Mathematical model of fluid filtration to the production wells in the reservoirs with discontinuities

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Abstract. Recently, it has been proved very important to study the filtration process in the reservoir with discontinuities of permeability. These discontinuities may be either natural, like tectonic fault, or artificial, like hydraulic fractures, and it may have various permeability values. This paper considers the steady-state flow process of incompressible fluid to the production well in a reservoir of constant height and permeability. There is a thin area in the reservoir with constant permeability k_f, which might be highly permeable crack or low permeable barrier. The characteristics of filtration process are studied for various k_f values. The nature of fluid flow to the wellbore is analyzed at different locations of the well and the crack for different values of the fracture conductivity in this paper and the analytical expression for skin effect is defined.

Keywords - Filtration theory, oil reservoir discontinuity, production wells, underground fluid mechanics
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1. Introduction

In spite of the most oil fields are on the final stage of the field development, there are a lot of ways to maintain and to increase the production. The modern level of science and technology allows extracting oil more efficiently, taking into account the individual characteristics of reservoir and the behavior of fluids. The main part of fluid flow occurs through the more permeable zones, therefore any deviation in the reservoir homogeneity acts on the production [1]. For example, in fractured reservoirs the main flow of oil to the well occurs through the fractures [2]. A characteristic feature of the development process of such reservoirs has the deviation in well productivity and rock permeability, significant dependence of Inflow Performance Relationship (IPR) curves on the pressure, etc. Oil filtration modelling in fractured reservoirs is also interesting from the point of view of application for hydraulic fracturing. Hydraulic fracturing is currently one of the most effective methods to increase oil production [3, 4]. Therefore these days especially important to study the filtration process in the reservoir with tectonic faults, hydraulic fractures (HF) and impermeable boundaries. This paper discusses the modelling of the fluid flow process to the well in the presence of cracks (inclusions) with different permeability, studies the impact of such inclusions on the nature of the fluid flow process to the production well. The task is modified by the representation of cracks in the section view of zero thickness but finite conductivity and by the difference of pressure above and below the section. The issues of flow modeling inside the fracture have been investigated in the article [5].

2. Problem formulation

Let us consider a plane stationary flow of incompressible fluid to the vertical production well in an isotropic porous medium. This process in the plane *(x, y)* is described by the equation of incompressibility and the Darcy's law of filtration [1]:

$$\begin{aligned} div V(x,y) &= 0, \\ V &= -(k/\mu) grad\ p \end{aligned} \quad (1)$$

where ***V****(x, y)* is the velocity vector of fluid filtration, *p(x, y)* is the pressure in the liquid, μ is the fluid viscosity and k is the reservoir permeability by the thickness h. In the works of other authors the high permeable area is usually represented by ellipse [6, 7], while, the different way of problem solution uses the integrals of Cauchy type [8]. Let us consider, that in the reservoir with the external boundary of radius R_c at the point $M(x_0, y_0)$ is placed the production well of radius r_w with a flow rate Q. Inside the external boundary there is a crack with length $2l$ and thickness 2δ $(\delta << l)$ and permeability k_f. Let us consider the crack is oriented along the axis x, and its center coincides with the origin plane *(x, y)* (Fig. 1).

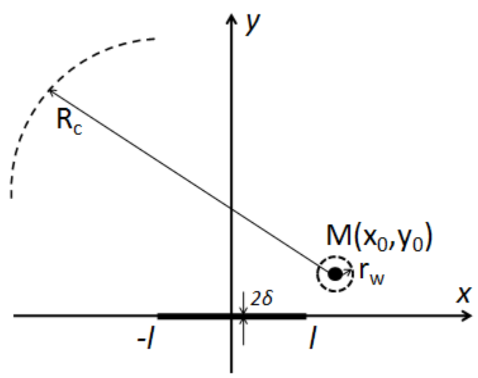

Figure 1 *Well and fracture places in the plane*

In the paper, published early by Astafiev and Fedorchenko [9], the problem was solved with assuming that the pressure is the same on the upper and lower banks of the crack. In this paper we consider the case with the difference of pressure.

