An Integrated Mathematical Model on Thermal Phenomena in the Cutting Process

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Abstract: - The paper presents theoretical and experimental research to unify the dynamic and thermal phenomena in a single comprehensive model of the cutting process identify parameters that characterize the state of the system and provide quick information on the rate at which to produce the wear for tool edges and how it can be influenced. Experimental and theoretical research on the temperature of the tool edge and the medium intensity of wear established similarities between the evolutions of the two phenomena that lead to the conclusion that by modeling the evolution of the thermal phenomenon can be determined the evolution of the medium intensity of wear.

Key-Words: - cutting process, heat conduction, tribosystem, intensity of wear, thermal phenomena, metal cutting, sources heat propagation modeling.

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1 Introduction

Machine parts, components of various devices, machines, machines, commonly known as parts, are essentially solid bodies bounded in space by several surfaces that are characterized by geometric shape, dimensions in different directions, degree of smoothness, and relative position. The layers of material whose dimensions in different directions are given by the difference between the dimensions of the initial surfaces and the processed surfaces and are included between these surfaces is called the addition of processing.

The technological process of machining by cutting is the process of generating surfaces by removing splinters, (Figure 1). It is the basis of the construction of machine tools, machines that generate surfaces by removing the addition of processing, removal carried out by the edge of a cutting tool that moves relative to the semi-finished part through a well-defined movement.

It generates a new surface created by the cutting tool edge during its relative movement to the blank. In metal cutting, due to the action of the edge of the tool pushed with a certain force into the processed material, a complex state of stresses and deformations is produced in the cutting area. A simplified diagram of the cutting process is shown in Figure 1 in which the machine tool through the cutting tool exerts a force P capable of overcoming the resistances arising in the processed material.

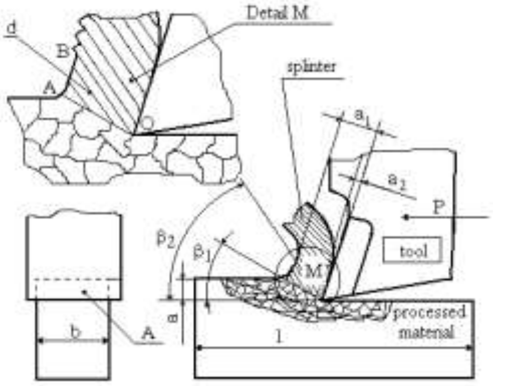


Fig. 1: Plastic deformation of the processed material

This diagram constitutes a basic physical model that is completed with other specific models from the fields of the theory of elasticity and plasticity, thermodynamics, tribology and thus forms a complex model, more or less detailed depending on the requirements or claims. The process of splinter formation is made up of a complex of physical and mechanical phenomena, each directly influencing the process of cutting manufacturing. The deformations occur as a result of the complex action of the cutting tool edge on the processed material, [1], [2], [3], [4].

The heating of the tribosystem, friction, and tool wear are the main processes and phenomena accompany the cutting process. As a result of the production of plastic deformations and friction between the elements of the tribological system, the energy dissipated on them is transformed into heat, an effect that becomes decisive with the increase in the values of the parameters of the cutting regime.

The mechanical work consumed in the cutting process, which is completely transformed into heat, generates the heat sources Q1, Q2, and Q3, Figure 2, ordered according to their intensity as follows:
Q1 > Q2 > Q3, [3], [4], [5], [6].

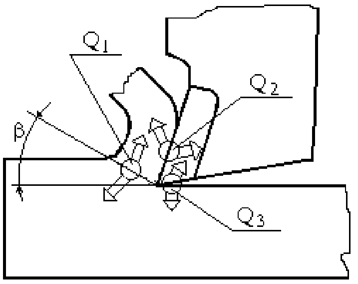


Fig. 2: Mechanical work heat dispersing

The mathematical model elaborated in the specialized literature corresponds in reality to a uniform and stationary thermal state, with the same temperature throughout the entire volume of the splinter and the cutting tool edge and constant over time, so quite far from reality even in a qualitative and phenomenological analysis. The considered hypothesis according to which, in the cutting zone, the thermal state is uniform and stationary, can be appreciated as a particular case which in reality is not possible.

For these reasons, it is necessary to develop an own model for heat sources, capable of leading to a correct determination of the real non-stationary and non-uniformly distributed thermal state.

As a result of the aforementioned processes that accompany the cutting process, friction, and heating, the wear phenomenon also appears in the tribological system, which is very important to know, especially concerning the cutting tool.

Through the wear of the cutting tool occurs the modification of the geometry of its active part with influence on the performance of the cutting process under optimal conditions.

