Computational Simulation: Multidisciplinary Teaching of Dynamic Models from the Linear Algebra Perspective

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Abstract: - The technological development has stablish a new learning perspective in the Linear Algebra Concepts. In progress is being made in multidisciplinary teaching in engineering careers, since the proposal is that students learn Math taking into account the benefits provided from the new technologies that analyse different aspects of an engineering system. In this paper are presented a classroom experience in the Computer and Multidisciplinary Laboratory of Basic Sciences where a fluid flow system is modelled when we want to deal with the Eigenvalues and Eigenvectors topic, content that belongs to Algebra and Analytic Geometrical program. The possibilities that the computational systems offers have unchained a reformulation of methodological focus of educative programs, since they propose to intensify learning through the acquisition of skills by students. The developed experience proposes a learning methodology that guarantees an academic knowledge according to the new developments, and assert that Engineering students approach new mathematical subjects trough adequate applications.

Key-Words: - Multidisciplinar Learning, Simulation, Eigenvectors, Modelling.

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1 Introduction

In the Lineal Algebra courses, in the field of engineering careers, the different reasons that obstacle the teaching learning process in the basic training of students.

Some of most relevant aspects that we can quote are: The few knowledge levels from the students bring from middle school, the difficult associated to the abstraction process which is required to Lineal Algebra study, and the lack of contextualization from the subject content and its relation with another Math or engineering courses.

Due to these issues, in the development of the Mathematics class, the formulation of simple models that address to the acquisition of new ways of thinking and reasoning process is essential and will be triggers of motivational situations.

Design a Mathematics class to Engineering careers requires to establish an appropriate learning strategy that allows a specific model situation related to engineering application field.

The Lineal Algebra must provide to the students the scientific basis for the management and use of concepts in the approach and solution of problems. In addition, it must provide them with the training of mental skills, reasoning and mathematical modelling. [1].

Mathematics teachers do not always have incorporated this vision, which agrees with the regulations agreed in the curriculum. In the actual normative proposes that student must acquire skills for diverse competences from basic subject cycle, to obtain a significant learning process

The Engineering mathematics learning purpose must find the correct equilibrium between mathematical model formulation and skills that the students need to solve the challenges that will arise in the area of applied technologies and in their future professional activity.

Motivation plays a fundamental role to achieve that the Engineering learning teaching process will be functional and relevant. For the engineers training and others professionals from the Engineering field are appealing use models and methods that mathematics offers, since these resources are the ones that must offer the most optimal solutions to the different models proposed to achieve the abstract thought.

The student must have a creative power that allows to impulse the mechanisms that internalize knowledge and not limit it to a mere informative aspect based on operational learning.

The use of Computational tools allows the students to explore, deduce, make conjectures, justify, test their arguments and thus build their own knowledge independently of the teacher's intervention.

These tools also make it possible for the teacher to concentrate on stimulating and guiding learning, but this new role requires greater activity from the teacher, since constant creativity is necessary in approaching the situations that arise in class.

2 Methodological criteria.

Computer and Multidisciplinary Laboratory of Basic Sciences is the physical space where a collaborative work field is generated, and for this purpose, it is designed a workshop class which is denominated as theoretical practical technological class where it is realized a practical work with the concepts of Eigenvalues and Eigenvectors subjects, with engineering applications.

The class characteristic are to establish the guidelines of an interactive engineering analysis work that could manifest in a production activity of new ideas.

The teaching learning methodology allows move forward first with the theoretical subject research, creating an environment where the student assume an active role, putting aside the fact of being a spectator, and permit to build the concepts by experimentation, and elaboration of conclusions

The mathematics teaching in the first years in the engineering career is needed for the student integral education, but the depth of its study is addressing to being limited.

It must give the knowledge with the goal of prepare and educate the student so they could find the tools that activate on a self-determined learning process.

Actually, the Mathematical programs in the University are oriented to the student, so they dispose of previous knowledge enough to allow build them a mental structures trending to achieve a functional and relevant learning.

That implies that the student could establish more complex relations in their learning process.

The use of computational resources provides an advantage in the teaching process, since the observation in a workshop classroom allows creating mental constructions trending to generate a new knowledge.

In this situation, must be taking account that the incorporation of computational tools are not limited to the problem of counting with tools that conforms those technologies: Equipment and computational software, but the most important is build an educational use and, in a strict sense, didactic of them. [2].

The use of semiotic representations that implies the management and conversion onto the mathematical language produces a disarticulation in thought that manifest in a relevant learning.

This approach to Mathematics is understood as a linguistic resource to describe and discern the processes that are seen in other disciplines, such as physics or chemistry, where almost all of its laws are stated with mathematical equations or with procedures that derive from them [3].

