Convection in Compressible Dusty Fluids

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Abstract: The aim of the present research was to study the thermosolutal convection in compressible fluids with suspended particles in permeable media. Following the linearized stability theory, Boussinesq approximation and normal mode analysis, it is found that that stable solute gradient introduces oscillatory modes which were non-existent in its absence. For the case of stationary convection, it is found that medium permeability and suspended particles have destabilizing effects whereas the stable solute gradient has a stabilizing effect on the system. This problem was further extended to include uniform rotation. In this case for stationary convection, the suspended particles are found to have destabilizing effect whereas stable solute gradient, rotation and compressibility have stabilizing effect on the system. The medium permeability has a destabilizing effect in the absence of rotation but has both stabilizing and destabilizing effects in the presence of rotation.

Key-words: Convection, Porous Medium, Rotation, Suspended Particles

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1 Introduction

The theoretical and experimental results on thermal convection in a fluid layer, under varying assumptions of hydrodynamics have been discussed in a treatise by Chandrasekhar [1]. The use of Boussinesq approximation has been made throughout which states that the density changes are disregarded in all other terms in the equations of motion except the external force term. The approximation is well justified in the case of incompressible fluids. Chandra [2] observed that in an air layer, convection occurred at much lower gradients than predicted if the layer depth was less than 7 mm and called this motion “columnar instability”. However, for layers deeper than 10 mm, a Be'nard-type cellular convection was observed. Thus there is a contradiction between the theory and experiment. In geophysical situations, the fluid is often not pure but contains suspended particles. The effect of particle mass and heat capacity on the onset of Be′nard convection has been considered by Scanlon and Segel [3]. They found that the critical Rayleigh number was reduced solely because the heat capacity of the pure fluid was supplemented by that of the particles. The effect of suspended particles was found to destabilize the layer i.e. to lower the critical temperature gradient. Palaniswamy and Purushotham [4] have considered the stability of shear flow of stratified fluids with fine dust and have found the effect of fine dust to increase the region of instability. Venetis [5] has investigated the boundary roughness of a mounted obstacle which is inserted into an incompressible, external and viscous flow field of a Newtonian fluid.

When the fluids are compressible, the equations governing the system become quite complicated. To simplify them, Boussinesq tried to justify the approximation for compressible fluids when the density variations arise principally from thermal effects. Spiegel and Venonis [6] have simplified the set of equations governing the flow of compressible fluids under the following assumptions:

(a) the depth of the fluid layer is much less than the scale height, as defined by them; and

(b) the fluctuations in temperature, density, and pressure, introduced due to motion, do not exceed their total static variations.

Under the above approximations, the flow equations are the same as those for incompressible fluids, except that the static temperature gradient is replaced by its excess over the adiabatic one and c_v is replaced by c_p. Using these approximations, Sharma [7] has studied the thermal instability in compressible fluids in the presence of rotation and a magnetic field. Hoshoudy and Kumar [8] have studied the Rayleigh-Taylor instability of a heavy fluid supported by a lighter one with suspended dust particles and small uniform general rotation. Compressibility effects on Rayleigh-Taylor instability of two plasmas layers are investigated by Hoshoudy et al. [9].

The investigation of double-diffusive convection is motivated by its interesting complexities as a double-diffusion phenomena as well as its direct relevance to geophysics and astrophysics. The conditions under which convective motion in double-diffusive convection are important (e.g. in lower parts of the Earth's atmosphere, astrophysics, and several geophysical situations) are usually far removed from the consideration of a single component fluid and rigid boundaries and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [10]. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The problem is of great importance because of its application to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the atmosphere. The thermosolutal convection problems also arise in oceanography, limnology and engineering. The onset of double-diffusive reaction-convection in fluid layer with viscous fluid, heated and salted from below subject to chemical equilibrium on the boundaries has been investigated by Gupta and Singh [11].

