Nonlinear Magnetic Levitation System Modelling and Switched Control

Peng Wen and **Adam Clarke**
Faculty of Health, Engineering and Science
University of Southern Queensland
Queensland, Australia
pengwen@usq.edu.au

Abstract—**This paper investigates the dynamic model of a nonlinear magnetic levitation system and its precise control using robust control technique. In this study, a magnetic levitation system model is first identified using piecewise linearization technique and numerical computation. Then a robust controller which could stand up to 50% displacement around the operating area is designed and investigated in simulation. Then this technique is extended to the whole range operation, using switched controller techniques. Finally, the performance of the designed controller was evaluated both in simulation and experiments. The results showed that the performance is consistently better off comparing with that of a traditional PID controller.**

Keywords— system modelling, switched control, robust control

I.   INTRODUCTION

Magnetic levitation systems can be applied to a wide area such as maglev passenger trains, levitation of wind tunnel model, frictionless bearings, etc. In particular, the magnetic levitation train reduces rolling friction between the locomotive and the railway, much higher speeds and less energy lost to friction can be achieved. For the Transrapid Maglev train in Germany, the gap between the train and the guidance coil is around 10cm [1]. Therefore the free play of the train is estimated to be around 1-2cm. It can be seen that for a train travelling at 500km/h (achievable by Maglev), keeping the train within a 2cm tolerance would require a precise control system. The goal is to ensure the train stays within the prescribed distances inside the guide rail while maintaining the safety of the passengers and cargo.

Due to open-loop instability and inherent nonlinearities associated with the electromechanical dynamics, the control problem is usually quite challenging to the control engineers. A lot of work has been done in the past decades and most of the research focuses on the nonlinearities of the system model and the uncertainties of physical parameters. Unfortunately, most of these techniques generally need linearization techniques and have a common drawback: exact information of the system parameters for a complete linearized model is always required [2]. This drawback often leads to the robustness problem of control systems [3]. In order to solve the robustness problem caused by the feedback linearization strategy, an integral sliding-mode control with H∞ method is proposed. Studies on applications of the integral sliding-mode control to the magnetic levitation systems have been published in [4-5]. These studies take use the advantages of the integral sliding-mode control: nominal performance dependent on sliding surface design and insensitivity to system uncertainties in the whole process [6–8]. However, both the conventional sliding-mode and integral sliding-mode controls are only robust to matched uncertainties or perturbations. Therefore, developments in recent years on improving the (integral) sliding-mode controls for robustness to unmatched uncertainties have been presented. These methods summarily include the following: dynamic sliding-surface design [4], a modified switching algorithm [6], and robust optimal controls [7-8]. In particular, [7-8] give a good control-design concept; that is, combining the integral sliding-mode control with another robust technique can effectively guarantee the stability of the systems with the presence of unmatched uncertainties.

In this study, we take the nonlinear system as a linear one for small degrees of motion. In this case, a process called piecewise linearization is applied to obtain the system parameters at each operation point. Once the system parameters have been identified, a switched controller is designed for robust deadbeat control. Finally, the proposed controller is implemented and tested in simulation and experiments.

This paper is presented in five sections. Following the Introduction is the Maglev and Model section which describes the system and identifies the system model. Section 3 presents the Robust Deadbeat Control Technique. Section 4 illustrates the simulation study, the system parameters selection, and the experiment results. The conclusion is drown in section 5.