3. Problem Solution

Suppose the crack is oriented along the *x*-axis, and its center coincides with the origin of the plane *(x,y)*. Next, we assume the borehole at the point $z_0=x_0+iy_0$, well flow rate *Q*, the radius of the drainage area R_C and well radius r_w (Fig. 1). Then, as have been described in work [9], the flow potential can be represented in the form:

$$\Phi(z) = q^{-1}\phi(z) = \ln(z - z_0) + \sum_{n=0}^{\infty} c_n z^{-n} \qquad (2)$$

where $q=\mu Q/(2\pi kh)$ is the modified flow rate, c_n is unknown coefficients in the expansion in a Laurent series of the disturbance caused by the presence of reservoir heterogeneity and decaying at infinity.

Because of $\delta<<l$, it was proposed [10, 11] to replace the ellipse with semi-axes *l* and *δ* by straight line section of zero thickness $(-1\leq\xi=x/l=\leq 1)$. Then the fluid flow in the fracture can be modeled as the following additional boundary conditions on the cut [10]:

$$\begin{cases} \alpha_0\sqrt{1-\xi^2}\,\dfrac{d}{d\xi}\mathrm{Re}(\Phi^+ + \Phi^-) = \mathrm{Im}(\Phi^+ - \Phi^-), \\ \beta_0\sqrt{1-\xi^2}\,\dfrac{d}{d\xi}\mathrm{Im}(\Phi^+ + \Phi^-) = -\mathrm{Re}(\Phi^+ - \Phi^-); \end{cases}$$
(3)

where Φ^+ and Φ^- is the flow potentials above and below the section, coefficient $\alpha_0=(\delta k_f)/(lk)$ is similar to F_{CD} for the hydraulic fractures [3] and $\beta_0=(\delta k)/(lk_f)$ is very important for the impermeable case.

We will look the function *Φ(z)* as the sum of even and odd function or as $\Phi(z)=\Phi_1(z)+\Phi_2(z)$, which are related as follows:

$\mathrm{Re}\Phi_1^+(z)=\mathrm{Re}\Phi_1^-(z)$; $\mathrm{Im}\Phi_1^+(z)=-\mathrm{Im}\Phi_1^-(z)$;

$\mathrm{Re}\Phi_2^+(z)=-\mathrm{Re}\Phi_2^-(z)$; $\mathrm{Im}\Phi_2^+(z)=\mathrm{Im}\Phi_2^-(z)$.

Then the boundary conditions (3) will be as:

$$\begin{cases} \alpha_0\sqrt{1-\xi^2}\,\dfrac{d}{d\xi}\mathrm{Re}(\Phi^+{}_1) = \mathrm{Im}(\Phi^+{}_1), \\ \beta_0\sqrt{1-\xi^2}\,\dfrac{d}{d\xi}\mathrm{Im}(\Phi^+{}_2) = -\mathrm{Re}(\Phi^+{}_2); \end{cases}$$
(4)

Mapping by the Zhukovsky function $z=l(v+v^{-1})/2$ the exterior of the section $-l<x<l$, $y=0$ on the exterior of a unit circle $|v|=1$, we can rewrite functions from the equation (2) as

$$\ln(z - z_0) = \ln\frac{l}{2} + \ln(\nu - \nu_0) - \sum_{n=0}^{\infty}\frac{1}{n}(\nu\nu_0)^{-n},$$
(5)

$$z^{-n} = \left(\frac{l}{2}\right)^{-n}\nu^{-n}\left(1+\frac{1}{\nu^2}\right)^{-n} = \left(\frac{l}{2}\right)^{-n}\nu^{-n}\sum_{m=0}^{\infty}k_m\nu^{-2m},$$
(6)

where k_n is new unknown coefficients in the expansion in a Laurent series of the disturbance in a variable ν caused by the presence of reservoir heterogeneity and decaying at infinity.

