Two Forty Phase Code Design with Good Discrimination Factor

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Abstract : Sequences with good discrimination factor are useful for radar and communication applications. In this paper Two Forty Phase sequences are synthesized using Modified Genetic Algorithm (MGA). Modified Genetic Algorithm is used as a statistical technique for obtaining approximate solutions to combinatorial optimization problems. This algorithm combines the good methodologies of the two algorithms like global minimum converging property of Genetic Algorithm (GA) and fast convergence rate of Hamming scan algorithm. The synthesized sequences have discrimination factor better than well-known Frank codes. The synthesized sequences also have complex signal structure which is difficult to detect and analyze by enemy electronics support measure.

Key words: Autocorrelation, Discrimination factor, Hamming scan, Polyphase codes, Genetic algorithm, Radar signal.

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1. Introduction

Sequences with low aperiodic autocorrelation sidelobe levels are useful for channel estimation, radar, and spread spectrum communication applications. Sequences achieving the minimum peak aperiodic autocorrelation sidelobe level one are called Barker Sequences[1,2].

The aperiodic autocorrelation function (ACF) of sequence S of length N is given by,

$$A(k)=\begin{cases}\sum_{n=0}^{N-k-1} s_n s_{n+k}^* ; & 0\le k\le N-1\\ \sum_{n=0}^{N+k-1} s_n\, s_{n-k}^* ; & -N+1\le k\le 0\end{cases} \quad .....(1)$$

If all the sidelobes of the ACF of any polyphase sequence are bounded by

$$|A(k)| \le 1,\ 1\le |k| \le N-1 \quad \ldots (2)$$

then the sequence is called a generalized Barker sequence or a polyphase Barker sequence.

If the sequence elements are taken from an alphabet of size M, consisting of the M^{th} roots of unity.

$$S_m=\exp\left\{2\pi i.\frac{m}{M}\right\}=:\exp(i\phi_m) \quad 0\le m\le M-1 \quad \cdots (3)$$

the sequence is alternatively named an M-phase Barker sequence. Polyphase signal has larger main lobe-to- peak sidelobe ratio over binary signal of the same code length. In addition, polyphase waveforms have a more complicated signal structure and thus, are more difficult to detect and analyze by an enemy's electronic support measures (ESMs). With the maturity of digital signal processing, the generation and processing of polyphase signals has become easy and less costly. Therefore, polyphase code is increasingly becoming a favorable alternative to the traditional binary code for radar signals and can be used as the basic code for radar signal design[2-9].

2.Two Forty Phase Code Design

The Two Forty Phase sequence of length N bits is represented by a complex number sequence

$$\left\{s(n)=e^{j\phi_m(n)}\ ,\quad n=1,2,...,N\right\} \quad \ldots \quad (4)$$

Where $\phi_m(n)$ is the phase of nth bit in the sequence and lies between 0 and 2π. If the number of the distinct phases available to be chosen for each bit in a code sequence is M, the phase for the bit can only be selected from the following admissible values:

$$\phi_m(n)\in\left\{0,\ \frac{2\pi}{M},\ 2\frac{2\pi}{M},...,\ (M-1)\frac{2\pi}{M}\right\} \quad .. \quad (5)$$
$$=\{\psi_1,\ \psi_2,.....,\psi_M\}.$$

For example if M = 4, then values of $\{\psi_1, \psi_2, \psi_3$ and $\psi_4\}$ will be 0, $\pi/2$, π and $3\pi/2$ respectively.

Considering a Two Forty Phase sequence S with code length N, one can concisely represent the phase values of S with the following 1 by N phase matrix:

$$S = [\phi_m(1), \quad \phi_m(2), \quad \phi_m(3),.... \quad \phi_m(N)\ ] \qquad \ldots (6)$$

where all the elements in the matrix can only be chosen from the phase set in eq. (5).

