Constructing the Continuity Concept

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Abstract: Key concepts in Mathematics, like derivative or integral, are connected with the notions of limit and continuity. Therefore, the construction of these concepts is relevant and should be well understood by the students. In this paper the construction of the continuity concept by the students is studied, without relying on the limit for that construction. With this purpose in mind, we start by defining the theoretical framework based on Abstraction in Context. After that, a continuity notion that does not depend on limit is presented. Finally, the results of a qualitative study performed with Public Management Quantitative Methods students, from the School of Technology and Management of the Polytechnic Institute of Leiria, is presented. The epistemic actions that were developed by the students while building the continuity notion are presented and analysed.

Key-Words: - abstraction in context, neighbourhood, limit, continuity

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1 Introduction

In Portugal, the notion of continuity is relatively well known by the majority of students entering higher education with Sciences or Economics basic formation (Mathematics A). Those students usually have a solid background in Mathematics, and therefore are not the target of the present study. However, nowadays many students ingress in scientific higher education courses (like engineering degrees) coming from professional schools, humanities courses, abroad courses or adult courses (Mathematics B or below). The majority of these last students need a preliminary course in Calculus before trying to assist to more advanced classes on Analysis, Algebra, Physics or Statistics. Even if the intuitive perceptions of limit and continuity at one point are reasonably apprehended by all the students, from a conceptual point of view, in the passage to the formal definition several issues might be detected. These difficulties are related with symbolism, formalism, graphic representations and with the notion of approximation.

Commonly, the pre-Calculus course for students with Mathematics B or below begins with a brief review of functions (what is a function, properties, linear function, quadratic function) and after that limits are introduced, usually without much success because limits notion is not very easy to understand. In fact, studies on the difficulty of the concept of limit abound in the literature. In [5] it is indicated that most students cannot understand the concept of limit, and therefore cannot calculate them. Moreover, it is indicated that most of the difficulties later encountered in concepts such as continuity, differentiation and integration come from difficulties with the concept and calculation of limits (still on this subject, see for example [14, 20]). Even when the technology was used to aid the calculation of limits, the results were unsatisfactory, which may be related to the lack of a unique strategy to calculate limits [13]. This last author also mentions that for most students limit is seen as a dynamic process and almost as "approaching without reaching", and not as a concrete value. In [4] it is indicated that, regardless of the limits being related to the continuity of functions or with sequences and series, their calculation and understanding is always complicated. Finally, [5] states that there are no works presenting effective strategies that allow overcoming the difficulties felt with the understanding and calculation of limits.

In this study, we intend to eliminate the dependency on the notion of continuity in relation to the notion of limit, starting by introducing the concept of continuity and only later the limit. The notion of a point's neighbourhood will be the basis of this approach. This perspective has the advantage that the concept of continuous function at one point or throughout its domain does not require, as a presupposition, the understanding of the limit concept, commonly regarded as more difficult. This way, the construction of knowledge by students can benefit. We do not intend to diminish the importance of the limit notion, but we believe that knowledge construction and academic success will benefit form this swap.

After that, we study the epistemic actions that surge throughout this process in order to understand how the notion of continuity is built by the students.

2 Abstraction in Context

Abstraction in Context (AiC) is a theoretical framework for studying students' processes of constructing abstract mathematical knowledge as it occurs in a context that includes specific mathematical and curricular components, as well as a particular learning environment. Seminal works by [6, 9, 15], among others, introduced this issue in the area of Mathematics Didactic. Essentially, "Abstraction" is a theoretical activity that contemplates a set of tasks performed by one or more individuals, motivated by a given problem integrated in a "Context", which encompasses the personal and social involvement of individuals. Currently, this methodology is used as a theoretical support in works that seek to understand the construction of mathematical knowledge, being mentioned in several studies that seek to understand how students reach concepts of various levels of difficulty [10, 12]. In those studies, the teacher introduces the subjects in various ways (practical problem, theoretical concept to deepen, notion to generalize) and helps students while they build their own considerations on the subject. It seeks to reflect the spirit of the Bologna process, with the key aspect being the autonomy of the student, accompanied by the constructive observations of the teacher. AiC as three stages, namely necessity, emergence and consolidation. The emergence process of new mathematical knowledge is the central part of the AiC and it is usually analysed under the epistemic actions *Recognizing* (R-Action), *Building-with* (B-Action) and *Constructing* (C-Action). Together they form the RBC model, to which we can join the *Consolidation* (Co). These are the epistemic actions which compose the RBC+Co model [9, 10, 12].

