Adaptive Control for Continuous Stirred Tank Reactor

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Abstract: - The Continuous Stirred Tank Reactor (CSTR) is one of the most important and central pieces of equipment in many chemical and biochemical industries. It exhibits complex nonlinear characteristics, and efficient control of the product concentration in a CSTR can be achieved only through an accurate model. Sometimes, conventional feedback controllers may not perform well online due to variations in process dynamics resulting from nonlinear actuators, changes in environmental conditions, and variations in the character of the disturbances. This paper presents the mathematical model of the CSTR and the design of a Model Reference Adaptive Control (MRAC) scheme using the MIT rule and the Lyapunov method for the adaptive mechanism. In addition, a comparative performance analysis of the two approaches for concentration control in a CSTR was conducted using MATLAB/Simulink. Finally, the simulation results showed that the MRAC using the MIT rule has better performance than the MRAC using the Lyapunov method.

Key-Words: - Adaptive control, MIT rule, Lyapunov, MATLAB, model reference, reactor, stirred tank, PID.

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1 Introduction

The chemical reactor stands as the indispensable core of countless industrial processes. It is the pivotal unit where raw materials—whether petroleum, natural gas, or biochemical feedstocks—are transformed into the valuable products that shape our modern world, from fuels and polymers to pharmaceuticals and specialty chemicals.

In essence, one could think of it as the beating heart of a chemical plant. Just as the heart sustains life by converting resources into energy, the reactor sustains the entire process by enabling the fundamental transformations that create value. Without this central component, operations across the chemical, petrochemical, and biochemical industries would simply cease[1], [2], [3], [4]. Basically, a chemical reactor is a device in whicha chemical reaction takes place. Chemical reactors can be classified according to different properties:

- Reaction phase.
- Operating modes.

According to the reaction phase chemical reactor can be classified as:

- Homogeneous reactor.
- Heterogeneous reactor.

According to the operating modes, chemical reactors can be classified as:

- Continuous stirred tank reactor.
- Batch stirred tank reactor.
- Semi-batch reactor.
- Tubular reactor.

While designing a chemical reactor following factor has to be considered:

- Overall size of reactor.
- Products emerging from the reactor.
- Temperature inside the reactor.
- Pressure inside the reactor.
- Rate of reaction.
- Activity and mode of catalyst.
- Stability and controllability of reactor[5], [6], [7].

For this study, the Continuous Stirred-Tank Reactor (CSTR) was selected as the benchmark system. This choice is motivated by its prominence as an educational model in university process control laboratories, where it serves to illustrate core principles of control engineering. The CSTR's relevance extends beyond the classroom; it embodies control challenges directly applicable to critical industries, including chemical manufacturing, pharmaceutical production (such as antidote preparation), and food processing. Its utility stems from a foundational duality: its operational principle is intuitively simple, involving the continuous feed and withdrawal of a perfectly mixed tank, yet it exhibits the complex nonlinear dynamics characteristic of real industrial processes. This combination makes the CSTR an ideal platform. It provides a tractable system for understanding fundamental concepts while enabling the exploration and application of a wide array of control strategies, from classical to advanced modern methods. The CSTR scheme is shown in Figure 1.

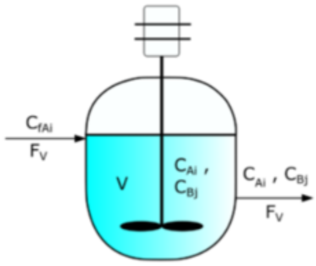

Fig. 1: Continuous stirred tank reactor scheme

For this study, the system comprises two well-mixed, isothermal chemical reactors configured in series. Key simplifying assumptions include constant and equal fluid density in both tanks, constant and equal volumetric flow rates throughout the system, and a constant inlet flow. Consequently, the liquid volume in each tank remains constant.

The primary control objective is to regulate the outlet concentration of the second tank (C_2). This is to be achieved by manipulating the outlet concentration of the first tank (C_1), effectively forcing C_2 to track a specified dynamic response based on the behavior observed in the first reactor [8], [9].

Adaptive control is a prominent strategy for designing advanced control systems that demand high performance and accuracy in the face of uncertainty. Among its various approaches, Model Reference Adaptive Control (MRAC) is a direct method. It operates by comparing the actual system output to the desired response of a pre-defined reference model. The discrepancy between these two drives an adjustment mechanism that continuously tunes specific controller parameters in real-time.

This capability gives adaptive controllers, like MRAC, a significant advantage over conventional fixed-gain controllers (such as the widely-used PID). While PID controllers are simple and effective for well-defined, constant processes, adaptive controllers excel in environments with unknown parameter variations, changing dynamics, or unpredictable disturbances.

Structurally, an adaptive controller features a dual-loop architecture:

The Outer (Primary) Loop: This is the standard feedback control loop that generates the control signal to drive the plant based on the tracking error.

