Assessing the Effectiveness of the APOS/ACE Method for Teaching Mathematics to Engineering Students

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Abstract: - The present work focuses on a classroom application for evaluating the effectiveness of the APOS/ACE instructional treatment for teaching mathematics to engineering students. Using linguistic (qualitative) grades, the assessment of the mean student performance is realized with the help of grey numbers and the assessment of their quality performance by calculating the Grade Point Average (GPA) index. A neutrosophic assessment method is also applied for evaluating the overall student performance, because the instructor had doubts about the accuracy of the qualitative grades assigned to some students.

Key-Words: - APOS/ACE instructional treatment, fuzzy assessment methods, grey numbers (GNs), GPA index, neutrosophic sets (NSs), neutrosophic triplets.

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1 Introduction

Computers have become nowadays a valuable tool for Education The animation of figures and representations, achieved by using the proper software, develop the students' imagination and enhance their problem-solving skills. In the forthcoming era of the Fourth Industrial Revolution computers will provide, through the advanced Internet of Things (IoT), a wealth of information for students and teachers.

Several didactic methods have been already developed in which computers play a dominant role. One of them is the APOS/ACE instructional treatment for teaching Mathematics, developed in the USA by Ed Dubinsky and his collaborators during the 1990's [1-3]. In earlier works we have applied this approach for teaching the rational numbers [4], the polar coordinates [5, 6] and the derivative [7, 8] at university level. The present paper focuses on a classroom application for the assessment of the effectiveness of the APOS/ACE approach for teaching Mathematics to engineering students using qualitative (linguistic) grades.

The rest of the paper is formulated as follows: Section 2 is devoted to a brief presentation of the basic principles of the APOS/ACE theory. The necessary mathematical background about fuzzy sets neutrosophic sets and grey numbers, needed for the purposes of this work, is presented in Section 3. The fuzzy methods used for the assessment of the effectiveness of the APOS/ACE instruction with qualitative grades are developed in Section 4 and the classroom application is presented in Section 5. The paper closes with the final conclusions and some hints for future research, which are included in the last Section 6.

2 The APOS/ACE Method

Dubinsky had already spent twenty- five years doing research in Functional Analysis and teaching undergraduate mathematics before starting on figuring out pedagogical strategies that help students to be more successful in learning mathematics. APOS is a theory based on Piaget's principle that an individual learns by applying certain mental mechanisms to build specific mental structures and uses these structures to deal with problems connected to the corresponding situations [9]. According to the APOS, these mechanisms involve *interiorization* and *encapsulation*, while the cognitive structures involve ***A****ctions*, ***P****rocesses*, ***O****bjects* and ***S****chemas*. The first letters of the last four words form the acronym APOS.

A mathematical concept begins to be formed as one applies transformations on certain entities to obtain other entities. A transformation is first conceived as an action. For example, if an individual can think of a function only through an explicit expression and can do little more than substitute for the variable in the expression and manipulate it, he (she) is considered to have an action understanding of functions.

As an individual repeats and reflects an action it may be interiorized to a mental process. A process performs the same operation as the action, but wholly in the mind of the individual, enabling her/him to imagine performing the transformation without having to execute each step explicitly. For example, an individual with a process understanding of a function thinks about it in terms of inputs, possibly unspecified, and transformations of those inputs to produce outputs.

If one becomes aware of a mental process as a totality and can construct transformations acting on this totality, then he/she has encapsulated the process into a cognitive object. In the case of functions, for example, encapsulation allows one to form sets of functions, to define operations on such sets, to equip them with a topology, etc. Although a process is transformed into an object by encapsulation, this is often neither easy nor immediate. This happens because encapsulation entails a radical shift in the nature of one's conceptualization, since it signifies the ability to think of the same concept as a mathematical entity to which new, higher-level transformations, can be applied. Many mathematical situations, however, require one to *de-encapsulate* an object back to the process that led to it. This cycle may be repeated one or more times, e.g. going back from a composite function to its component functions for the better understanding of the rule of derivation of a composite function, going back from the derivative to the initial function in order to understand the process of the integration of a function, etc.

A mathematical topic often involves many actions, processes and objects that need to be organized into a coherent framework that enables the individual to decide which mental processes to use in dealing with a mathematical situation. Such a framework is called a schema. In the case of functions, for example, the schema structure is used to recognize the need of using a specific function in a given mathematical or real-world situation.

