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Robust Control of Robot Manipulator based on QFT and H_∞

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***Abstract*— In this paper robust controllers have been obtained by using QFT and H_∞ control techniques, to control movements of waist, shoulder and elbow of a 6-DOF manipulator. An uncertain experimental linear model of the robot, that represents the non-linearity of the dynamics system, was used to design the controllers. First, H_∞ controllers were obtained by using weighting functions and then robust PID-controllers and pre-filters for each link of the robot were obtained by using QFT control methodology, in order to compare the dynamical performance of both types of robust controllers.**

***Keywords*—robot dynamics; robust control; robustness; uncertainty**

I. Introduction

ADVANCES in computing, communications and electronics have allowed that robotics and related technologies such as cybernetics and mechatronics may have an amazing growth in last decades. For this reason, in recent years the concept of robot has gone from a science fiction movie to become a reality as a standalone machine, with great potential for use in applications, since welding robots in the automotive industry, robots used in medicine for the study of the human body, until teleoperated arms in space shuttles, among others [1].

The main advantages supported for using this type of mechanism in the industry include reducing the production costs due to the increase of accuracy, productivity, quality, and flexibility compared to specialized machines. Therefore, the objective of the control system in robot manipulators is to maximize the accuracy, repeatability and speed of execution of tasks, taking into account the physical limitations of the actuators and establishing a commitment to the needs of each application practice.

In order to ensure an efficient dynamic performance in a robotic manipulator with nonlinearity conditions, coupled dynamics, disturbances and unmodeled dynamics, this paper proposes to use two robust control strategies that take into account these characteristics: Quantitative Feedback Theory (QFT) proposed by Horowitz [2] and the design of controllers using H_∞ technique [3].

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Quantitative Feedback Theory has proved a very powerful methodology for the design of feedback systems controllers. Its effectiveness is due to its ability to tune robust controllers for systems where the plant presents uncertainty and / or there are disturbances acting on the plant. In the case of MIMO systems, as in the field of robotics, the classical idea of the QFT technique is based on fixed point theory wherein the multivariable system is broken down into single loop MISO systems and coupling effects are treated as interference to the entrance of the plant [4].

The H_∞ control theory proposed by Zames in 1981, is widely recognized as an indispensable method of designing robust control systems. The optimum design H_∞ is based on the minimization of an objective function specific to the dynamic constraints of the system, so that the controller meets the design objectives proposed. This approach has been widely discussed for both its stability robustness and disturbance attenuation capability in linear control systems and nonlinear time-invariant systems [5].

Motivated by the above discussion, PID controllers design based on control theory QFT and the design of H_∞ controllers is proposed for a robotic system with parametric uncertainty. Finally the performance of robust controllers is compared designed based on the calculation and analysis of some behavioral indices.

This paper is organized as follows. In Section II, the model description of the manipulator and the model with parameter uncertainty are given. Section III presents the design of QFT and H_∞ controllers. In Section IV, simulations are performed to confirm the robust performances of the proposed controllers for robot manipulator under parameter uncertainty. In Section V, the conclusions are presented.

II. DYNAMIC MODEL OF THE MANIPULATOR

A. *Robot Dynamics*

According LaGrange's theory [1], [6], the dynamics of a manipulator with n-degrees-of-freedom is represented by:

$$\tau = M(q) * \ddot{q} + V(q, \dot{q}) * \dot{q} + F(\dot{q}) + G(q) + \tau_p \quad (1)$$

Where:

$M(q)$: Inertia matrix [Kg-rad], of dimensions $n \times n$.

$V(q, \dot{q})$: Centrifugal and Coriolis forces matrix [Kg-rad/s], of dimensions $n \times 1$.

$F(\dot{q})$: $n \times 1$ vector modeling joint friction in [N].

$G(q)$: $n \times 1$ Gravitational torques vector in [N*m].

τ_p: Disturbance that considers unmodeled dynamics of the system in [N*m].
τ: Generalized force vector joints in [N*m].