More specifically, it is considered that Mathematics education is the social, heterogeneous and complex system in which it is necessary to distinguish at least three components or fields:

a) The practical and reflexive action on the Mathematics learning and teaching process.
b) The educational technology that proposes to develop resources and materials, using the available scientific knowledge
c) Scientific research, which tries to understand the functioning of the teaching of mathematics as a whole, as well as that of the specific didactic systems (formed by the teacher, the students and mathematical knowledge).

Those three fields are interested in the same object: the functioning of the didactic systems, and they even have a common ultimate goal: the improvement of the teaching and learning of Mathematics.

But the temporal perspective, the goals, the available resources, the operating rules and restrictions to which they are subjected, are intrinsically different.

The world of practical action is the own teacher's field, who is in charge of one or several groups of students to whom he tries to teach mathematics [4].

Understanding an engineering problem means converting this problem into a physical or chemical problem and translating it into mathematical terms.

In the teaching of subjects such as Algebra and Analytical Geometry, a certain level of concern is distinguished by the scant interest of the students regarding how the contents are presented in the class and how the appropriation of the knowledge that will be used in advanced courses is carried out.

Some students, however, state that this problem is caused by the minimal understanding they have of the concepts and the way in which they are presented during classes. They also state that teacher's exhibit concepts with a certain amount of generalization and abstraction and this attitude do not conform to student expectations.

In this paper, a class is designed with the purpose of incorporating learning methodologies aimed at generating intrinsic mechanisms in students that allow them to discover knowledge and achieve independence skills for reasoning and induction.

3 Objectives

The implemented actions to develop the programmed activities have got as goal the realization of a laboratory experience where the students perform a self-managed and collaborative project, to conceptualize the Eigenvalues and Eigenvectors subject.

The task is developed by the design of an engineering situation that study the flow of a fluid when transit across two water tanks.

It pretends with the experience to convert the Algebra classroom into a workshop class where the student experiment a learning process generated by interaction techniques between the teacher as passive subject and the student as active protagonist of knowledge,

This line of work allows the class to be organized contemplating a transformation of learning that leads the student to abandon the central place that he has historically had within the classroom to occupy another space in the class dynamics; necessary space to interact with their peers and with the work proposal [6].

Meanwhile, the first objective of a teacher is to improve the learning of his students, so he will be mainly interested in the action that can produce an immediate effect on his teaching.

The second component, which we have called technological (or applied research) is prescriptive, since it is more involved with the elaboration of devices for action and is the proper field of curriculum designers, writers of school manuals and teaching materials.

Finally, the scientific research (basic, anaclitic and explanatory) is particularly involved with the theory elaboration and it is used usually at universities institutions [4].

In this experience, the use of a valuable tool such as symbolic calculation is proposed, which is used as a connector of mathematical functions applied to real situations, which allows validating the student's own skills for the development of basic capacities.

4 Pedagogical Bases in Classroom

In the actual paper is described a classroom experience of Algebra and Analytical Geometry course, in the Chemical Engineering career, where is pretended that students integrate knowledge when modelling a system of fluid flow in the development of the Eigenvalues theme and Eigenvectors..

A theoretical practical technological class is designed that takes place in a three-hour session with different stages, with which the aim is to lead students to learn the proposed topic.

The stages designed to carry out the class are: assembly of work groups, presentation of a problematic situation, review and subsequent selection of bibliographic material on the subject, review of the contents developed in the theory class, modelling of the situation, and resolution of the problem and conclusions.

Moving from a frontal class to one focused on learning is probably a central task at the moment, but this requires thinking about the use of ICT not as a mere substitute for information.

Before, the information was entrusted to the teacher (front class), now online programs are developed that contain all the information that is required. This vision changes the medium and probably makes it a little more attractive, but it does not fundamentally transform educational work.

In this sense, the ordering principle of the learning task starts from the didactic foundations, learning is built if it is taken into account that it needs to be built as a personal work project, with the effort that it implies on the part of the students, to which is necessary to allow the previous knowledge of the students, their notions, pre-notions and prejudices to emerge, and at the same time open a space for doubt, for questioning, so that the student can build an enigma [3]..

The interconnection between Lineal Algebra concepts, the real cases in the engineering field, and the anticipation to knowledge that after will be more complex in the courses that involve about basic and applied technologies provide students with versatility when dealing with increasingly sophisticated models in the specific subjects of their career.

5 Teaching Learning Sequence Process.

Students work in the Laboratory class designing the systems on their worksheet and doing the calculations on the computer.