3 The Notion of Neighbourhood

We mentioned, in the Introduction, that there are advantages by introducing the notion of continuity before the notion of limit. It is now important to see how this process can be implemented in practice. It should be noted that this is not a new idea, even in the Portuguese context. In [21], a support manual for mathematics teachers of the 10.º grade, continuity is not introduced with the limits. However, this perspective is only available for the teachers and not for the students, and is provided to teachers solely as "general culture" information. Later, in the 12.º grade, students work on continuity using the definitions of Cauchy and Heine [22], but usually consider that a function is continuous at a point a belonging to its domain when the limit in that point is equal to the value of the function in that point.

Many students justify a function continuity in a given point referring only to the equality of the lat-

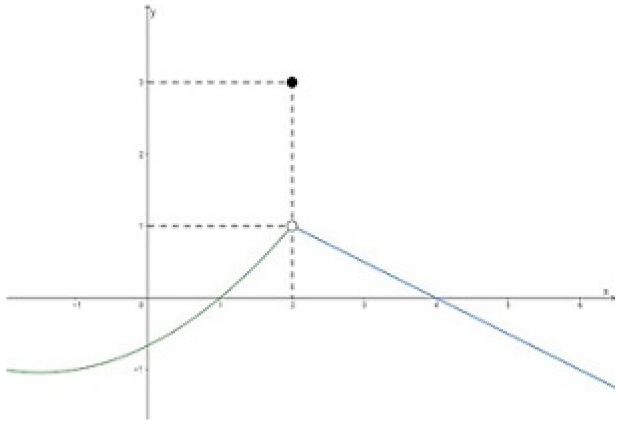

Figure 1: Continuity at $x = 2$ where $\lim_{x\to 2} f(x) \neq f(2)$.

eral limits without mentioning the function value in that point (see Figure 1).

But the main issue is the fact that in 12.º grade the limit notion is based on accumulation points, meaning that x cannot be equal to a, and therefore there is no limit at isolated points (see Figure 2). Of course, we can define the limit considering (or not) isolated points, but this causes, in many cases, confusion among the students.

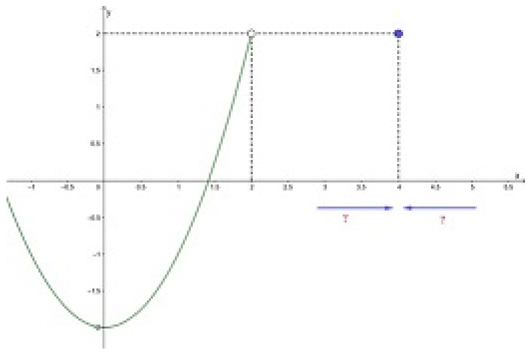

Figure 2: Continuity at $x = 4$ where $\lim_{x\to 4} f(x)$ doesn't exist for limit notions based on accumulation points.

The following question arises: "How to introduce the concept of continuity without using limits?". Following [11] line of thought, we will base the concept of continuity on the notion of a point's neighbourhood. The notion of a point's neighbourhood is easy to understand because it can be seen as a symmetrical range around a point (see Figure 3).

In a formal level, the notion of a point's neighbourhood will be defined as follows.

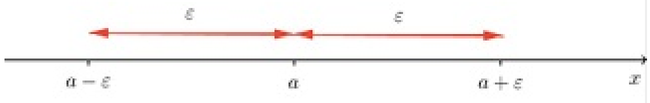

Figure 3: Neighbourhood of point a with radius ϵ

Definition 1 Neighbourhood at a point.
Let a be a real number and ϵ a positive real number. The neighbourhood of a is the set of real numbers whose distance to a is less then ϵ, that is,

$$V_\varepsilon(a) = \{x \in \mathbb{R} : |x-a| < \varepsilon\} =]a-\varepsilon; a+\varepsilon[\quad (1)$$

Thus, students have to understand the behaviour of a certain f function around the vicinity of a, which seems to us a simpler and more elegant way of introducing the concept of continuity at a point, leading to a set of advantages that will be tackled later.

4 The Concept of Continuity

The notion of continuity that is usually taught, both in secondary and in higher education, is based on the concept of limit. However, historically the notion of continuity pre-date limit's notion - at least in the perspective of a function "without holes" [23]. This author considers that the term limit, in the most current sense of the term, was only defined in the nineteenth century, while the concept of continuity was already well established in the eighteenth century. In 1748, Euler published "Introductio in Analysin Infinitorum", where most of the mathematical concepts used nowadays were defined (of course, with a very different notation and presentation). In that work, Euler indicates that a continuous function is formed by a single analytical expression, while discontinuous functions are formed by more than one analytical expression [18]. Naturally, this notion of continuous function lacks, in the light of current knowledge, from mathematical rigorous, and denotes some flaws. A simple counter-example is when a function is defined by branches but still is continuous throughout the domain. Nevertheless, Euler's work was pioneer for the time and had a great impact on the later development of mathematics.