Unmodeled dynamics, meanwhile, are included in the pair of joint τ_p [7], which contains all types of external action. In regard to the actuator [8], it is a DC motor with negligible inductance which will result in *n* uncoupled equations [9], one for each joint:

$$u - k_b.\dot{q}_m = R_a.i \tag{2}$$

$$\tau_m = J_m.\ddot{q}_m + B.\dot{q}_m + R.\tau \tag{3}$$

Given that $\tau_m = K_v.i$ [11] and collecting terms, the following expression is obtained:

$$J_m.\ddot{q}_m + \left(B + \frac{K_v.K_b}{R_a}\right).\dot{q} + R.\tau = K_m.u \tag{4}$$

Where in this case the articular variables shown represent positions, velocities and accelerations of rotation of the corresponding motor shaft [8], while R is a multiplicative term reduction due to coupling between the actuator and the manipulator.. Because of these reducers, it is obtained that $q_i = R.q_{mi}$, where q_i joint variables of the manipulator and q_{mi} represent the motor joint variables. Thus, introducing the dynamic equation of the manipulator in the motor equation, is obtained:

$$J_m.\ddot{q} + B_m.\dot{q} + R^2\tau = K_m.R.u \tag{5}$$

$$\begin{aligned}(J_m + R^2.M(q)) * \ddot{q} + (B_m + R^2.V(q,\dot{q})) * \dot{q} \\ + R^2.F(\dot{q}) + R^2.G(q) + R^2.\tau_p \\ = K_m.R.u\end{aligned} \tag{6}$$

However, considering the large amount of reducers it can perform the next approximation to obtain a linear model uncertainty:

$$J_{ef}.\ddot{q} + B_{ef}.\dot{q} + \tau_d = K_R u \tag{7}$$

Where τ_d is the disturbance due to the system performance and other friction joints herein, and secondly $K_R = K_m.R$. Thus, as shown in Figure 1 the system has a linear model of each joint which shows the variation of three parameters: K_R, J_{ef} y B_{ef} and a disturbance that can be modeled.

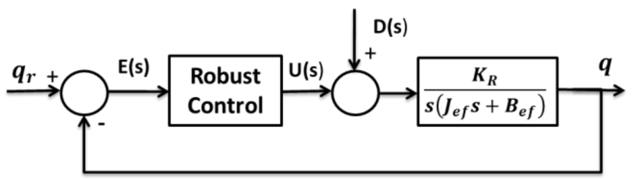

Fig.1. Simplified dynamic model.

B. Parametric Identification

To determine the uncertainty associated with the dynamic models, a step signal was injected to each joint at different operating points. Therefore the uncertain linear matrix transfer function is a below:

$$P(s) = \begin{bmatrix} P_{11}(s,\propto) & 0 & 0 \\ 0 & P_{22}(s,\propto) & 0 \\ 0 & 0 & P_{33}(s,\propto) \end{bmatrix} \tag{8}$$

Where:

$$P_{11} = \frac{q_1}{u_1} = \frac{K_{11}}{s(T_1 s + 1)};$$
$$K_{11} \in [0.825 - 0.975];\ T_1 \in [0.05 - 0.4]; \tag{9}$$

$$P_{22} = \frac{q_2}{u_2} = \frac{K_{22}}{s(T_2 s + 1)};$$
$$K_{22} \in [0.48 - 0.75];\ T_2 \in [0.05 - 0.2]; \tag{10}$$

$$P_{33} = \frac{q_3}{u_3} = \frac{K_{33}}{s(T_3 s + 1)};$$
$$K_{33} \in [1.035 - 1.665];\ T_3 \in [0.04 - 0.2]; \tag{11}$$

III. CONTROLLERS DESIGN

A. H_∞ Controllers Design

The controllers design method H_∞ is related to the minimization of the peak value in the frequency response of some closed-loop function [10]. For that purpose the weighting functions W_i is introduced in the system in order to reflect the design goals and previous information about input and output signals. These signals are bounded because in order to obtain the robust controller H_∞, the $\|.\|_\infty$ norm of each function has its upper limit unit [11].

