A Hybrid Model for Decision Making Utilizing TFNs and Soft Sets as Tools

MICHAEL GR. VOSKOGLOU
Mathematical Sciences, School of Technological Applications
University of Peloponnese (ex T.E.I. of Western Greece)
Meg. Alexandrou 1, 26334 Patras
GREECE

Abstract: - Decision making is the process of evaluating multiple alternatives to choose the one satisfying in the best way the existing goals. Frequently, however, the boundaries of those goals and/or of the existing constraints are not sharply defined due to the various forms of uncertainty appearing in the corresponding problems. Since the classical methods cannot be applied in such cases, several methods for decision making under fuzzy conditions have been proposed. In this work a new hybrid decision making method is developed utilizing triangular fuzzy numbers (TFNs) and soft sets as tools, which improves an earlier method of Maji, Roy and Biswas, which uses soft sets only. The importance of this improvement is illustrated by an application concerning the decision for the purchase of a house satisfying in the best possible way the goals put by the candidate buyer.

Key-Words: - Decision making, fuzzy logic, fuzzy set (FS), triangular fuzzy number (TFN), soft sets, intelligent computing, soft computing

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1 Introduction

Decision making is the process of evaluating, with the help of suitable criteria, multiple alternatives to choose the one satisfying in the best way the existing goals. In decision making under fuzzy conditions the boundaries of the goals and/or of the existing constraints are not sharply defined. Several methods for decision making in such cases have been proposed, e.g. [1-5], etc. Maji et al. [6] used a tabular form of a *soft set* in the form of a binary matrix to introduce a type of parametric decision making method. Their method, however, replaces the characterizations (parameters; e.g. beautiful) of the elements of the set of the discourse (houses in their example) with the binary elements 0, 1 representing the truth values of those parameters. In other words, although their method starts from a fuzzy framework (soft sets), it uses bivalent logic for making the required decision (e.g. beautiful or not beautiful)! Consequently, this approach could lead to a wrong decision, when some (or all) of the parameters have not a bivalent texture; e.g. the parameter “wooden” characterizing a house has a bivalent texture, but not the parameter “beautiful”.

In this work, in order to tackle this problem, we modify the method of Maji et al. by using *triangular fuzzy numbers (TFNs)* instead of the binary elements 0, 1. The rest of the paper is organized as follows: In section 2 the basic information about TFNs and soft sets is given which is necessary for the understanding of the paper. In section 3 the modified decision making method is presented and compared with the method of Maji et al. through a suitable example. The paper closes with the final conclusions and some hints for future research included in section 4.

2 Preliminaries

The frequently appearing in real world, in science and in everyday life uncertainty is due to several reasons, like randomness, imprecise or incomplete data, vague information, etc. Probability theory has been proved sufficient for dealing with the cases of uncertainty due to randomness. During the last 50-60 years, however, various mathematical theories have been introduced for tackling effectively the other forms of uncertainty, including fuzzy sets [7], intuitionistic fuzzy sets [8], neutrosophic sets [9], rough sets [10] and several others [11]. The combination of two or more of those theories gives in many cases better results for the solution of the corresponding problems and that is what we are going to attempt here for decision making

2.1 Triangular Fuzzy Numbers

Definition 1: Let U be the universal set of the discourse, then a *fuzzy set (FS)* A on U is defined with the help of its *membership function m*: $U \to [0,1]$ as the set of the ordered pairs

$$A = \{(x, m(x)): x \in U\} \quad (1)$$

The real number $m(x)$ is called the *membership degree* of x in A**.** The greater is $m(x)$, the more x satisfies the characteristic property of A. Many authors, for reasons of simplicity, identify a fuzzy set with its membership function.

A crisp subset A of U is a fuzzy set on U with membership function taking the values $m(x)$=1, if x belongs to A, and 0 otherwise.