Therefore, and as we mentioned in the Introduction, there seems to be pedagogical advantages in introducing continuity first, and and only after that limit. In a simpler and more intuitive notion, the concept of continuity based on the notion of a point's neighbourhood can be defined as [11].

Definition 2 Continuity at one point.
Let f be a real function defined in $D \subset \mathbb{R}$ and $a \in D$ a point in the domain. The function f is continuous in a if

$$\forall \delta > 0, \exists \varepsilon > 0 : \forall x \in \mathbb{R}, \\ (x \in D \cap V_\varepsilon(a) \Rightarrow f(x) \in V_\delta(f(a))). \quad (2)$$

Note that in this definition we do not remove a from its own neighbourhood, and therefore the function is continuous at isolated points.

5 The Pedagogical Proposal

As already stated, we changed the teaching order of the concepts of limit and continuity. Therefore, we seek to verify whether the cutting of dependence on the notion of continuity in relation to the notion of limit leads to a better understanding of continuity. Also, we seek to identify the epistemic actions that arise during the abstraction process and that are relevant for the construction of mathematical knowledge, as well as the difficulties felt by the students, specially at the level of the application of symbolic language and interconnection of concepts. The research questions associated with the pedagogical proposal are as follows:

- what epistemic actions are possible to identify, during students abstraction process, related with the constructing (*C-Action*) of new mathematical knowledge, namely:
  - while developing an understanding of the problems;
  - identify the need to use other mathematical concepts or previous constructions;
  - implement intermediate strategies and solutions;
  - organize knowledge and ideas;
  - construct the concept of continuity;
- how do these epistemic actions are sequenced and related?

Throughout an experimental study, we tried to answer the previews questions. The final goal is to present a detailed description of the entire knowledge building system. The experimental study was implemented in a Quantitative Methods curricular unity (QM). This course belongs to the Public Management degree from School of Technology and Management, Polytechnic Institute of Leiria (ESTG). The students who attend to QM usually did not have Mathematics A in high school, and therefore their mathematical background is similar to engineering students that must attend to a preliminary course in Calculus (see Introduction). Even more, QM has similar contents to the preliminary course in Calculus in ESTG. For these reasons, we believe that the results can be extended to the less prepared engineering students attending preliminary curricular unity in Calculus.

6 Research Methodology and Data Collect

The methodology adopted in this work is qualitative and interpretive. According to [7], qualitative research is fundamentally concerned with processes and dynamics, and is dependent on the researcher. Bogdan and Biklen [1], in one of the best-known works on this subject, state that the multiple ways of interpreting experiences depend on the relationships between the various actors in the learning process. In the Portuguese context, [16, 17] studied the methodology to apply in Mathematics classes. Thus, we sought to understand the process of knowledge construction for the involved students, through the analysis of the evolution of their productions in class (oral participation, resolution of exercises, etc.). Later, some case studies that seemed more important for understanding the process of knowledge construction were analysed in detail. About 15 students participated in the study, but not all of them performed all the tasks. Students that were repeating the course had one of the weekly lessons partially overlapped, which conditioned attendance. The collected data (concerning continuity notion) was produced by students frequenting QM, during two classes of 120 minutes each. In general, students seemed rather motivated with the aims of the study and did their best to participate.

At the end of the class that preceded the beginning of the continuity study, the pedagogical proposal was briefly presented to the QM students by the researcher. Note that the researcher responsible for the study implementation attended to the classes, but was not the teacher, even if sometimes he helped the teacher. This is a procedure already considered has advantageous by some researchers [8], mainly because this way the researcher can try to collect the "best" data and at the same time the teacher can struggle for students' success. After this introductory procedure, all students had to answer, individually and in paper, to a first question. With the answers to that question, we intended to identify the students' preliminary knowledge about the notion of continuity. In subsequent classes the process was similar. The professor distributed tasks, of increasing difficulty, about the continuity of a function, both in a point or in its domain. Each exercise was first projected onto the board, without any major considerations being made about its resolution. The students answered to the questions on paper, and after that the answers were discussed in class, being subject to correction after the discussion phase. This iterative process sought to contribute to the process of construction of mathematical, abstract and advanced knowledge by the students. After performing a set of tasks, the students answered again to the initial question, trying to assess whether or not they were able to understand the continuity notion. Moreover, we tried to understand if they evolved in relation to their initial knowledge. Only after this phase the formal notion of continuity was debated, and later written in accordance with Definition 2.