In recent years, the investigations of flow of fluids through porous media have become an important topic due to the recovery of crude oil from the pores of reservoir rocks. The problem of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground-water hydrology and astrophysics. The development of geothermal power resources holds increased general interest in the study of the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat transfer mechanism in the development of young oceanic crust (Lister [12]). Generally it is accepted that comets consist of a dusty "snowball" of a mixture of frozen gases which, in the process of their journey, changes from solid to gas and vice-versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context. A mounting evidence, both theoretical and experimental, suggests that Darcy's equation provides an unsatisfactory description of the hydrodynamic conditions, particularly near the boundaries of a porous medium. Beavers et al. [13] have experimentally demonstrated the existence of shear within the porous medium near surface, where the porous medium is exposed to a freely flowing fluid, thus forming a zone of shear-induced flow field. The Darcy's equation however, cannot predict the existence of such a boundary zone, since no macroscopic shear term is included in this equation (Joseph and Tao [14]). To be mathematically compatible with the Navier-Stokes equations and physically consistent with the experimentally observed boundary shear zone mentioned above, Brinkman proposed the introduction of the term $\frac{\mu}{\varepsilon}\nabla^2\vec{V}$ in addition to $-\left(\frac{\mu}{k_1}\right)\vec{V}$ in the equations of fluid motion. The elaborate statistical justification of the Brinkman equations has been presented by Saffman [15] and Lundgren [16]. Stommel and Fedorov [17] and Linden [18] have remarked that the length scales characteristics of double-diffusive convecting layers in the ocean could be sufficiently large for Earth's rotation to become important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid flowing through a porous medium, and the distortion plays an important role in the extraction of energy in the geothermal regions. Brakke [19] explained a double-diffusive instability that occurs when a solution of a slowly diffusing protein is laid over a denser solution of more rapidly diffusing sucrose. Nason et al. [20] found that this instability, which is deleterious to certain biochemical separations, can be suppressed by rotation in the ultracentrifuge. Sharma and Sharma [21] and Sharma and Kumari [22] have considered the thermosolutal convection in porous medium under varying assumptions of hydrodynamics and hydromagnetics. Misra et al. [23] have studied the numerical simulation of double-diffusive laminar mixed convection flow in a lid-driven porous cavity. Thermosolutal instability of magneto-hydrodynamic flow through porous medium has been studied by Choudhary [24]. Choudhary and Bhattacharjee [25] presents the study of three-dimensional flow and the injection/suction on an oscillatory flow of a visco-elastic incompressible fluid through a highly porous medium bounded between two infinite horizontal porous plates. Harfash and Alshara [26] have studied the problem of double diffusive convective movement of a reacting solute in a viscous incompressible occupying a plane layer in a saturated porous medium and subjected to a vertical magnetic field. Sriveni and Ratnam [27] have considered the double diffusive mixed convective heat and mass transfer flow of a viscous fluid through a porous medium in a rectangular duct. Coupled parallel flow of fluid with pressure-dependent viscosity through an inclined channel underlain by a porous layer of a variable permeability and variable thickness has been studied by Zaytoon and Hamdan [28]. Kumar and Gupta [29] have investigated the instability of the plane interface between two viscoelastic superposed conducting fluids in the presence of suspended particles and variable horizontal magnetic field through porous medium.

Keeping in mind the importance in geophysics, astrophysics and various applications mentioned above, the thermosolutal convection in compressible fluids with suspended particles in a porous medium, in the absence and presence of a uniform rotation, separately, has been considered in the present paper.

2 Formulation of the Problem and Basic Equations

Here we consider an infinite horizontal, compressible fluid-particle layer of thickness d bounded by the planes $z = 0$ and $z = d$ in a porous medium of porosity ε and permeability k_1. This layer is heated from below and subjected to a stable solute gradient such that steady adverse temperature gradient $\beta(= |dT/dz|)$ and a solute concentration gradient $\beta'(= |dC/dz|)$ are maintained.