Zadeh [12] has also introduced FNs as follows:

Definition 2: A FN is a FS A on the set ***R*** of real numbers with membership function m: $\boldsymbol{R} \to [0, 1]$, such that:

- A is *normal*, i.e. there exists x in ***R*** such that $m(x) = 1$,
- A is *convex*, i.e. all its *a-cuts* $A^a = \{x \in U: m_A(x) \geq \alpha\}$, α in [0, 1], are closed real intervals, and
- Its membership function $y = m(x)$ is a piecewise continuous function.

Arithmetic operations between FNs have been introduced in two, equivalent to each other, methods: (i) By applying the Zadeh's extension principle [13, Section 1.4, p.20], which provides the means for any function f mapping a crisp set X to a crisp set Y to be generalized so that to map fuzzy subsets of X to fuzzy subsets of Y and (ii) with the help of the α-cuts of the corresponding FNs and the representation-decomposition theorem of Ralesscou - Negoita [14, Theorem 2.1, p.16) for FS. With the second method the fuzzy arithmetic is turned to the arithmetic of the closed real intervals.

In practice, however, the previous two general methods of fuzzy arithmetic requiring laborious calculations are rarely used in applications, where the utilization of simpler forms of FNs is preferred. For general facts on FNs we refer to the book [15].

TFNs is the simplest form of FNs. A TFN is defined as follows:

Definition 3: Let a, b and c be real numbers with $a < b < c$. Then the TFN (a, b, c) is a FN with membership function:

$$y = m(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b] \\ \dfrac{c-x}{c-b}, & x \in [b,c] \\ 0, & x < a \text{ or } x > c \end{cases}$$

Proposition 1: The coordinates (X, Y) of the *centre of gravity (COG)* of the graph of the TFN *(a, b, c)* are calculated by the formulas $X = \frac{a+b+c}{3}$, $Y = \frac{1}{3}$.

Proof: The graph of the TFN (a, b, c) is the triangle ABC of Fig. 1, with A (a, 0), B(b, 1) and C (c, 0).

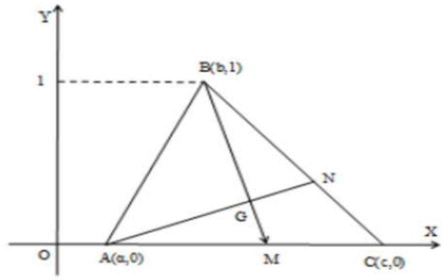

Figure 1: Graph and COG of the TFN (a, b, c)

Then, the COG, say G, of ABC is the intersection point of its medians AN and BM. The proof of the Proposition is easily obtained by writing down the equations of AN and BM and by solving the linear system of those two equations.-

The x-coordinate of the COG of a TFN A= (a, b, c) is usually considered as the crisp representative of A (*defuzzification* of A). We write then

$$D(A) = \frac{a+b+c}{3} \quad (2)$$

It can be shown [15] that the two previously mentioned general methods of defining arithmetic operations between FNs reduce to the following simple rules for the addition and subtraction of TFNs:

Let A = (a, b, c) and B = (a_1, b_1, c_1) be two TFNs. Then

$$A + B = (a+a_1, b+b_1, c+c_1) \quad (3), \text{ and}$$
$$A - B = A + (-B) = (a-c_1, b-b_1, c-a_1) \quad (4),$$

where $-B = (-c_1, -b_1, -a_1)$ is defined to be the opposite of B.

On the contrary, the product and the quotient of two TFNs, although they are FNs, need not be TFNs.

Further the scalar product of a real number k with a TFN A(a, b, c) is defined by

kA = (ka, kb, kc), if k>0 and kA = (kc, kb, ka), if k<0 **(5)**

2.2 Soft Sets

Molodstov [16] introduced in 1999 the concept of soft set for tackling the uncertainty created by a set of parameters characterizing the elements of the set of the discourse.
Let E be a set of parameters, let Q be a subset of E and let g: Q → P(V) be a map from Q to the power set P(V) of the universal set V. Then the soft set on V defined with the help of g and Q and denoted by (g, Q) is the set of ordered pairs:

$$(g, Q) = \{(q, g(q)): q \in Q\} \quad (6)$$

The characterization "soft" is related to the fact that the form of the set (g, Q) depends on the parameters. For example, let V = {H_1, H_2, H_3, H_4, H_5, H_6} be a set of houses and let E = {e_1, e_2, e_3, e_4, 5_5} be the set of the parameters e_1=beautiful, e_2=wooden, e_3=in the country, e_4=modern and e_5=cheap. Consider the subset Q = {e_1, e_2, e_3, e_5} of E and assume that H_1, H_2, H_6 are the beautiful, H_2, H_3, H_5, H_6 are the wooden, H_3, H_5 are the country houses and H_4 is the unique cheap house. Then a map g: Q → P(V) is defined by g(e_1) = {C_1, C_2, C_6}, g(e_2) = { H_2, H_3, H_5, H_6}, g(e_3) = {H_3, H_5}, g(e_5) = { H_4} and the soft set

$$(g, Q) = \{(e_1, \{H_1, H_2, H_6\}), (e_2, \{H_2, H_3, H_5, H_6\}), (e_3, \{H_3, H_5\}), (e_5, \{H_4\})\} \quad (7)$$

A FS on V, with membership function *y=m(x)* is a soft set (g, [0, 1]) on V with g(a)={$x \in$ V: $m(x) \geq a$}, for each *a* in [0, 1].
The use of soft sets instead of FSs has, among others, the advantage that, by using the parameters, one overpasses the existing difficulty of defining properly the membership function of a FS. In fact, the definition of the membership function is not unique, depending on the observer's subjective criteria [13]. In case of determining the FS of the cheap houses under sale in a certain area of a city, for example, the membership function could be defined in several ways, according to the financial power of each candidate buyer.
Maji et al. [17] introduced a tabular representation of soft sets in the form of a binary matrix in order to be stored easily in a computer's memory. For example, the tabular representation of the soft set (5) is given in Table 1.

Table 1: Tabular representation of the soft set (g, Q)

	e_1	e_2	e_3	e_5
H_1	1	0	0	0
H_2	1	1	0	0
H_3	0	1	1	0
H_4	0	0	0	1
H_5	0	1	1	0
H_6	1	1	0	0

Soft sets have found important applications to several sectors of the human activity like decision making, parameter reduction, data clustering and data dealing with incompleteness, etc. [18]. One of the most important steps for the theory of soft sets was to define mappings on soft sets, which was achieved by Kharal & Ahmad [19] and was applied to the problem of medical diagnosis in medical expert systems. But fuzzy mathematics has also significantly developed at the theoretical level providing important insights even into branches of classical mathematics like algebra, analysis, geometry, topology etc. For example, one can extend the concept of topological space, the most general category of mathematical spaces, to fuzzy structures and in particular can define soft topological spaces and generalize the concepts of convergence, continuity and compactness within such kind of spaces [20]. The present author has recently used soft sets as tools in assessment problems [21].

The Hybrid Decision Making Method

3.1 The Method of Maji et al.

Maji et al. [17] used the tabular form of a soft set described in section 2.2 as a tool for decision making in a parametric manner. Here, we use the example of section 2.2 to highlight their method.
For this, assume that one is interested to buy a beautiful, wooden, country and cheap house by choosing among the six houses of the previous example. Forming the tabular representation of the soft set (g, Q) (Table 1) the *choice value* of each house is calculated by adding the binary elements of the corresponding row of it. The houses H_1 and H_4 have, therefore, choice value 1 and all the others have choice value 2. Consequently, the buyer must

choose one of the houses H_2, H_3, H_5 or H_6, which is not a very helpful decision.

3.2 The New Hybrid Method

Observe now that the parameters e_1 and e_5 have not a bivalent texture. In fact, how beautiful is a house depends on the subjective opinion of each person, whereas its low or high price depends on the financial ability of the candidate buyer. In other words, the previous decision could not be the best one.

