Design of optimal IIR digital filter using Teaching-Learning based optimization technique

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Abstract: - In this paper an Enhanced Teaching-Learning Based Optimization (ETLBO) algorithm is employed to design stable digital infinite impulse response (IIR) filter using L_p-norm error criterion. The original TLBO algorithm has been remodeled by merging the concept of opposition-based learning and migration for selection of good candidates and to maintain the diversity, respectively. The multiobjective IIR digital filter design problem considers minimizing the L_p-norm approximation error and minimizing the ripple magnitude simultaneously while satisfying stability constraints on the coefficients of the filter. Weighted sum method and p-norm method are applied to solve the multicriterion optimization problem . Best weight pattern is searched using evolutionary search method that minimizes the performance criteria simultaneously. The validity of the method is demonstrated for the design of low pass (LP), high pass (HP), band pass (BP) and band stop (BS) IIR filters. The comparison of simulation results with other existing methods show that the proposed ETLBO algorithm is superior in terms smaller L_1-norm error, L_2 -norm error and smaller pass band and stop band ripples.

Key-Words: - IIR filter, TLBO, magnitude response, stability, L_p-approximation error.

1 Introduction

Filters are mainly used for extorting informative part of the signal and to remove undesirable component of the signal. Extorting of signal is required when a noise or some disturbance contaminates a signal [1]. Digital filters have attracted the attention of researchers due to large number of application like data communication, video processing, radar and optical communications, speech processing and many more. In terms impulse response, digital filters are widely categorized as infinite impulse response (IIR) and finite impulse response (FIR) filters [2]. The selection of digital filter for an application is a tedious task involving finding of optimum structure in order to satisfy certain parameters of frequency response, namely ripples in pass band, transition band width and attenuation in stop band. Digital IIR filters are preferred over FIR digital filters because of higher computational efficiency and accurate frequency selectivity. The two problems with the design of IIR digital filter are [3-4]: (i) tendency of the filter to become unstable (ii) filter error surface is multimodal in nature due to which conventional design optimization algorithms may stuck at local minima. The stability problem is handled by imposing stability constraints on the filter coefficients. Numerous evolutionary and meta-heuristic optimization algorithms have been successfully applied to handle the non-differentiable and multimodal error surface of digital IIR filter. Some evolutionary optimization algorithms recently applied for digital IIR filter are: genetic algorithms [5-10], immune algorithm [11], particle swarm optimization [12-14], seeker-optimization-algorithm [15], predator-prey optimization [16], heuristic search method (HSM) [17], two-stage ensemble evolutionary algorithm [18], gravitation search algorithm [19] and many more.

The main drawbacks of the above algorithms are slow convergence towards optimal solution and the requirement of algorithm specific controlling parameters in addition to regular controlling parameters like size of population, number of iterations, group size etc. In order to overcome the above drawbacks, teaching-learning based optimization (TLBO) algorithm developed by Rao et al. [20-21] has been applied to design the digital IIR filters. TLBO is a heuristic search method inspired by the learning behavior of the students in a class. There is no need to tune any algorithm specific controlling parameters in order to implement TLBO, thus making it more robust.

The intent of this paper is to introduce enhancement in original TLBO to improve its exploration and exploitation capabilities, by initializing with good candidates and maintaining the diversity. The concept of opposition-based learning is employed for initialization and evolution of population. Further migration has been applied to maintain the diversity and search space exploration, and avoid premature convergence. The unique combination of broad exploration and further exploitation yields a powerful option to solve multimodal optimization problems that designs IIR filters. The multicriterion optimization problem of digital IIR filter design is converted into scalar constrained optimization problem using weighting p-norm method. The weighting technique is used to generate non-inferior solutions, which allow explicit trade-off between conflicting objective levels. Evolutionary search technique is employed to search for the weightage pattern. The paper analyzes the performance of Enhanced TLBO (ETLBO) algorithm for designing digital IIR filters using L_p-approximation error criterion and the obtained results are compared with hierarchical genetic algorithm (HGA) [8], hybrid taguchi genetic algorithm (HTGA) [10], taguchi immune algorithm (TIA) [11] and heuristic search method (HSM) [17] for the low-pass (LP), high-pass (HP), band-pass (BP), and band-stop (BS) filters for validation.

The paper is structured as follows. The digital IIR filter design problem is formulated in Section 2. Section 3 elaborates the implementation of ETLBO algorithm for the digital IIR filter design. The performance of ETLBO is evaluated and compared with the design obtained by various researchers in Section 4. Finally, Section 5 concludes the outcomes of the work.

