computational theory to emulate logical operations, and independently on different branches?

5. Conclusion

Thus, in the given paper we propose to consider the problem of complex behavior and synchronization for a new class of objects, namely for chains and networks with presumed multivaluedness (e.g. for systems with strong anticipation). The assumed multivaluedness of the solutions leads to new interesting properties within the framework of existing concepts. However, new properties may also emerge (e.g. non-heterogeneous multivalued synchronization) which are very promising for further research. There is also a need for applications of multivalued analogs of groups, geometries, and symmetries of multivalued objects, including multivalued flows and semi-flows, and methods of operator theory. Also new may be the consideration of blow-up solutions in multivalued cases, including blow-up of selected branches of the solution.

We have described only the first results of investigations and only some new presumed forms of research problems. However, the obvious mathematical novelty of the proposed problems and the probable great importance in applications (e.g. in social systems, computation, signal processing theory, consciousness research, etc.) lead to the need for further development of the investigations.

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