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FLR Effect on Stability of a Plasma in Porous Medium

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***Abstract*---The Rayleigh-Taylor instability of an infinitely conducting plasma in porous medium is investigated taking into account the finiteness of ion Larmor radius (FLR) in the presence of a horizontal magnetic field. Here we considered the perturbations propagating along the ambient magnetic field. It is established that, real part of n is negative, where n is the growth rate of disturbance, so that instability does not arise in the form of increasing amplitude, i.e. overstability. To obtain an approximate solution of the problem, a variational principle is used. Further, the case of two semi-infinitely extending plasmas of constant densities separated by a horizontal interface is considered. It is found that the system is stable for all wave numbers for potentially stable configuration and unstable (for some wave numbers) for potentially unstable configuration. Also for the said disturbances, the inclusion of FLR and porosity does not change the criteria of stability or instability.**

Keywords--- Finite Larmor Radius Effect, horizontal magnetic field, porous medium, Rayleigh-Taylor instability.

I.INTRODUCTION

When two fluids of different densities are superposed one over the other (or accelerated towards each other), the instability of the plane interface between the two fluids, when it occurs, is called Rayleigh-Taylor instability. In general, it is derived from the character of the equilibrium of an incompressible stratified heterogeneous fluid. Hide [1] has treated the character of the equilibrium of a viscous, incompressible, rotating fluid of variable density and found that rotation stabilizes the potentially unstable arrangement for certain wave-number range. He has considered the directions of angular velocity vector and gravity vector (in the direction of the vertical) to be inclined. In another study, Hide [2] has studied the case of a viscous, incompressible, electrically conducting fluid of variable density in the presence of a vertical magnetic field and found that magnetic field considerably stabilizes the configuration and it is possible to have oscillatory motions in the presence of magnetic field even if the configuration is thoroughly unstable (density wise). Chandrasekhar [3] has given a detailed account of the instability of the plane interface between two incompressible and viscous fluids of different densities when the lighter is accelerated into the heavier. The influence of viscosity on the stability of the plane interface separating two electrically conducting, incompressible superposed fluids of uniform densities, when the whole system is immersed in a uniform horizontal magnetic field, has been studied by Bhatia [4]. He has carried out the stability analysis for two fluids of high viscosities and different uniform densities. Hoshoudy and El-Ansary [5] have studied the effect of viscosity and homogeneous horizontal magnetic field on Rayleigh-Taylor instability of a heavy fluid supported by a lighter one. A good account of stability problems has also been given by Joseph and Renardy [6] in their study of two-fluid dynamics.

The properties of ionized space and laboratory magnetic fluids (plasmas) have been intensively investigated theoretically and experimentally in the past sixty years. One of the key aspects studied in this context is the stability of plasma structures. Usually, instabilities can be divided into two categories: macro- and micro-instabilities. Macro-instabilities occur with low frequencies compared to the plasma and cyclotron frequency and they are studied within the framework of magnetohydrodynamics (MHD). Physicists have understood the behaviour of macro-instabilities and they showed how to avoid the most destructive of them, but small-scale gradient driven micro-instabilities are still a serious obstacle for having a stable plasma for a large range of parameters. Micro-instabilities are described by models which include, e.g. finite Larmor radius (FLR) and collisionless dissipative effects in plasmas. Time and length scales of micro-instabilities are comparable to the turbulent length scales and the length scales of transport coefficients. In general, the FLR effect is neglected. However, when the Larmor radius becomes comparable to the hydromagnetic length of the problem (e.g. wavelength) or the gyration frequency of ions in the magnetic field is of the same order as the wave frequency, finiteness of the Larmor radius must be taken into account. Strictly speaking, the space and time scale for the breakdown of hydromagnetics are on the respective scales of ion gyration about the field, and the ion Larmor frequency. In the present paper, we explore the effect of FLR on Rayleigh-Taylor instability of a plasma. Finite Larmor radius effect on plasma instabilities has been the subject of many investigations. In many astrophysical plasma situations such as in solar corona, interstellar and interplanetary plasmas the assumption of zero Larmor radius is not valid. The stability effects of the finiteness of the ion Larmor radius (FLR) for perturbations transverse to a horizontal magnetic field is well established. Using kinetic equations of plasma, Rosenbluth et al. [7] studied the effect of FLR for several astrophysical situations such as slowly rotating plasma, mirror machines etc. Roberts and Taylor [8] derived the magnetohydrodynamic equations taking the effect of FLR in the form of 'magnetic viscosity' in the off-diagonal elements in the pressure tensor term. Hernegger [9] investigated the stabilizing effect of FLR on thermal instability and showed that thermal criterion is changed by FLR for wave propagation perpendicular to the magnetic field. Sharma [10] investigated the stabilizing effect of FLR on thermal instability of rotating plasma. Ariel [11] discussed the stabilizing effect of FLR on thermal instability of conducting plasma layer of finite thickness surrounded by a non-conducting matter. Bhatia and Chhonkar [12] investigated the stabilizing effect of FLR on the instability of a rotating layer of self-gravitating plasma incorporating the effects of viscosity and Hall current. Marcu and Ballai [13] showed the stabilizing effect of FLR on thermosolutal stability of a two-component rotating plasma. Kaothekar and Chhajlani [14] investigated the problem of Jeans instability of self-gravitating rotating radiative plasma with finite Larmor radius corrections.