2 Problem Formulation

To design a digital IIR filter a set of optimum filter coefficients are searched in order to meet various objectives summarized below:

- Minimize the absolute error L_1-norm of magnitude response.
- Minimize the squared error L_2-norm of magnitude response.
- Minimize the magnitude pass band ripples.
- Minimize the magnitude stop band ripples.

The IIR digital filter is denoted by the following transfer function:

$$H(z) = \frac{\sum_{k=0}^{M} a_k z^{-k}}{1 + \sum_{k=0}^{M} b_k z^{-k}} \tag{1}$$

$$H(\omega, x) = A \times \left(\prod_{l=1}^{M} \frac{1 + a_{1l} e^{-jw}}{1 + b_{1l} e^{-jw}} \right) \times \left(\prod_{m=1}^{N} \frac{1 + c_{1m} e^{-jw} + c_{2m} e^{-2jw}}{1 + d_{1m} e^{-jw} + d_{2m} e^{-2jw}} \right) \tag{2}$$

where

Vector $X=[a_{11}, b_{11},..., a_{1M}, b_{1M}, c_{11}, c_{21}, d_{11}, d_{21},..., c_{1N}, c_{2N}, d_{1N}, d_{2N}, A]^T$ of dimension $S\times 1$, with $S=2M+4N+1$ denotes the filter coefficients. A represents the gain of the filter. The main goal of the design algorithm of digital IIR filter is to search for filter coefficients a_k and b_k such that the magnitude response error in terms of L_p-norm [10-11] and ripples in pass band and stop band are minimized. The magnitude response is specified at K evenly distributed discrete points of frequency in pass band and stop band. Absolute magnitude response error is represented by $E_1(x)$ and squared magnitude response error is denoted by $E_2(x)$:

$$E_1(x) = \sum_{i=0}^{K} \left| H_I(\omega_i) - \left| H(\omega_i, x) \right| \right| \tag{3}$$

$$E_2(x) = \sum_{i=0}^{K} \left(\left| H_I(\omega_i) - \left| H(\omega_i, x) \right| \right| \right)^2 \tag{4}$$

Ideal magnitude response $H_I(\omega_i)$ of IIR digital filter is stated as:

$$H_I(\omega_i) = \begin{cases} 1, & for\, \omega_i \in passband \\ 0, & for\, \omega_i \in stopband \end{cases} \tag{5}$$

The pass band and stop band ripples to be minimized are denoted by $\delta_p(x)$ and $\delta_s(x)$ respectively:

$$\delta_p(x) = \max_{\omega_i} \left\{ \left| H(\omega_i, x) \right| \right\} - \min_{\omega_i} \left\{ \left| H(\omega_i, x) \right| \right\} \quad for\ \omega_i \in passband \tag{6}$$

and

$$\delta_s(x) = \max_{\omega_i} \left\{ \left| H(\omega_i, x) \right| \right\} \quad for\ \omega_i \in stopband \tag{7}$$

Accumulating all above mentioned criteria's, the multicriterion constrained optimization problem is formulated as below:

$$\begin{aligned} &Minimize\ F_1(x) = E_1(x) \\ &Minimize\ F_2(x) = E_2(x) \\ &Minimize\ F_3(x) = \delta_p(x) \\ &Minimize\ F_4(x) = \delta_s(x) \end{aligned} \tag{8a}$$

Subject to: the stability constraints

$$1 + b_{1l} \geq 0 \ \ (l = 1, 2, ..., M) \tag{8b}$$

$$1 - b_{1l} \geq 0 \ \ (l = 1, 2, ..., M) \tag{8c}$$

$$1 - d_{2m} \geq 0 \ (m = 1, 2, ..., N) \tag{8d}$$

$$1 + d_{1m} + d_{2m} \geq 0 \ (m = 1, 2, ..., N) \tag{8e}$$

$$1 - d_{1m} + d_{2m} \geq 0 \ (m = 1, 2, ..., N) \tag{8f}$$

IIR digital filter design task is a multi-objective optimization problem (MOOP) because several objectives are optimized simultaneously as shown in Eq. (8a). The multiobjective constrained optimization task for the design of IIR digital filter is converted into a scalar constrained optimization problem by using weighting method as defined below:

$$\text{Minimize } f(x) = \sum_{k=1}^{L} \alpha_k \, F_k(x) \tag{9}$$

Subject to: The satisfaction of stability constraints stated in Eqs. (8b) to (8f).

where $F_k(x)$ is the k^{th} objective function and α_k is nonnegative real number called weight assigned to k^{th} objective function. This approach yields meaningful results when solved many times for different values of α_k ($k=1,2,...,L$). The p-norm weighting patterns are either presumed on the basis of decision maker's intuition or simulated with suitable step size variation. Weighted sum technique causes problem when the lower boundary of function space is not convex [22], because not every non-inferior solution will have a supporting hyper-plane. In this paper weight Pattern search based on evolutionary search method is applied to search the normalized weights, α_k ($k=1,2,...,L$) assigned to participating objectives.