A more practical approach to design Two Forty Phase sequences with properties in eq. (2) is to numerically search the best phase sequences by minimizing a cost function that measures the degree to which a specific result meets the design requirements. For the design of Two Forty Phase sequences used in radar and communication the cost function is based on the sum of square of autocorrelation side lobe peaks. Hence, from eq. (1) the cost function can be written as,

$$E = \sum_{k=1}^{N-1} |A(k)| \qquad ..... \quad (7)$$

The minimization of cost function in eq. (7) generates a Two Forty Phase sequences that are automatically constrained by eq. (2). In this optimization we have minimize the autocorrelation sidelobe energy.

3. Discriminating Factor (DF)

The discriminating factor **(**DF**)** as defined by Golay is ratio of mainlobe peak value to the magnitude of sidelobe peak value of Autocorrelations function of sequence S. The DF, mathematically is defined as follows[10].

$$DF = \frac{r(0)}{\max_{k \neq 0} |r(k)|} \qquad \ldots \qquad (8)$$

The denominator is a measure of the peak sidelobe value and is related to the L_∞ norm of the sidelobes.

4. Hamming Scan Algorithm

The Hamming scan algorithm is a traditional greedy optimization algorithm, which searches in the neighborhood of the point in all directions to reduce the cost function and has fast convergence rate[11-13]. This algorithm mutates element of sequence one by one. The Mutation is a term metaphorically used for a change in an element in the sequence. For example if a phase value of a Two-Forty Phase sequence is ψ_m ($1 \leq m \leq 240$), i.e., one term in eq. (6), it is replaced with phase ψ_i, i =1, 2,…, 240, $i \neq m$, and the cost for each ψ_i change is evaluated. If the cost is reduced due to a change in phase value, the new phase value is accepted; otherwise, the original phase value is retained. The same procedure is performed for all phase values of sequence, i.e., every term of eq. (6). This process is recursively applied to the matrix until no phase changes are made. A single mutation in a sequence results in a Hamming distance of one from the original sequence. The Hamming scan algorithm mutates all the elements in a given sequence one by one and looks at all the first order-Hamming neighbors of the given sequence. Thus, Hamming scan performs recursively local search among all the Hamming-1 neighbors of the sequence and selects the one whose objective function value is minimum.

5. Genetic Algorithm (GA)

GA technique, introduced by John Holland at University of Michigan proved efficient and powerful tool to find optimal or near optimal solutions for complex multivariable nonlinear functions [14-15]. The major advantage of the GA algorithm over the traditional “greedy” optimization algorithms is the ability to avoid becoming trapped in local optima during the search process.

The genetic algorithm creates a population of solutions and applies genetic operators such as crossover and mutation to evolve the solutions in order to find the best one(s). The three most important aspects of using genetic algorithms are: (1) definition of the objective function, (2) definition and implementation of the genetic representation, and (3) definition and implementation of the genetic operators. Once these three have been defined, the generic genetic algorithm should work fairly well. But the limitation of GA is slow convergence rate. This limitation is overcome by in modified Genetic algorithm.

6. Modified Genetic Algorithm (MGA)

Modified Genetic Algorithm is proposed as a statistical technique for obtaining approximate solutions to combinatorial optimization problems. The proposed algorithm is a combination of Genetic Algorithm (GA) and Hamming Scan algorithms. It combines the good methodologies of the two algorithms like global minimum converging property of GA algorithm and fast convergence rate of Hamming scan algorithm. The demerit of Hamming scan algorithm is that it gets stuck in the local minimum point because it has no way to distinguish between local minimum point and a global minimum point. Hence it is sub-optimal. The drawback in Genetic algorithm is that it has a slow convergence rate because even though it may get closer to the global minimum point, it may skip it because of the methodology it employs. The MGA overcomes these drawbacks. It is quite effective to combine GA with Hamming Scan (HSA) Algorithm. GA tends to be quite good at finding generally good global solutions, but quite inefficient at finding the last few mutations to find the absolute optimum. Hamming Scan are quite efficient at finding absolute optimum in a limited region. Alternating MGA improve the efficiency of GA while overcoming the lack of robustness of HSA. MGA are introduced as a computational analogy of adaptive systems. They are modeled loosely on the principles of the evolution via natural selection, employing a population of individuals that undergo selection in the presence of variation-inducing operators such as mutation and recombination. A fitness function is used to evaluate individuals, and reproductive success varies with fitness.