The flow through porous media is of considerable interest for petroleum engineers, for geophysical fluid dynamicists and has importance in chemical technology and industry. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. Among the applications in engineering disciplines one can find the food processing industry, chemical processing industry, solidification and centrifugal casting of metals. Such flows has shown their great importance in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoirs; in chemical engineering for filtration and purification processes and in the field of agriculture engineering to study the underground water resources, seepage of water in river beds. The problem of Rayleigh-Taylor instability in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The study of Rayleigh-Taylor instability of fluids saturated porous media, has diverse practical applications, including that related to the materials processing technology, in particular, the melting and solidification of binary alloys. The development of geothermal power resources has increased general interest in the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat-transfer mechanism in young oceanic crust (Lister, [15]). Generally it is accepted that comets consists of a dusty 'snowball' of a mixture of frozen gases which in the process of their journey changes from solid to gas and vice- versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context (McDonnel, [16]). The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region. Vaghela and Chhajlani [17] studied the stabilizing effect of FLR on magneto-thermal stability of resistive plasma through a porous medium with thermal conduction. Vyas and Chhajlani [18] pointed out the stabilizing effect of FLR on the thermal instability of magnetized rotating plasma incorporating the effects of viscosity, finite electrical conductivity, porosity and thermal conductivity. Thus FLR effect is an important factor in the discussion of Rayleigh-Taylor instability and other hydromagnetic instabilities.

In the present work, we study the effects of finite Larmor radius on the Rayleigh-Taylor instability through porous medium of two semi-infinitely extending plasmas of constant densities separated by a plane interface at $z = 0$. The case of a uniform horizontal magnetic field and longitudinal perturbations is considered. A variational principle is developed from which the approximate solutions are obtained.

II. FORMULATION OF THE PROBLEM

Here we consider an incompressible and perfectly conducting plasma of variable density and viscosity stratified in a gravitational field $\vec{g}(0, 0, -g)$. The system is acted on by a uniform horizontal magnetic field $\vec{H}(H, 0,0)$. Then equations of motion for the plasma through the porous medium are

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\varepsilon}(\vec{q}.\nabla)\vec{q}\right] = -\nabla\vec{P}+\rho\vec{g}+\frac{1}{4\pi}(\nabla\times\vec{H})\times\vec{H} - \frac{\mu}{k_1}\vec{q}\ , \quad (1)$$

where ε is the porosity of the medium, $\vec{q}$ is the velocity vector and $\vec{P}$ is the stress tensor. Magnetic permeability of the medium is assumed to be unity. The equation of continuity of matter for an incompressible plasma is

$$\nabla.\vec{q} = 0. \quad (2)$$

Since the plasma is incompressible and diffusion effects (which tend to change the density of a plasma particle) are neglected, the density of a particle moving with plasma will remain constant and hence

$$\frac{\partial \rho}{\partial t}+\frac{1}{\varepsilon}(\vec{q}.\nabla)\rho = 0. \quad (3)$$

Maxwell's equations yield

$$\varepsilon\frac{\partial \vec{H}}{\partial t} = \nabla\times(\vec{q}\times\vec{H})+\varepsilon\eta\nabla^2\vec{H}, \quad (4)$$

and

$$\nabla.\vec{H} = 0, \quad (5)$$

where η is the electrical resistivity. For a perfect conductor ($\eta = 0$), (4) gives

$$\varepsilon\frac{\partial \vec{H}}{\partial t} = \nabla\times(\vec{q}\times\vec{H}). \quad (6)$$