The digital IIR filter design requires the satisfaction of stability constraints. The stability constraints to be imposed on the coefficients of IIR digital filter as stated in Eqs. (8b) to (8f) are obtained by employing Jury method [23]. The value of filter coefficients are updated with a random variation as given below in order to satisfy the stability constraints The care is taken that the amount of variation is small enough so that it should not change the characteristic of the population.

$$b_{1l} = \begin{cases} b_{1l}(1-r)^2; & (1+b_{1l}) < 0 \\ & or\ (1-b_{1l}) < 0 \\ b_{1l} & ; Otherwise \end{cases} \tag{10a}$$

$$d_{2m} = \begin{cases} d_{2m}(1-r)^2; & (1-d_{2m}) < 0 \\ d_{2m} & ; Otherwise \end{cases} \tag{10b}$$

$$d_{1m} = \begin{cases} d_{1m}(1-r)^2; & (1+d_{1m}+d_{2m}) < 0 \\ & or\ (1-d_{1m}+d_{2m}) < 0 \\ d_{1m} & ; Otherwise \end{cases} \tag{10c}$$

where r is a uniform random number whose value varies between [0, 1].

3 IIR Filter Design Using ETLBO

ETLBO based on the noble concept of teaching-learning [20-21] is a recently developed population based optimization technique. The unique feature of ETLBO is that it requires to tune few control parameters.

The growth of every society to a great extent is influenced and dependent upon the quality of teachers in the society. ETLBO efficiently explores the knowledge base of a teacher to increase the know-how of learners / students. A teacher puts his best effort in order to increase the mean score of all learners in each allotted subject towards its own mean score. So, the mean fitness of the class is increased by the teacher according to his / her own capability. The learners also further improve their knowledge base by interacting and sharing information with each other.

In the implementation of ETLBO for the design of digital IIR filter NL the number of learners in a class represent the population and each learner has been assigned S subjects (filter coefficients). The i^{th} learner is represented as $X_i = [x_{i1}, x_{i2}, ...x_{iS}]$ and $f(X_i)$ represent the fitness function for i^{th} learner.

$$class = \begin{bmatrix} X_{11} & X_{12} & .. & .. & X_{1S} \\ X_{21} & X_{22} & .. & .. & X_{2S} \\ . & . & X_{ij} & .. & . \\ . & . & .. & .. & . \\ X_{NL1} & X_{NL2} & .. & .. & X_{NLS} \end{bmatrix} \begin{matrix} \to f(X_1) \\ \to f(X_2) \\ \to f(X_i) \\ .. \\ \to f(X_{NL}) \end{matrix}$$

- *Intitialization of Class*

The marks for all the subjects of learners in a class are initialized with the help of random search. Global search is applied to explore the starting point and then the starting point is perturbed in local search space to record the best starting point. The search process is started by initializing the variable x_{ij}^{t} using Eq. (11):

$$x_{ij}^{t} = x_j^{\min} + R()(x_j^{\max} - x_j^{\min}) \quad (i = 1,2,\ldots,NL;\ j = 1,2,\ldots,S) \tag{11}$$

where

R is a uniform random generated number between (0,1).

S is number of subjects allotted to each learner.

NL number of learners in a class.

t is the iteration counter.

$x_j^{\max}$ and $x_j^{\min}$ are the maximum and minimum values of j^{th} decision variable (filter coefficient) of vector X.

- *Opposition-based Learning*

The theory of opposition-based learning [24] is applied to enhance the convergence rate of ETLBO . The notion behind opposition-based learning is to select better current candidate solution by comparing the current population and its opposite population. The opposition-based learning is applied using Eq. (12) to record the alternative starting point and starting point x_{ij}^{t} is further explored using:

$$x_{i+NL,j}^{t} = x_j^{\max} - R()(x_j^{\max} - x_j^{\min}) \quad (i = 1,2,\ldots,NL;\ j = 1,2,\ldots,S) \tag{12}$$

Out of $2\times NL$ learners, best NL learners constitute a class to initiate the process. For the global search, best learner is selected out of class of learners.