Let ρ, μ, p and $\vec{V}(u, v, w)$ denote respectively the density, viscosity, pressure and filter velocity of the pure fluid: $\overrightarrow{V_d}(\overline{x}, t)$ and $N(\overline{x}, t)$ denote filter velocity and number density of the particles, respectively. If g is acceleration due to gravity, $K = 6\pi\rho\nu\epsilon'$where ε' is the particle radius, $\overrightarrow{V_d} = (l, r, s)$, $\overline{x} = (x, y, z)$ and $\overrightarrow{\lambda_1} = (0,0,1)$, then the equation of motion and continuity for the fluid are

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \vec{V}}{\partial t} + \frac{1}{\varepsilon}(\vec{V}.\nabla)\vec{V}\right] = -\nabla p - \rho g\overrightarrow{\lambda_1} + \left(\frac{\mu}{\varepsilon}\nabla^2 - \frac{\mu}{k_1}\right)\vec{V} + \frac{KN}{\varepsilon}\left(\overrightarrow{V_d} - \vec{V}\right), \quad (1)$$

$$\left(\varepsilon\frac{\partial}{\partial t} + \vec{V}.\nabla\right)\rho + \rho\nabla.\vec{V} = 0 \ . \quad (2)$$

Since the distances between particles are assumed to be quite large compared with their diameter, the interparticle relations, buoyancy force, Darcian force and pressure force on the particles are ignored. Therefore the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial \overrightarrow{V_d}}{\partial t} + \frac{1}{\varepsilon}(\overrightarrow{V_d}.\nabla)\overrightarrow{V_d}\right] = KN\left(\vec{V} - \overrightarrow{V_d}\right), \quad (3)$$

$$\varepsilon\frac{\partial N}{\partial t} + \nabla.\left(N\overrightarrow{V_d}\right) = 0 \ . \quad (4)$$

Let c_v , c_p , c_{pt} , T, C and q denote respectively the heat capacity of fluid at constant volume, heat capacity of fluid at constant pressure, heat capacity of particles, temperature, solute concentration and "effective thermal conductivity" of the clean water. Let c_v' , c_{pt}' and q' denote the analogous solute coefficients. When particles and the fluid are in thermal and solute equilibrium, the equations of heat and solute conduction give

$$[\rho c_v\varepsilon + \rho_s c_s(1-\varepsilon)]\frac{\partial T}{\partial t} + \rho c_v\left(\vec{V}.\nabla\right)T + mNc_{pt}\left(\varepsilon\frac{\partial}{\partial t} + \overrightarrow{V_d}.\nabla\right)T = q\nabla^2 T, \quad (5)$$

$$[\rho c_v'\varepsilon + \rho_s c_s'(1-\varepsilon)]\frac{\partial C}{\partial t} + \rho c_v'(\vec{V}.\nabla)C + mNc_{pt}'\left(\varepsilon\frac{\partial}{\partial t} + \overrightarrow{V_d}.\nabla\right)C = q'\nabla^2 C \ , \quad (6)$$

where ρ_s , c_s are the density and heat capacity of the solid matrix, respectively.

Spiegel and Venonis [6] have expressed any state variable (pressure, density or temperature), say X, in the form

$$X = X_m + X_0(z) + X'(x,y,z,t) \ ,$$

where X_m stands for the constant space distribution of X , X_0 is the variation in X in the absence of motion, and $X'(x,y,z,t)$ stands for the fluctuations in X due to the motion of the fluid. Following Spiegel and Veronis [6], we have

$$T(z) = -\beta z + T_0 \ ,$$

$$p(z) = p_m - g\int_0^z (\rho_m + \rho_0)dz \ ,$$

$$\rho(x) = \rho_m[1 - \alpha(T - T_m) + \alpha'(C - C_m) + \alpha''(p - p_m)] \ ,$$

$$\alpha = -\left(\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right), \alpha' = \left(\frac{1}{\rho}\frac{\partial\rho}{\partial C}\right), \alpha'' = \left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right) \ .$$

Thus p_m , ρ_m stand for the constant space distribution of p and ρ and T_0 , ρ_0 stand for the temperature and density of the fluid at the lower boundary (and in the absence of motion