The inclusion of the weighting functions in a general feedback configuration is presented in Fig. 2. Where, the input signals are respectively the signal reference (r), noise (n) and disturbance to the output (d), while the weighted outputs of the system are z_S, z_U and z_T.

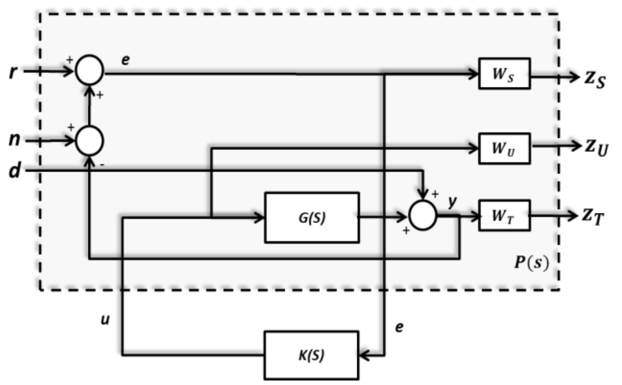

Fig. 2. General configurations for H_∞ control problems

The H_∞ design problem is to find all admissible compensators $K_\infty(s)$ that can stabilize internally the system and to minimize the next norm:

$$\|T_{zw}\|_\infty = \mathrm{Sup}_w \bar{\sigma}[T_{zw}] \quad (12)$$

The function $W_T(s)$ is used to achieve a robust system when uncertainties are presented by the inaccuracy of the linearized model and variation of the parameters of the plant. It usually takes low values at low frequencies and high values at high frequency values. To ensure a bandwidth of around 26 rad/s and provide good robust stability control system, the following weighting function is chosen:

$$W_T(s) = \frac{1.3 * (1 + {}^{s}/_{20})(1 + {}^{s}/_{50})(1 + {}^{s}/_{80})}{(1 + {}^{s}/_{120})^3} \quad (13)$$

The function $W_S(s)$ provides an adequate attenuation for perturbations of low frequency and a precise monitoring of the step slogans. Modeling errors and the bandwidth of the actuators generally impose this weighting function to take low values at high frequency.

To ensure a phase margin greater than 41° and a gain margin above 17.65dB the following weighting function is selected:

$$W_S(s) = \frac{\sqrt{0.5}s + 15.1}{s + 15.1 * 10^{-4}} \quad (14)$$

The function $W_U(s)$ is intended to reduce the over oscillation of temporal response affecting the speed of the same. Likewise, the inclusion of W_U allows avoiding numerical problems in the calculation of the controller. However, W_U=1 can present excellent results.

B. QFT Controllers Design

QFT is a control strategy that explicitly proposes the use of feedback to reduce the effects of the uncertainty of the plant and meet the desired performance specifications (Figure 3). This method allows designing a controller for a estimated uncertainty of the plant, and for a given set of perturbations and specifications.

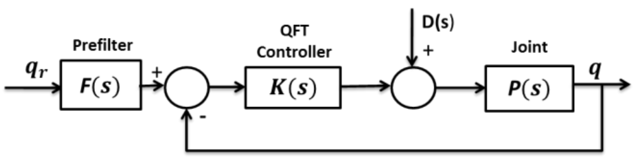

Figure 3. Diagram of QFT Control.

The specifications used to design the link controllers are:

Robust Stability:

$$\frac{q(s)}{q_r F(s)} = \left|\frac{P(s)K(s)}{1 + P(s)K(s)}\right| \leq 1.3 \quad (15)$$

Tracking Performance:

$$T_L(s) \leq \left|\frac{F(s)P(s)K(s)}{1 + P(s)K(s)}\right| \leq T_U(s) \quad (16)$$

Where

$$T_L(s) = \frac{200}{(s+4)(s+5)(s+20)}; \quad T_U(s) = \frac{3.2(s+20)}{s^2 + 8s + 64}$$

Fig. 4 shows plant uncertainty in Nichols chart for the first link and Figure 5 depicts the robust stability bounds at this frequencies.