II.   MAGLEV SYSTEM AND MODEL

The Maglev development system shown in Fig. 1 is a one-degree-of-freedom (vertical) control system. There are two coils in the machine. The bottom coil is effective for the bottom half of the glass rod, and the top coil is effective for the top half (with minor cross-coupling). SISO (Single Input, Single Output) operation uses the bottom coil and one magnet, and the top coil is not energised. Fig. 2 shows the interaction of forces which are exerted on the magnets (MIMO configuration), all the variables and constants are provided in the ECP manual.
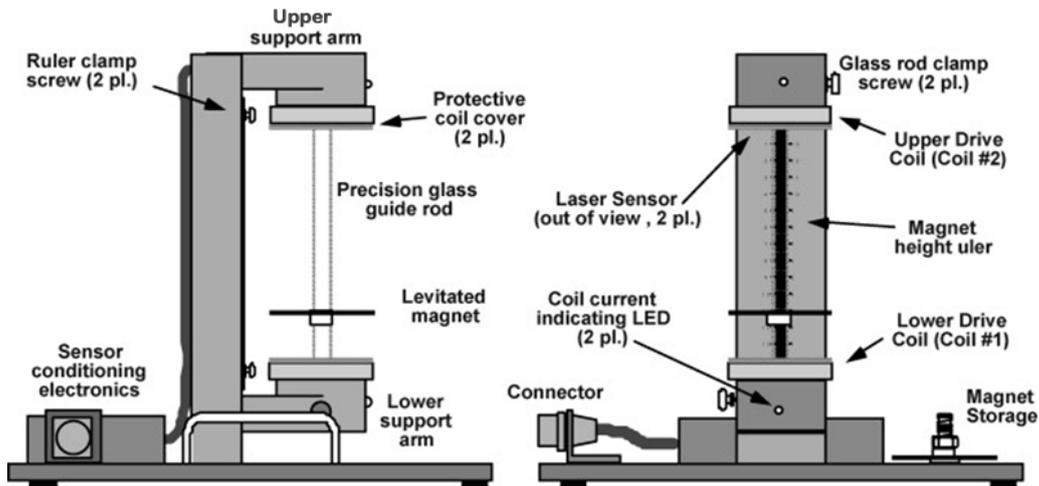
Fig. 1. ECP Model 730 Representation (Source: Educational Control Products 2002

This study focuses on the control of a single magnet using the bottom coil, in a SISO configuration. Therefore all forces related to coil 2 and magnet 2 is null. The following equation shows the summation of forces in the system.

$$m\ddot{y}_1 = F_{U_{11}} - mg \qquad (1)$$

$$F_{U_{11}} = \frac{I_1}{a(y_1+b)^4} \qquad (2)$$

Note from the equation above, the coil current I is directly proportional to the control effort U. The control effort is the output of the real time control algorithm, and the IO controller converts this into an actual current. From (2), it can be seen that as the distance increases between the coil and magnet, a much greater control effort (current) is required to produce the same force [9]. This creates nonlinearities in the control system. The nonlinear relationship between the distance of the magnet from the coil, and the force imparted on the magnet must be compensated for while the plant is modelled as a linear system. Therefore, a linear approximation of this nonlinear characteristic must be made. As ECP manual states that there may be up to a 10% variance in the force/distance/current equation for any Maglev machine. For the accurate realisation of the Maglev system parameters, this relationship must be numerically calculated for each plant. To obtain the force/distance/current relationship, the machine must be energised with different coil efforts (proportional to current) and the height at which the magnet settles is recorded. Via numerical analysis, the constants a and b are calculated as 1.64 and 6.2 respectively.

Fig.3 shows how close the estimated curve follows the measured data points.

Taylor's series expansion method is applied to the above system, and results in a linearized system for small excursions about the operating point. The equation which represents the control system in terms of differentials is:

$$\frac{d^2 y_1}{dt^2} + \frac{c_1}{m}\frac{dy_1}{dt} + \frac{c}{m(y_{12}+d)^4} + \frac{U_2}{ma(y_c-y_1+b)^4} + g + \frac{U_1}{ma(y_1+b)^4} = 0$$