The Inner (Adaptation) Loop: This is the defining feature. It monitors performance and dynamically adjusts the controller's parameters according to an adaptation law to minimize the error between the plant and the reference model outputs [10].

In this research, a MRAC using MIT and Lyapunov methods for a continuous stirred tank reactor is proposed to control the concentration of the linear CSTR. The performance of the MRAC using the MIT rule will be compared with a MRAC using by Lyapunov method using MATLAB Simulink. Finally, the task of this project is to design and select the best controller for the system that can control the concentration of the CSTR.

2 Mathematical Model

The first step in studying the dynamic behaviour and control of a Continuous Stirred-Tank Reactor (CSTR) is to develop a foundational mathematical model. This model, based on mass and component balance equations, serves as the cornerstone for all subsequent analysis and design work. It is essential to note that high nonlinearity is an inherent characteristic of any industrial CSTR, a fundamental trait that necessitates advanced control approaches [11], [12]. Perfect mixing is assumed inthe CSTR, and the change in volume due to reaction is negligible, and the density is the same in both tanks.

The experimental setup features two feed tanks supplying distinct reactants to a Continuous Stirred-Tank Reactor (CSTR). The CSTR is equipped with a mechanical stirrer to ensure perfect mixing of its contents. Product samples for analysis are collected from the reactor's top outlet.

Based on standard overflow tank assumptions, the liquid volume in each feed tank remains constant. Furthermore, all volumetric flow rates—including the constant inlet flow—are assumed to be equal and steady throughout the system [3]. In the Figure 2, two CSTRs are connected in series.

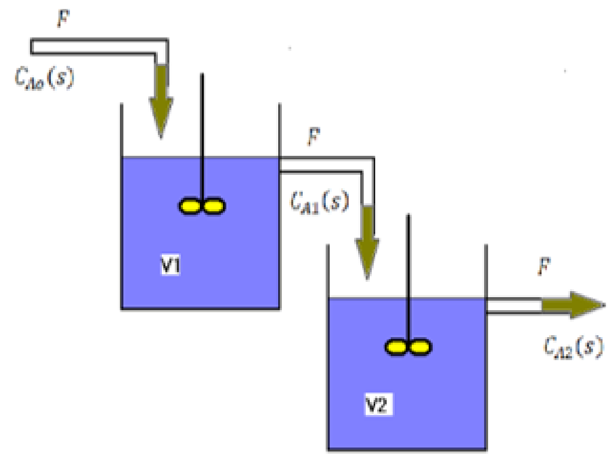

Fig. 2: The simple concentration process control

The value of the concentration in the second tank is desired, but it depends on the concentration in the first tank. Therefore, the component balances in both tanks are formulated. The transfer function of the first tank can be obtained as

$$V_1 \frac{dC_{A1}}{dt} = FC_{A0} - FC_{A1} - V_1 K C_{A1} \quad (1)$$

Where V_1 is the volume of the first tank, F is the flow, C_{A0} is the inlet concentration of the first tank, C_{A1} is the outlet concentration of the first tank and inlet concentration of the second tank, and K is the reaction rate.

Equation (1) can be rearranged to be

$$\frac{dC_{A1}}{dt} + \frac{1}{\tau_1} C_{A1} = \frac{F}{V_1} C_{A0} \quad (2)$$

Where $\tau_1 = \frac{V_1}{F+KV_1}$ is the time constant of the first tank.

By taking the Laplace transform and rearranging Equation (2), the transfer function of the first tank can be expressed as

$$\frac{C_{A1}(s)}{C_{A0}(s)} = \frac{K_{P1}}{(\tau_1 s + 1)} \quad (3)$$

Where $K_{P1} = \frac{F}{F+KV_1}$ is the gain of the transfer function of the first tank. The transfer function of the second tank can be derived

$$V_2 \frac{dC_{A2}}{dt} = FC_{A1} - FC_{A2} - V_2 K C_{A2} \quad (4)$$

Where $V_2$and C_{A2} are the volume and the inlet concentration of the second tank, respectively. Equation (4) can be rearranged to be

$$\frac{dC_{A2}}{dt} + \frac{1}{\tau_2} C_{A2} = \frac{F}{V_2} C_{A2} \quad (5)$$

Where $\tau_2 = \frac{V_2}{F+KV_2}$ is the time constant for the second tank.

By taking the Laplace transform and rearranging Equation (5), the transfer function of the second tank can be obtained.

$$\frac{C_{A2}(s)}{C_{A1}(s)} = \frac{K_{P2}}{(\tau_2 s + 1)} \quad (6)$$

Where $K_{P2} = \frac{F}{F+KV_2}$ is the gain of the transfer function of the second tank. The transfer function of the whole system can be obtained according to the following assumptions of parameters[13].