Further the opposition-based learning is also employed for generating new learners after the completion of learner phase using Eq. (13):

$$x_{i+NL,j}^{t} = x_j^{U} + x_j^{L} - x_{ij} \quad (i = 1,2,\ldots,NL;\ j = 1,2,\ldots,S) \tag{13}$$

where

$$x_j^{U} = \max\left\{ x_{ij};\ (i = 1,2,\ldots,NL)\right\} \ (j = 1,2,\ldots,S)$$
$$x_j^{L} = \min\left\{ x_{ij};\ (i = 1,2,\ldots,NL)\right\} \ (j = 1,2,\ldots,S)$$

- *Teacher Phase*

The best learner is selected among all the learners in a class based upon the fitness function value calculated using Eq. (9) and act as teacher xt_j^t for current iteration t. The mean (μ_j) for S subjects allotted to the students is evaluated and a randomly weighted differential vector ($Diff_j$) from current mean and various desired mean vectors [25] is calculated as shown below:

$$\mu_j^{t} = \frac{1}{NL}\sum_{i=1}^{NL}\left(x_{i,j}^{t}\right) \qquad (j = 1,2,\ldots,S) \tag{14}$$

$$Diff_j^{\,t} = R()\times(xt_j^{t} - (T_f \times \mu_j^{t}) \quad (j = 1,2,\ldots,S) \tag{15}$$

where

μ_j^t is mean of j^{th} subject for all learners of a class; xt_j^t is the score of the teacher in j^{th} subject; T_f is the teaching factor; R is a uniform generated random number between {0,1}.

The teaching factor (T_f) is one of the vital aspect that facilitates the convergence of ETLBO. In this paper the value of T_f is heuristically selected as 1 or 2 as shown below:

$$T_f = ROUND(1.0 + R()) \tag{16}$$

The weighted differential vector ($Diff_j$) generated using Eq. (15) is added to current score of learners in different subjects to generate new learners:

$$xnew_{ij}^{t} = x_{ij}^{t} + Diff_j^{\,t} \quad (j = 1,2,\ldots,S) \tag{17}$$

The newly generated learner with a better fitness value replaces the existing learner in the class.

- *Learner Phase*

The knowledge acquired by the learners in teacher phase is further disseminated among learners themselves through sharing of notes, discussions and presentations. The second phase of ETLBO emulates this sharing of knowledge by learners among themselves. Two target learners namely i and m are selected randomly such that $i \neq m$. The resultant new learners after sharing / exchange of know-how are generated as follows:

$$xnew_{ij}^{t} = \begin{cases} x_{ij}^{t} + R()\times(x_{ij}^{t} - x_{mj}^{t});\ f(X_i) < f(X_m) \\ x_{ij}^{t} + R()\times(x_{mj}^{t} - x_{ij}^{t});\ Otherwise \end{cases} \tag{18}$$

where

$(j = 1,2,\ldots,S)$

- *Migration*

The decrease in the ability of exploration of search space by learners may lead to premature convergence. In order to increase the diversity of the learners random individuals are introduced into each generation from the global search space. In order to increase the exploration of the search space, it is randomly selected $0.3NL$ learners to start migration operation. The j^{th} subject score of i^{th} learner is randomly regenerated as**:**

$$x_{ij}^{t} = \begin{cases} G_j + R()\times(x_j^{\min} - G_j) & ;\beta < \dfrac{(G_j - x_j^{\min})}{(x_j^{\max} - x_j^{\min})} \\ G_j + R()\times(x_j^{\max} - G_j) & ;Otherwise \end{cases} \tag{19}$$

where $j=1,2,...,S$, G_j is the global best marks, R and β are uniform random number.

At the end of each iteration if the function value obtained by the best learner is better than the function value of global best than value of teacher is updated for next iteration.

- *Evolutionary Weight Pattern Search*

The optimal weight pattern is obtained by performing evolutionary search. One weight assigned to an objective is considered dependent weight required to meet the equality constraint required to ensure normalized weight pattern and search is performed on rest of weights. So, in this method, 2^{L-1} feasible solutions are generated for (L-1) weights assigned to participating objectives except weight assigned to dependent objective. A (L-1) dimensional hypercube of side Δ_i is formed around the point. w_i^C represents weight pattern that is assigned to objectives from the current point in the hyperspace. The better feasible solution is obtained from objective function of the filter design performance index. Another hypercube is formed around the better point, to continue the iterative process. All the corners of the hypercube represented in binary *(L-1)* bits equivalent code, generated around the current set of assigned weight pattern of units, are explored for the desired solution, simultaneously. Table 1 shows the weight pattern for 4-objectives where 3 bits code is considered to represent the corners of the 3-dimensional hypercube (Figure 1) because one weight is taken as dependent/slack weight.