That is, the series from the equation (2) can be presented in the form:

$$\sum_{n=0}^{\infty} c_n z^{-n} = c_0 + \sum_{m=1}^{\infty} a_m \nu^{-m} \qquad (7)$$

Then if we substitute (5)-(7) in the equation (2), the potential in a new variable *v* can be written as:

$$\Phi(\nu) = \ln(\nu - \nu_0) + \sum_{n=0}^{\infty} a_n \nu^{-n},$$
(8)

where $lv(z)=z+(z^2-l^2)^{1/2}$, $lv(z_0)=z_0+(z_0^2-l^2)^{1/2}$, $|v|>1$, a_n is new unknown coefficients in the expansion in a Laurent series of the disturbance in a variable ν caused by the presence of reservoir heterogeneity and decaying at infinity.

Let us consider the variable *v* in the form $v=\xi+i\eta=\rho e^{i\theta}$, where $\xi=\rho\cos\theta$ and $\eta=\rho\sin\theta$. We consider the segment $-1<\xi<1$, therefore $\rho=1$. The upper part of the section will be $\eta=0_+$, that is $0<\theta<\pi$, while the lower part of the cut will be $\eta=0_-$, that is $-\pi<\theta<0$. Therefore, the system (4) can be presented in the next form:

$$\begin{cases} \alpha_0\left(\dfrac{d}{d\theta}\mathrm{Re}(\Phi^+{}_1)\right) = -\mathrm{Im}(\Phi^+{}_1), \\ \beta_0\left(\dfrac{d}{d\theta}\mathrm{Im}(\Phi^+{}_2)\right) = \mathrm{Re}(\Phi^+{}_2); \end{cases}$$
(9)

As it is given that $\rho=1$, we can rewrite $v=e^{i\theta}$, that is $d/d\theta=ivd/dv$. Thus the system (9) is presented as:

$$\begin{cases} \mathrm{Re}\left(\alpha_0 i\nu\dfrac{d\Phi^+{}_1}{d\nu}\right) = -\mathrm{Im}(\Phi^+{}_1), \\ \mathrm{Im}\left(\beta_0 i\nu\dfrac{d\Phi^+{}_2}{d\nu}\right) = \mathrm{Re}(\Phi^+{}_2); \end{cases}$$
(10)

Also, taking into account that $\mathrm{Re}(iz)=-\mathrm{Im}(z)$, and $\mathrm{Im}(iz)=\mathrm{Re}(z)$, then the system of equations (10) can be rewritten in the form:

$$\begin{cases} \operatorname{Im}\left(\alpha_0 \nu \dfrac{d\Phi^+{}_1}{d\nu} - \Phi^+{}_1\right) = 0, \\ \operatorname{Re}\left(\beta_0 \nu \dfrac{d\Phi^+{}_2}{d\nu} - \Phi^+{}_2\right) = 0. \end{cases} \quad (11)$$

The first equation in the system (11) is equivalent to the equation

$$\alpha_0 \nu \frac{d\Phi_1(\nu)}{d\nu} - \Phi_1(\nu) = \overline{\alpha_0 \nu \frac{d\Phi_1(\nu)}{d\nu}} - \overline{\Phi_1(\nu)} \quad (12)$$

and the second equation in the system (11) is equal to the equation

$$\beta_0 \nu \frac{d\Phi_2(\nu)}{d\nu} - \Phi_2(\nu) = -\overline{\beta_0 \nu \frac{d\Phi_2(\nu)}{d\nu}} - \overline{\Phi_2(\nu)}, \quad (13)$$

It remains to split $\Phi(\nu)$ into even and odd functions, that is, to find $\Phi_1(\nu)$ and $\Phi_2(\nu)$. So, if $\nu=e^{i\theta}$, we can rewrite (4) for even and odd functions separately in the form:

$$\Phi(\theta) = \ln(-\nu_0) - \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{\nu}{\nu_0}\right)^n + \sum_{n=0}^{\infty} a_n \nu^{-n}, \quad (14)$$

$$\Phi(-\theta) = \Phi(\bar{\nu}) = \Phi\left(\frac{1}{\nu}\right) = \ln(-\nu_0) - \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{1}{\nu\nu_0}\right)^n + \sum_{n=0}^{\infty} a_n \nu^n, \quad (15)$$