To tackle this problem, we replace in Table 1 the binary elements 0, 1 for the parameters e_1 and e_5 by the TFNs G_1 = (0.85, 0.925, 1), G_2 = (0.75, 0.795, 0.84), G_3 = (0.6, 0.67, 0.74], G_4 = (0.5, 0.545, 0.59) and G_5 = (0, 0.245, 0.49). Assume further that the candidate buyer, after studying the existing information about the six houses, decided to use Table 2 for making the final decision.

Table 2: Revised tabular representation of the soft set (g, Q)

	e_1	e_2	e_3	e_5
H_1	G_1	0	0	G_3
H_2	G_1	1	0	G_5
H_3	G_3	1	1	G_3
H_4	G_4	0	0	G_1
H_5	G_4	1	1	G_3
H_6	G_1	1	0	G_4

From Table 2 we calculate the choice value V_i of the house Hi, i=1, 2, 3, 4, 5, 6 as follows:
V_1=D(G_1+G_3), or by (3) V_1= D(1.45, 1.595, 1.74) and finally by (2) $V_1=\frac{1.45+1.595+1.74}{3}$ =1.595.
Similarly V_2=1+D(G_1+G_5)=2.17,
V_3=2+D(G_3+G_3)=3.34, V_4=D(G_4+G_1)=1.47,
V_5=2+D(G_4+G_1)=3.215, V_6=1+D(G_1+G_4)=2.47.
Therefore, the right decision is to buy the house H_3.

3.3 Remarks

I) The choice of the extreme entries of the TFNs G_i, i=1, 2, 2, 4, 5, was made according to the accepted standards for the linguistic grades excellent (A), very good (B), good (C), almost good (D) and unsatisfactory (F) respectively. Their choice however is not unique and could slightly differ from case to case according to the goals of the decision maker. The middle entry of each TFN is equal to the mean value of its two extreme entries.
II) If a parameter takes the value 0 (or 1) for all elements of the set of the discourse, then it can be withdrawn from the tabular form of the corresponding soft set, since it does not affect the final decision. The resulting table in those cases is called a *reduct table* of the soft set. When all the parameters having the previous property have been withdrawn, then the resulting reduct table is called the *core* of the soft set [17]. Obviously the core of a soft set is contained in all reduct tables of it.
III) An analogous to the previous one decision making method could be introduced by using *grey numbers* instead of TFNs [22].

3.4 Weighted Decision Making

Frequently, the goals put by the decision-maker have not the same importance. In this case, and in order to make the proper decision, weight coefficients must be assigned to each parameter.

For instance, assume that in the previous example the candidate buyer has assigned the weight coefficients 0.9 to e_1, 0.7 to e_2, 0.6 to e_3 and 0.5 to the parameter e_5. Then, the weighted choice values of the houses are calculated with the help of Table 2 as follows:
V_1=D(0.9G_1+0.5G_3), or by (2) and (3) V_1=1.46.
Similarly V_2=0.7+D(0.9G_1+0.5G_5) =1.655,
V_3=0.7+0.6+D(0.9G_3+0.5G_3)=2.238,
V_4=D(0.9G_4+0.5G_1)=0.953,V_5=0.7+0.6 +D(0.9G_4+0.5G_3)=2.1255,
V_6=0.7+W(0.9G_1+0.5G_4)=1.805.
Consequently, the right decision is again to buy the house H_3.

4. Discussion and Conclusions

In this work a parametric decision making method was introduced using TFNs and soft sets as tools. As it was illustrated by the given example, this hybrid approach is very useful in cases where one or more parameters have not a bivalent texture (beautiful and cheap houses in our example), because it enables one to make the best decision, which is possibly not possible to be succeeded by using only soft sets, as the method of Maji et al. suggests.

The combination of two or more of the theories that have been developed for tackling efficiently the various forms of the existing in real world, everyday life and science uncertainty, appears in general to be an effective tool for obtaining better results, not only for decision making, but also for assessment and for a variety of other human activities. Consequently this is a promising area for future research.

References:

[1] Bellman, R.A., Zadeh, L.A., Decision Making in Fuzzy Environment, *Management Science*, 17, 1970, pp. 141-164.