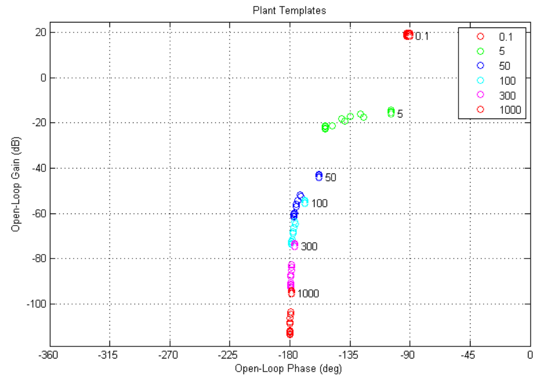

Fig. 4. Uncertainty Templates for Arm 1.

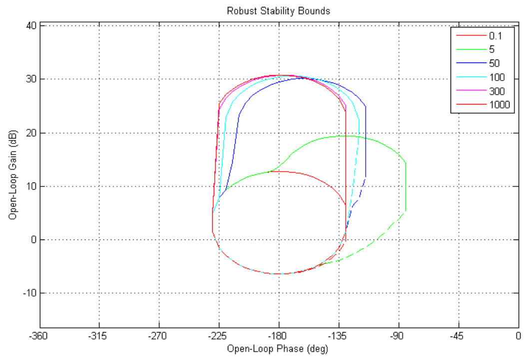

Fig. 5. Robust stability bounds for Arm1.

The overshoot and the setting time specifications (M_p=5% and T_s=1s) are given in the form of upper and lower bounds in frequency domain, usually based on simple second-order models to represent the status of damped condition (See Fig. 6)

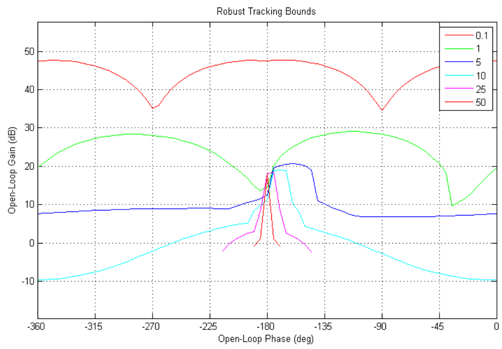

Fig. 6. Robust tracking stability for Link 1

The design of pre-filter guarantees the satisfaction of tracking specification. In Fig. 7 pre-filter shaping of open loop transfer function for the first link is shown.

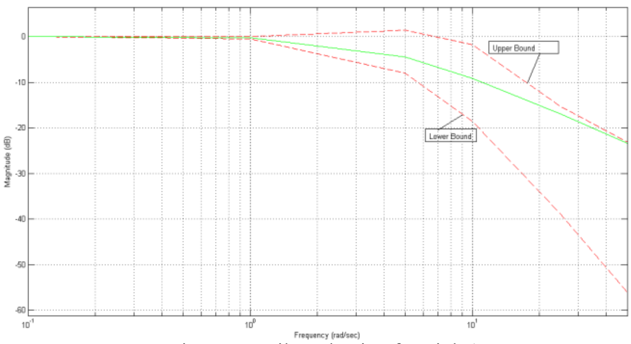

Fig. 7. Pre-Filter Shaping for Link 1

IV. Simulation

In order to verify the validity of these both kinds of control algorithm, those are put forward by this paper.

A. H_∞ Controller

Based on the weighting functions presented above, the next set of H_∞ controllers were obtained for each link:

Link 1:

$$K_1(s) = \frac{108.9s^3 + 10^4 s^2 + 3.22 * 10^4 s + 2.067 * 10^{-6}}{s^4 + 112.9s^3 + 2284s^2 + 1.292 * 10^4 s + 19.51}$$

Link 2:

$$K_2(s) = \frac{5.426s^3 + 688.6s^2 + 646310^4 s + 5.887 * 10^{-5}}{s^4 + 129.7s^3 + 1533s^2 + 3540s + 1.618}$$