If we take into account that $\nu=e^{i\theta}$, the equations (14) and (15) can be rewritten:

$$\Phi(\theta) = \ln(-\nu_0) - \sum_{n=1}^{\infty} \frac{1}{n}(\nu_0)^{-n}(\cos n\theta + i\sin n\theta) + \sum_{n=0}^{\infty} a_n(\cos n\theta - i\sin n\theta), \quad (16)$$

$$\Phi(-\theta) = \ln(-\nu_0) - \sum_{n=1}^{\infty} \frac{1}{n}(\nu_0)^{-n}(\cos n\theta - i\sin n\theta) + \sum_{n=0}^{\infty} a_n(\cos n\theta + i\sin n\theta) \quad (17)$$

The real parts of the equations (16) and (17) are:

$$\operatorname{Re}\Phi(\theta) = \operatorname{Re}(-\nu_0) - \sum_{n=1}^{\infty} \frac{1}{n}(\operatorname{Re}\nu_0^{-n}\cos n\theta - \operatorname{Im}\nu_0^{-n}\sin n\theta) + \sum_{n=0}^{\infty}(\operatorname{Re}a_n\cos n\theta + \operatorname{Im}a_n\sin n\theta),$$

$$\operatorname{Re}\Phi(-\theta) = \operatorname{Re}(-\nu_0) - \sum_{n=1}^{\infty} \frac{1}{n}(\operatorname{Re}\nu_0^{-n}\cos n\theta + \operatorname{Im}\nu_0^{-n}\sin n\theta) + \sum_{n=0}^{\infty}(\operatorname{Re}a_n\cos n\theta - \operatorname{Im}a_n\sin n\theta).$$

That is, to satisfy $\operatorname{Re}\Phi(\theta)=\operatorname{Re}\Phi(-\theta)$ it is necessary $\operatorname{Im}\nu_0^{-n}=0$, $\operatorname{Im}a_n=0$, and $\nu_0^{-n}=\rho_0^{-n}(\cos n\theta_0 - i\sin n\theta_0)$. Therefore the condition $\operatorname{Re}\Phi^+=\operatorname{Re}\Phi^-$ will be executed if $\operatorname{Im}a_n=0$, $\theta_0=0, \pi$. In this case $\operatorname{Im}\Phi^+=-\operatorname{Im}\Phi^-$.

Consequently, $\Phi_1(\nu)=\operatorname{Re}\Phi(\nu)$ is real and $\Phi_2(\nu)=i\operatorname{Im}\Phi(\nu)$ is image, and so the flow potential:

$$\Phi(\nu) = \left[\ln(\rho_0) - \sum_{n=1}^{\infty} \frac{1}{n}(\nu_0)^{-n}\cos n\theta + \sum_{n=0}^{\infty} a_n\cos n\theta\right] - i\sum_{n=1}^{\infty} \frac{1}{n}(\nu_0)^{-n}\sin n\theta - i\sum_{n=0}^{\infty} a_n\sin n\theta \quad (18)$$

Now let us consider general case when $\nu_0=\rho_0 e^{i\theta}$ and $a_n=a_n^{(\alpha)}+ia_n^{(\beta)}$. Thus, the flow potential in this case would be:

$$\Phi(\theta) = \pm i\pi + \ln\rho_0 + i\theta - \sum_{n=1}^{\infty} \frac{1}{n}\rho_0^{-n} e^{-in\theta}\nu^n + \sum_{n=1}^{\infty}(a_n^{(\alpha)} + ia_n^{(\beta)})\nu^{-n} =$$

$$\left[\ln\rho_0 + i\pi - \sum_{n=1}^{\infty} \frac{\cos n\theta_0}{n\rho_0^n}\nu^n + \sum_{n=1}^{\infty} a_n^{(\alpha)}\nu^{-n}\right] + i\left[\theta_0 + \sum_{n=1}^{\infty} \frac{\sin n\theta_0}{n\rho_0^n}\nu^n + \sum_{n=1}^{\infty} a_n^{(\beta)}\nu^{-n}\right] = \varphi_1 + i\varphi_2$$