Let $\delta\rho, \delta\vec{P}, \vec{q}(u, v, w)$ and $\vec{h}(h_x, h_y, h_z)$ denote, respectively, the perturbations in density ρ, stress tensor $\vec{P}$, velocity (initially zero) and magnetic field $\vec{H}$. Then the linearized hydromagnetic perturbation equations relevant to the problem are

$$\frac{\rho}{\varepsilon}\frac{\partial \vec{q}}{\partial t} = -\nabla\delta\vec{P}+\frac{1}{4\pi}(\nabla\times\vec{h})\times\vec{H}+\vec{g}(\delta\rho)-\frac{\mu}{k_1}\vec{q}\ , \quad (7)$$

$$\frac{\partial}{\partial t}(\delta\rho) = -\frac{1}{\varepsilon}(\vec{q}.\nabla)\rho, \quad (8)$$

$$\frac{\partial \vec{h}}{\partial t} = \frac{1}{\varepsilon}(\vec{H}.\nabla)\vec{q}, \quad (9)$$

$$\nabla.\vec{q} = 0 \text{ and } \quad \nabla.\vec{h} = 0. \quad (10)$$

For the magnetic field along x-axis, $\vec{P}$ taking into account the FLR effects has the following components

$$\left.\begin{aligned} P_{xx} = p, \quad P_{xy} = P_{yx} &= -2\rho\nu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right), \\ P_{xz} = P_{zx} &= 2\rho\nu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right), \\ P_{yz} = P_{zy} &= \rho\nu\left(\frac{\partial v}{\partial y}-\frac{\partial w}{\partial z}\right), \\ P_{yy} &= p-\rho\nu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right), \\ P_{zz} &= p+\rho\nu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right), \end{aligned}\right\} \quad (11)$$

where p is the scalar part of the pressure and $\rho\nu = \frac{NTk^*}{4\omega_H}$; ω_H is the ion-gyration frequency, k^* is the Boltzman constant, while N and T denote, respectively, the number density and temperature of ions.

Analyzing the disturbances in terms of longitudinal modes, we seek the solutions of (7) - (10) in which $x-t$ dependence is given by

$$exp(ikx+nt), \quad (12)$$

where k denotes the wave number of disturbance and n is the growth rate of disturbance.

Using (11) and (12), (7) - (10) can be written as

$$n\rho u = -ik\varepsilon(\delta p)-2ik\nu\varepsilon D(\rho\nu) - \frac{\mu}{k_1}\varepsilon u, \quad (13)$$

$$n\rho v = -2\rho\nu\varepsilon(D^2+k^2)w+\nu\varepsilon D(\rho Dw)+\frac{ik\varepsilon Hh_y}{4\pi} - \frac{\mu}{k_1}\varepsilon v, \quad (14)$$

$$n\rho w = -\varepsilon D(\delta p)+2\rho\nu\varepsilon k^2 v-\nu\varepsilon D(\rho Dv) + \frac{gw}{n}(D\rho)+\frac{H\varepsilon}{4\pi}(ikh_z-Dh_x) - \frac{\mu\varepsilon}{k_1}w, \quad (15)$$

$$n\delta\rho = -\frac{w}{\varepsilon}(D\rho), \quad (16)$$

$$\left.\begin{aligned} \varepsilon nh_x &= ikHu\ , \\ \varepsilon nh_y &= ikHv\ , \\ \varepsilon nh_z &= ikHw\ , \end{aligned}\right\} \quad (17)$$

$$iku+Dw = 0, \quad (18)$$

and

$$ikh_x+Dh_z = 0, \quad (19)$$

where

$$D = \frac{d}{dz}.$$

Eliminating δp from (13) and (14), and using (14) and (16) - (19), we obtain the following pair of equations in w and v

$$n^2[\rho k^2 w - D(\rho D w)] - g k^2 (D\rho) w - \frac{H^2 k^2}{4\pi}(D^2 - k^2) w + \frac{\varepsilon n}{k_1}[\mu k^2 w - D(\mu D w)] - v \varepsilon n k^2 [2(D^2 + k^2)(\rho v) - D(\rho D v)] = 0, \quad (20)$$

and

$$\left[n\rho + \frac{H^2 k^2}{4\pi n} + \frac{\mu\varepsilon}{k_1}\right] v = -v\varepsilon[2\rho(D^2 + k^2) w - D(\rho D w)]. \quad (21)$$

BOUNDARY CONDITIONS

On a boundary, vertical motion is not possible, thus

$$w = 0, \quad (22)$$

on a boundary free or rigid.