1- The flow rate is constant for the whole system

$$F = 0.085\ m^3/min$$

2- The volume of the two tanks is the same

$$V_1 = V_2 = V = 1.05\ m^3$$

3- Reaction rate $K = 0.04\ min^{-1}$

Since the time constants and the gains are equal for both tanks, they can be computed as follows:

$$\tau = \frac{V}{F + KV} = 8.25\ min$$

$$K = \frac{F}{F + KV} = 0.669$$

The transfer function of the combined two tanks with the assumed parameters can be obtained

$$G(s) = \frac{C_{A2}(s)}{C_{A0}(s)} = \frac{K_P^2}{(\tau_2 s + 1)^2} \quad (7)$$

$$G(s) = \frac{0.4462}{245000s^2 + 990.8s + 1} \quad (8)$$

Figure 3 shows the block diagram of theopen-loop combined two-tank system.

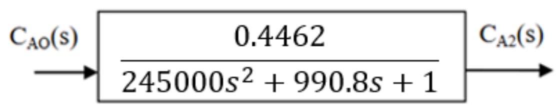

Fig. 3: The block diagram of the open-loop system

Mathematical Analyzer Transmitter:

Assume that the measuring element converts the concentration to an electrical signal.

The composition sensor transmitter can be approximated by a first-order transfer function[14]

$$\frac{C_{Am}(s)}{C_{A2}(s)} = \frac{K_m}{\tau_m s + 1}$$

This instrument has negligible dynamics when $\tau >> \tau_m$. A useful approximation is to set $\tau_m = 0$

$$\frac{C_{Am}(s)}{C_{A2}(s)} = K_m$$

Let $K_m = 1$.

The block diagram of the overall system is shown in Figure 4.

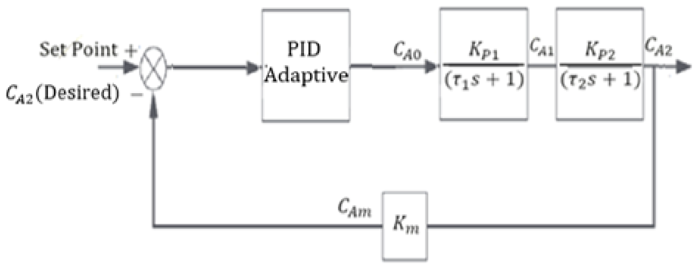

Fig. 4: block diagram of the overall system

2.1 System Analysis

After finding the mathematical model of the reactor, it is necessary to analyse the performance of the system in terms of stability and speed of response.

To know the stability of the system, there are several methods, such as: Root locus and bode plot.

The root locus method is popular with control system engineers because it lets them quickly and graphically determine how to modify controller poles, zeros, and gain to position dominant closed-loop system poles in the desired locations.

The root locus of the CSTR system is shown in Figure 5.

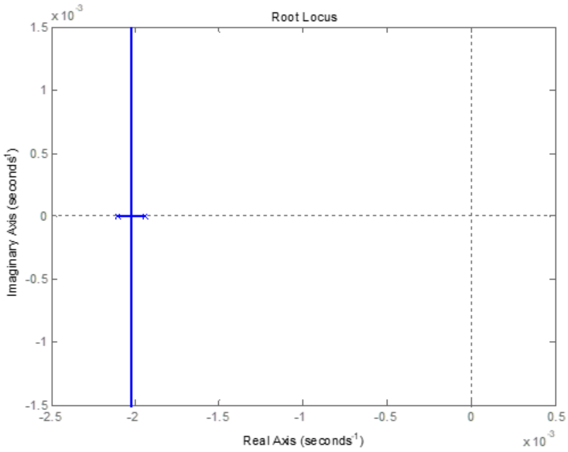

Fig. 5: The root locus of the CSTR system

From Figure 5: The root loci do not cross the imaginary axis. Therefore, the system is stable for any value of K.

A Bode plot is a graph commonly used in control system engineering to determine the stability of a control system. A Bode plot maps the frequency response of the system through two graphs – the Bode magnitude plot (expressing the magnitude in decibels) and the Bode phase plot (expressing the phase shift in degrees).

The bode plot of the CSTR system is shown in Figure 6.

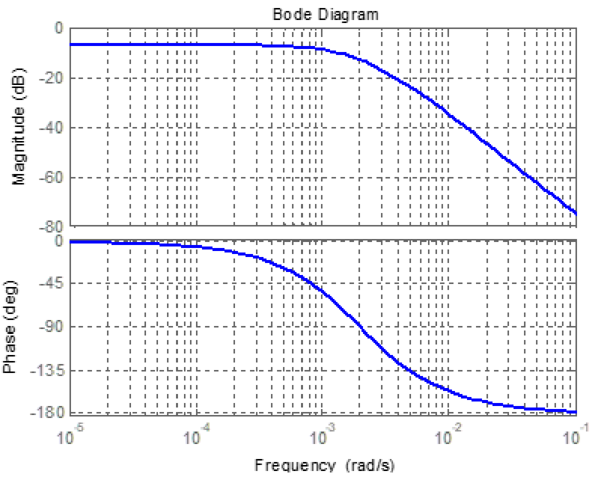

Fig. 6: The bode plot of the CSTR system

From Figure 6: The phase curve is notcrossed $-180°$. Therefore, the system is stable for any value of K.