The real and the image parts of the coefficients φ_1 and φ_2 are:

$$\operatorname{Re}\varphi_1 = \ln\rho_0 - \sum_{n=1}^{\infty} \frac{\cos n\theta_0}{n\rho_0^n}\cos n\theta - \sum_{n=1}^{\infty} a_n^{(\alpha)}\cos n\theta;$$

$$\operatorname{Im}\varphi_1 = \pi - \sum_{n=1}^{\infty} \frac{\cos n\theta_0}{n\rho_0^n}\sin n\theta + \sum_{n=1}^{\infty} a_n^{(\alpha)}\sin n\theta;$$

$$\operatorname{Re}\varphi_2 = -\left[\sum_{n=1}^{\infty} \frac{\sin n\theta_0}{n\rho_0^n}\sin n\theta - \sum_{n=1}^{\infty} a_n^{(\beta)}\sin n\theta\right]$$

$$\operatorname{Im}\varphi_2 = \theta_0 + \left[\sum_{n=1}^{\infty} \frac{\sin n\theta_0}{n\rho_0^n}\cos n\theta + \sum_{n=1}^{\infty} a_n^{(\beta)}\cos n\theta\right]$$

Let find $a_n^{(\alpha)}$ and $a_n^{(\beta)}$ from the condition (6) as:

$$a_n^{(\alpha)} = \frac{n\cdot\alpha_0 - 1}{n\cdot\alpha_0 + 1}\frac{\cos n\theta_0}{n\rho_0^n}; \quad (19)$$

$$a_n^{(\beta)} = -\frac{n\cdot\beta_0 - 1}{n\cdot\beta_0 + 1}\frac{\sin n\theta_0}{n\rho_0^n}. \quad (20)$$

If we substitute (19) and (20) in the equation (2), we will find:

$$\Phi(\nu) = \ln(\nu - \nu_0) + \sum_{n=1}^{\infty} \left[\frac{n \cdot \alpha_0 - 1}{n \cdot \alpha_0 + 1} \frac{\cos n\theta_0}{n\rho_0^n} - i \frac{n \cdot \beta_0 - 1}{n \cdot \beta_0 + 1} \frac{\sin n\theta_0}{n\rho_0^n} \right] \nu^{-n} . \quad (21)$$

In a case, when $\theta_0=0, \pi$, or if the well is located on the x axis, this solution coincides with the solution obtained in the works written by Astafiev and Fedorchenko this solution coincides with the solution obtained in the works written by [9]:

$$\Phi(\nu) = \ln(\nu - \nu_0) + \sum_{n=1}^{\infty} \frac{1}{n} \left[\frac{n \cdot F_{cd} - 1}{n \cdot F_{cd} + 1} \left(\nu \overline{\nu_0}\right)^{-n} \right] + C_0 .$$

where $F_{cd}=k_f\delta/kl$ is the dimensionless fracture conductivity [3].