7. Ambiguity Function(AF)

The radar signal detection is actually based on ambiguity function(AF) and cross ambiguity function(CAF) rather than the function of autocorrelation and cross correlation. Ambiguity Function is used to assess the properties of the transmitted waveform with respect to its target resolution, accuracy of measurement, ambiguity and response to Doppler's effect and clutter. Transmit waveform specifies the sensor's ability to resolve targets as a function of delay (τ) and Doppler (ν). The ideal transmit signal would produce an ambiguity function with zero value for all non-zero delay and Doppler (i.e., a "thumbtack"), indicating that the responses from dissimilar targets are perfectly uncorrelated. It is well known that if the ambiguity function about the origin is sharply peaked, then simultaneous range and velocity resolution capability is good Ambiguity function $|\chi(\tau,\nu)|$ which is the time response of a filter matched to a given finite energy signal when the signal is received with delay τ and a Doppler shift ν relative to the nominal values (zeros) predicted from the filter. It may be defined as[2]

$$|\chi(\tau,\nu)| = \left| \int_{-\infty}^{\infty} s(t)\, s^*(t+\tau) \exp(j2\pi\nu t)\, dt \right| \quad ..(9)$$

Where transmitted signal is s(t). AF and Cross AF is used to design radar signal rather than ACF or CCF. The effect of Doppler frequency shift on the performance of the design radar signals can be demonstrated using Ambiguity function. If the ambiguity function is thumbtack than range and velocity resolution capability can be achieved simultaneous.

A noise like waveform has an ambiguity function called thumbtack. Pure noise waveforms are seldom employed in radar; but constant – amplitude noise like waveforms have been used to produce a thumbtack ambiguity diagram. The thumbtack has the advantage that the time - delay and frequency measurement accuracies are independently determined, respectively, by the bandwidth of the modulation and the duration of the pulse.

8. Results Analysis

Two Forty Phase sequences are designed using the MGA, the length of the sequence, N, is varied from 16 to 484. The Table-I shows the Comparison of Discrimination Factors of Two Forty Phase synthesized sequences with Frank codes. In the table I, column 1 shows sequence length, N, column 2 shows DF of Frank codes and column 3 shows the DF of Two Forty Phase sequences. The fig (1) is the graphical representation of comparison of DF of Frank codes with synthesized two forty phase sequences whose values are shown in table 1. As shown in fig (1) the DF of Two forty phase sequences are far better than well-known Frank codes. Apart from having better DF the synthesized sequences have a complicated signal structure than Frank and binary codes as shown in fig (2) and (3), and thus, are more difficult to detect and analyze by an enemy's electronic support measures (ESMs). Fig.(4) shows the comparison of Frank and Two Forty phase sequence autocorrelation function of sequence of length N=36. As shown in the figure autocorrelation sidelobes of Two Forty phase sequence is around 5 dB lower than Frank code. Figs (5) and (6) show the Ambiguity function of Frank code and two forty phase codes respectively. As shown in the fig (6) side lobes of two forty phase sequence is better. Hence, it can be observed from analysis that two forty phase not only have complex signal structure but also have better DF than Frank codes.