It is in this task where collaborative work, together with the ability to build problem situations, can allow the student to carry out information search activities, analysis, and construction of their own responses. Ultimately, to reconstruct knowledge from the conceptual structures that it already possesses [3].

It is proposed to study a first order system consisting of two interconnected tanks as shown in Fig.1.

The interest in this type of system is formulate a model that represents the outlet flow fluid in the tank Nº2 in function of time, when it is applied in the fluid entrance in tank N° 1, a variation of unitary step of $10\frac{m^3}{h}$ type.

In the cross-sectional areas for the first and second tank are $2m^2$ and $4m^2$ respectively, and the hydraulic resistances for both tanks are $0{,}5\frac{h}{m^2}$.

Being: Flow fluid that get into tank Nº1; Flow fluid that get into tank N° 2 coming from the tank N° 1; fluid flow coming out of tank Nº 3; level reached in the tank N° 1, level reached in the tank N° 2; is the hydraulic resistance of the fluid leaving the tank N° 1 and is the hydraulic resistance of the fluid leaving the tank N° 2 [5].

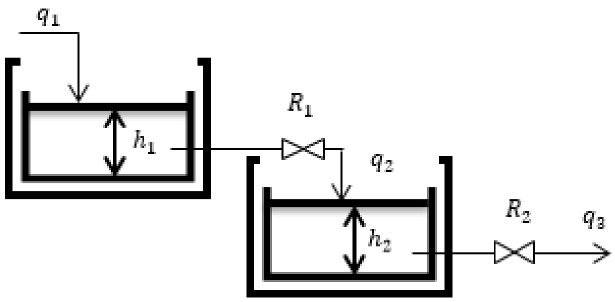

Fig.1 Studied System Scheme.

According to the data provided, the work of the students is guided by the following instructions:

1) Which model rules the behaviour of the Fig.1 system?
2) Which differential equation system is equivalent to the represented equation in the proposed model?
3) Which are the Eigenvalues and Eigenvectors linked to the differential equation system?
4) ¿Which is the fundamental Matrix associated to the system?
5) Find the equation that determines the outlet flow evolution in time function for the indicated values in the question and graphic it.
6) How is the system behaviour?
7) Could the system represent another different behaviours?
8) at what value the outlet flow in extremely long times trends?
9) Is it exists some relative extreme, or inflexion point in the graphic that could you infer as a system characteristic?

Before starting to analyse Fig.1, the student define the hydraulic resistance R parameter, which relate the liquid height on a tank with respect to the outlet flow rate, as indicated in equation (1)

$$R = \frac{h}{q} \tag{1}$$

They also define the time constant, T, expressed in the equation (2), which is the product between hydraulic resistance and the cross-sectional area in a tank.

$$T = RA \tag{2}$$

Then they perform the energy balance of the dynamic system of Fig. 1. In tank Nº 1, the net flow is the product of the cross-sectional area times the velocity, it is indicated in equation (3)

$$q_1(t) - q_2(t) = A_1\frac{dh_1}{dt} \tag{3}$$

With the relationships established in equation (1) they can determine the speed in tank Nº1, which is expressed by means of equation (4)

$$\frac{dh_1}{dt} = A_1R_1\frac{dq_2(t)}{dt} \tag{4}$$

Resulting in the net flow for tank Nº 1, the one identified by the first order differential equation (5)

$$q_1(t) - q_2(t) = A_1R_1\frac{dq_2(t)}{dt} \tag{5}$$

Now if they apply a similar energy balance for tank Nº2, the net flow in tank Nº 2 is that expressed in equation (6)

$$q_2(t) - q_3(t) = A_2\frac{dh_2}{dt} \tag{6}$$

According to the relationships defined above, in equation (7), the velocity of the fluid in the tank is indicated N°2

$$\frac{dh_2}{dt} = R_2\frac{dq_3(t)}{dt} \tag{7}$$

Equation (8) is the first order differential equation, which indicates the net flow for tank N° 2

$$q_2(t) - q_3(t) = A_2R_2\frac{dq_3(t)}{dt} \tag{8}$$

If $T_1 = A_1R_1$ and $T_2 = A_2R_2$ are time constant for the tanks N° 1 and N° 2 respectively, it is designated with $A = T_1T_2$ and $B = T_1+T_2$, so relating the equations (5) and (8), may obtain the ordinary differential equation for constant coefficients that determine the system model of Fig.1 which is expressed in the equation (9) [5].

$$A\frac{d^2q_3(t)}{dt^2}+B\frac{dq_3(t)}{dt}+q_3(t)=q_1(t) \quad (9)$$

The starter conditions for the proposed model are: outlet initial flow is null $q_3(0)=0$; and the initial outlet speed flow is also null$\frac{dq_3(0)}{dt}=0$.