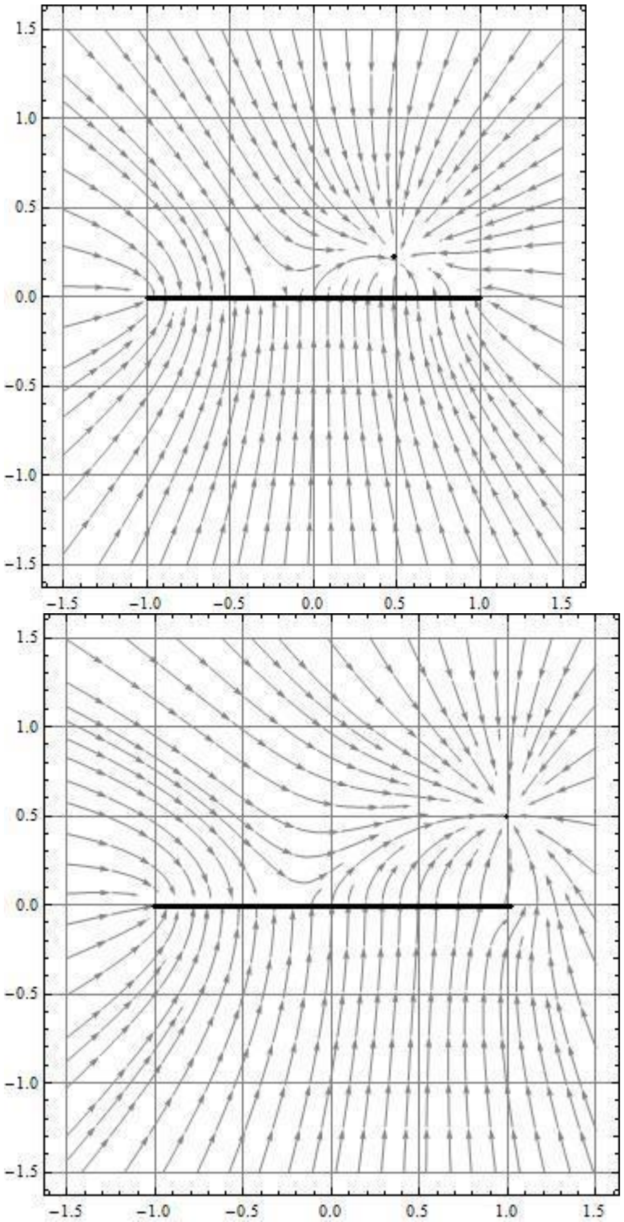

Figure 2 *Streamlines of the fluid flow to the well, located at the point (0.2, 0.5)-left and at the point (0.5, 1)-right for the values of $\alpha_0=\infty$; $\beta_0=0$.*

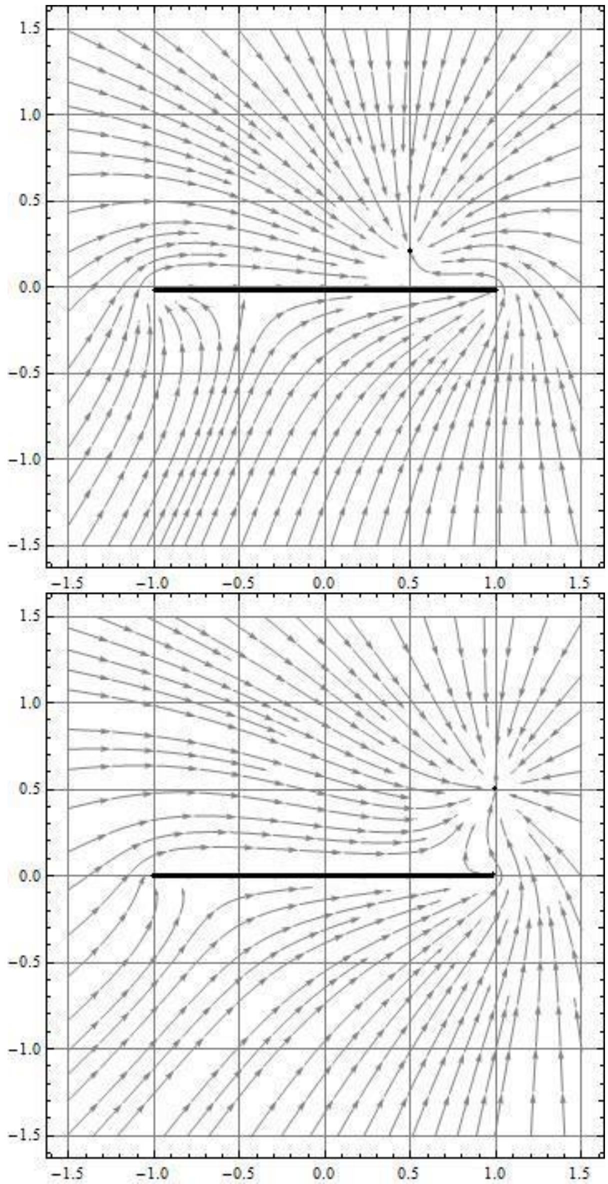

Figure 3 *Streamlines of the fluid flow to the well, located at point (0.2, 0.5)-left and at point (0.5, 1)–right for the values of $\alpha_0= 0$; $\beta_0=\infty$.*

The nature of fluid flow to the wellbore at different locations of the crack and the well for different values of the coefficients α_0 and β_0 are shown in Fig.2 and Fig. 3. As we can see, the obtained flow potential equation allow us to solve the problem for any well and fracture location and for different fracture conductivity. On the Fig. 2 there is permeable fracture, and if we place the well in the center of the fracture, we can see the hydraulic fracturing case. On the Fig. 3 there is the impermeable fracture, which acts like impermeable boundary.