The closed loop step response of the CSTR system is shown in Figure 7.

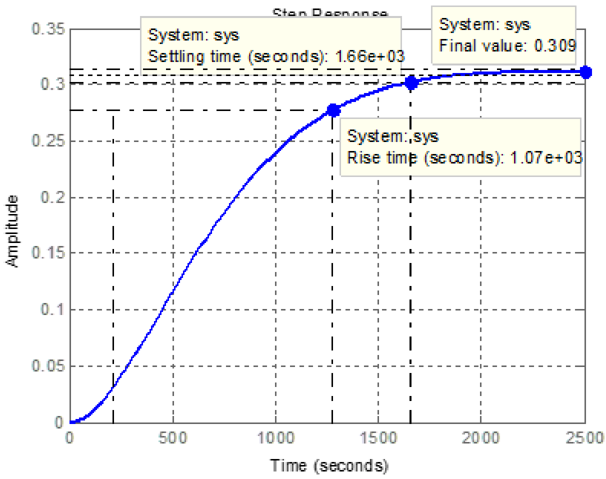

Fig. 7: The closed loop step response of the CSTR system

Table 1. illustrates the performance specifications that define the system response for the step response input for the system. The final value is 0.309, but the desired value is 1 to make the error steady state zero. From this case, the response has a high error compared with the desired value

Table 1. Performance specification of step response for the system without a controller

Overshoot	Rise Time	Settling Time	Steady State Error
---	17.83 min	27.67 min	0.691

According to the results in Table 1, it is necessary to design a controller to improve the system performance.

3 Design of Model Reference Adaptive Controller:

The design of a Model Reference Adaptive Control (MRAC) system involves two key elements: selecting a reference model that defines the ideal dynamic response, and designing an adaptive controller. The core objective of this controller is to minimize the error between the plant's actual output and the reference model's desired output.

This is achieved through a continuous adaptation mechanism. Regardless of the plant's true or changing internal parameters, the MRAC system dynamically adjusts the controller's coefficients. This adjustment forces the overall closed-loop system's behavior to converge with that of the prescribed reference model. Consequently, the specific values of the plant's parameters become less critical to achieving the desired performance [15], [16], [17].

Two types of MRAC design methods will be discussed in this research. They are:

- Gradient Method / MIT Rule.
- Lyapunov Method .

The basic block diagram of the MRAC system is shown in Figure 8, $y_m(t)$ is the output of the reference model, and $y(t)$ is the output of the actual plant, and difference between them is denoted by $e(t)$

$$e(t) = y(t) - y_m(t)$$

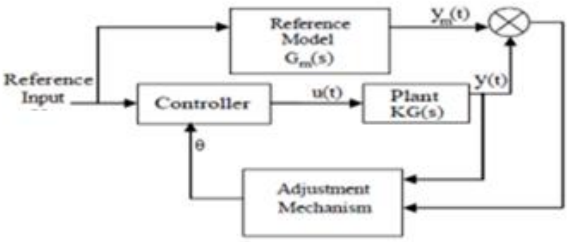

Fig. 8: Model reference adaptive control system

3.1 Design PID MRAC using MIT rule for CSTR

A PID (Proportional – Integral – Derivative) controller is an instrument used by control engineers to regulate temperature, flow, pressure, concentration, speed, and other process variables in industrial control systems. PID controllers use a control loop feedback mechanism to control process variables and are the most accurate and stable controllers. The transfer function in the Laplace domain of the PID controller is

$$\frac{U(s)}{E(s)} = \left(K_P + \frac{K_i}{s} + K_d s\right)$$

The objective is to design a controller to control the concentration of the second tank, by adjusting the values K_P, K_i, and K_d to achieve a desired response described by a reference model as:

$$Y_m(s) = \frac{2.5 \times 10^{-6}\, R(s)}{s^3 + 0.048s^2 + 0.000348s + 2.5 \times 10^{-6}}$$

The required performance specifications of the reference model are the follows:

1.
o offset or steady state error should be observed.
2. Overshoot equals 15%.
3.
good transient response should be obtained (the settling time less than or equal to 25 minutes).