Serial numbers of hypercube corners in decimal are converted into their binary equivalent code. The deviation from the current centre point is obtained by replacing 0's with -Δ and 1's with +Δ in code associated with hypercube corners.

As the number objectives are increased, the number of hypercube corners increases exponentially. The process of exploring the better solution from all corners of the hypercube becomes time consuming, which needs some efficient search technique that should explore all the corners of the hypercube with minimum number of function evaluations and comparisons. The weights are generated as given below:

$$w_i^j = w_i^c + \Delta_i \, (i = 1,2,\ldots,L; i \neq k \;\; ; j = 1,2,\ldots,2^L) \tag{20}$$

where Δ_i is the distance of the corners of the hypercube from the point around which the hypercube is generated.

$$\Delta_i = \frac{w^{\max} - w^{\min}}{\delta_2} \qquad \text{where } w_i \in \{0,1\} \tag{21}$$

The weight of kth objective is calculated as:

$$w_k^j = 1 - \sum_{i=1}^{L} w_i^j \qquad (i \neq k) \tag{22}$$

$$w_{\min}^j = \min\{ w_k^j \, (k = 1,2,\ldots,L)\} \quad (j = 1,2\ldots,2^{L-1}) \tag{23}$$

The weights α_k^j are obtained as

$$\alpha_k^j = \frac{w_k^j}{w_{\min}^j} \quad (k = 1,2,\ldots,L; \quad j = 1,2\ldots,2^{L-1}; w_{\min}^j \neq 0) \tag{24}$$

The normalized weights are generated as described above and the best function value is designated as global best. The above procedure is repeated with incremented *t* value until the value of *t* reaches the maximum value of iterations specified.

4 Results and Comparisons

In this section LP and HP IIR digital filter design examples of HGA [8], HTGA [10] TIA [11] and HSM [17] are considered to investigate the performance of filter designed with ETLBO algorithm. The design specification in terms of pass band and stop band cut-off frequencies are considered as given in Table 2. The intent is to design the IIR digital filter by minimizing the objective function as given in Eq. (9) while meeting the stability constraints given by Eqs. (8b) to (8f).

For the design of IIR digital filter 200 evenly distributed points are chosen in the frequency span $[0,\pi]$. ETLBO considers L_1-norm error, L_2-norm error, pass band ripples and stop band ripples for designing IIR digital filter. In most of the cases the above mentioned criteria's are conflicting in nature. Depending upon the specification of the filter the weightage to be given to each criteria has to be decided by the designer. Weights are adjusted using evolutionary search method. In the purposed heuristic approach larger value of weights w_3 and w_4 are chosen to obtain small ripple magnitude of both pass-band and stop-band. The weights w_1, w_2, w_3 *and* w_4 are set to be 1, 1, 6.6, and 11.4, respectively, for the LP, HP, BP and BS filter. The results obtained by employing ETLBO are given and compared with HGA [8], HTGA [10], TIA [11] and HSM [17] in Tables 3-6. The magnitude response diagrams of LP, HP BP and BS digital IIR filters designed with the proposed ETLBO are presented in Figure 2. The optimized value of numerator and denominator coefficients of LP, HP, BP and BS filters, obtained by employing ETLBO are given by Eq. (25), Eq. (26), Eq. (27) and Eq. (28) respectively.

Table 1: Generation of weight pattern at hypercube corners for 4 - objectives

Hyper cube Corners	Possible combinations of 3-bits $C_2\, C_1\, C_0$	Distance of hypercube corners from centre point $w_1^c,\ w_2^c,\ w_3^c$	Pattern of weight generation vector at the hypercube corners		
0	0 0 0	[illegible]	[illegible]	[illegible]	[illegible]
1	0 0 1	[illegible]	[illegible]	[illegible]	[illegible]
2	0 1 0	[illegible]	[illegible]	[illegible]	[illegible]
3	0 1 1	[illegible]	[illegible]	[illegible]	[illegible]
4	1 0 0	[illegible]	[illegible]	[illegible]	[illegible]
5	1 0 1	[illegible]	[illegible]	[illegible]	[illegible]
6	1 1 0	[illegible]	[illegible]	[illegible]	[illegible]
7	1 1 1	[illegible]	[illegible]	[illegible]	[illegible]