Table –1

Comparison of Discrimination Factor of Two Forty Phase synthesized sequences with Frank codes

Sequence Length	Frank codes	Two Forty phase Sequences
16	11.3	16
25	15.5	23.7
36	18.0	30.6
49	21.8	28.8
64	24.4	42.9
81	28.1	40.2
100	30.9	43.0
121	34.4	45.0
144	37.3	46.1
169	40.7	47.1
196	43.6	55.2
225	47.0	65.2
256	50.0	62.5
289	53.32	64.0
400	62.62	66.2
441	65.92	70.83
484	68.62	73.56

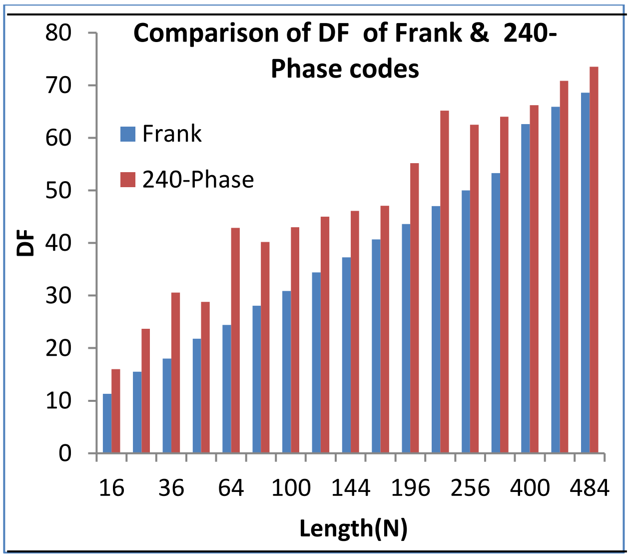

Fig (1) Comparison of Discrimination Factor of Frank codes and Two Forty Phase sequences

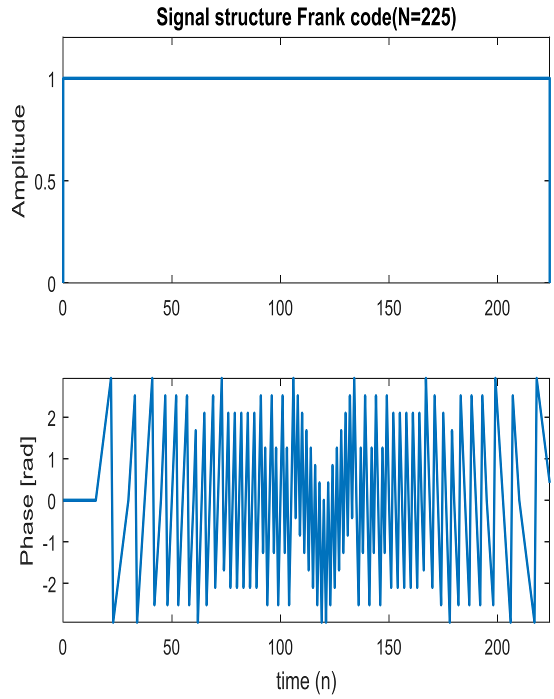

Fig (2) Signal structure of Frank Code of length (N=225).

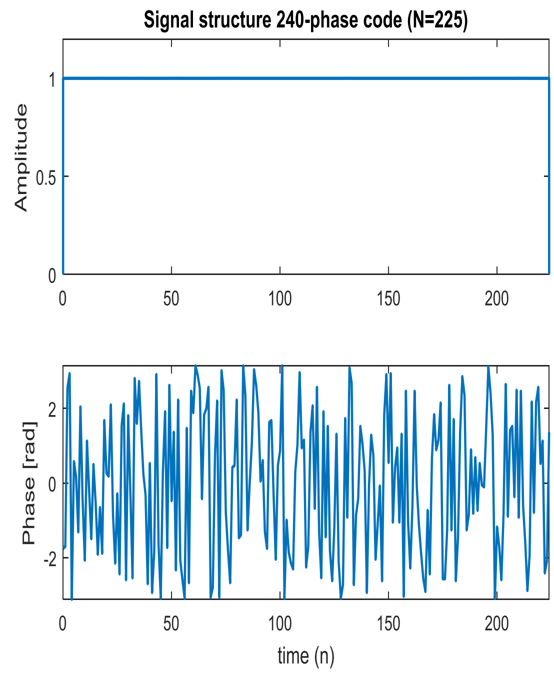

Fig (3) Signal structure of Two forty phase Code of length (N=225).