If the plasma is bounded by two rigid boundaries which are both ideally conducting, no disturbance within it can change the electromagnetic quantities outside. This merely leads to the boundary condition (22). A boundary condition on v can be prescribed by preluding the presence of surface charge or surface current at the rigid boundaries which are perfectly conducting. Thus we choose

$$v = 0, \quad (23)$$

at a surface bounded by an ideal conduction.

If the plasma is confined between two free boundaries, the tangential stresses

$$P_{xz} = 2\rho v i k v + \frac{ikH^2 w}{4\pi n}$$

and $P_{yz} = -\rho v D w$ vanish. Hence

$$v = Dw = 0, \quad (24)$$

at a free boundary. Should there be discontinuities in the density as in the case of two superposed plasma layers of different densities, we require the continuity of the vertical component of velocity, tangential stresses and pressure at interface. Thus

$$w,\ \rho D w,\ \rho v,$$

and the total pressure must be continuous as at the interface.

III. DISCUSSION

THEOREM I: A necessary and sufficient condition for δn^2 to be zero to the first order for all small arbitrary variations δw and δv (connected by (42)) in w and v which is compatible with the boundary conditions is that w and v should be the solutions of the eigenvalue problem governed by (20) and (21).

PROOF**:** Let n_i and n_j denote the two characteristic values, and let the solutions belonging to these characteristics values be distinguished by the subscripts i and j. Multiplying (20) for i by w_j and integrating with respect to z over the whole vertical extent of the plasma (denoted by $\int_L$), then with the help of (21) and boundary conditions, we obtain

$$n_i^2 \int_L \rho\left(w_i w_j + \frac{1}{k^2} D w_i D w_j\right) dz + \frac{H^2 k^2}{4\pi} \int_L \left(w_i w_j + \frac{1}{k^2} D w_i D w_j\right) dz - g\int_L (D\rho) w_i w_j dz + \frac{\varepsilon n_i}{k_1} \int_L \mu\left(w_i w_j + \frac{1}{k^2} D w_i D w_j\right) dz + n_i n_j \int_L \rho v_i v_j dz + \frac{H^2 k^2 n_i}{4\pi n_j} \int_L v_i v_j dz + \frac{\varepsilon n_i}{k_1} \int_L \mu v_i v_j dz = 0. \quad (25)$$

Taking $i = j$ and suppressing the subscripts, we obtain the following variational formulation of the problem

$$n^2(I_1 + I_4) + n(I_6 + I_7) - gI_2 + I_3 + I_5 = 0, \quad (26)$$

where

$$I_1 = \int_L \rho\left[w^2 + \frac{1}{k^2}(Dw)^2\right] dz, \quad (27)$$

$$I_2 = \int_L (D\rho) w^2 dz, \quad (28)$$

$$I_3 = \frac{H^2 k^2}{4\pi} \int_L \left[w^2 + \frac{1}{k^2}(Dw)^2\right] dz, \quad (29)$$

$$I_4 = \int_L \rho v^2 dz, \quad (30)$$

$$I_5 = \frac{H^2 k^2}{4\pi} \int_L v^2 dz, \quad (31)$$

$$I_6 = \frac{\varepsilon}{k_1} \int_L \mu\left[w^2 + \frac{1}{k^2}(Dw)^2\right] dz, \quad (32)$$

$$I_7 = \frac{\varepsilon}{k_1} \int_L \mu v^2 dz. \tag{33}$$

Consider the effect on n of an arbitrary variation δw and δv in w and, respectively, to satisfy the boundary conditions (22) and (23) of the eigen-value problem, (26) after first order approximation can be written as

$$\delta n^2(I_1 + I_4) + n^2(\delta I_1 + \delta I_4) + \delta n(I_6 + I_7) + n(\delta I_6 + \delta I_7) - g\delta I_2 + \delta I_3 + \delta I_5 = 0, \tag{34}$$

where $\delta I_s (s = 1 \ to\ 7)$ are the corresponding variations in $I_s (s = 1 \ to\ 7)$. After repeated integrations by parts, we find that these latter variations are given by