Since density variations are mainly due to variations in temperature and solute concentration, equations (1) – (6) must be supplemented by the equation of state

$$\rho(z) = \rho_m[1 - \alpha(T - T_m) + \alpha'(C - C_m)] \ . \quad (7)$$

Let $\delta\rho$, δp , θ , γ , $\vec{V}$, $\overrightarrow{V_d}$ and N denote the perturbations in fluid density ρ, pressure p, temperature T, solute concentration C, fluid velocity (0,0,0) and particle number density N_0 , respectively. Then the linearized perturbation equations, under the Spiegel and Veronis [6] assumptions, are

$$\frac{1}{\varepsilon}\frac{\partial\vec{V}}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p - g\left(\frac{\delta\rho}{\rho_m}\right)\overrightarrow{\lambda_1} + \left(\frac{\nu}{\varepsilon}\nabla^2 - \frac{\nu}{k_1}\right)\vec{V} + \frac{KN_0}{\rho_m\varepsilon}\left(\overrightarrow{V_d} - \vec{V}\right) \ , \quad (8)$$

$$\nabla.\vec{V} = 0 \ , \quad (9)$$

$$mN_0\frac{\partial\overrightarrow{V_d}}{\partial t} = KN_0\left(\vec{V} - \overrightarrow{V_d}\right) \ , \quad (10)$$

$$\varepsilon\frac{\partial N}{\partial t} + \nabla.\left(N_0\overrightarrow{V_d}\right) = 0 \ , \quad (11)$$

$$(E + h\varepsilon)\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{c_p}\right)(w + h's) + \kappa\nabla^2\theta \ , \quad (12)$$

$$(E' + h'\varepsilon)\frac{\partial\gamma}{\partial t} = \beta'(w + h's) + \kappa'\nabla^2\gamma \ . \quad (13)$$

Here

$$E = \varepsilon + (1-\varepsilon)\frac{\rho_s c_s}{\rho_m c_v} \ , \ E' = \varepsilon + (1-\varepsilon)\frac{\rho_s c_s'}{\rho_m c_v'} \ ,$$
$$h = f\frac{c_{pt}}{c_v} \ , h' = f\frac{c_{pt}'}{c_v'} \ ,$$

$$f = \frac{mN_0}{\rho_m} \ , \kappa = \frac{q}{\rho_m c_v} \ , \ \kappa' = \frac{q'}{\rho_m c_v'} \ and \ \delta\rho = -\rho_m(\alpha\theta - \alpha'\gamma).$$

Using d , d^2/κ , κ/d , $\rho\nu\kappa/d^2$, βd and $\beta' d$ to denote the length, time, velocity, pressure, temperature and solute concentration scale factors, respectively, the linearized dimensionless perturbation equations become

$$p_1^{-1}\frac{\partial\overrightarrow{V^*}}{\partial t^*} = -\nabla^*\delta p^* + R\theta^*\overrightarrow{\lambda_1} - S\gamma^*\overrightarrow{\lambda_1} + \left(\frac{1}{\varepsilon}\nabla^{*^2} - \frac{1}{P}\right)\overrightarrow{V^*} + \omega\left(\overrightarrow{V_d^*} - \overrightarrow{V^*}\right) \ , \quad (14)$$

$$\nabla^*.\overrightarrow{V^*} = 0 \ , \quad (15)$$

$$\left(\tau\frac{\partial}{\partial t^*} + 1\right)\overrightarrow{V_d^*} = \overrightarrow{V^*} \ , \quad (16)$$

$$\left(\frac{\partial M}{\partial t^*} + \nabla^*.\overrightarrow{V_d^*}\right) = 0 \ , \quad (17)$$

$$(E + h\varepsilon)\frac{\partial\theta^*}{\partial t^*} = \frac{G-1}{G}(w^* + hs^*) + \nabla^{*^2}\theta^* \ , \quad (18)$$

$$(E' + h'\varepsilon)\frac{\partial\gamma^*}{\partial t^*} = (w^* + h's^*) + \frac{1}{\lambda}\nabla^{*^2}\gamma^* \ , \quad (19)$$

where

$$P=\frac{k_1}{d^2},G=\frac{c_p\beta}{g},p_1=\frac{\varepsilon\nu}{\kappa},R=\frac{g\alpha\beta d^4}{\nu\kappa}, S=\frac{g\alpha'\beta'd^4}{\nu\kappa'},M=\frac{\varepsilon N}{N_0},\omega=\frac{KN_0d^2}{\rho_m\nu\varepsilon},$$

$$\tau=\frac{m\kappa}{Kd^2},f=\frac{mN_0}{\rho_m}=\tau\omega p \ and \ \lambda=\frac{\kappa}{\kappa'},$$

and starred $(*)$ quantities are expressed in dimensionless form. Hereafter, we suppress the stars for convenience.