The implementation of the APOS theory as a framework for learning and teaching mathematics involves three stages. First, a theoretical analysis, called *genetic decomposition* (GD) of the concepts under study, is performed. The GD comprises a description that includes actions, processes and objects and the order in which it may be best for learners to experience them. Then instructional sequences based on the GD are developed and implemented and finally data are collected and analysed in order to test and refine the GD and the pedagogical strategies that have been employed.

The main contribution obtained from an APOS analysis is the increased understanding of an aspect of human thought. However, explanations offered by such analyses are limited to descriptions of the thinking that an individual may be capable of and not of what really happens in an individual's mind, since this is probably unknowable. Moreover, the fact that one possesses a certain mental structure does not mean that he/she will necessarily apply it in a given situation. This depends on other factors regarding managerial strategies, prompts, emotional state, etc.

The APOS theory has important consequences for education. Simply put, it says that the teaching of mathematics should consist in helping students use the mental structures they already have to develop an understanding of as much mathematics as those available structures can handle. For students to move further, teaching should help them to build new, more powerful, structures for handling more and more advanced mathematics.

Dubinsky and his collaborators realized that for each mental construction that comes out from an APOS analysis, one can find a computer task such that, if a student engages in that task, he (she) is fairly likely to build the mental construction that leads to the learning of the corresponding mathematical topic. As a consequence, the pedagogical approach based on the APOS analysis, known as the *ACE teaching cycle*, is a repeated cycle of three components: *Activities on the computer (**A**)*, *Classroom discussion **(C)*** and *exercises **(E)*** done outside the class.

In applying the ACE cycle the mathematical topic under consideration is divided into smaller subtopics and each iteration of the cycle corresponds to one of the above subtopics. The computer activities, which form the first step of the ACE approach, are designed to foster the students' development of the appropriate mental structures. The students do all of their work in computer laboratories divided in cooperative groups.

In the classroom the teacher guides the students to reflect on the computer activities and their relation to the mathematical concepts being studied. They do this by performing mathematical skills without using the computers. They discuss their results and listen to explanations, by fellow students or the teacher, of the mathematical meanings of what they are working on.

The homework exercises are fairly standard problems related to the topic being studied. Students reinforce the knowledge obtained in the computer activities and classroom discussions by applying it in solving these problems.

The implementation of the ACE cycle and its effectiveness in helping students make mental constructions and learn mathematics has been reported in several research studies of Dubinsky's team, e.g. [3, 10, 11],etc.

3 Mathematical Background

3.1 Fuzzy Sets and Logic

The development of human science and civilization owes a lot to Aristotle's (384-322 BC) *bivalent logic (BL)*, which was in the center of human reasoning for centuries. BL is based on the *"Principle of the Excluded Middle",* according to which each proposition is either true or false. Opposite views, however, appeared also early in the human history supporting the existence of a third area between true and false, where these two notions can exist together; e.g. by Buddha Siddhartha Gautama (India, around 500 BC), by Plato (427-377 BC), more recently by the Marxist philosophers, etc. Integrated propositions of multi-valued logics reported, however, only during the early 1900's, mainly by Lukasiewicz and Tarski [12, Section 2]. According to the Lukasiewicz's *"Principle of Valence"* propositions are not only either true or false, but they may have intermediate truth-values too.

Zadeh, replacing the characteristic function of a crisp subset of the universe U with the *membership function* m: $U\rightarrow [0, 1]$, introduced in 1965 the concept of *fuzzy set (FS)* [13], in which each element x of U has a membership degree m(x) in the unit interval. The closer m(x) to 1, the better x satisfies the characteristic property of the corresponding FS. For example, if A is the FS of the tall men of a country and m(x) = 0.8, then x is a rather tall man. On the contrary, if m(x) = 0.4, then x is a rather short man. Formally, a FS A in U can be written as a set of ordered pairs in the form

$$F = \{(x, m(x)): x \in U\} \quad (1)$$

Zadeh also introduced, with the help of FS, the infinite-valued in the unit interval *fuzzy logic (FL)* [14], on the purpose of dealing with the existing in the everyday life partial truths. FL, in which truth values are modelled by numbers in the unit interval, embodies the Lukasiewicz's "Principle of Valence".

Uncertainty can be defined as the shortage of precise knowledge or complete information on the data that describe the state of a situation. It was only in a second moment that FS theory and FL were used to embrace uncertainty modelling. This happened when membership functions were reinterpreted as possibility distributions [15, 16]. Zadeh [15] articulated the relationship between possibility and probability, noticing that what is probable must preliminarily be possible.