The transfer function of the CSTR system is:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\alpha_1}{s^2 + 0.004s + \alpha_2}$$

Where: $\alpha_1 = 0.00000182$, and $\alpha_2 = 0.000004$.

$$U(s) = \left(K_P + \frac{K_i}{s} + K_d s\right)\left(R(s) - Y(s)\right)$$

$$Y(s) = \frac{\alpha_1(K_d s^2 + K_P s + K_i)R(s)}{s^3 + (0.004 + \alpha_1 K_d)s^2 + (\alpha_2 + \alpha_1 K_P)s + \alpha_1 K_i}$$

$$e = Y(s) - Y_m(s)$$

$$\frac{\partial e}{\partial K_p} = \frac{\alpha_1 s(r-y)}{s^3 + 0.048s^2 + 0.000384s + 2.5 \times 10^{-6}}$$

$$\frac{\partial e}{\partial K_i} = \frac{\alpha_1 (r-y)}{s^3 + 0.048s^2 + 0.000384s + 2.5 \times 10^{-6}}$$

$$\frac{\partial e}{\partial K_d} = \frac{\alpha_1 s^2(r-y)}{s^3 + 0.048s^2 + 0.000384s + 2.5 \times 10^{-6}}$$

$$\frac{dK_P}{dt} = -\gamma_1 e \frac{\partial e}{\partial K_p}$$

$$K_P = -\gamma_1 e \frac{\alpha_1 (r-y)}{s^3 + 0.048s^2 + 0.000384s + 2.5 \times 10^{-6}}$$

$$\frac{dK_i}{dt} = -\gamma_2 e \frac{\partial e}{\partial K_i}$$

$$K_i = \frac{-\gamma_2}{s} e \frac{\alpha_1 (r-y)}{s^3 + 0.048s^2 + 0.000384s + 2.5 \times 10^{-6}}$$

$$\frac{dK_d}{dt} = -\gamma_1 e \frac{\partial e}{\partial K_d}$$

$$K_d = -\gamma_3 s e \frac{\alpha_1 (r-y)}{s^3 + 0.048s^2 + 0.000384s + 2.5 \times 10^{-6}}$$

3.2 Design PID MRAC using the Lyapunov method for CSTR

The transfer function of the CSTR system is:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\alpha_1}{s^2 + 0.004s + \alpha_2}$$

Where: $\alpha_1 = 0.00000182$, and $\alpha_2 = 0.000004$.

$$U(s) = \left(K_P + \frac{K_i}{s} + K_d s\right)\left(R(s) - Y(s)\right)$$

$$Y(s) = \frac{\alpha_1(K_d s^2 + K_P s + K_i)}{s^3 + (0.004 + \alpha_1 K_d)s^2 + (\alpha_2 + \alpha_1 K_P)s + \alpha_1 K_i} R(s)$$

$$Y_m(s) = \frac{2.5 \times 10^{-6}}{s^3 + 0.048s^2 + 0.000348s + 2.5 \times 10^{-6}} R(s)$$

$$\dddot{y} = -(0.004 + \alpha_1 K_d)\ddot{y} - (\alpha_2 + \alpha_1 K_P)\dot{y} - \alpha_1 K_i y + \alpha_1 K_d \ddot{r} + \alpha_1 K_P \dot{r} + \alpha_1 K_i r$$

$$\dddot{y_m} = -0.048\ddot{y_m} - 0.000348\dot{y_m} - 2.5 \times 10^{-6} y_m + 2.5 \times 10^{-6} r$$

$$\dddot{e} = \dddot{y} - \dddot{y_m}$$

$$\dddot{e} = -(\alpha_1 K_d - 0.044)\ddot{y} - (\alpha_1 K_P - 0.000344)\dot{y} - (\alpha_1 K_i - 2.5 \times 10^{-6})y + \alpha_1 K_d \ddot{r} + \alpha_1 K_P \dot{r} + (\alpha_1 K_i - 2.5 \times 10^{-6})r - 0.048\ddot{e} - 0.000348\dot{e} - 2.5 \times 10^{-6} e$$

$$\dddot{e} = -0.048\ddot{e} - 0.000348\dot{e} - 2.5 \times 10^{-6} e + \alpha_1 K_d \ddot{r} + \alpha_1 K_P \dot{r} + (0.044 - \alpha_1 K_d)\ddot{y} + (0.000344 - \alpha_1 K_P)\dot{y} + (\alpha_1 K_i - 2.5 \times 10^{-6})(r - y)$$

$$\dddot{e} = -0.048\ddot{e} - 0.000348\dot{e} - 2.5 \times 10^{-6} e + \alpha_1 K_d(\ddot{r} - \ddot{y}) + \alpha_1 K_P(\dot{r} - \dot{y}) + (\alpha_1 K_i - 2.5 \times 10^{-6})(r - y) + 0.044\ddot{y} + 0.000344\dot{y}$$

$$\dddot{e} = -0.048\ddot{e} - 0.000348\dot{e} - 2.5 \times 10^{-6} e + x_1(\ddot{r} - \ddot{y}) + x_2(\dot{r} - \dot{y}) + x_3(r - y) + 0.044\ddot{y} + 0.000344\dot{y}$$

$$v = \lambda_1 \ddot{e}^2 + \lambda_2 \dot{e}^2 + \lambda_3 e^2 + \lambda_4 x_1^2 + \lambda_5 x_2^2 + \lambda_6 x_3^2$$