After discuss and propose solutions to the raised questions, determine that Equation (9) can be expressed by the differential system equation [7], [8].

$$\begin{cases} \dfrac{dq_3(t)}{dt}=x(t) \\ \dfrac{dx(t)}{dt}=\dfrac{q_1(t)}{A}-\dfrac{q_3(t)}{A}-\dfrac{B}{A}q_3(t) \end{cases} \quad (10)$$

The Equation (10) can be represented by the equation matrix (11)

$$\begin{pmatrix} \dfrac{dq_3(t)}{dt} \\ \dfrac{dx(t)}{dt} \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ \dfrac{-1}{A} & \dfrac{-B}{A} \end{pmatrix}\begin{pmatrix} q_3 \\ x \end{pmatrix}+\begin{pmatrix} 0 \\ \dfrac{q_1}{A} \end{pmatrix} \quad (11)$$

According to the data supplied to the student, the Equation (11) is transformed into Equation (12).

$$\begin{pmatrix} \dfrac{dq_3(t)}{dt} \\ \dfrac{dx(t)}{dt} \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ \dfrac{-1}{2} & \dfrac{-3}{2} \end{pmatrix}\begin{pmatrix} q_3 \\ x \end{pmatrix}+\begin{pmatrix} 0 \\ 5 \end{pmatrix} \quad (12)$$

From Equation (12) we observe that system matrix characteristic (10) is

$$X=\begin{pmatrix} 0 & 1 \\ \dfrac{-1}{2} & \dfrac{-3}{2} \end{pmatrix} \quad (13)$$

Then, calculate the eigenvectors from the expressed matrix on (13) by the use of characteristic equation expressed on (14)

$$det\left[\begin{pmatrix} 0 & 1 \\ \dfrac{-1}{2} & \dfrac{-3}{2} \end{pmatrix}-\lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]=0 \quad (14)$$

Resulting:

$$\begin{pmatrix} -\lambda & 1 \\ \dfrac{-1}{2} & \dfrac{-3}{2}-\lambda \end{pmatrix}=0 \quad (15)$$

From Equation (15), the eigenvalues results: $\lambda=-1$ and $\lambda=\frac{-1}{2}$, and if

$$v=\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (17)$$

is an eigenvector of X corresponding to each of the obtained λ, if only v is a trivial solve of matrix equations (18) and (20).

In the case of assuming that $\lambda=\frac{-1}{2}$ the eingenvector is a particular solution for equation (18)

$$\begin{pmatrix} \dfrac{1}{2} & 1 \\ \dfrac{-1}{2} & -1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (18)$$

In this case,

$$v=\begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad (19)$$

Now if we replace by $\lambda=-1$ the eingenvector is a particular solve from Equation (20)

$$\begin{pmatrix} 1 & 1 \\ \dfrac{-1}{2} & \dfrac{-1}{2} \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (20)$$

In this case,

$$v=\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (21)$$

The expressed eingenvectors in expressions (19) and (21) can be used to build a new U Matrix, as observed in (22)

$$U=\begin{pmatrix} 2 & 1 \\ -1 & -1 \end{pmatrix} \quad (22)$$

Matrix (22) allows determining the Equation (9) sol, resulting [7], [8]:

$$q_3(t)=10+2c_1e^{\frac{-t}{2}}+c_2e^{-t} \quad (23)$$

From the solution of Equation (9), and applying the boundary conditions established in the particular solution of equation (23), they obtained equation (24) that represents the outlet flow

$$q_3(t)=10-20e^{\frac{-t}{2}}+10e^{-t} \quad (24)$$

The students made Fig.2 that shows the graph that determines the evolution of the outlet flow as a function of time, they also noted that at the beginning the system drains slowly, while in a time of 3 hours the evacuation is significant and in 6 hours there is a meagre flow to be drained.

In a 9 hours lapse, practically all the fluid was evacuated from tank No. 2. In addition, they determined that the graph in Fig.2 does not have relative extremes. Instead, it has an inflection point at the abscissa point 1.38 (represents 1 hour and 23 minutes from the beginning of the exit of the fluid from tank No. 2), and at that moment only 25% of the total flow was drained [9].