Eliminating $\overrightarrow{V_d}$ from equation (14) with the help of equation (16) and then eliminating $u,v,\delta p$ from the three scalar equations of (14), and using equation (15), we obtain

$$\left[L_1-L_2\left(\frac{1}{\varepsilon}\nabla^2-\frac{1}{P}\right)\right]\nabla^2 w = L_2[R\nabla_1^2\theta-S\nabla_1^2\gamma], \tag{20}$$

$$L_2\left[(E+h\varepsilon)\frac{\partial}{\partial t}-\nabla^2\right]\theta=\left(\frac{G-1}{G}\right)\left(\tau\frac{\partial}{\partial t}+H\right)w, \tag{21}$$

$$L_2\left[(E'+h'\varepsilon)\frac{\partial}{\partial t}-\frac{1}{\lambda}\nabla^2\right]\gamma=\left(\tau\frac{\partial}{\partial t}+H'\right)w, \tag{22}$$

where

$$L_1=p_1^{-1}\left(\tau\frac{\partial^2}{\partial t^2}+F\frac{\partial}{\partial t}\right), L_2=\left(\tau\frac{\partial}{\partial t}+1\right),\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},$$

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2},F=f+1,H=h+1,H'=h'+1.$$

Analyzing the perturbations into normal modes by seeking solutions in the form of functions of x,y and t

$$[w,\theta,\gamma]=[W(z),\Theta(z),\Gamma(z)]\exp\left(ik_x x+ik_y y+nt\right), \tag{23}$$

where n is, in general, complex, and $k=\left(k_x^2+k_y^2\right)^{1/2}$ is the wave number of disturbance.

Eliminating θ,γ between equations (20) – (22) and using expression (23), we obtain

$$\left[L_1+\frac{L_2}{P}-\frac{L_2}{\varepsilon}(D^2-k^2)\right][D^2-k^2-n(E+h\varepsilon)][D^2-k^2-\lambda n(E'+h'\varepsilon)](D^2-k^2)W$$
$$=\left(\frac{G-1}{G}\right)(\tau n+H)Rk^2[D^2-k^2-\lambda n(E'+h'\varepsilon)]W-\lambda(\tau n+H')Sk^2[D^2-k^2-n(E+h\varepsilon)]W, \tag{24}$$

where

$$L_1=p_1^{-1}(\tau n^2+Fn), L_2=\tau n+1 \ \ and \ \ D=\frac{d}{dz}.$$

3 Some Important Theorems

Theorem I: The stable solute gradient introduces oscillatory modes in the system while in its absence principle of exchange of stabilities is satisfied.

Proof: Let

$$U=(D^2-k^2)W \ \ and \ \ X=\left[L_1+\frac{L_2}{P}-\frac{L_2}{\varepsilon}(D^2-k^2)\right]U. \tag{25}$$

In terms of X, the equation satisfied by W is

$$[D^2-k^2-n(E+h\varepsilon)][D^2-k^2-\lambda n(E'+h'\varepsilon)]X=k^2\left(\frac{G-1}{G}\right)R(\tau n+H)$$
$$[D^2-k^2-\lambda n(E'+h'\varepsilon)]W-\lambda k^2S(\tau n+H')[D^2-k^2-n(E+h\varepsilon)]W. \tag{26}$$

Consider the case of two free surfaces having uniform temperature and solute concentration. The boundary conditions appropriate for the problem are

$$W = D^2W = 0, \Theta = \Gamma = 0 \; at \; z = 0 \; and \; 1. \tag{27}$$

Multiplying equation (26) by X^*, the complex conjugate of X, integrating over the range of z and using the boundary conditions (27), we obtain