Probability theory used to be for a long period the unique tool in hands of the specialists for dealing with problems connected to uncertainty. Probability, however, was proved to be suitable only for tackling the cases of uncertainty which are due to *randomness* [17]. Randomness characterizes events with known outcomes which, however, cannot be predicted in advance, e.g. the games of chance. FSs, apart from randomness, tackle also successfully the uncertainty due to *vagueness*, which is created when one is unable to distinguish between two properties, such as "a good player" and "a mediocre player". For general facts on FSs and the connected to them uncertainty we refer to the book [18].

3.2 Neutrosophic Sets

Several generalizations and extensions of the theory of FSs have been developed during the last years for the purpose of tackling more effectively all the forms of the existing in real world uncertainty. The most important among them are briefly reviewed in [19].

Atanassov in 1986, considered, in addition to Zadeh's membership degree, the degree of *non-membership* and extended FS to the notion of *intuitionistic FS (IFS)* [20]. Smarandache in 1995, inspired by the frequently appearing in real life neutralities - like <friend, neutral, enemy>, <win, draw, defeat>, <high, medium, short>, etc. - generalized IFS to the concept of *neutrosophic set* (NS) by adding the degree of *indeterminacy* or *neutrality* [21]. The word "neutrosophy" is a synthesis of the word "neutral΄ and the Greek word "sophia" (wisdom) and means "the knowledge of the neutral thought". The simplest form of a NS is defined as follows:

Definition 1: A *single valued NS (SVNS)* A in the universe U is of the form

$$A = \{(x,T(x),I(x),F(x)): x \in U,\ T(x),I(x),F(x) \in [0,1],\ 0 \leq T(x)+I(x)+F(x) \leq 3\} \quad (2)$$

In equation (2) T(x), I(x), F(x) are the degrees of *truth* (or membership), *indeterminacy* (or neutrality) and *falsity* (or non-membership) of x in A respectively, called the *neutrosophic components* of

x. For simplicity, we write A<T, I, F>. Indeterminacy is defined to be in general everything that exists between the opposites of truth and falsity [22].

Example 1: Let U be the set of the players of a soccer club and let A be the SVNS of the good players of the club. Then each player x is characterized by a *neutrosophic triplet* (t, i, f) with respect to A, with t, i, f in [0, 1]. For example, x(0.7, 0.1, 0.4) ∈ A means that there exists a 70% belief that x is a good player, but at the same time there exist a 10% doubt about it and a 40% belief that x is not a good player. In particular, x (0, 1, 0) ∈ A means that we do not know absolutely nothing about the quality of player x (new player).

If the sum T(x) + I(x) + F(x) < 1, then it leaves room for incomplete information about x, if it is equal to 1 for complete information and if it is >1 for *inconsistent* (i.e. contradiction tolerant) information about x. A SVNS may contain simultaneously elements leaving room to all the previous types of information. All notions and operations defined on FSs are naturally extended to SVNSs [23].

Summation of neutrosophic triplets is equivalent to the union of NSs. That is why the neutrosophic summation and implicitly its extension to neutrosophic scalar multiplication can be defined in many ways, equivalently to the known in the literature neutrosophic union operators [24]. For the needs of the present work, writing the elements of a SVNS A in the form of neutrosophic triplets and considering them simply as ordered triplets we define addition and scalar product as follows:

Definition 2: Let (t_1, i_1, f_1), (t_2, i_2, f_2) be in A and let k be a positive number. Then:

- The *sum* $(t_1, i_1, f_1) + (t_2, i_2, f_2) = (t_1+ t_2, i_1+ i_2, f_1+ f_2)$ (2)
- The *scalar product* $k(t_1, i_1, f_1) = (kt_1, k i_1, kf_1)$ (3)

Remark 1: Summation and scalar product of the elements of a SVNS A with respect to Definition 2 need not be closed operations in A, since it may happen that $(t_1+ t_2)+(i_1+ i_2)+(f_1+ f_2)>3$ or $kt_1+k i_1 + kf_1>3$. With the help of Definition 2, however, one can define in A the *mean value* of a finite number of elements of A as follows:

Definition 3: Let A be a SVNS and let (t_1, i_1, f_1), (t_2, i_2, f_2), …., (t_k, i_k, f_k) be a finite number of elements of A. Assume that (t_i, i_i, f_i) appears n_i times in an application, i = 1,2,…., k. Set $n = n_1+n_2+….+n_k$. Then the *mean value* of all these elements of A is defined to be the element (t_m,i_m,f_m) of A calculated by

$$\frac{1}{n}[n_1(t_1, i_1,f_1)+n_2(t_2,i_2,f_2)+….+n_k(t_k,i_k, f_k)] \quad (4)$$

3.3 Grey Numbers

The theory of *grey systems* [25] introduces an alternative way for managing the uncertainty in case of approximate data. A grey system is understood to be any system which lacks information, such as structure message, operation mechanism or/and behavior document.