Table 1: Epistemic Actions for the RBC model

R-action
Interpret Acquired structure Neighbourhood
B-action
Strategies Previous construction application Intermediate solutions Justification
C-action
Reorganization Continuity at a point { Left-continuity / Right-continuity Continuity on the domain Continuity on an interval Communication

Given that it would be very difficult to record all the discussion that took place in class, when solving the proposed tasks, the lessons were recorded and later viewed. All the appropriate authorizations were obtained, and it was ensured that the recordings would only be used for academic purposes. With this procedure it was possible to collect a large amount of data, since that information came with the written component (individual answers to the questions) but also with the oral component (video recording of the discussion of the problems).

Considering that it wouldn´t be possible to consistently analyse the production of a large number of respondents, two students were selected for detailed analysis (AF and DV). These students were selected considering that some students didn´t answer all the proposed questions and, among those who answered all the questions, some did it in an incipient way.

According with the RBC+Co model introduced in previous questions, the categories and subcategories concerning the epistemic model [19] can be seen in Table 1. Note that *Consolidation* (Co) does not have subcategories because it will only appear when applying continuity construction in other problems. Table 1

Table 2: Epistemic actions for the R-action

Interpret
Piecewise function Quadratic function
Acquired structure
Quadratic formula Domain Graphical representation
Neighbourhood
At a point

will be used as an artefact [2, 3, 24] by the researchers since it is the primary tool to understand students production. We will not analyse this table here, because in the next section its utility will be highlighted. Note that throughout the text references to the lines of Table 1 will appear, but to avoid repetitions they will not be explicit.

7 Results

In this section we focus our attention in one of the questions that were presented during the classes. The selected question is relevant since it deals with continuity in an isolated point but also in an interval, for a function with different expressions in parts of its domain. Therefore, we will explain the epistemic actions that occur throughout the process.

Question Let f be a real function of a real variable x defined as

$$f(x) = \begin{cases} 2, & x = 3 \\ x^2 - x - 2, & x \leq 2. \end{cases} \quad (3)$$

Verify if f is continuous on its domain.

In the remaining subsections, the transcriptions from the students (AF and DV), professor (P) and researcher (R) interventions are identified by letter and number. For example, P3 refers to the third intervention in the discussion, in this case performed by the teacher.

7.1 R-action

The students began to *Recognize* that they were dealing with a piecewise function, with a quadratic function in the second branch. The epistemic actions Interpret and Acquired Structure occur almost simultaneously, because students acknowledge the need of graphical representation, that rely on quadratic formula and domain. Clearly, there is a strong interlink between these actions. Finally, the students needed to recall Neighbourhood at a point in order to study continuity. In a nutshell, the epistemic actions and their subcategories can be found in Table 2, with the class discussion in Table 3.

A graphical representation from the relations between the different epistemic actions and class productions can be seen in Figure 4. Figure 4 and remaining figures are at the end of the document in a bigger size so that they can be properly read.

Table 3: Class discussion for the R-action

P1: for this function, what have you been doing? AF2: calculating the zeros. P3: using? AF4: quadratic formula. .... P7: and the vertex? DV8: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. .... P15: after that, what can we do? AF16: the graph! R17: the students represent graphically f function. P18: quadratic function is just when? AF19: $x \leq 2$. DV20: $x = 3$ is missing! [R: referring to the domain]. .... P31: remember, for non continuity, what happens? DV32: neighbourhood “catch” different values.

7.2 B-action

In the *Building-with* phase, students initial Strategy was the graphical representation of the quadratic function, that they identify as a parabola (Previous construction application). To obtain the roots and vertex, quadratic formula and vertex formula were used as a preamble for graphical representation ($x \leq 2$). The presented calculus are the Justification. We believe that the achieved graphical representation can be seen as an Intermediate solution, where we can also verify the Previous construction application for the isolated point. After that, and from the graphic analysis (here seen as a Justification), students obtained the function domain, that also represents an Intermediate solution. The written domain is another Justification. The epistemic actions for the *Building-with* phase are summed up in Table 4, while student’s discussion about this subject is presented in Table 5.

A graphical representation from the relations between the different epistemic actions and class productions can be seen in Figure 5.

Table 4: Epistemic actions for the B-action

Strategies
Calculation of function roots Calculation of parabola vertex Graphical representation
Previous construction application
Use of quadratic formula Use of vertex formula Use of isolated point
Intermediate solutions
Obtained graphical representation Obtained function domain
Justification
Written calculus for the function roots and vertex Graphical representation Written domain

Table 5: Class discussion for the B-action

P1: for this function, what have you been doing? AF2: calculating the zeros. .... P5: results are $x = -1$ or $x = 2$, right? AF6: yes! P7: and the vertex? .... P13: Ok! The results are?! DV14: 0.5 and -2.25 [R: Referring to x and y]. .... R17: the students represent graphically f function. .... P23: so, the domain is? AF24: $]-\infty; 2] \cup \{3\}$.