$$(3)$$

Applying Taylor's series expansion to (3) to get the linearized equation, then the system can be represented in the state-space form.
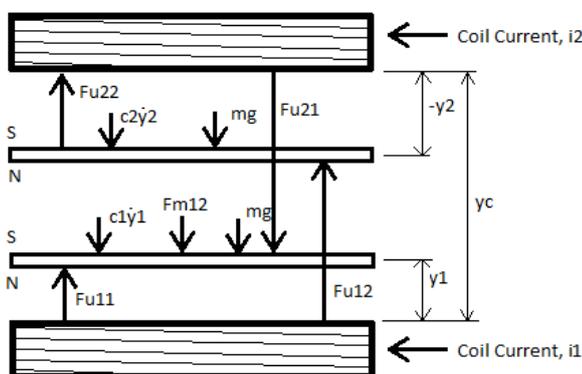
Fig. 2. Free-body diagram with force interactions [9]
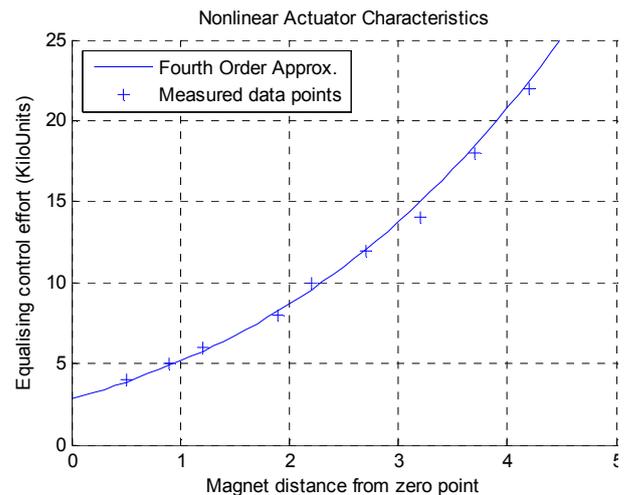
Fig. 3. Representation of the estimated curve against the experimental data

$$x = \begin{bmatrix} y_1 \\ \dot{y}_1 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 \\ \frac{4U_1}{ma(y_{1_0}+b)^5} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{ma(y_{1_0}+b)^4} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(4)

Equation (4) gives a linear approximation about the desired operating point (y_{1_0}) and the corresponding equalising control effort (U_1).

The linearization theorem exhibits the highest accuracy when the magnet is close to the linearization point. However, the accuracy degrades exponentially as the magnet moves away from point. Therefore it is deemed that this model of the system is accurate only for small movements about the linearization point.

III.   ROBUST DEADBEAT CONTROL TECHNIQUE

The robust deadbeat control technique for a known system was proposed by Dawes, Ng, Dorf and Tam (1994) [10]. This technique includes the robust control abilities of a PID controller, and applies feedback poles and constants which result the deadbeat response properly. Fig. 4 shows a diagram of the proposed robust deadbeat controller.

In Fig. 4, a 'zero' is added in a feedback loop $(1 + K_b s)$. Through this feedback loop, the error between the desired position and the current position is calculated. A PID controller is then applied to this error signal, making the system robust. A state variable feedback constant is applied to the output of the controller. Dawes, Ng, Dorf and Tam (1994) claimed that the robust deadbeat controller could reject up to a 50% change in the system dynamics before the response of the controller was affected. Therefore it is assumed that for small movements of the magnet, the controller is able to reject the nonlinear error. Note that the procedures for applying this controller to different order plants differ slightly. The following procedure outlines the method which applies a robust deadbeat controller to a second order plant, as the approximation for the Maglev characteristics is second order [10]:

1. Approximate the plant transfer function $G_P(s)$
2. Let K=1 (or any other arbitrary number)
3. Have the characteristic equation of the system equal

$$C.E = s^3 + \alpha w_n s^2 + \beta w_n^2 s + w_n^3 \qquad (5)$$

4. Find the alpha, beta gains based on the order of the plant
5. Compare the characteristic equation above to the closed loop transfer function of the controller and plant, finding X, Y etc.
6. Simulate the system with different K values until the best response is found

The next step is to calculate the controller parameters specific to the Maglev plant. The design of the controller must use an approximation of the plant in order to achieve a deadbeat response. The approximation for the preliminary design is made to be about a magnet position of 2cm. As stated, linearizing the plant about 2cm will cause the plant to be accurate at this position; however the accuracy will degrade as the magnet moves away from this position. (6) shows the result of applying Taylor's theorem to the plant for a magnet position of 2cm.