$$\dot{v} = 2\lambda_1 \ddot{e}\dddot{e} + 2\lambda_2 \dot{e}\ddot{e} + 2\lambda_3 e\dot{e} + 2\lambda_4 x_1 \dot{x_1} + 2\lambda_5 x_2 \dot{x_2} + 2\lambda_6 x_3 \dot{x_3}$$

$$\dot{v} = -0.096\lambda_1 \ddot{e}^2 - 0.000696\lambda_1 \dot{e}\ddot{e} - 5 \times 10^{-6}\lambda_1 e\ddot{e} + (2\lambda_1 \ddot{e}(\ddot{r} - \ddot{y}) + 2\lambda_4 \dot{x_1})x_1 + 2\lambda_1 \ddot{e}(\dot{r} - \dot{y}) + 2\lambda_5 \dot{x_2})x_2 + (2\lambda_1 \ddot{e}(r - y) + 2\lambda_6 \dot{x_3})\, x_3 + 5.86\dot{y} + 3.14\dot{y} + 2\lambda_2 \dot{e}\ddot{e} + 2\lambda_3 e\dot{e}$$

$$\lambda_1 = 2873.5\lambda_2$$

$$2\lambda_1 \ddot{e}(\ddot{r} - \ddot{y}) + 2\lambda_4 \dot{x_1} = 0$$

$$\dot{x_1} = \frac{-2\lambda_1 \ddot{e}(\ddot{r} - \ddot{y})}{2\lambda_4}$$

$$\dot{x_1} = -\dot{\gamma_1}\ddot{e}(\ddot{r} - \ddot{y})$$

Where $\dot{\gamma_1} = 2\lambda_1/2\lambda_4 = 2873.5\lambda_2/\lambda_4$

$$2\lambda_1 \ddot{e}(\dot{r} - \dot{y}) + 2\lambda_5 \dot{x_2} = 0$$

$$\dot{x_2} = \frac{-2\lambda_1 \ddot{e}(\dot{r} - \dot{y})}{2\lambda_5}$$

$$\dot{x_2} = -\dot{\gamma_2}\ddot{e}(\dot{r} - \dot{y})$$

Where $\dot{\gamma_2} = 2873.5\lambda_2/\lambda_5$

$$2\lambda_1 \ddot{e}(r - y) + 2\lambda_6 \dot{x_3} = 0$$

$$\dot{x_3} = \frac{-2\lambda_1 \ddot{e}(r - y)}{2\lambda_6}$$

$$\dot{x_3} = -\dot{\gamma_3}\ddot{e}(r - y)$$

Where $\dot{\gamma_3} = -2873.5\lambda_2/\lambda_6$

$$x_1 = K_d = -\dot{\gamma_1} e s^3(r - y)$$

$$x_2 = K_P = -\dot{\gamma_2} e\, s^2(r - y)$$

$$x_3 = K_i = -\dot{\gamma_3} e\, s(r - y)$$

4 Results and Discussion

This section presents the simulation results using MRAC by MIT rule and the Lyapunov method for the CSTR system. The performance of these controllers is evaluated. Then, compared in terms of speed response, disturbance rejection, and sin-wave tracking. The performance index, Integral Time Absolute Error (ITAE), is computed to demonstrate a performance comparison between the two controllers. Note that a justification that having a smaller value of ITAE, which means better performance.

4.1 Simulation Results of PID MRAC using MIT rule for CSTR

The PID using the MIT rule is designed for a CSTR system to control the concentration in the second reactor. The controller parameters are tuned based on the trial-and-error tuning method.

In the simulation, the adaptation gains are found:

$\gamma_1 = -1.$

$\gamma_2 = -0.003.$

$\gamma_3 = -0.0001.$

Using these adaptation gains, the output response of concentration at the second reactor is shown in Figure 9.

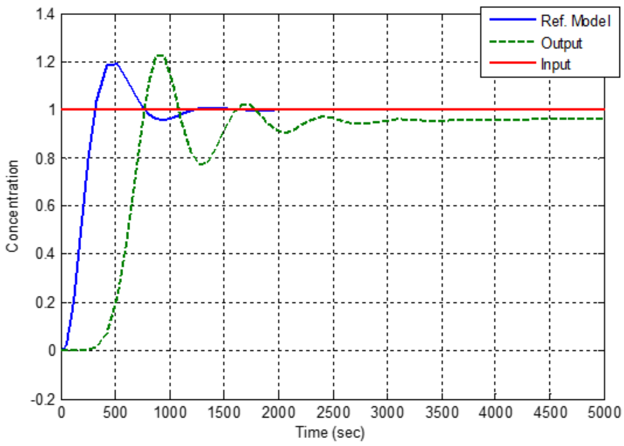

Fig. 9: Output response of concentration for the second reactor(MIT rule)

From Figure 9, it can be seen that the overshoot is approximately 23%. Also, the performance of the PID using the MIT rule can be evaluated by means of the performance index, Integral Time Absolute error (ITAE). In this case, ITAE is obtained with a value of 875.6.