7.3 C-action

The *Constructing* phase starts with the Reorganization of previously obtained Intermediate solutions, mainly Analysing obtained domain and Analysing obtained graphical representation. The main issue, as expected, occurred when dealing with *Constructing* the Continuity on the domain (see AF26 in Table 7). Only after teacher's intervention (see P31 and subsequent interventions in Table 7) it was possible for the students to *Construct* Continuity at an isolated point, after the teacher had remembered Neighbourhood notion. By the other hand, the *Construction* of Continuity on an interval was easily achieved by the students (see P28 and DV29 in Table 7). Therefore, Continuity at an isolated point and Continuity on an interval preceded Continuity on the domain, and a little "push" from the teacher was required in order for the students to achieve the late construction. Finally, Communication is only expressed when the students answer to question, that is, when they write the domain (Table 5, P23 and AF 24) and say where the function is continuous (Table 7, DV37).

Table 6: Epistemic actions for the C-action

Reorganization
Analysing obtained graphical representation Analysing obtained domain
Continuity on the domain
Continuity at an isolated point Continuity on an interval
Communication

Table 7: Class discussion for the C-action

R25: Although most students identify the domain, most of them does not answer to the question. AF26: Professor! It isn't continuous, right? It stops! R27: AF refers to the "leap" from the point (2; 0) to the point (3; 2), the isolated point. P28: The main question is: this function is continuous where? DV29: From minus infinity until 2. .... P31: remember, for non continuity, what happens? DV32: neighbourhood "catch" different values. P33: right. When we calculate the neighbourhood, we must obtain different images for non continuity, ok? Here [R: referring to $x = 2$] is there any continuity issue? DV34: no. P35: And here? [R: referring to $x = 3$]. Who believes that it is continuous and who says that it isn't? AF36: It is continuous! The neighbourhood is empty and far from 3. DV37: so it is continuous in the entire domain!

A graphical representation from the relations between the different epistemic actions and class productions can be seen in Figure 6.

8 Conclusion and Future Work

In this paper we studied the *Construction* of continuity concept from university students, using RBC methodology as a theoretical framework. Considering the epistemic actions, we can conclude that the abstraction process was always triggered by the *R-Action*, when students recognized the need of using previous constructions (quadratic and piecewise functions, domain, neighbourhood, ...). After that, students used the previous constructions in order to obtain intermediate solutions (graphical representation and domain). These epistemic actions belong to the the *B-action*. Finally, the *C-action* was always initiated by the reorganization of the constructions developed in the *B-action*. Construction may not be isolated since some constructions are related to each other, in many cases promoting new constructions associated to the concept under study (that is, continuity on an interval and continuity at an isolated point promoted the construction on the domain). Because traditional notion of continuity relies on the limit notion, that is usually complicated to apprehend by the students, we defined continuity in a coherent way using neighbourhood. In Figure 7 the reader can see, in a schematic way, all the major relations between *R-action*, *B-action* and *C-action*. Future work should focus in consolidation (*Co-Action*). To perform that task, different questions must be dealt to the students to check if they can use continuity construction in another context. Also, the bridge between limit and continuity using neighbourhood would also be an interesting topic of study.

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Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

Ana Santos was the main researcher and conducted the pedagogical experience.
Miguel Felgueiras was the class teacher and developed the class questions.
Ana Santos designed the figures presented in this text.
Miguel Felgueiras was the main writer of this article.
Both authors read the manuscript.