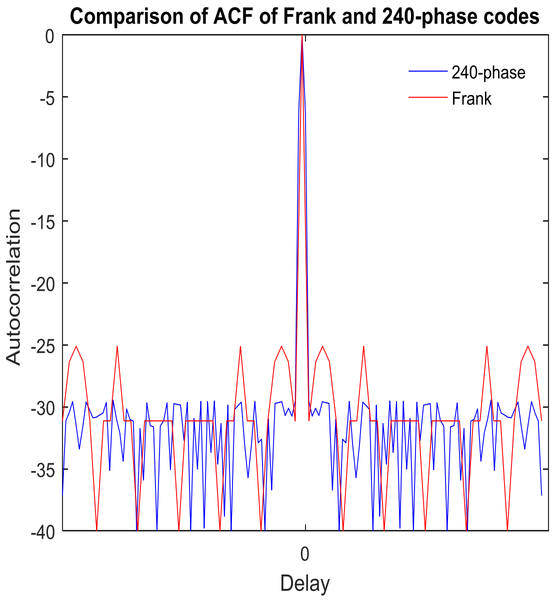

Fig (4) Autocorrelation function of Two Forty phase and Frank code

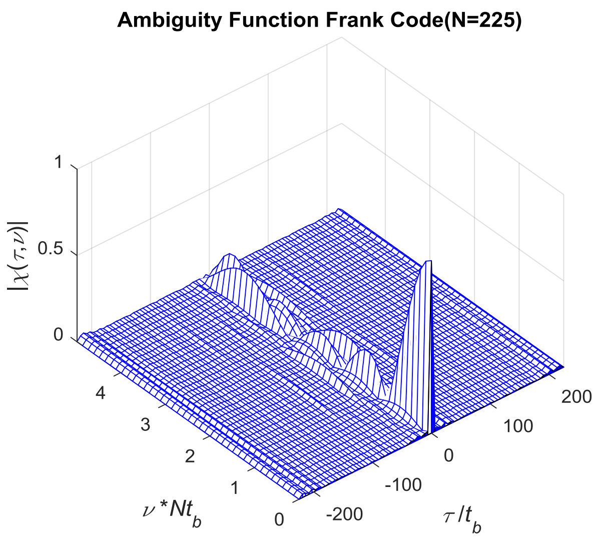

Fig (5) Ambiguity function of Frank code (N=225)

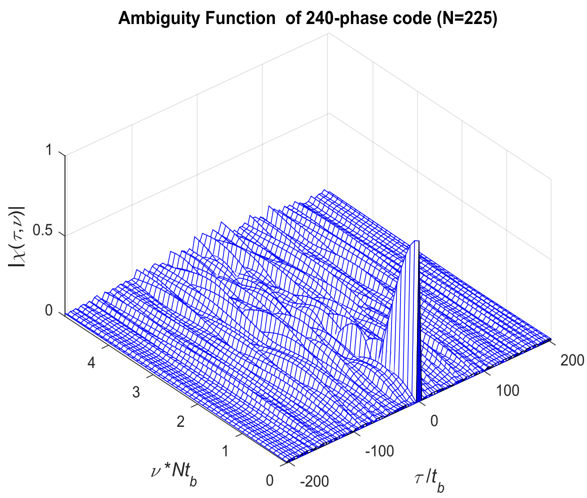

Fig (6) Ambiguity function of Two Forty phase (N=225)

9. Conclusions

An effective Modified Genetic algorithm has been used for designing the Two Forty Phase coded sequences with good Discrimination Factor. The synthesized sequences can be used in radar systems and spread spectrum communications for significantly improving performance of the system. The Two Forty Phase sequences are designed upto a length of 484. The synthesized results presented in this paper not only have better DF but also have more complicated signal structure which is difficult to detect and analyze by an enemy's electronic support measures (ESMs). Hence, it can be concluded that the design results are very useful for radar as well as spread spectrum communication systems.

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