Link 3:

$$K_3(s) = \frac{18.19s^3 + 2201s^2 + 2.06 * 10^4 s + 2.675 * 10^{-3}}{s^4 + 132.3s^3 + 2415s^2 + 1.465 * 10^4 s + 6.701}$$

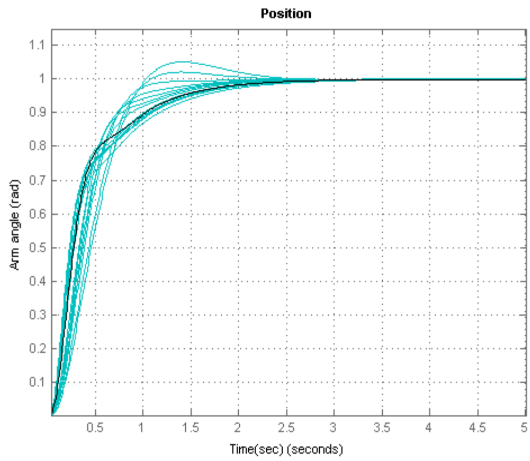

Figure 8. Tracking stability for Link 1 (H_∞ Controller)

B. QFT Controllers

PID controllers were obtained from QFT robust control methodology. The controller transfer function for the articulation QFT *i* is:

$$K_i(s) = K_P + \frac{K_I}{s} + K_D s \quad (17)$$

And the first order pre-filter associated with this controller is given by:

$$F_i(s) = \frac{a}{s + a} \quad (18)$$

Table I lists the parameter of PID controllers and the pre-filter for each link, obtained by QFT, and in Fig. 9 the response of the link 1 is shown.

Table I.
Parameters of Advanced PID-Controllers

Link	K_P	K_I	K_D	a
Link 1	43.5	8.5	5	3.815
Link 2	70	20	5	3.0
Link 3	31.6874	11	1.76	3.1

The stability margin validation-curve and the tracking performance validation-curve are shown respectively in Fig. 10 and 11.

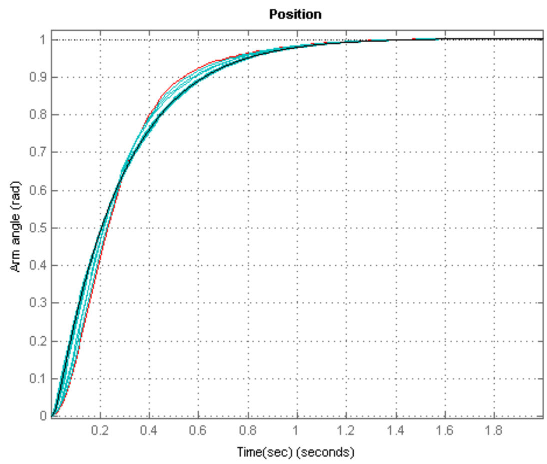

Fig. 9. Tracking stability for Link 1 (*QFT* Controller)

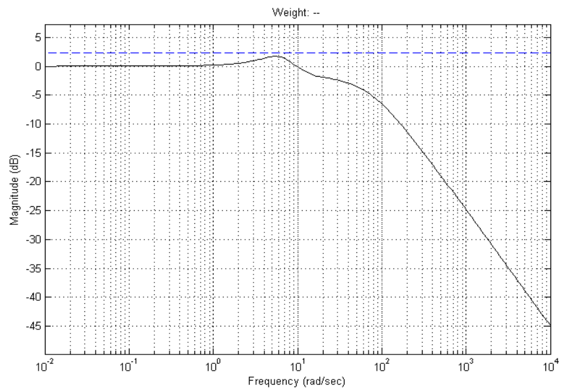

Fig. 10. Stability margin validation-curve for link 1.