Closed real intervals are used for performing the necessary calculations in grey systems. In fact, a closed real interval [x, y] could be considered as representing a real number T, termed as a GN, whose exact value in [x, y] is unknown. We write then T ∈ [x, y]. A GN T, however, is frequently accompanied by a *whitenization function* f: [x, y] → [0, 1], such that, if f(a) approaches 1, then a in [x, y] approaches the unknown value of T. If no whitenization function is defined, it is logical to consider as a representative crisp approximation of the GN T the real number

$$V(T) = \frac{x+y}{2} \quad (3)$$

The arithmetic operations on GNs are introduced with the help of the known arithmetic of the real intervals [26]. In this work we are going to make use only of the addition of GNs and of the scalar multiplication of a GN with a positive number, which are defined as follows:

Definition 2: Let $A \in [x_1, y_1]$, $B \in [x_2, y_2]$ be two GNs and let k be a positive number. Then:

- The sum: A+B is the GN $A+B \in [x_1+y_1, x_2+y_2]$ (4)
- The scalar product kA is the GN $kA \in [kx_1, ky_1]$ (5)

4 Fuzzy Assessment Methods with Qualitative Grades

In many cases it is a common practice to assess the student performance by using qualitative (linguistic) instead of numerical grades. A widely accepted scale of such grades is the following: A=excellent, B=very good, C=good, D=mediocre and F=unsatisfactory. Here we present a series of fuzzy assessment methods {see also [27]) that we are going to use in this work for assessing the overall performance of a student group.

4.1 Mean Performance

In case of using qualitative grades the mean performance of a student group cannot be evaluated with the classical method of calculating the mean value of the student individual grades. To overcome this difficulty, we assign to each grade a GN, denoted for simplicity with the same letter, as follows: A = [85, 100], B = [75, 84], C = [60, 74], D = [50, 59],
F = [0, 49]. The choice of the above GNs, although it corresponds to generally accepted standards, is not unique. For example, for a more strict assessment, one may choose A= [90, 100], B = [80, 89], C= [70, 79], D = [60, 69], F = [0, 59], etc. Such changes, however, does not affect the generality of our method

Assume now that, from the n in total students of the group, n_X obtained the grade X=A, B, C, D, F. It is logical then to accept that the crisp approximation V(M) of the GN

$$M = \frac{1}{n}(n_A A + n_B B + n_C C + n_D D + n_F F) \qquad (5)$$

can be used for estimating the mean performance of the student group.

4.2 Quality Performance

A very popular in USA and other countries method for evaluating the quality performance of a group is the use of the *Grade Point Average (GPA)* index [28, p.125], which is calculated by the formula

$$GPA = \frac{0n_F + n_D + 2n_C + 3n_B + 4n_A}{n} \qquad (6)$$

In other words, the GPA index is a weighted average in which greater coefficients (weights) are assigned to the higher grades. Note that, since in the worst case ($n=n_F$) is GPA=0 and in the ideal case ($n=n_A$) is GPA=4, we have in general that

$$0 \leq GPA \leq 4 \qquad (7)$$

When two groups have the same GPA index, however, this method is not sufficient to show which of them performs better. In such cases the *Rectangular Fuzzy Assessment Model (RFAM)*, which is based on the *Center of Gravity (COG) defuzzification technique* can be used [27, pp. 126-130].

4.3 Neutrosophic Assessment

Frequently in practice the teacher has doubts about the grades assigned to some students, either because he/she had not the opportunity to evaluate their skills explicitly during a course, or because they didn’t clarify their answers properly in a written test. In such cases, the most suitable method for assessing the overall performance of a student group is to use NSs as tools. Considering, for example, the NS of the good students of the group, one introduces neutrosophic triplets characterizing the individual performance of each student and then calculates the mean value of all these triplets with the help of equation (4) in order to obtain the proper conclusions about the group’s overall performance. In order to have complete information for each student’s performance, the sum of the component of each triplet must be equal to 1.