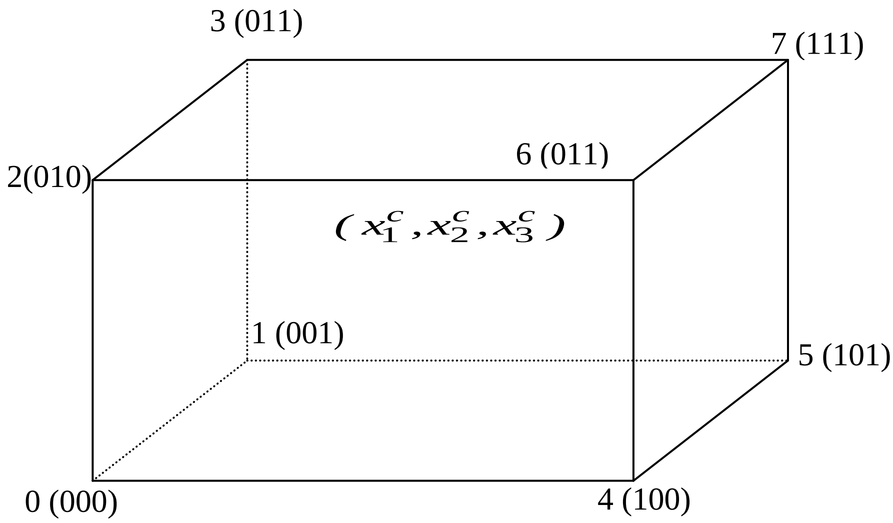

Figure 1: Three dimensional hypercube representing filter coefficients

$$H_{LP}(z) = 0.042107\frac{(z+0.999965)(z^2-0.618377z+1.000066)}{(z-0.671649)(z^2-1.404777z+0.755318)} \tag{25}$$

$$H_{HP}(z) = 0.055899\frac{(z-0.979040)(z^2+0.663209z+1.000136)}{(z+0.615937)(z^2+1.343876z+0.725876)} \tag{26}$$

$$H_{BP}(z) = 0.028077\left(\frac{(z^2-0.178804z-0.990762)}{(z^2-0.629070z+0.774990)}\right)\times\left(\frac{(z^2+0.152085z-0.853690)}{(z^2+0.005788z+0.549454)}\right) \times\left(\frac{(z^2-0.080277z-0.948735)}{(z^2+0.633009z+0.773338)}\right) \tag{27}$$

$$H_{BS}(z) = 0.4382301\left(\frac{(z^2+0.436324z+0.999220)}{(z^2-0.816543z+0.538778)}\right)\times\left(\frac{(z^2-0.436462z+1.00049)}{(z^2+0.816614z+0.538894)}\right) \tag{28}$$

Table 2: Prescribed design conditions on LP, HP, BP and BS filters

Filter type	Pass-band	Stop-band	Maximum Value of $\lvert H(\omega, x)\rvert$
Low-Pass	$0 \le \omega \le 0.2\pi$	$0.3\pi \le \omega \le \pi$	1
High-Pass	$0.8\pi \le \omega \le \pi$	$0 \le \omega \le 0.7\pi$	1
Band-Pass	$0.4\pi \le \omega \le 0.6\pi$	$0 \le \omega \le 0.25\pi$ $0.75 \le \omega \le \pi$	1
Band-Stop	$0 \le \omega \le 0.25\pi$ $0.75 \le \omega \le \pi$	$0.4\pi \le \omega \le 0.6\pi$	1

Table 3: Design results for LP filter employing minimization of $E_1(X) + E_2(X) + \delta_p(X) + \delta_s(X)$

Method	Order	L_1-norm error	L_2-norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
ETLBO	3	4.0482	0.4154	$0.9117 \le \lvert H(e^{j\omega})\rvert \le 1.006$ (0.0943)	$\lvert H(e^{j\omega})\rvert \le 0.1217$ (0.1217)
HSM [17]	3	4.1145	0.4107	$0.9246 \le \lvert H(e^{j\omega})\rvert \le 1.011$ (0.0871)	$\lvert H(e^{j\omega})\rvert \le 0.1510$ (0.1238)
TIA [11]	3	4.2162	0.4380	$0.9012 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0988)	$\lvert H(e^{j\omega})\rvert \le 0.1243$ (0.1243)
HTGA [10]	3	4.2511	0.4213	$0.9004 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0996)	$\lvert H(e^{j\omega})\rvert \le 0.1247$ (0.1247)
HGA. [8]	3	4.3395	0.5389	$0.8870 \le \lvert H(e^{j\omega})\rvert \le 1.009$ (0.1139)	$\lvert H(e^{j\omega})\rvert \le 0.1802$ (0.1802)