$$G_p(s) = \frac{1111}{s^2 + 4s + 478.5} \qquad (6)$$

This equation is a linear approximation of the plant parameters. The characteristic equation of the plant is to be compared with (5). Dawes, Ng, Dorf and Tam (1994) state that for a second order system, the following constants are to be used [10]:

$$\alpha = 1.90, \beta = 2.20, Tr_{90} = 3.48, T_s = 4.04$$

Combining (10) and (12), the characteristic equation is described as:
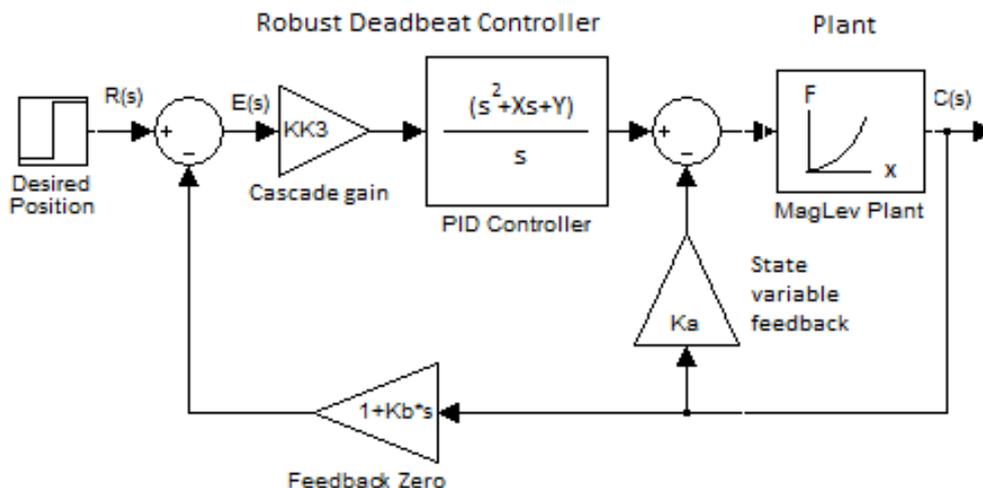
Fig. 4.   Deadbeat Control Technique [10].

$$C.E = s^3 + 73.104s^2 + 3256.9s + 56961 \qquad (7)$$

The closed loop transfer function of the controller and plant is described as:

$$G(s) = \frac{G_c(s)G_P(s)}{1+G_c(s)G_P(s)KK_3+G_c(s)G_P(s)(1+K_b)} \qquad (8)$$

Substituting the plant and controller parameters, the entire system transfer function with the controller variables is:

$$\frac{1110KK_3(s^2 + Xs + Y)}{s^3 + \dfrac{s^2(1110KK_3 + 1110KK_3K_bX + 4)}{1110KK_3K_b + 1} + \dfrac{s(1110K_a + 1110KK_3X + 1110KK_3K_bY + 479)}{1110KK_3K_b + 1} + \dfrac{1110KK_3Y}{1110KK_3K_b + 1}} \qquad (9)$$

The characteristic equation is the entire denominator of the above equation. Note that the s-variable orders (s^2, s etc.) have been separated, which assists in the following comparison between the characteristic equations. Comparing (7) and (8), we have

$$\frac{(1110KK_3 + 1110KK_3K_bX + 4)}{1110KK_3K_b + 1} = 73.104$$

$$\frac{(1110K_a + 1110KK_3X + 1110KK_3K_bY + 479)}{1110KK_3K_b + 1} = 3256.9$$

$$\frac{1110KK_3Y}{1110KK_3K_b + 1} = 56961$$

By choosing arbitrary K, K3 and TS values, the variables X, Y and Ka can be found using simultaneous solutions. Note that Kb is directly related to the $T_{desired}$ value. It can be seen that varying both K and K3 will result in dramatic changes in the control constants, and that these values must be optimised in order to get the best response.

IV.   SIMULATION AND EXPERIMENT STUDY

A Simulink model is designed to accept constants/parameters from the Matlab script in order to automate the entire simulation process. The model is also able to pass data from the input, output and other key points of interest back to the Matlab variable workspace. This helps in plotting the data, finding the settling time and system diagnostics. Fig. 5 shows the basic Simulink model which was used to simulate the whole system including controller and plant.

In the model, it can be seen that the basic controller is broken down into sections, each being created using certain blocks in Simulink. The feedback zero, $(1 + sK_b)$, is converted to the summation of the output and the Kb constant multiplied by the derivative of the output. This signal is then negated from the input, creating an error signal. The PID controller inside the robust deadbeat controller is a simple summation of the proportional, integral and derivative of the error signal, multiplied by the respective gains. The cascade gain, Ka, is then negated from the output of the PID controller, giving the final control signal which is fed into the Maglev plant simulation block. This completes the controller design in Simulink. The constants/parameters are pre-computed, and Simulink handles the actual step-by-step simulation. Fig. 6 shows the simulation results.