A basic characteristic of cutting tools is wear resistance with direct implications on some basic parameters of production (optimal cutting speed, reduction of tool consumption, reduction of energy consumption, productivity, precision, etc.). Wear resistance is one of the main criteria for optimizing the geometry of cutting tools. The wear of the tools is progressive and manifests itself in several aspects (increase in temperature, damage to the surfaces to be processed, increase in cutting forces) which ultimately lead to the removal from service of the cutting tool.

In the cutting process, the different types of wear occur rarely separately, they usually occur simultaneously, with one or another type of wear predominating depending on the cutting conditions, Figure 3.

The cutting tool wear manifests under many aspects (cutting forces increase, temperature increase in cutting tool, processed area deterioration) finally leading to their stop functioning and it is progressive wear. Figure 3 presents more tool edge wear types.

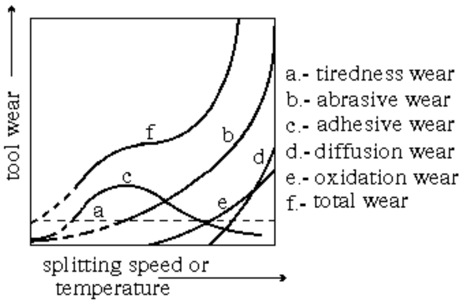


Fig. 3: The influences of tool wear with splitting speed or temperature

This diagram results abrasive wear has the highest influence on the total wear of cutting tool edges.

Friction has a predominant role in heating the elements of the tribological system workpiece tool to be processed, therefore with a decisive influence on the wear of the cutting tool. As the cutting speed can vary within very wide limits and considering some insufficiently in-depth research both quantitatively and qualitatively regarding the influence of the speed, I consider that it is necessary to undertake research on the coefficient of friction for each specific case of cutting, cutting tool edge material and material to be processed, to highlight the fact that also in this case the friction coefficient is non-Colombian type.

As a general characteristic, it can be said that it is a dry friction, on very small contact surfaces, with high and unevenly distributed contact pressures and at high temperatures.

The temperature of the splinter increases very much as a result of supplying it with energy coming exclusively from friction between the splinter and the cutting tool and from the intercrystalline and intercrystalline frictions that occur in the process of splinter formation and detachment. As the temperature increases and the splinter becomes more plastic, certain areas of it even reach the liquid phase, the frictions decrease in intensity leading to the release of less energy, therefore at lower temperatures of the splinter. These temperatures make the splinter more solid, and more intense friction occurs which tends to increase the temperature again, and so on. So there is a combination of effects with opposite tendencies, the result being an equilibrium temperature below the melting temperature of processed material. The increase in speed, especially at high cutting speeds, leads to a feed-back type chain according to Figure 4.

For the complete tribological study of the cutting process, very important is the heating phenomenon of each element that participates in the process (splinter, cutting edge, piece), their temperature being the factor with the greatest influence on the behavior of the tribosystem.

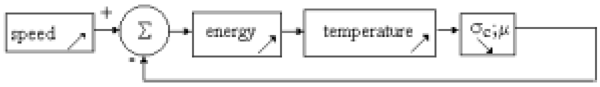


Fig. 4: The feedback influence of temperature increasing on the mechanical characteristics of the processed material

2 The Research and Experiment Methodology

From the many physical models that describe the mechanism of the appearance of cutting efforts, we have adapted for theoretical research the orthogonal free-cutting model developed by Merchant [4]. In the evaluation of the external forces, of interaction with the tool edge, we considered the friction between the splinter and the tool edge as a non-Colombian dry friction, so with a variable coefficient of friction, much closer to reality, [4], [5], [6], [7], [8].

The heat comes from non-conservative mechanical work consumed through plastic deformation in the area of the shear plane and from the non-conservative mechanical friction work on the front and back surfaces of the tool, [4], [9], [10].

Heat propagation modeling: the heat produced by the sources presented above propagates in a non-homogeneous environment consisting of a splinter, tool edge, and tool body, each of them with different calorific coefficients (thermal conductivity, specific heat) both as value and temperature dependence.

Modeling of heat propagation must be done to highlight the non-stationary regime, respectively by considering the equations of heat transfer by conduction, convection, and radiation, which are systems of differential equations with partial derivatives with variable coefficients and algebraic equations.

Assuming known thermal intensities from the sources, respectively the volumetric density of power in each source, Q1, Q2, Q3, having the unit of measurement [W/m^3], can be 3odelled the heating phenomenon in splinter, tool, and work piece. First of all, interested in the temperature in the area of the tool edge, the edge being powered thermally mainly by the energy resulting from the mechanical work of friction between the splinter and the front face of the tool. Edge heating is mostly done by thermal conduction. The thermal state at a given moment is deduced by solving the heat transfer problem in the tool edge.