The disturbance rejection for the CSTR system was considered in this scope of work, where the applied disturbance signal that affects the system was assumed as a concentration of $0.1\ \mathrm{mol/m^3}$ at time 4000 sec.

According to Figure 10, the response takes roughly 27 minutes for the system to settle back to the commanded set-point. The ITAE is computed to be 976.6.

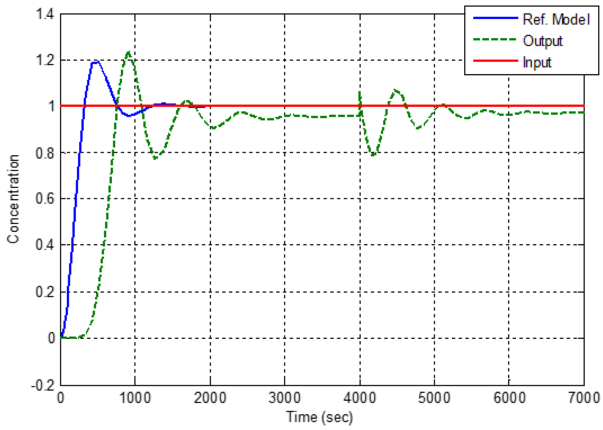

Fig. 10: Response of the PID using the MIT rule in the presence of load disturbance

The designed PID using the MIT rule was tested in terms of sine wave tracking performance as shown in Figure 11.

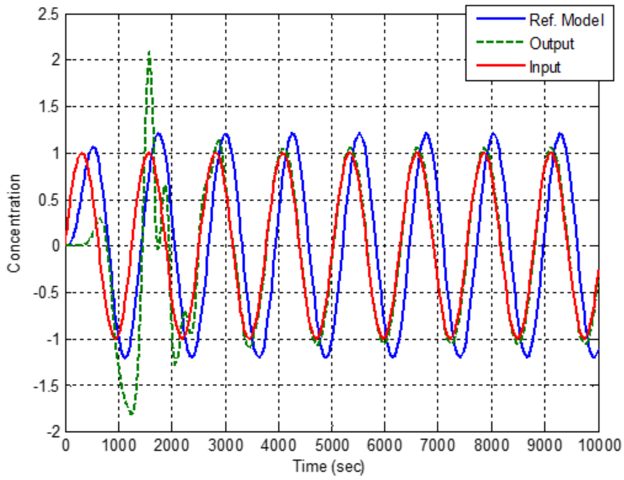

Fig. 11: Response of the system with PID MIT rule in terms of sine wave tracking performance

According to Figure 11, it seems that although PID using the MIT rule can track the commanded sine wave. The performance of the PID using the MIT rule in terms of sine wave tracking can also be computed by ITAE. In this case, ITAE is computed as 1977.

4.2 Simulation Results of PID using Lyapunov method for CSTR System

In this section, PID using the Lyapunov method has been designed to control the concentration in the second reactor, and the results will be demonstrated. In the simulation, the adaptation gains are found:

$$\dot{\gamma_1} = -1.$$

$$\dot{\gamma_2} = -0.1.$$

$$\dot{\gamma_3} = -1350.$$

Using these adaptation gains, the output response of concentration at the second reactor is shown in Figure 12.

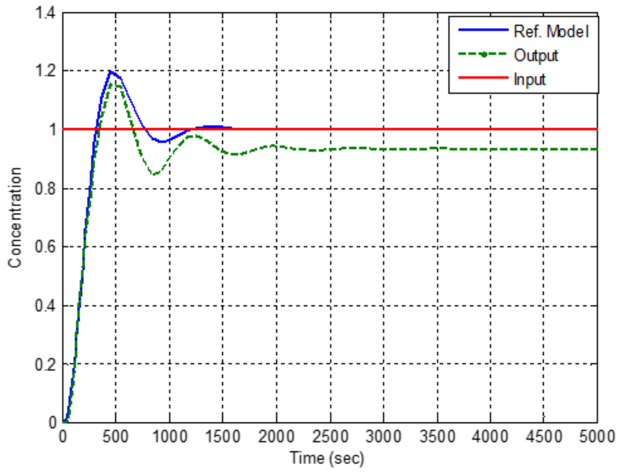

Fig. 12: Output response of concentration for the second reactor(Lyapunov method)

As can be seen from Figure 12, the output response of concentration in the second reactor has a fast transient response with an acceptable overshoot of about 16%. The system has 21 21-minute settling time. But, the system has a steady state error and equals 0.06. The performance index ITAE using this controller is 525.4.

To check the robustness of the controllers, the system is subjected to a load or disturbance, where the applied disturbance signal that affects the system is assumed to be a concentration of 0.1 mol/m^3 at time 4000 sec. The output responses under the effect of the disturbance signal are shown in Figure 13.