$$\frac{1}{2}\delta I_1 = \int_L \left[\rho w - \frac{1}{k^2} D(\rho D w)\right] \delta w dz, \tag{35}$$

$$\frac{1}{2}\delta I_2 = \int_L (D\rho) w \, \delta w dz, \tag{36}$$

$$\frac{1}{2}\delta I_3 = \frac{H^2 k^2}{4\pi} \int_L \left(w - \frac{1}{k^2} D^2 w\right) \delta w dz, \tag{37}$$

$$\frac{1}{2}\delta I_4 = \int_L \rho v \delta v dz, \tag{38}$$

$$\frac{1}{2}\delta I_5 = \frac{H^2 k^2}{4\pi} \int_L v \delta v \, dz, \tag{39}$$

$$\frac{1}{2}\delta I_6 = \frac{\varepsilon}{k_1} \int_L \left[\mu w - \frac{1}{k^2} D(\mu D w)\right] \delta w \, dz \tag{40}$$

and

$$\frac{1}{2}\delta I_7 = \frac{\varepsilon}{k_1} \int_L \mu v \delta v \, dz. \tag{41}$$

Furthermore, δw and δv are connected by the relation

$$\delta n \left[\rho - \frac{H^2 k^2}{4\pi n^2}\right] v + n \left[\rho + \frac{H^2 k^2}{4\pi n^2} + \frac{\mu \varepsilon}{k_1 n}\right] \delta v = -\nu\varepsilon[2\rho(D^2 + k^2)\delta w - D(\rho D \delta w)]. \tag{42}$$

If we substitute for I_s and $\delta I_s (s = 1 \ to\ 7)$ in (34) and make use of (42), we obtain after repeated integrations by parts,

$$\delta n^2 \left[I_1 + \frac{1}{n^2} I_5 + \frac{1}{2n}(I_6 + I_7)\right] + \frac{2}{k^2} \int_L \Big[n^2\{\rho k^2 w - D(\rho D w)\} - g k^2 (D\rho) w - \frac{H^2 k^2}{4\pi}(D^2 - k^2) w + \frac{\varepsilon n}{k_1}\{\mu k^2 w - D(\mu D w)\} - \nu\varepsilon k^2 n\{2(D^2 + k^2)(\rho v) - D(\rho D v)\}\Big] \delta w \, dz = 0. \tag{43}$$

We observe that the quantity occurring as a factor of δw under the integral sign vanishes if and only if (20) is satisfied. Thus a necessary and sufficient condition for δn^2 to be zero to the first order for all small arbitrary variations δw and δv (connected by (42)) in w and v which is compatible with the boundary conditions is that w and v should be the solutions of the eigenvalue problem governed by (20) and (21). A variational procedure of solving for the characteristic values is, therefore, possible.

THEOREM II**:** If oscillatory modes exist they should be stable.

PROOF**:** From (25), we have

$$n_i \int_L \left(w_i w_j + \frac{1}{k^2} D w_i D w_j\right) dz - \frac{g}{n_i} \int_L (D\rho) w_i w_j \, dz + \frac{H^2 k^2}{4\pi n_i} \int_L \left(w_i w_j + \frac{1}{k^2} D w_i D w_j\right) dz + \frac{\varepsilon}{k_1} \int_L \mu \left(w_i w_j + \frac{1}{k^2} D w_i D w_j\right) dz + n_j \int_L \rho v_i v_j \, dz + \frac{H^2 k^2}{4\pi n_j} \int_L v_i v_j \, dz + \frac{\varepsilon}{k_1} \int_L \mu v_i v_j dz = 0. \tag{44}$$

Interchanging i and j and noting that the above integrals are symmetric in i and j, we obtain

$$n_j \int_L \left(w_i w_j + \frac{1}{k^2} D w_i D w_j\right) dz - \frac{g}{n_j} \int_L (D\rho) w_i w_j \, dz + \frac{H^2 k^2}{4\pi n_j} \int_L \left(w_i w_j + \frac{1}{k^2} D w_i D w_j\right) dz + \frac{\varepsilon}{k_1} \int_L \mu \left(w_i w_j + \frac{1}{k^2} D w_i D w_j\right) dz + n_i \int_L \rho v_i v_j \, dz + \frac{H^2 k^2}{4\pi n_i} \int_L v_i v_j \, dz + \frac{\varepsilon}{k_1} \int_L \mu v_i v_j dz = 0. \tag{45}$$

Let us consider two solutions characterized by n and n^* (the complex conjugate of n). We expect that the corresponding solutions will also be the complex conjugates of each other. Hence if $n_i = n, n_j = n^*$, then $w_i = w, w_j = w^*, v_i = v \text{ and } v_j = v^*$.