The heat sources Q1 and Q2 depend on some constants of material such as the coefficient of friction, and the unit flow stress of the catted metal layer, which in turn are dependent on temperature and strain rate. These will create, in the physical-mechanical model developed that feedback effect, difficult to highlight in the models so far.

Assuming that the thermal intensities from the sources are known, respectively the volume density of power in each source, Q1, Q2, and Q3, the heating phenomenon in the splinter, tool, and piece can be modeled. First of all, the temperature in the area of the tool edge is of interest, the edge is thermally fed mainly by the energy resulting from the mechanical work of friction between the splinter and the rake face. The heating of the cutting edge is mostly done by thermal conduction. The thermal state at a given time is deduced by solving the heat transfer problem in the tool edge.

The solving of heat propagation in a transient regime and a very heterogeneous environment leads to knowing at any time the temperature at any point in the investigated environment.

Knowing that:

$$\Delta Q = C \cdot \Delta \theta \tag{1}$$

where:

ΔQ – is the variation in the amount of heat;
C – is the caloric capacity, $C = m \cdot c$,
m – being the mass and c – the specific heat;
$\Delta\theta$ – temperature variation.

By differentiating relation (1) concerning time, the variation of the amount of heat as a function of time is obtained:

$$\frac{\partial Q}{\partial t} = m \cdot c \cdot \frac{\partial \theta}{\partial t} \tag{2}$$

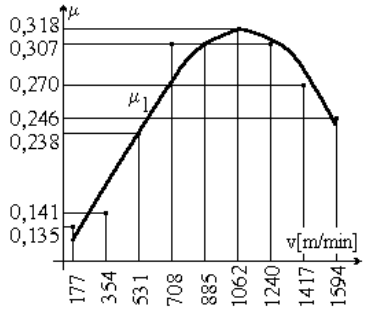


Fig. 5: The variation curve of the coefficient of friction with the relative speed

If the material is anisotropic and inhomogeneous:

$$\frac{\partial}{\partial t}(\rho \cdot c \cdot \theta) = \frac{\partial}{\partial x}\left(\lambda_x \cdot \frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \cdot \frac{\partial \theta}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \cdot \frac{\partial \theta}{\partial z}\right) \tag{3}$$

where:

ρ - *material density (kg/m3);*

c – specific heat of the material (J/kgK);

λ_x, λ_y, λ_z- thermal conductivity of the material (W/mK).

The integration of the differential equation (3) is analytically very difficult and the specialized literature does not provide exact solutions for any practical case. However, to obtain an analytical result where ρ = ct; c = ct; λ_x = ct; λ_y = ct; λ_z = ct; a solution of the form is proposed:

$$\theta(t,x,y,z) = T(t) \cdot F(x,y,z)$$

which replaced in the heat equation (3) leads to:

$$\rho \cdot c \cdot \frac{\partial T}{\partial t} \cdot F(x,y,z) = T(t) \cdot \left[\lambda_x \cdot \frac{\partial^2 F(x,y,z)}{\partial x^2}\right] + \\ + T(t) \cdot \left[\lambda_y \cdot \frac{\partial^2 F(x,y,z)}{\partial y^2} + \lambda_z \cdot \frac{\partial^2 F(x,y,z)}{\partial z^2}\right] \tag{4}$$

The final solution is:

$$\theta(t,x,y,z) = \sum_i e^{-\frac{i^2}{\rho \cdot c} \cdot t} \cdot i \cdot \left(A_{xi} \cdot \cos\frac{i \cdot x}{\sqrt{\lambda_x}} + B_{xi} \cdot \sin\frac{i \cdot x}{\sqrt{\lambda_x}}\right) + \\ + \left(A_{yi} \cdot \cos\frac{i \cdot y}{\sqrt{\lambda_y}} + B_{yi} \sin\frac{i \cdot y}{\sqrt{\lambda_y}}\right) + \\ \left(A_{zi} \cdot \cos\frac{i \cdot z}{\sqrt{\lambda_z}} + B_{zi} \cdot \sin\frac{i \cdot z}{\sqrt{\lambda_z}}\right) \tag{5}$$

The integration constants A_{xi}, B_{xi}, A_{yi}, B_{yi}, A_{zi}, B_{zi} are determined from limit conditions on the edge surface.