5 The Classroom Application

The purpose of the following classroom application was to evaluate the effectiveness of the APOS/ACE approach for teaching mathematics to engineering students. The subjects were the first term students of two departments of the School of Engineering of my university during the teaching of the course “Higher Mathematics I”, which includes Complex Numbers, Differential and Integral Calculus in one variable and elements from Linear Algebra. According to the grades obtained in the PanHellenic examination for entrance in Higher Education, the potential of the two departments in mathematics was about the same. The course’s instructor was also the same person, but the teaching methods followed were different. Namely, the APOS/ACE approach was applied for teaching the course to the 60 students of the first department (experimental group), whereas the classical method with lectures on the board was applied for the 60 students of the second department (control group).

The results of the final examination, after the end of the course, were the following:

- Department I: A: 9 students, B: 15, C: 18, D: 12, F: 6
- Department II: A: 12, B: 15, C: 9, D: 12, F: 12

Therefore, applying the assessment methods of Section 4, one evaluates the performance of the two departments as follows:

Mean performance

By equation (5) one finds that

$$M_I = \frac{1}{60}\,(9[85,100]+15[75,84]+18[60,74]+12[50,59]+$$

$+6[0,49]) = \frac{1}{60}[3570,4994] \approx [59.5,83.23]$.

Therefore, equation (3) gives that $V(M_I) \approx 71.36$, which shows that the experimental group demonstrated a good (C) mean performance.

In the same way one finds that $V(M_{II}) \approx 62.56$, which shows that the control group also demonstrated a good (C) mean performance, which, however, was 8.8% worse than that of the experimental group.

Quality performance

Equation (6) gives that

$GPA_I = \frac{12+2*18+3*15+4*9}{60}$ =2.12and similarly

GPA_{II}=2.05, which shows that the experimental group demonstrated a slightly better quality performance. In fact, with the help of equation (7) it is easy to check that the superiority of the experimental group in this case is only 0.07*25=1.75%.

Note that, some of the student answers in the final examination were not clearly presented or well justified. As a result, the instructor was not quite sure for the accuracy of the grades assigned to them. For this reason, we decided to apply the neutrosophic method of section 4.3 too for the assessment of the two departments' overall performance. For this, starting from the students with the higher grades, let us denote by S_i, i=1,2,….,60, the students of each department. Considering the NS of the good students, we assigned neutrosophic triplets to all students of the two departments as follows:

- Department I: S_1-S_{32}: (1,0,0), S_{33}-S_{38}: (0.8,0.1,0.1), S_{39}-S_{42}: (0.7,0.2,0.1), S_{43}-S_{46}: (0.4,0.2,0.4), S_{47}-S_{50}: (0.3,0.2,0.5), S_{51}-S_{53}: (0.2,0.2,0.6), S_{54}-S_{55}: (0.1,0.2,0.7), S_{56}-S_{57}: (0,0.2,0.0.8), , S_{578}-S_{60}: (0,0,1).
- Department II: S_1-S_{31}: (1,0,0), S_{32}-S_{35}: (0.8,0.1,0.1), S_{36}: (0.7,0.1,0.2), S_{35}-S_{43}: (0.4,0.1,0.5), S_{44}-S_{46}: (0.3,0.2,0.5), S_{47}-S_{50}: (0.2,0.2,0.6), S_{51}-S_{52}: (0.1,0.2,0.7), S_{53}-S_{58}: (0,0.3,.0.7), , S_{59}-S_{60}: (0,0,1).

Then, by equation (4), the mean value of the neutrosophic triplets of Department I is equal to

$\frac{1}{60}$[32(1,0,0)+6 (0.8,0.1,0.1)+ 4(0.7,0.2,0.1)+4 (0.4, 0.2,0.4)+ 4(0.3,0.2,0.5)+3(0.2,0.2,0.6)+ 2(0.1,0.2, 0.7)+2(0,0.2,0.0.8)+3 (0,0,1)≈(0.72, 0.07, 0.21). In the same way one finds that the mean value of the neutrosophic triplets of Department II is equal to (0.65,0.08,0.27).

Thus, the probability for a random student of Department I to be a good student is 72%, but at the same time there exists a 7% doubt about it and a 21% probability to be not a good student. Also, the probability for a random student of Department II to be a good student is 65%, with a 8% doubt about it and a 27% probability to be not a good student. Consequently, the experimental group, despites the doubts of the instructor for the grades assigned to the students, demonstrated a better overall performance.

6 Conclusion

The classroom application presented in this work demonstrated a superiority of the experimental (APOS/ACE) group with respect to the control group. This superiority was significant concerning the two groups' mean and overall (in terms of the neutrosophic method) performance, but rather negligible concerning their quality performance. This gives a strong indication that the application of the APOS/ACE method benefits more the mediocre and the weak in mathematics students, but less the good students. Much more experimental research is needed, however, for obtaining safer conclusions.

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