Table 4: Design results for HP filter employing minimization of $E_1(X) + E_2(X) + \delta_p(X) + \delta_s(X)$

Method	Order	L_1-norm error	L_2-norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
ETLBO	3	4.4939	0.4478	$0.9894 \le \lvert H(e^{j\omega})\rvert \le 1.004$ (0.0497)	$\lvert H(e^{j\omega})\rvert \le 0.1571$ (0.1571)
HSM [17]	3	4.6635	0.4439	$0.9584 \le \lvert H(e^{j\omega})\rvert \le 1.008$ (0.0504)	$\lvert H(e^{j\omega})\rvert \le 0.1477$ (0.1477)
TIA [11]	3	4.7144	0.4509	$0.9467 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0533)	$\lvert H(e^{j\omega})\rvert \le 0.1457$ (0.1457)
HTGA [10]	3	4.8372	0.4558	$0.9460 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0540)	$\lvert H(e^{j\omega})\rvert \le 0.1457$ (0.1457)
HGA. [8]	3	14.5078	1.2394	$0.9224 \le \lvert H(e^{j\omega})\rvert \le 1.003$ (0.0779)	$\lvert H(e^{j\omega})\rvert \le 0.1819$ (0.1819)

Table 5: Design results for BP filter employing minimization of $E_1(X) + E_2(X) + \delta_p(X) + \delta_s(X)$

Method	Order	L_1-norm error	L_2-norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
ETLBO	6	1.2762	0.1789	$0.9886 \le \lvert H(e^{j\omega})\rvert \le 1.005$ (0.0171)	$\lvert H(e^{j\omega})\rvert \le 0.0525$ (0.0525)
HSM [17]	6	1.4360	0.2052	$0.9896 \le \lvert H(e^{j\omega})\rvert \le 1.004$ (0.0147)	$\lvert H(e^{j\omega})\rvert \le 0.0627$ (0.0627)
TIA [11]	6	1.6119	0.2191	$0.9806 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0194)	$\lvert H(e^{j\omega})\rvert \le 0.0658$ (0.0658)
HTGA [10]	6	1.9418	0.2350	$0.9760 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0234)	$\lvert H(e^{j\omega})\rvert \le 0.0711$ (0.0711)
HGA. [8]	6	5.2165	0.6949	$0.8956 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.1044)	$\lvert H(e^{j\omega})\rvert \le 0.1772$ (0.1772)

Table 6: Design results for BS filter employing minimization of $E_1(X)+E_2(X)+\delta_p(X)+\delta_s(X)$

Method	Order	L_1-norm error	L_2-norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
ETLBO	4	3.6000	0.4579	$0.9668 \le \lvert H(e^{j\omega})\rvert \le 1.006$ (0.0429)	$\lvert H(e^{j\omega})\rvert \le 0.1107$ (0.1107)
HSM [17]	4	3.7699	0.4532	$0.9652 \le \lvert H(e^{j\omega})\rvert \le 1.008$ (0.0434)	$\lvert H(e^{j\omega})\rvert \le 0.1060$ (0.1060)
TIA [11]	4	4.1275	0.4752	$0.9560 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0440)	$\lvert H(e^{j\omega})\rvert \le 0.1164$ (0.1164)
HTGA [10]	4	4.5504	0.4824	$0.9563 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.0437)	$\lvert H(e^{j\omega})\rvert \le 0.1013$ (0.1013)
HGA. [8]	4	6.6072	0.7903	$0.8920 \le \lvert H(e^{j\omega})\rvert \le 1.000$ (0.1080)	$\lvert H(e^{j\omega})\rvert \le 0.1726$ (0.1726)