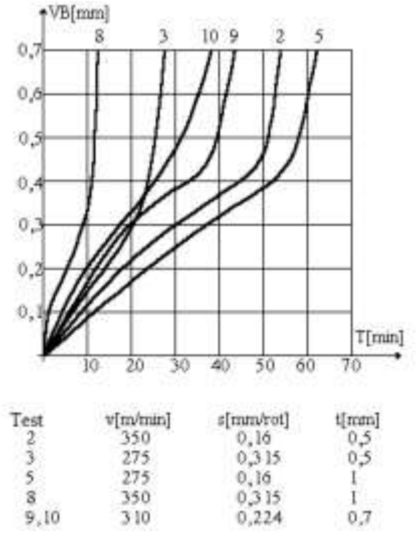
Solution (5) is valid for equation (3), without a heat source in the studied volume. If the thermal sources mentioned above are also considered, equation (3) becomes:

$$\rho \cdot c \cdot \frac{\partial \theta}{\partial t} = \left(\frac{\partial^2 \theta}{\partial x^2} \cdot \lambda_x + \frac{\partial^2 \theta}{\partial y^2} \cdot \lambda_y + \frac{\partial^2 \theta}{\partial z^2} \cdot \lambda_z\right) + R(t,x,y,z) \tag{6}$$

In the mathematical model, the heat exchanges in the volume and on the surfaces of the elements of the tribosystem are realized by knowing some limit conditions. These conditions are extremely difficult to describe analytically and therefore numerical integration is preferred, the most suitable being the finite difference method. Thus, the differential equation is transformed into an algebraic equation by approximating derivatives with finite differences, time is divided into equal time increments τ and space into equal space increments δ_x, δ_y, δ_z, resulting in a network of nodes in a space with 4 dimensions, in which the temperature is defined.

The friction coefficient used in the calculation program was determined experimentally. The conclusion was that in this case, the friction is non-coulombian. Its dependence on speed is shown in Figure 5. The coefficient of friction used in the calculation program was determined using a stand made physically.

The experimental results obtained for the wear of cutting tools were synthesized in wear diagrams, as shown in Figure 6 for a series of cases of processing. These wear curves, continuous over time, allowed the study of its evolution in correlation with the proposed mathematical modeling.



Test	v[m/min]	s[mm/rot]	t[mm]
2	350	0,16	0,5
3	275	0,315	0,5
5	275	0,16	1
8	350	0,315	1
9,10	310	0,224	0,7

Fig. 6: VB wear

By solving the mathematical model with the help of the developed specialized program, the image of the thermal fields is obtained. Assessments can be made regarding the wear and therefore the durability of the cutting tools. With the data obtained, a curve can be drawn as a function of temperature and speed, Figure 7.

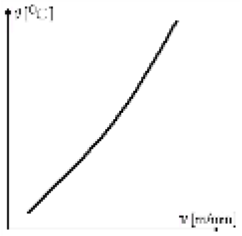


Fig. 7: The variation of temperature on cutting speed

The similarity between the shape of the curve $\theta^0C = f(v)$, Figure 7, and the shape of the curves, Figure 8 can be observed. So, on the path of physical-mathematical modeling, data were obtained that are comparable to the experimental data.

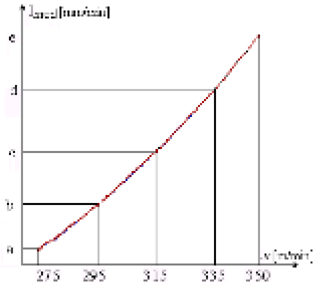


Fig. 8: The variation of medium intensity on cutting

A direct relationship: $I_{med}=f(\theta^0C)$ can be deduced from the relationships $I_{med}=f(v)$ and the relationship $\theta^0C = f(v)$.

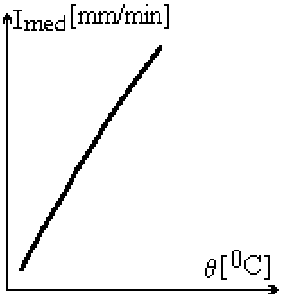


Fig. 9: The variation of medium intensity on maximum temprature

From the graphic representation of this relationship, Figure 9, it is noticeable the good proportionality between the average rate of wear, I_{med}, and the maximum temperature in the cutting process, a fact found experimentally and presented in the specialized literature.

3 Conclusion

The mathematical model proposed for determining the thermal state of the cutting tool edge depending on the main elements of the cutting regime can highlight the role of different parameters of the cutting regime on the thermal state of the cutting tool edge.

The complete and correct research of the thermal phenomena in the cutting area is possible only with the consideration of the feedback relationship between the elements that physically and phenomenological compose the tribosystem studied and with the consideration of the movement of the splinter over time, having as an effect, on the one hand, a continuous feed with cold material layers of the splinter formation zone, on the other hand, a heat evacuation by physical transport of the heated splinter.

Experimental research on tool edge temperature and medium wear rate has established similarities between the evolution of the two phenomena.

So, the evolution of the medium wear rate can be determined by modeling the evolution of the thermal phenomenon by applying a proportionality constant. This constant that can be determined experimentally for the different couple splinter-tool edge.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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