4. Coefficient of well productivity

The main value, which can express effectiveness of HF, is the skin factor (can be measured only by well test analysis). The skin factor reflects any physical or mechanical impact decreasing flow to the well [12, 13]. Firstly, A.F.

Van Everdingen and W. Hurst [12] introduced in practice the term skin-effect to evaluate the near wellbore condition. According to the authors, change of buttonhole pressure as the result of increasing or decreasing permeability is proportional to skin effect. In that way, the skin effect expresses the value of additional pressure drawdown as a consequence of a deviation from the radial flow [14]. Mainly the damaged by drilling zone causes the decrease of permeability near wellbore. However, the permeability of skin-area may be increased in case of hydraulic fractures, and negative skin effect can be imposed if a successful hydraulic fracture is created.

All of the components of petroleum production system can be condensed into the productivity index, which can be presented in the next form [3]:

$$J = \frac{q}{p_c - p_w} = (\ln\frac{R_c}{r_w} + S)^{-1} \tag{22}$$

where p_c – pressure on the external boundary, p_w – pressure on the well bottom, S – skin factor. The case when the well is placed on the fracture and pressure is equal by both sides from the discontinuity was considered in [15]. Let us find the value of the skin factor for more general case from the equation (21) for the potential $\varphi(z)$ from the following conditions:

$p_c=\mathrm{Re}(z)$,
$z=z_c=z_0+R_c e^{i\Theta}$,

$p_w=\mathrm{Re}\varphi(z)$,
$z=z_w=z_0+r_w e^{i\Theta}$.

We can rewrite the coefficients in the equation for flow potential in such form:

$$\frac{n\cdot\alpha_0 - 1}{n\cdot\alpha_0 + 1} = 1 - \frac{2}{n\alpha_0 + 1};$$

$$\frac{n\cdot\beta_0 - 1}{n\cdot\beta_0 + 1} = 1 - \frac{2}{n\beta_0 + 1}.$$

So we can get potential from the (21) in the next form:

$$\Phi(v) = \ln(v - v_0) - 2\sum_{n=1}^{\infty}\frac{1}{n}\left[\frac{1}{\alpha_0+1}\frac{\cos n\theta_0}{n\rho_0^n} - i\frac{1}{\beta_0+1}\frac{\sin n\theta_0}{n\rho_0^n}\right]v^{-n} + \sum_{n=1}^{\infty}\frac{1}{n}\left[\frac{\cos n\theta_0}{n\rho_0^n} - i\frac{\sin n\theta_0}{n\rho_0^n}\right]v^{-n} + C$$

Consequently we can find pressure on the external boundary as the real part of the flow potential:

$$p_c = q\,\mathrm{Re}\left\{\ln\frac{2}{l}R_c e^{i\theta} - 2\ln\left(1 - \frac{1}{v(z)v_0}\right) - 2\sum_{n=1}^{\infty}\frac{1}{n}\left[\frac{1}{\alpha_0+1}\frac{\cos n\theta_0}{n\rho_0^n} - i\frac{1}{\beta_0+1}\frac{\sin n\theta_0}{n\rho_0^n}\right]v(z)^{-n} + C\right\}.$$