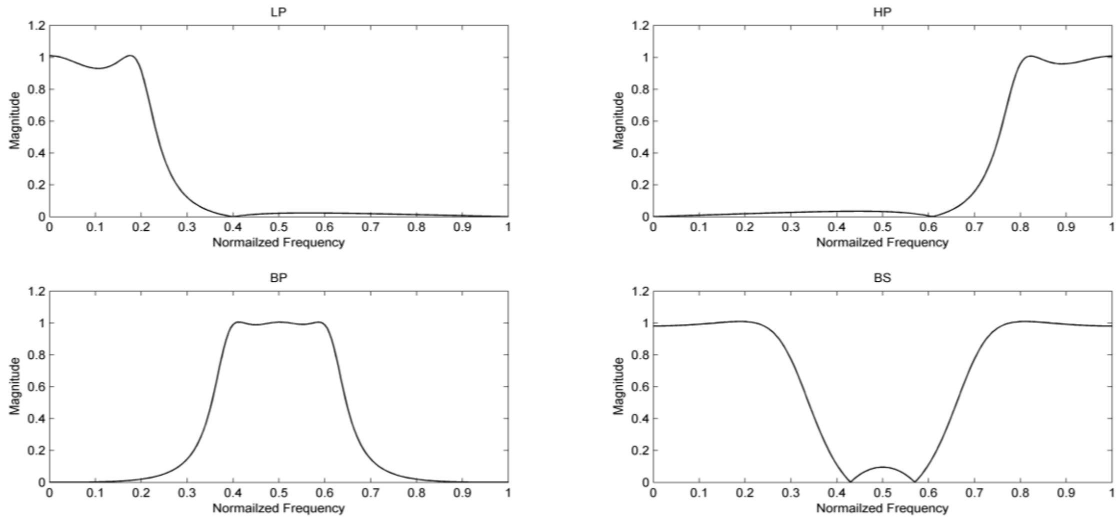

Figure 2: Magnitude response of LP, HP, BP and BS IIR filter using ETLBO approach employing $E_1(X)+E_2(X)+\delta_p(X)+\delta_s(X)$ criterion.

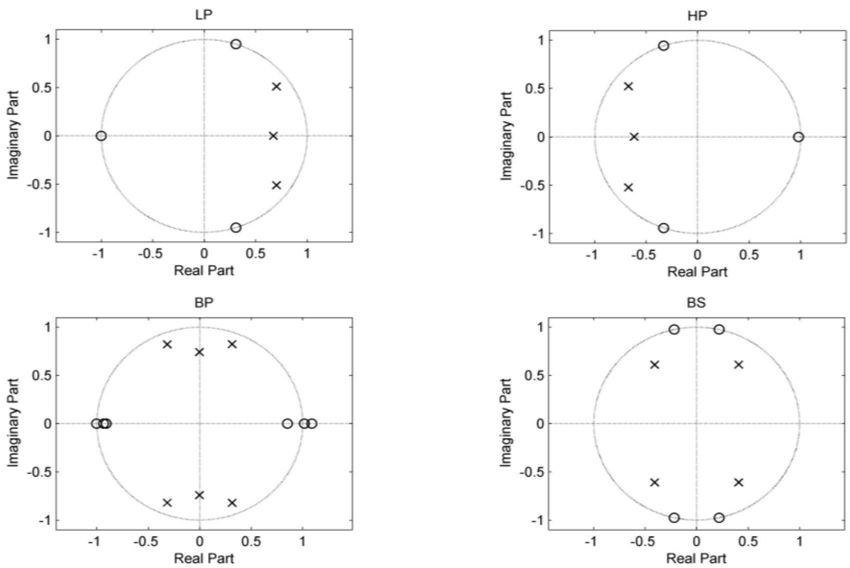

Figure 3: Pole-Zero of LP, HP, BP and BS IIR filter using ETLBO approach employing $E_1(X)+E_2(X)+\delta_p(X)+\delta_s(X)$ criterion

The scrutinizing of the results presented in Tables 3-6 reveal that ETLBO obtains smaller L_1-norm approximation errors, the smaller L_2–norm approximation errors, and better magnitude performances in both pass-band and stop-band than HGA [8], HTGA [10], TIA [11] and HSM [17]. The designed LP, HP, BP and BS IIR digital filter with ETLBO are tested for stability by drawing pole-zero diagrams shown in Figure 3. It can be observed from Figure 3 that the designed filters follow the stability constraints imposed in the design procedure as all the poles lie inside the unit circle. The stability of filter is not influenced by the zeros lying outside the unit circle.

5 Conclusion

In this paper a heuristic algorithm ETLBO is successfully applied to design digital IIR filter and gives substantial improvement in terms of results and convergence. The performance of the original TLBO is enhanced with the introduction of the concept of opposition-based learning and migration for starting with good population of learners and maintain the diversity of the learners, respectively. ETLBO is very feasible to design the digital IIR filters, particularly when the complicated constraints, the design requirements, and the multiple criteria are all involved. The designed optimal filters meet the stability criterion, gives better performance in terms of L_p-approximation error for magnitude response and ripples in pass band and stop band in comparison to existing methods. The advantage of applied ETLBO algorithm is that it do not requires to tune any algorithm-specific parameters.

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