Fig. 6 shows the movement from magnet rest to 2cm, and the response is shown in the top figure. The settling time is 181ms. When observing the initial current surge at <50ms, the current limiter clamps the machine to maximum current, which is quite acceptable, as the inertia of the magnet must be overcome.

As the controller is only designed for movements about 2cm, it is known that the linearity error will increase as the desired position moves away from 2cm. The oscillations appear when an 1 cm deviation from the linearized point is generated. When the desired position is moved more than 2cm from the linearization point, the plant becomes almost unstable and exhibits violent oscillations.

The initial implantation of the controller on the plant using the optimised simulation values failed. The magnet was found to vibrate about the control point, with movements of up to ±1.5cm. It was found that the plant controller was critically stable, as the oscillation amplitudes were consistent over time.
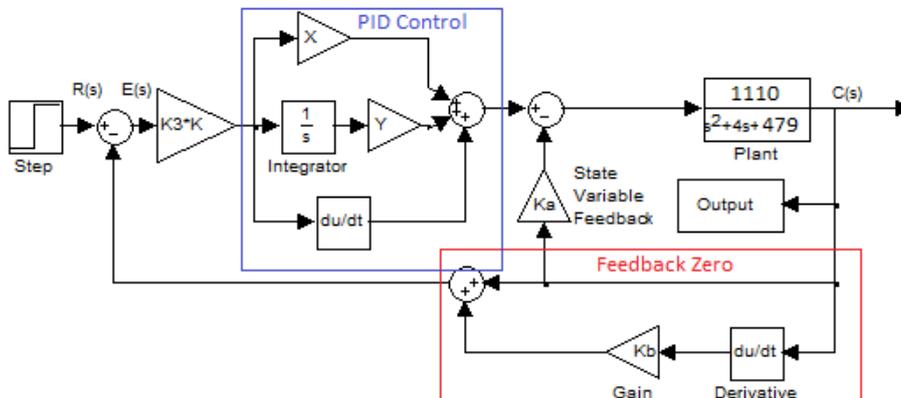
Fig. 5.   Robust deadbeat controller simulation model
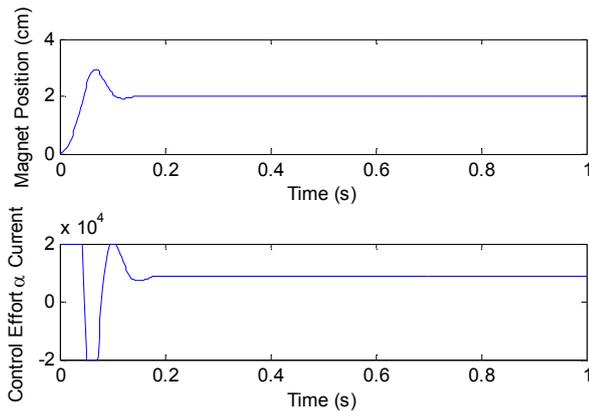
Fig. 6.   Best Simulink response for a 2cm movement

Fig. 7.   Step response comparison for small magnet movements

This response was anticipated, as the plant is subjected to many different environmental and machine issues which could not be investigate in simulation. Diagnostics were undertaken to identify the issue, and solutions to obtain a better response were found through filtering out sensor noise, adding thermal compensation and changing settling time specification.

Fig. 7 shows the responses of an switched adaptive deadbeat controller designed using piecewise linearization techniques and the response of an optimised PID controller. The red line is the input to the system which is applied to both the switched adaptive deadbeat controller and the PID controller. This line is about 2cm, and deviates by ±1cm, which shows a good range of responses. This provides a good test for the controllers, as it causes the switched deadbeat controller to switch between controllers. The blue line indicates the response of the best PID controller, as shown by ECP. It can be seen that there is a large overshoot, which is present for a significant amount of time. The response here is quite slow, as the controller does take time for correction. The settling time is 550ms on average, and the stability of the system is quite good as there are no significant oscillations present.