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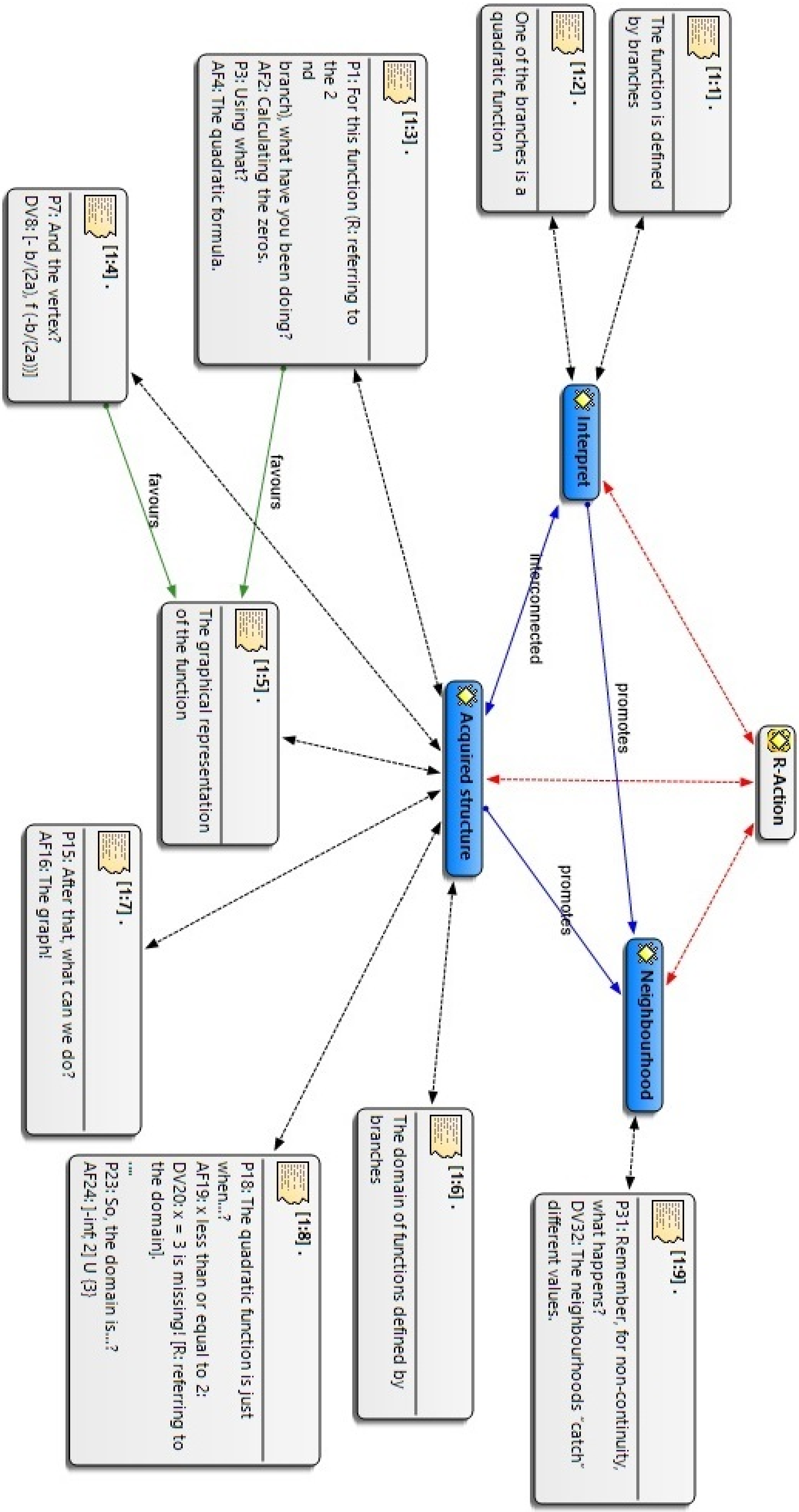