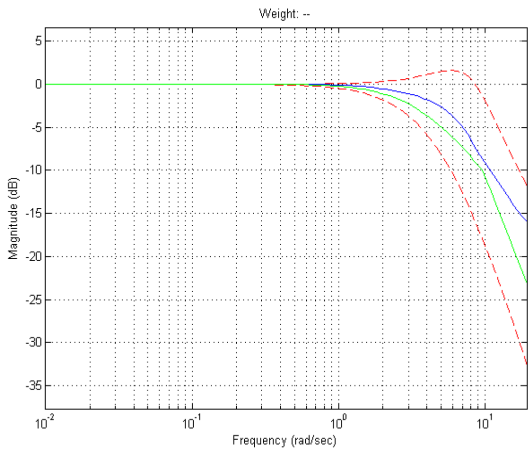

Fig. 11. Tracking performance validation-curve for link 1.

C. Performance Indices

In several automatic control applications such as adaptive systems, optimization of parameters and optimal control design, the quantitative measurement of control system performance becomes necessary. The performance indices are quantitative measures of the control system that compares the quality of the controller action and the effort necessary to achieve control.

For our analysis the indices ISE, IAE and ITAE is calculated into four operating points for each link of the robot in order to compare the performance of H_∞ and QFT controllers designed. Table II shows the results for H_∞controllers while the results of QFT controllers are shown in Table III.

TABLE II
PERFORMANCE INDICES FOR H_∞ CONTROLLERS

Arm	Parameters	ISE	IAE	ITAE
Link 1	$K_p = 0.825;\ T_P = 0.05$	0.2921	0.5630	0.3660
	$K_p = 0.825;\ T_P = 0.4$	0.4532	0.6653	0.3591
	$K_p = 0.975;\ T_P = 0.05$	0.2536	0.4765	0.2697
	$K_p = 0.975;\ T_P = 0.4$	0.4142	0.6200	0.3261
Link 2	$K_p = 0.48;\ T_P = 0.05$	0.4815	0.9665	1.0070
	$K_p = 0.48;\ T_P = 0.2$	0.5527	0.9667	0.8651
	$K_p = 0.75;\ T_P = 0.05$	0.3177	0.6192	0.4363
	$K_p = 0.75;\ T_P = 0.2$	0.3889	0.6194	0.3450
Link 3	$K_p = 1.035;\ T_P = 0.04$	0.2364	0.4489	0.2465
	$K_p = 1.035;\ T_P = 0.20$	0.3142	0.4490	0.1758
	$K_p = 1.665;\ T_P = 0.04$	0.2553	0.4182	0.1882
	$K_p = 1.665;\ T_P = 0.20$	0.1682	0.2797	0.1075

TABLE III
PERFORMANCE INDICES FOR QFT CONTROLLERS

Arm	Parameters	ISE	IAE	ITAE
Link 1	$K_p = 0.825;\ T_P = 0.05$	0.0010	0.0441	0.0942
	$K_p = 0.825;\ T_P = 0.4$	0.0051	0.0665	0.0948
	$K_p = 0.975;\ T_P = 0.05$	0.0007	0.0373	0.0795
	$K_p = 0.975;\ T_P = 0.4$	0.0038	0.0555	0.0795
Link 2	$K_p = 0.48;\ T_P = 0.05$	0.0028	0.0758	0.1636
	$K_p = 0.48;\ T_P = 0.20$	0.0055	0.0802	0.1579
	$K_p = 0.75;\ T_P = 0.05$	0.0012	0.0485	0.1038
	$K_p = 0.75;\ T_P = 0.20$	0.0024	0.0508	0.1000
Link 3	$K_p = 1.035;\ T_P = 0.04$	0.0006	0.0351	0.0751
	$K_p = 1.035;\ T_P = 0.2$	0.0014	0.0719	0.0367
	$K_p = 1.665;\ T_P = 0.04$	0.0006	0.0227	0.0444
	$K_p = 1.665;\ T_P = 0.2$	0.0002	0.0218	0.0465

The simulation results show that the PID controllers designed using QFT approach presented better results than robust H_∞ controllers.

V. CONCLUSION

This paper has developed the control system for a manipulator with parametric uncertainty via H_∞ control approach and the design of advanced PID controllers from QFT approach. Although both controllers showed satisfactory results, the simulation results showed better dynamic performance by robust QFT controllers.

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