$$\begin{aligned} I_1 + n[(E+h\varepsilon) + \lambda(E'+h'\varepsilon)]I_2 &+ \lambda n^2(E+h\varepsilon)(E'+h'\varepsilon)I_3 = k^2\left(\frac{G-1}{G}\right)R(\tau n + H) \\ \left(L_1^* + \frac{L_2^*}{P}\right)[I_4 + \lambda n(E'+h'\varepsilon)I_5] &- \lambda k^2 S(\tau n + H')\left(L_1^* + \frac{L_2^*}{P}\right)[I_4 + \lambda n(E+h'\varepsilon)I_5] + \\ +k^2\frac{L_2^*}{\varepsilon}\left[\left(\frac{G-1}{G}\right)R(\tau n+H) - \lambda S(\tau n + H')\right]I_6 &+ \\ +k^2\lambda n\frac{L_2^*}{\varepsilon}\left[\left(\frac{G-1}{G}\right)R(\tau n+H)(E'+h'\varepsilon) - S(\tau n+H')(E+h\varepsilon)\right]I_7 &, \end{aligned} \tag{28}$$

where

$$I_1 = \int_0^1 (|D^2X|^2 + 2k^2|DX|^2 + k^4|X|^2)dz, \quad I_2 = \int_0^1 (|DX|^2 + k^2|X|^2)dz, \quad I_3 = \int_0^1 (|X|^2)dz,$$

$$I_4 = \int_0^1 (|U|^2)dz, \quad I_5 = \int_0^1 (|DW|^2 + k^2|W|^2)dz, \quad I_6 = \int_0^1 (|DU|^2 + k^2|U|^2)dz,$$

$$I_7 = \int_0^1 (|D^2W|^2 + 2k^2|DW|^2 + k^4|W|^2)dz \; . \tag{29}$$

The integrals $I_1 - I_7$ are all positive definite.

Putting $= in_0$, where n_0 is real, into equation (28) and equating imaginary parts, we obtain

$$n_0^2 = \frac{\left\{\begin{array}{c} [(E+h\varepsilon)+\lambda(E'+h'\varepsilon)]I_2 + k^2\left[\begin{array}{c}\left(\frac{G-1}{G}\right)R\left(\frac{HF}{p_1}+\frac{\tau h}{P}\right)- \\ \lambda S\left(\frac{H'F}{p_1}+\frac{\tau h'}{P}\right)\end{array}\right]I_4 + \\ \lambda k^2\left[S(E+h\varepsilon)H' - \left(\frac{G-1}{G}\right)R(E'+h'\varepsilon)H\right] \\ \left(\frac{I_5}{P}+\frac{I_7}{\varepsilon}\right) + \frac{\tau k^2}{\varepsilon}\left[\left(\frac{G-1}{G}\right)Rh - \lambda Sh'\right]I_6 \end{array}\right\}}{\left\{\begin{array}{c} \lambda k^2\left[\begin{array}{c} -\left(\frac{G-1}{G}\right)R(E'+h'\varepsilon)\left\{\frac{\tau(f-h)}{p_1}+\frac{\tau^2}{P}\right\} + \\ S(E+h\varepsilon)\left\{\frac{\tau(f-h)}{p_1}+\frac{\tau^2}{P}\right\}\end{array}\right]I_5 + \\ \frac{k^2\tau^2}{p_1}\left\{\left(\frac{G-1}{G}\right)R - \lambda S\right\}I_4 + \\ \frac{k^2\tau^2\lambda}{\varepsilon}\left[-\left(\frac{G-1}{G}\right)R(E'+h'\varepsilon) + S(E+h\varepsilon)\right]I_7 \end{array}\right\}}, \tag{30}$$

or

$$n_0 = 0 \; . \tag{31}$$

In the absence of stable solute gradient, equations (30) and (31) become

$$n_0^2 = -\frac{\left(\frac{G}{G-1}\right)(E+h\varepsilon)I_2 + k^2R\left\{p_1^{-1}HF + \frac{\tau h}{P}\right\}I_4 + \frac{k^2\tau}{\varepsilon}RhI_6}{k^2\tau^2Rp_1^{-1}I_4} \; , \tag{32}$$

or

$$n_0 = 0 \; . \tag{33}$$

Since the integrals are positive definite and n_0 is real. **It follows that $n_0 = 0$ and the principle of exchange of stabilities is satisfied, in the absence of stable solute gradient. In the presence of stable solute gradient, the principle of exchange of stabilities is not satisfied and oscillatory modes come into play. The stable solute gradient, thus, introduces oscillatory modes which were non-existent in its absence.**

Theorem II: For the case of stationary convection, the medium permeability and suspended particles have destabilizing effects, whereas the stable solute gradient has a stabilizing effect on the system.