Figure 4: R-Action

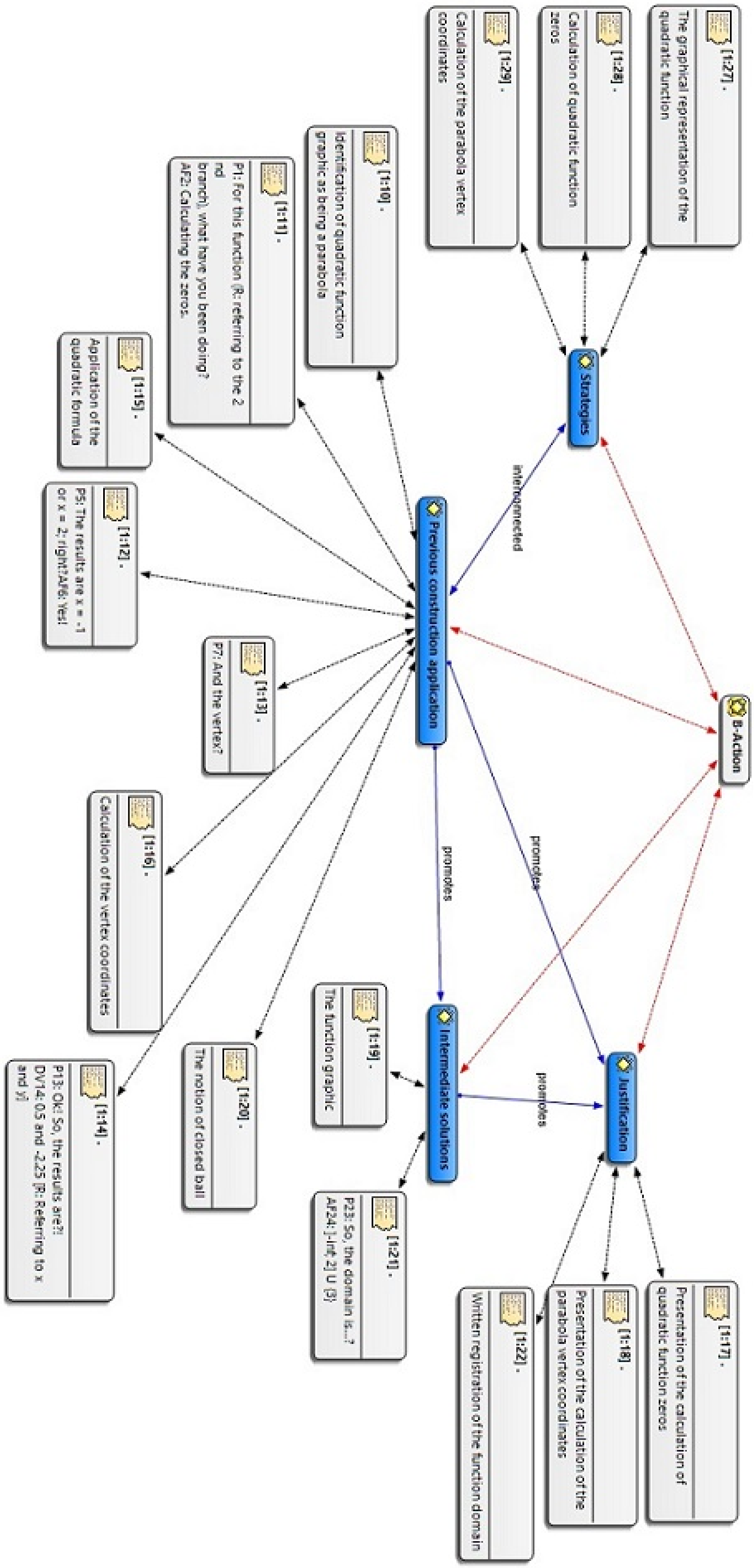

Figure 5: B-Action

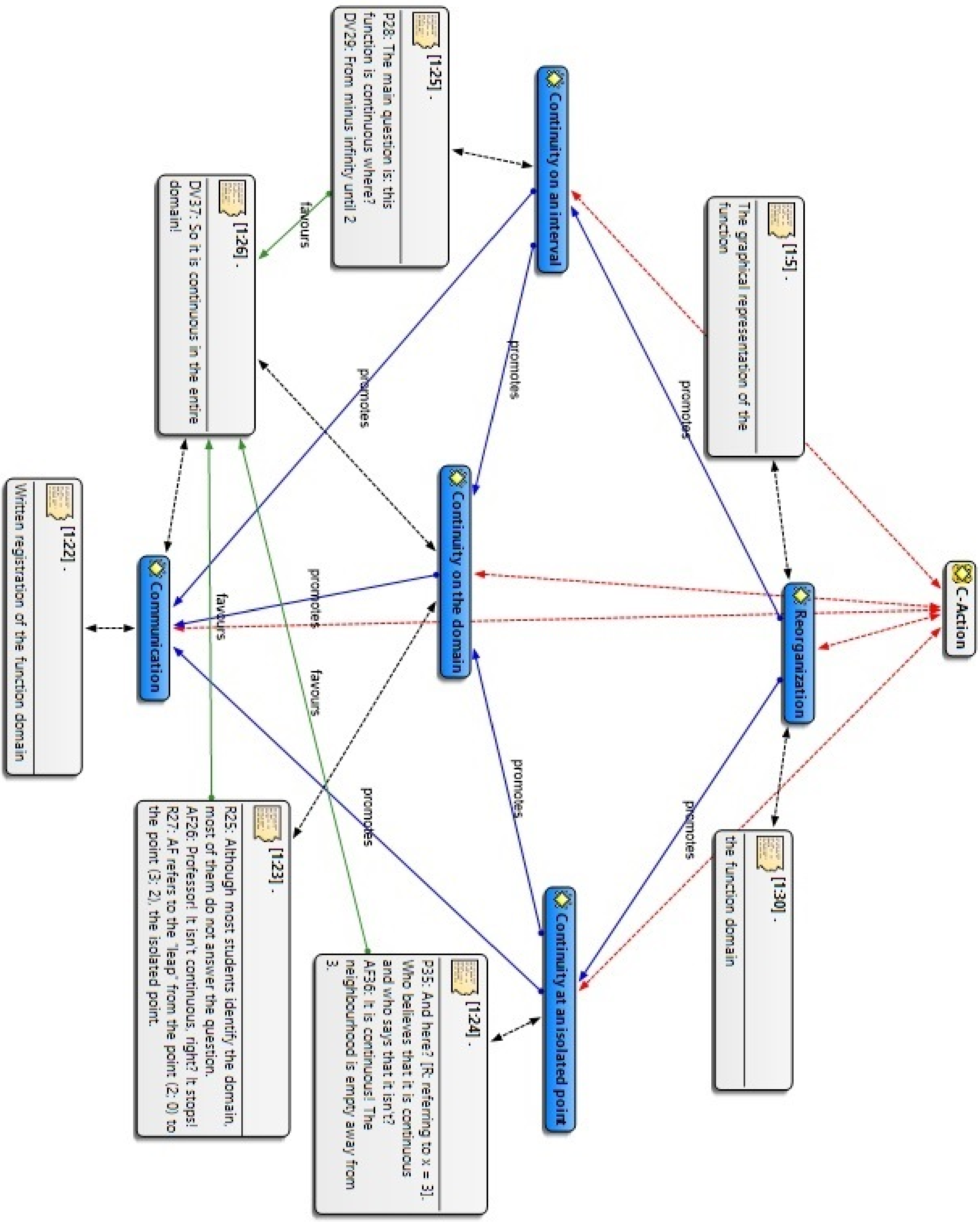