Taking into account that $l/R_c<<1$, or $lv(z_c)=z_c+(z_c^2-l^2)^{1/2}=2z_c$, we can write p_c from the (21) as

$$p_c / q = (\ln(2R_c / l) + O(l / R_c) \approx \ln(2R_c / l).$$

Similarly, considering that $l/r_w>>1$ or $lv(z_w)=z_w+(z_w^2-l^2)^{1/2}=lv_0$, we get equation for p_w from the (21):

$$p_w / q = \ln\frac{2r_w}{l} - 2\ln\left|1 - \frac{1}{v_0^2}\right| - 2\,\mathrm{Re}\sum_{n=1}^{\infty}\frac{1}{n}\left[\frac{\cos^2 n\theta_0}{n\alpha_0+1} - \frac{\sin^2 n\theta_0}{n\beta_0+1}\right]\frac{e^{-in\theta_0}}{\rho_0^n}.$$

Consequently, skin factor can be expressed from (22) as:

$$S = -\ln\frac{R_c}{r_w} + \frac{p_c - p_w}{q},$$

or

$$S = \ln\left|1 - v_0^{-2}\right| + 2\sum_{n=1}^{\infty}\frac{1}{n}\left[\frac{\cos^2 n\theta_0}{n\alpha_0+1} - \frac{\sin^2 n\theta_0}{n\beta_0+1}\right]\rho_0^{-n}. \tag{23}$$

Particularly, from the (23) we can conclude that, for a highly permeable fracture $\alpha_0=\infty;\ \beta_0=0$:

$$S = 2\ln\left|1 - v_0^{-2}\right| - 2\sum_{n=1}^{\infty}\frac{1}{n}\frac{\sin^2 n\theta_0}{\rho_0^{2n}};$$

and for an impermeable fracture $\alpha_0=0;\ \beta_0=\infty$:

$$S = 2\ln\left|1 - v_0^{-2}\right| + 2\sum_{n=1}^{\infty}\frac{1}{n}\frac{\cos^2 n\theta_0}{\rho_0^{2n}}.$$

Additionally, when the well is placed on the x axis or for $\theta_0=0, \pi$, skin factor is:

$$S = 2\ln\left(1 - \rho_0^{-2}\right) + 2\sum_{n=1}^{\infty}\frac{1}{n}\frac{\rho_0^{-2n}}{n\alpha_0+1};$$

$$S = \begin{cases} 2\ln\left(1-\rho_0^{-2}\right) < 0, & \alpha_0=\infty, \beta_0=0, \\ 2\ln\left(1-\rho_0^{-2}\right) + 2\sum_{n=1}^{\infty}\frac{1}{n}\rho_0^{-2n} = 0, & \alpha_0=0, \beta_0=\infty. \end{cases}$$

Therefore, impermeable fracture for such well location gives skin $S=0$, while highly permeable gives

$S=2\ln(1-\rho_0^{-2})<0$.

For a case than $\theta_0=\pi/2$, skin factor is:

$$S = 2\ln\left(1+\rho_0^{-2}\right)+2\sum_{k=1}^{\infty}\left(\frac{1}{2k}\frac{\rho_0^{-4k}}{2k\alpha_0+1}-\frac{1}{2k-1}\frac{\rho_0^{-4k+2}}{(2k-1)\beta_0+1}\right);$$

$$S = \begin{cases} 2\ln\left(1+\rho_0^{-2}\right)-2\sum_{k=1}^{\infty}\left(\frac{1}{2k-1}\rho_0^{-2(2k-1)}\right)<0, & \alpha_0=\infty, \beta_0=0, \\ \ln\frac{\left(1+\rho_0^{-2}\right)}{\left(1-\rho_0^{-2}\right)}>0, & \alpha_0=0, \beta_0=\infty. \end{cases}$$

5. Conclusion

In this work the formulation and solution of the problem of fluid flow to the well at the presence of a crack of different conductivity has been done. The solution obtained by the replacement of ellipse like approximation to the section view of zero thickness but finite conductivity. More general boundary conditions were considered taking into account the pressure difference above and below the section. Thus we obtained more general equation for the flow potential which coincides with previous solutions. This s solution is suitable for any cases of various well and crack places and for different values of a fracture permeability and allow us to analyze the nature of fluid flow.

The skin factor is one of the most important parameter for the evaluation of well productivity, which can reflect additional pressure dropdown as the result of a deviation from the radial flow. Therefore in the last part, the equation for skin effect is defined for different values of fracture conductivity and various well-crack locations.

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