Proof: When instability sets in as stationary convection, the marginal state will be characterized by $n = 0$ and equation (24) reduces to

$$\left[\frac{1}{P}-\frac{1}{\varepsilon}(D^2-k^2)\right](D^2-k^2)^2W = \left(\frac{G-1}{G}\right)k^2RHW - \lambda k^2SH'W . \quad (34)$$

Considering the case of two free boundaries, it can be shown that all the even order derivatives of W vanish on the boundaries and hence the proper solution of equation (34) characterizing the lowest mode is

$$W = W_0 \sin \pi z \ , \quad (35)$$

where W_0 is a constant. Substituting the solution (35) in equation (34), we obtain

$$R = \frac{\left(\frac{G}{G-1}\right)\left[\left(\frac{1}{P}+\frac{\pi^2+k^2}{\varepsilon}\right)(\pi^2+k^2)^2 + \lambda k^2 H'S\right]}{k^2H} . \quad (36)$$

If R_c denotes the critical Rayleigh number in the absence of compressibility and $\overline{R_c}$ stands for the critical Rayleigh number in the presence of compressibility, then we find that

$$\overline{R_c} = \left(\frac{G}{G-1}\right)R_c \ .$$

Since critical Rayleigh number is positive and finite, so $G > 1$ and we obtain a stabilizing effect of compressibility as its result is to postpone the onset of double-diffusive convection in a fluid-particle layer of porous medium.

It is evident from equation (36) that

$$\frac{dR}{dP} = -\left(\frac{G}{G-1}\right)\frac{(\pi^2+k^2)^2}{k^2HP^2} \ , \quad (37)$$

$$\frac{dR}{dH} = -\left(\frac{G}{G-1}\right)\frac{\left(\frac{1}{P}+\frac{\pi^2+k^2}{\varepsilon}\right)(\pi^2+k^2)^2 + \lambda k^2H'S}{k^2H^2} \ , \quad (38)$$

and

$$\frac{dR}{dS} = \lambda\left(\frac{G}{G-1}\right)\frac{H'}{H} \ . \quad (39)$$

The medium permeability and suspended particles have thus destabilizing effects, whereas the stable solute gradient has a stabilizing effect on the thermosolutal convection in compressible fluids with suspended particles in a porous medium.

4 Effect of Rotation

Formulation of the Problem: In this section, we consider the same problem as that studied above except that the system is in a state of uniform rotation $\vec{\Omega}(0,0,\Omega)$. The Coriolis force acting on the particles is also neglected under the assumptions made in the problem. The linearized non-dimensional perturbation equations of motion for the fluid are

$$p_1^{-1}\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p + \omega(l-u) + T_A^{1/2}v + \left(\frac{1}{\varepsilon}\nabla^2 - \frac{1}{P}\right)u, \quad (40)$$

$$p_1^{-1}\frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p + \omega(r-v) - T_A^{1/2}u + \left(\frac{1}{\varepsilon}\nabla^2 - \frac{1}{P}\right)v, \quad (41)$$

$$p_1^{-1}\frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p + \omega(s-w) + R\theta - S\gamma + \left(\frac{1}{\varepsilon}\nabla^2 - \frac{1}{P}\right)w, \quad (42)$$

where $T_A = \frac{4\Omega^2 d^4}{\varepsilon^2 \nu^2}$ is the non-dimensional number accounting for rotation, and equations (14) – (19) remain unaltered.

Eliminating $\overrightarrow{V_d}(l,r,s)$ with the help of (16) and then eliminating $u, v, \delta p$ between equations (40) – (42), we obtain

$$\left(L_1 + \frac{L_2}{P} - \frac{L_2}{\varepsilon}\nabla^2\right)^2 \nabla^2 w + L_2^2 T_A \frac{\partial^2 w}{\partial z^2} = L_2\left(L_1 + \frac{L_2}{P} - \frac{L_2}{\varepsilon}\nabla^2\right)\nabla_1^2(R\theta - \lambda S\gamma). \quad (43)$$