Then, from (44) and (45) by addition and subtraction, we have

$$Re(n)\left[\overline{I_1} + \overline{I_5} - \frac{g}{|n|^2}\overline{I_2} + \frac{H^2k^2}{4\pi|n|^2}\overline{I_3} + \frac{H^2k^2}{4\pi|n|^2}\overline{I_4}\right] + \overline{I_6} + \overline{I_7} = 0, \tag{46}$$

and

$$Im(n)\left[\overline{I_1} - \overline{I_5} + \frac{g}{|n|^2}\overline{I_2} - \frac{H^2k^2}{4\pi|n|^2}(\overline{I_3} - \overline{I_4})\right] = 0, \tag{47}$$

where

$$\left.\begin{aligned}
\overline{I_1} &= \int_L \rho\left(|w|^2 + \tfrac{1}{k^2}|Dw|^2\right)dz,\\
\overline{I_2} &= \int_L (D\rho\,|w|^2)dz,\ \ \overline{I_3} = \int_L \left(|w|^2 + \tfrac{1}{k^2}|Dw|^2\right)dz,\\
\overline{I_4} &= \int_L |v|^2 dz,\ \ \overline{I_5} = \int_L \rho|v|^2 dz,\\
\overline{I_6} &= \tfrac{\varepsilon\mu}{k_1}(|w|^2 + \tfrac{1}{k^2}|Dw|^2)dz,\\
\overline{I_7} &= \int_L \tfrac{\varepsilon\mu}{k_1}|v|^2 dz.
\end{aligned}\right\} \tag{48}$$

Integrals $\overline{I_s}(s = 1 \text{ to } 7)$ are all positive.

If n is complex, $Im(n) \neq 0$, hence (47) gives

$$\overline{I_1} - \overline{I_5} + \frac{g}{|n|^2}\overline{I_2} - \frac{H^2k^2}{4\pi|n|^2}(\overline{I_3} - \overline{I_4}) = 0, \tag{49}$$

so that (46) now gives

$$2Re(n)\left[\overline{I_1} + \frac{H^2k^2}{4\pi|n|^2}\overline{I_4}\right] = -(\overline{I_6} + \overline{I_7}) = -\frac{\varepsilon}{k_1}\int_L \mu\left\{|w|^2 + \frac{1}{k^2}|Dw|^2 + |v|^2\right\}dz \tag{50}$$

From (50), it follows that $R_e(n)$ is negative, which implies that if oscillatory modes exist; they should be stable, thus ruling out possibility of overstability.

IV. DISCUSSION ON THE CASE OF TWO SEMI-INFINITELY EXTENDING PLASMAS OF CONSTANT DENSITIES SEPARATED BY A HORIZONTAL PLANE

We consider the case when two semi-infinitely extending plasma layers of constant densities ρ_1 and ρ_2, and uniform viscosities μ_1 and μ_2 are separated by a horizontal boundary at $z = 0$. The subscripts 1 and 2 distinguish the lower and upper plasma layers, respectively.

We choose the following trial function for $w(z)$,

$$w(z) = \begin{cases} Ae^{+kz} & z < 0; \\ Ae^{-kz} & z > 0, \end{cases} \tag{51}$$

which is consistent with the boundary conditions (22) - (24). Here the same constant has been chosen to ensure the continuity of w at $z = 0$.

The value of v in the two regions can be calculated from (21) and noting that ρ and μ are constant, we have

$$v(z) = \begin{cases} Z_1e^{+kz} & z < 0; \\ Z_2e^{-kz} & z > 0, \end{cases} \tag{52}$$

where

$$Z_{1,2} = \frac{-3v\varepsilon k^2 nA}{n^2 + k^2V_{1,2}^2 + \frac{v'\varepsilon n}{k_1}}, \tag{53}$$

$$V_1^2 = \frac{H^2}{4\pi\rho_1} \text{ and } V_2^2 = \frac{H^2}{4\pi\rho_2}. \tag{54}$$

We assume that the kinematic viscosities in both the regions to be equal i.e. $v_1 = \frac{\mu_1}{\rho_1} = \frac{\mu_2}{\rho_2} = v_2 (= v')$ [Chandrasekhar [3], p. 443], as the simplifying assumption does not obscure any of the essential features of the problem.