Figure 6: C-Action

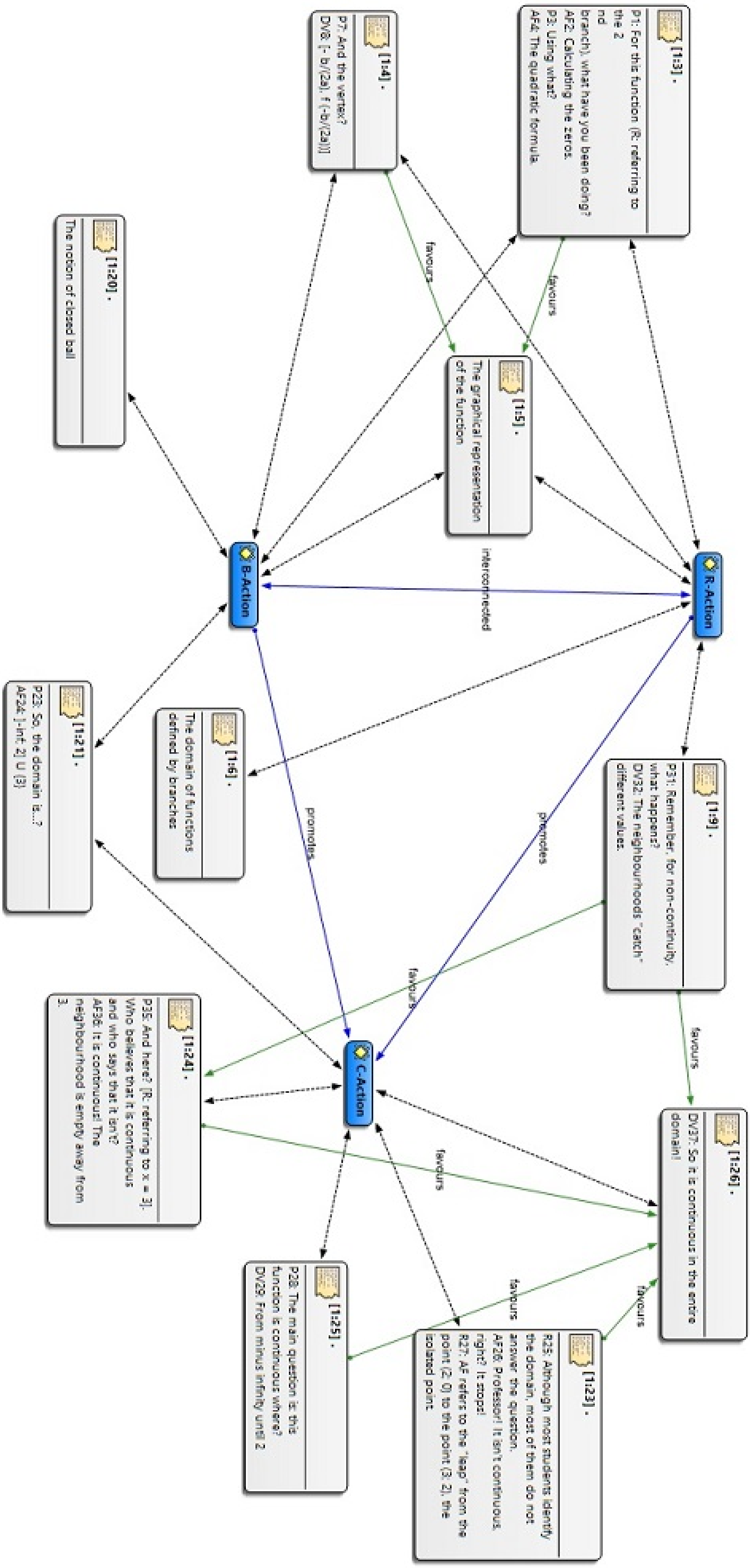

Figure 7: Relations between the epistemic actions.