Eliminating θ and γ between equations (21), (22) and (43) and using expression (23), we get

$$[D^2-k^2-n(E+h\varepsilon)][D^2-k^2-\lambda n(E'+h'\varepsilon)]\left[\left\{L_1+\frac{L_2}{P}-\frac{L_2}{\varepsilon}(D^2-k^2)\right\}^2(D^2-k^2)+L_2^2T_AD^2\right]W=\left\{L_1+\frac{L_2}{P}-\frac{L_2}{\varepsilon}(D^2-k^2)\right\}k^2\left[\left(\frac{G-1}{G}\right)\{D^2-k^2-\lambda n(E'+h'\varepsilon)\}(\tau n+H)R-\lambda\{D^2-k^2-n(E+h\varepsilon)\}(\tau n+H')S\right]W\ . \quad (44)$$

Theorem III: For the case of stationary convection, the suspended particles have a destabilizing effect, whereas the rotation and stable solute gradient have stabilizing effects on the system under consideration. The medium permeability has both stabilizing and destabilizing effects, depending on the rotation parameter.

Proof: For the stationary convection $n=0$ and equation (44) reduces to

$$(D^2-k^2)\left[\left\{\frac{1}{P}-\frac{(D^2-k^2)}{\varepsilon}\right\}^2(D^2-k^2)+T_AD^2\right]W=k^2\left\{\frac{1}{P}-\frac{(D^2-k^2)}{\varepsilon}\right\}\left[\left(\frac{G-1}{G}\right)RH-\lambda SH'\right]W\ . \quad (45)$$

Considering again the case of two free boundaries with constant temperature and solute concentration and using the proper solution (35), we obtain from equation (45)

$$R=\left(\frac{G}{G-1}\right)\left[\frac{\pi^2+k^2}{k^2H}\left\{\frac{(\pi^2+k^2)\left(\frac{1}{P}+\frac{\pi^2+k^2}{\varepsilon}\right)^2+\pi^2T_A}{\left(\frac{1}{P}+\frac{\pi^2+k^2}{\varepsilon}\right)}\right\}+\lambda S\frac{H'}{H}\right]. \quad (46)$$

It is evident from equation (46) that

$$\frac{dR}{dT_A}=\left(\frac{G}{G-1}\right)\frac{\pi^2(\pi^2+k^2)}{\left(\frac{1}{P}+\frac{\pi^2+k^2}{\varepsilon}\right)k^2H}\ , \quad (47)$$

$$\frac{dR}{dH}=-\left(\frac{G}{G-1}\right)(\pi^2+k^2)\left[\frac{\left(\frac{1}{P}+\frac{\pi^2+k^2}{\varepsilon}\right)^2(\pi^2+k^2)+\pi^2T_A}{\left(\frac{1}{P}+\frac{\pi^2+k^2}{\varepsilon}\right)k^2H^2}+\lambda S\frac{H'}{H^2}\right], \quad (48)$$

$$\frac{dR}{dS}=\lambda\left(\frac{G}{G-1}\right)\frac{H'}{H}\ . \quad (49)$$

Therefore the suspended particles have a destabilizing effect, whereas the rotation and stable solute gradient have stabilizing effects on the system under consideration.

Equation (46) also yields

$$\frac{dR}{dP}=\left(\frac{G}{G-1}\right)\frac{(\pi^2+k^2)}{k^2H}\left[-\frac{\pi^2+k^2}{P^2}+\frac{\pi^2T_A}{P^2\left(\frac{1}{P}+\frac{\pi^2+k^2}{\varepsilon}\right)^2}\right]. \quad (50)$$

If

$$T_A > \left(1 + \frac{k^2}{\pi^2}\right)\left(\frac{1}{P} + \frac{\pi^2 + k^2}{\varepsilon}\right)^2 ,$$

then dR/dP is positive.

If

$$T_A < \left(1 + \frac{k^2}{\pi^2}\right)\left(\frac{1}{P} + \frac{\pi^2 + k^2}{\varepsilon}\right)^2 ,$$

then dR/dP is negative.

Thus **the medium permeability has both stabilizing and destabilizing effects, depending on the rotation parameter**, whereas in the absence of rotation as concluded earlier from equation (36) that medium permeability always has destabilizing effect.

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