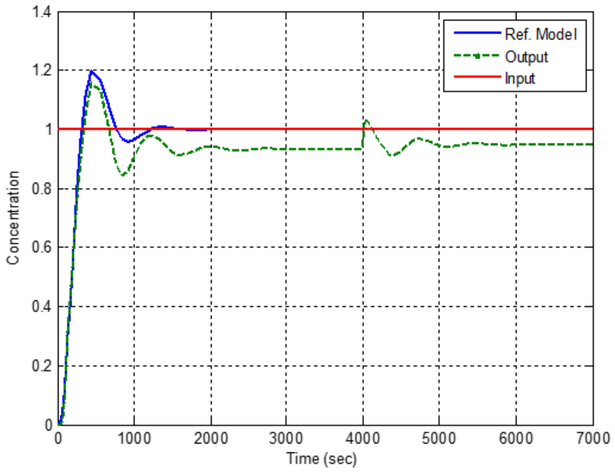

Fig. 13: Response of PID using Lyapunov method in the presence of load disturbance

The designed PID MRAC using the Lyapunov method was tested in terms of sine wave tracking performance as shown in Figure 14.

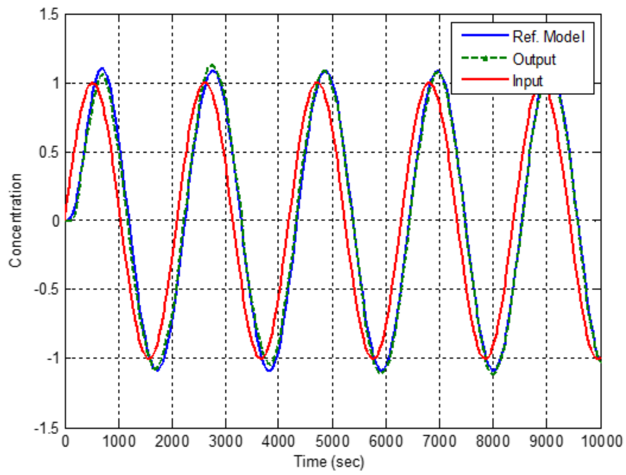

Fig. 14: Response of the system with PID Lyapunov method in terms of sine wave tracking performance

According to Figure 14, it seems that although PID using the Lyapunov method can track the commanded sine wave with a small error. The performance of the PID using the Lyapunov method in terms of sine wave tracking can also be computed by ITAE. In this case, ITAE is computed as 2930.

4.3 Comparison between the MIT rule and the Lyapunov method

The objective of simulating the two controllers on the CSTR system is to find out which one performs best in controlling the concentration of the second reactor. The Control techniques and schemes of the two controllers have been illustrated, and their performances are evaluated in terms of speed of response, disturbance rejection, and sine wave commanded.

The performance characteristics of output responses for both controllers that have been designed to control concentration in the second reactor are summarized as shown in Table 2.

Table 2. Performance characteristics of output responses using MIT and Lyapunov methods

Performance Specifications	MIT rule	Lyapunov method
Rise Time	12.5 min	5 min
Overshoot	23 %	15.5 %
Settling Time	30 min	21 min
Steady State Error	0.03	0.06

The two controllers provide a suitable response; the Lyapunov method is more stable, as it exhibits a smaller overshoot and shorter settling time, and is faster than the MIT rule. But the MIT rule is more accurate than the Lyapunov method.

5 Conclusion

The continuous stirred tank reactor (CSTR) is an important piece of equipment used in many applications. It has widespread application in industry and embodies many features of other types of reactors. In order to design an efficient controller for the product concentration in a CSTR, an accurate mathematical model of the system was first derived. Then, several controllers were implemented. The trial-and-error technique was used to tune the parameters of the investigated controllers. From the simulation results, the PID controller using the MIT rule exhibited a steady-state error smaller than that of the Lyapunov-based controller. Furthermore, the simulation results show that the reduction in rise time, settling time, and maximum overshoot is remarkable for the Lyapunov rule compared to the MIT rule. Thus, the Lyapunov rule confirms the stability of the system. The PID controller using the Lyapunov rule also has a faster response speed than the MIT rule and can track the command sine wave perfectly. Hence, it can be concluded that between the PID using the MIT rule and the PID using the Lyapunov method, the better controller for the CSTR system is the PID using the Lyapunov rule.

To advance beyond the limitations of manual tuning, a clear direction for subsequent research is the implementation of intelligent optimization frameworks. The trial-and-error approach is inherently inefficient and highly dependent on the designer's experience. Therefore, we strongly recommend adopting metaheuristic optimization techniques, specifically Particle Swarm Optimization (PSO) or Genetic Algorithms (GA), for PID parameter tuning. These algorithms can systematically explore the parameter space to find global or near-global optima that satisfy multiple, potentially competing, performance criteria (e.g., rise time, settling time, integral error). This would not only streamline the design process but also ensure consistently superior and reproducible system performance.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

Khalid Mustafa contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.
Abtihal Abdulqadir was responsible for the results and discussion section.

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