To evaluate the integrals $I_s(s = 1 \text{ to } 7)$ in (26), we divide the region of integration into three parts $(i) -\infty < z < -\varepsilon$ (ii) $\varepsilon < z < \infty$ $(iii) -\varepsilon < z < \varepsilon$, and then pass it over to the limit $\varepsilon \to 0$. On substituting their values in (26), we obtain the following dispersion relation between n and k,

$$n^2 - gk(\alpha_2 - \alpha_1) + k^2V_A^2 + \frac{n\varepsilon v'}{k_1}(\alpha_1 + \alpha_2) + \frac{g}{2}v^2\varepsilon^2k^4n^2\left\{\frac{\alpha_1}{n^2 + k^2V_1^2 + \frac{v'\varepsilon n}{k_1}} + \frac{\alpha_2}{n^2 + k^2V_2^2 + \frac{v'\varepsilon n}{k_1}}\right\} = 0, \tag{55}$$

where

$$V_A = \left[\frac{H^2}{2\pi(\rho_1 + \rho_2)}\right]^{1/2} \quad (56)$$

can be termed as mean Alfven's velocity in the medium and

$$\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1 + \rho_2}. \quad (57)$$

Letting

$$n = \frac{g}{V_A} n^*, \qquad k = \frac{g}{V_A^2} k^*$$

and omitting the asterisks for simplicity, so that the (55) takes the following dimensionless form

$$A_6 n^6 + A_5 n^5 + A_4 n^4 + A_3 n^3 + A_2 n^2 + A_1 n + A_0 = 0, \quad (58)$$

where

$$A_6 = 4\,, A_5 = 4U(1 + \alpha_1 + \alpha_2)\,,$$

$$A_4 = 2k^2\left(\frac{\alpha_1 + \alpha_2}{\alpha_1\alpha_2}\right) + U^2[1 + 4(\alpha_1 + \alpha_2)] + 4[k^2 - k(\alpha_2 - \alpha_1)] + 2L(\alpha_1 + \alpha_2)k^4,$$

$$A_3 = U(\alpha_1 + \alpha_2)\left[Lk^4 + \frac{k^2}{\alpha_1\alpha_2} + 2k^2\left(\frac{\alpha_1 + \alpha_2}{\alpha_1\alpha_2}\right) + U^2\right] + 4U[k^2 - k(\alpha_2 - \alpha_1)],$$

$$A_2 = \frac{k^4}{\alpha_1\alpha_2} + 4[k^2 - k(\alpha_2 - \alpha_1)] + Lk^6\left(\frac{\alpha_1}{\alpha_2} + \frac{\alpha_2}{\alpha_1}\right),$$

$$A_1 = \left[\frac{k^4}{\alpha_1\alpha_2} + \frac{k^2}{\alpha_1\alpha_2}\{k^2 - k(\alpha_2 - \alpha_1)\}\right]U(\alpha_1 + \alpha_2)\,,$$

$$A_0 = \frac{k^4}{\alpha_1\alpha_2}[k^2 - k(\alpha_2 - \alpha_1)]\,, \quad (59)$$

$$L = \frac{9\nu^2 g^2 \varepsilon^2}{V_A^6}\,, \quad (60)$$

is a non-dimensional number measuring the relative importance of FLR effects and magnetic field.

Here

$$U = \frac{\nu' \varepsilon V_A}{k_1 g}\,, \quad (61)$$

is a non-dimensional number measuring the relative importance of porosity and magnetic field.

For the potentially stable configuration ($\alpha_2 < \alpha_1$), all the coefficients of (58) are positive. So no positive real root or complex root with negative real part exists. Therefore, **the medium is stable for all the wave numbers.**

For the potentially unstable configuration ($\alpha_2 > \alpha_1$), the absolute term in (58) is negative, if

$$0 < k < k', \quad (62)$$

where

$$k' = \alpha_2 - \alpha_1\,. \quad (63)$$

Therefore (58) possesses at least one real root which is positive leading to an instability of the configuration.

Also we see that k' is independent of L, a measure of FLR effect and U. Hence we conclude that for longitudinal perturbations, the stability criterion is independent of magnetic viscosity and porosity.

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