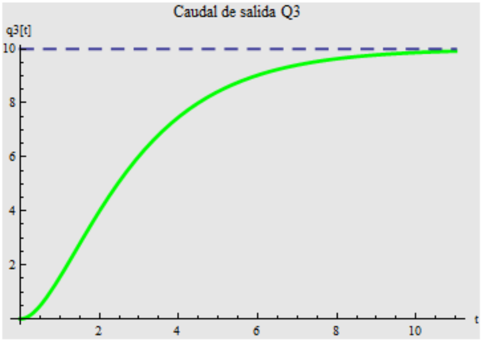

Fig.2 Outlet Flow

By system characteristics, and with the analysis done in Fig.2 and Equation (7), the students conclude that the system is hyper-damping, and the outlet flow is bounded in $10\frac{m^3}{h}$.

In Fig.2 graphic, the students indicated with a dot line the asymptote that determines the outlet flow value, they verified this result calculating with the software

$$\lim_{x->\infty} q_3(t) = 10\frac{m^3}{h} \qquad (25)$$

that allows prove the value for the amount outlet flow from tank N°2 is $10\frac{m^3}{h}$.

From the analysis carried out, the students determined that the expression:

$$(T_1 + T_2)^2 - 4T_1T_2 \qquad (26)$$

is the discriminant of the characteristic polynomial of Equation (9), and also verifies that

$$(T_1 + T_2)^2 - 4T_1T_2 = (T_1 - T_2)^2 \qquad (27)$$

The students concluded that both sides of Equation (26) are always positive, and therefore the system is over damped. In addition, they stated that it is impossible for the system to be under damped since the relationship in equation (25) will never be negative.

They confirmed that the system will not be critically damped either because in the relationship expressed in (25) the T_1 and T_2 constants would be the same and that would imply a null output flow.

In Fig.3 they observed the behaviour of the system taking different values of T_2, with values of T_1 constants and less than T_2.

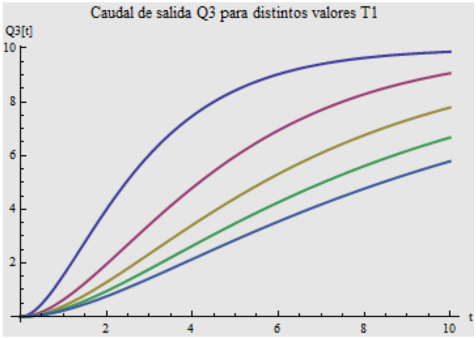

Fig.3 Outlet flow to $T_2 > T_1$ (with a constant T_1)

Analogously, in Fig.4 they show the inverse situation, they analyse the behaviour of the system taking different T_1 values, with constant T_2 values less to T_1.

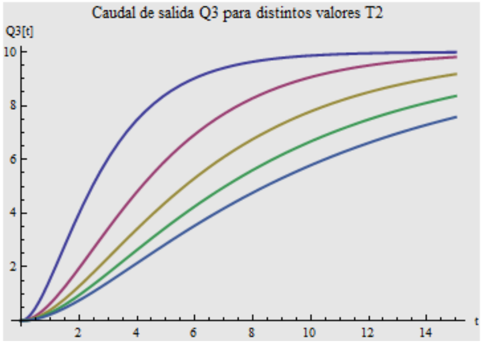

Fig.4 outlet flow for $T_1 > T_2$ (with T_2constant)

In all cases, if they extend the domain in Fig.3 y 4 graphics, they check that when time trends to infinite the outlet flow trends to$10\frac{m^3}{h}$.

It is suggest as additional work to them analysing another liquid flow system and taking the same considerations given is this case. Also, it is suggest to then to compare both models, to obtain conclusions about it.

6 Conclusion

The collaborative environment developed in the Basic Sciences Laboratory and the applied teaching methodology provided students with a series of strategies and skills to solve the proposed activity, which resulted in increased interest and motivation.

The teachers who carry out the experience facilitated the development and articulation of the concepts through a didactic situation of little complexity that offered the students to integrate the knowledge in a functional and meaningful learning.

The visualization carried out with the support of computational tools allows exploring the concepts of the subject Eigenvalue and Eigenvectors, and discovering the relationships that these concepts have with others of the same subject and with the model presented.

The use of technology made it possible to make the role of the modes of representation more explicit, in particular, the way in which the complementarity between the graphic, the numerical, the symbolic, the algebraic and the simulation revealed by the software used, helped to develop the processes of theoretical conceptualization and the systems studied.

In this way, the description of the construction process followed by the students has made it possible to relate aspects of particular to reflective abstraction derived from the modes of representation in the construction of knowledge.

The proposal presented in class is based on integrated and systemic learning. This experience is different from the one presented in the rest of the courses when the subject Eigenvalues and Eigenvectors is treated, it is based on permanent dialogue, the affinity of criteria and fundamentally on the active participation of the main protagonist of the educational fact that is the student.

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