[2] Alcantud, J.C.R. (Ed.), *Fuzzy Techniques for Decision Making* (reprint of the special issue published in *Symmetry*), MDPI, Basel, Switzerland, 2018.

[3] Fahad Kh. A.O.H. Alazemi, Mohd Khairol Anuar Bin, Mohd Ariffin , Faizal Bin Mustapha & Eris Elianddy bin Supeni, A Comprehensive Fuzzy Decision-Making Method for Minimizing Completion Time in Manufacturing Process in Supply Chains, *Mathematics*, 9, 2021, article 2019.

[4] Khan, A., Yang, M.-S., Haq, M., Shah, A. A & Arif, M., A New Approach for Normal Parameter Reduction Using σ-Algebraic Soft Sets and Its Application in Multi-Attribute Decision Making, *Mathematics*, 10, 2022, article 1297.

[5] Zhu, B. & Ren, P., Type-2 fuzzy numbers made simple in decision making, *Fuzzy Optimization and Decision Making*, 21, 2022, pp.175-195.

[6] Maji, P.K., Roy, A.R., Biswas, R., An Application of Soft Sets in a Decision Making Problem, *Computers and Mathematics with Applications*, 44, 2002, pp. 1077-1083.

[7] Zadeh, LA. Fuzzy Sets, *Information and Control*, 8, 1965, pp. 338-353.

[8] Atanassov, K.T., Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*, 20(1), 1986, pp.87-96.

[9] Smarandache, F., *Neutrosophy / Neutrosophic probability, set, and logic*, Proquest, Michigan, USA, 1998.

[10] Pawlak, Z. (1991}, *Rough Sets: Aspects of Reasoning about Data*, Kluer Academic Publishers, Dordrecht.

[11] Voskoglou, M.Gr., Fuzzy Systems, Extensions and Relative Theories, *WSEAS Transactions on Advances in Engineering Education*, 16, 2019, pp. 63-69

[12] Zadeh, L.A.**,** The Concept of a Linguistic Variable and its Application to Approximate Reasoning, Parts 1–3. *Information Science*, 8, 1975, pp.199–249 and 301–357, 9, pp.43-80.

[13] Klir, G. J. & Folger, T. A., *Fuzzy Sets, Uncertainty and Information*, Prentice-Hall, London, 1988.

[14] Sakawa, M., *Fuzzy Sets and Interactive Multiobjective Optimization,* Plenum press, NY and London, 1993.

[15] Kaufmann, A. & Gupta, M., *Introduction to Fuzzy Arithmetic*, Van Nostrand Reinhold Company, New York, 1991.

[16] Molodtsov, D. , Soft Set Theory-First Results, *Computers and Mathematics with Applications*, 37(4-5), 1999, pp. 19-31.

[17] Maji, P.K., Biswas, R., & Ray, A.R., Soft Set Theory, *Computers and Mathematics with Applications*, 45, 2003, pp. 555-562,.

[18] Tripathy, B.K., Arun, K.R. (2016), Soft Sets and Its Applications, in J.S. Jacob (Ed.), *Handbook of Research on Generalized and Hybrid Set Structures and Applications for Soft Computing*, IGI Global, Hersey, PA, 2016, pp. 65-85.

[19] Kharal, A. & Ahmad, B., Mappings on Soft Classes, *New Mathematics and Natural Computation*, 7(3), 2011, pp. 471-481.

[20] Shabir, M. & Naz M. (2011), On Soft Topological Spaces, *Computers and Mathematics with Applications*, 61, 2011, pp.1786-1799.

[21] Voskoglou, M.Gr., Using Soft Sets for a Parametric Assessment of Problem Solving Skills, *International Journal of Circuits, Systems and Signal Processing*, 16, 2022, pp. 882-886.

[22] Voskoglou, M.Gr., A Combined Use of Soft Sets and Grey Numbers in Decision Making, *Journal of Computational and Cognitive Engineering*, accepted for publication, 2022