V.   CONCLUSION

This paper studied the model and dynamic control of a magnetic levitation system based on an ECP Magnetic Levitation plant, and proposed a new adaptive switched controller. This study firstly investigated the system dynamics and identified its nonlinearity. Then a system model was developed using linearization technique. A robust deadbeat controller was designed and simulated based on the developed model. The initial simulations found that this model works only in a small range around the operating point, and a piece-wise system model and a switch controller were required, due to the nonlinear dynamics of the plant. While applying the designed switch controller to the plant, some real-world problems such as noise and control errors were encountered.
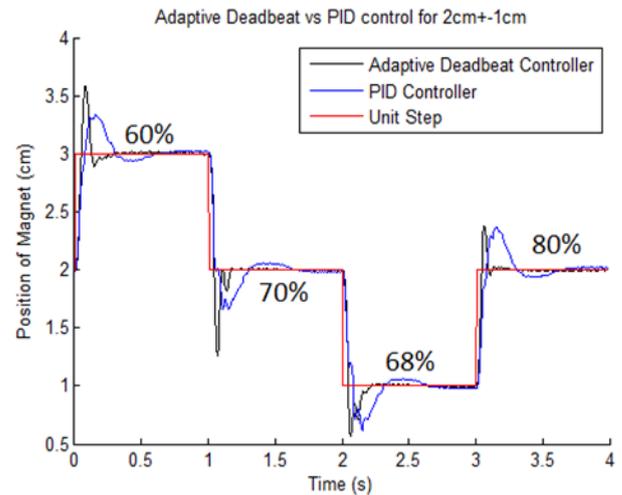
Digital signal processing techniques were applied to solve these problems. In the final testing, the performance of the designed controller was evaluated by comparing with an optimised PID controller. The results showed that the performance is consistently 60% better in settling time than that of a traditional PID controller. Other performance includes 66% better off in disturbance rejection, and 30% increasing in bandwidth in frequency response.

VI.   REFERENCES

[1] Lee Hyung-Woo, Kim Ki-Chan, and Lee Ju, Review of Maglev Train Technologies, IEEE Transaction on Magnetics, Vol. 42, No.7, pp. 1917-1925, July 2006.

[2] Chen, Mei-Yung, Wang Ming-Jyh, and Fu Li-Chen, "A novel dual-axis repulsive maglev guiding system with permanent magnet: modelling and control design," IEEE/ASME Trans. Mechatronics., Vol. 8, No. 1, pp. 77–86, March 2003.

[3] Sastry, Shankar, Nonlinear Systems: Analysis, Stability, and Control, Springer, New York, U.S.A. 1999.

[4] Yang, Z. J., K. Miyazaki, S. Kanae, K. Wada,"Robust position control of a magnetic levitation system via dynamic surface control technique," IEEE Trans. Ind. Electron., Vol. 51, No. 1, pp. 26–34 (2004).

[5] Lee, J. H., P. E. Allaire, G. Tao, X. Zhang, "Integral sliding mode control of a magnetically suspended balance beam: analysis, simulation, and experiment," IEEE Trans. Ind. Electron., Vol. 6, No. 3, pp. 338–346 (2001).

[6] Castanos, F. and L. Fridman, "Design of integral sliding manifolds for multi-model uncertain systems via LMI." Proc. Int. Workshop Variable Struct. Syst., Italy, pp. 63–67 (2006).

[7] Bejarano, F. J., L. Fridman, A. Poznyak, "Output integral sliding mode with application to the LQ optimal control," Proc. Int. Workshop Variable Struct. Syst., Italy, pp. 68–73 (2006).

[8] Castanos, F. and L. Fridman, "Analysis and design of integral sliding manifolds for systems with unmatched perturbations," IEEE Trans. Autom. Control, Vol. 51, No. 5, pp. 853–858 (2006).

[9] Parks, T 1999, Manual for Model 730 Magnetic Levitation System, Educational Control Products, California, America.

[10] J. Dawes, L. Ng, R. Dorf, and C. Tam, "Design of deadbeat robust systems," Glasgow, UK, pp1597-1598, 1994.