

Passive control of a Microbial Electrolysis Cell for the production of hydrogen from agricultural wastewater

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Abstract: - This work presents a local Port-Hamiltonian-based control strategy for a microbial electrolysis cell aimed at regulating the substrate concentration through the dilution rate. The proposed approach combines an equilibrium-based formulation with a damping injection term inspired by the IDA-PBC methodology. A nonlinear model including substrate, anodophilic biomass, methanogenic biomass, oxidized mediator concentration, and current dynamics is considered. The controller performance is evaluated under changes in the operating point and variations in the inlet substrate concentration. The simulation results show accurate substrate regulation, bounded control effort, and a consistent internal response of the electrochemical and biological variables, supporting the usefulness of the proposed formulation as a control-oriented framework for microbial electrolysis cells.

Key-Words: -Microbial electrolysis cells, nonlinear control, passivity-based control, Port-Hamiltonian systems, substrate regulation, hydrogen production, bioelectrochemical systems.

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1 Introduction

The conversion of pollutants present in wastewater into electricity and valuable products has become one of the major challenges in recent years. Bioelectrochemical systems take advantage of the unique ability of certain microorganisms to oxidize the metabolic substrates found in wastewater and release electrons through a process known as anodic respiration, [1]. As a result of bacterial activity, the chemical energy stored in the organic matter of wastewater is transformed into electrical energy. In this process, microorganisms oxidize the substrates to produce electrons, which are then transferred to the anode. At the same anode, protons are also generated and diffuse through a membrane toward the cathode, [2].

Microbial electrolysis cells (MECs) are an innovative and promising approach for hydrogen

production from organic matter, including wastewater and other renewable resources. The earliest studies date back to 2005 at Pennsylvania State University, [3]. MECs use electrochemically active bacteria to oxidize organic matter and generate CO_2 , electrons, and protons. The bacteria transfer the electrons to the anode, while the protons are released into the solution. The electrons then travel through an external circuit to the cathode, where they combine with the free protons. However, this process does not occur spontaneously and requires an external voltage input of about 0.2 V under controlled biological conditions ($pH = 7$, $T = 30^\circ C$, $P = 1$ atm). Although MECs do require an energy input, they operate at much lower voltages (0.2–0.8 V) than conventional water electrolysis (1.23–1.8 V), [4].

Despite the considerable interest in microbial electrolysis cells, their dynamic behavior remains difficult to control because of the nonlinear

coupling between microbial growth, substrate conversion, and electrochemical phenomena, [5]. The need for adequate mathematical descriptions of MEC behavior has also been emphasized in recent modeling studies, [6]. In this work, a local Port-Hamiltonian-based control strategy is proposed for a microbial electrolysis cell model, using the dilution rate as the manipulated input. The main objective is to regulate the substrate concentration around desired operating conditions while preserving a coherent internal behavior in the biological and electrochemical states.

Recently, some studies have highlighted the need for advanced control strategies capable of addressing the nonlinear and complex dynamics in MECs. A robust controller for a batch-fed MEC was developed considering parameter uncertainty, measurement noise, and external disturbances, [7]. A data-driven approach was adopted, proposing deep learning-based models and controllers to regulate current in MEC systems and comparing their performance with conventional control strategies, such as PI, using integral error and control effort metrics, [8]. On the other hand, in renewable hydrogen production systems, passivity-based control methods have attracted interest because they take advantage of the physical structure of nonlinear interconnected systems. A strategy incorporating energy storage was applied, and closed-loop stability was analyzed in the Lyapunov sense, [9]. Thus, the application of passivity-based and port-Hamiltonian control to MEC dynamics remains relatively limited. This motivates the present study to develop a control law for substrate regulation in the MEC model.

This paper revisits the previously described nonlinear MEC model, [5] and develops a control law for substrate regulation in a continuous-flow MEC. The main contributions of this study are the development of a passivity-based control law with damping injection, using the dilution rate as the manipulable variable.

2 Methodology

2.1 Model

The nonlinear microbial electrolysis cell model adopted in this study was originally developed in a previous work, [5]. The model is used here as the process description for controller design and is not presented as a new mathematical model. It was selected because it captures the main interactions among substrate consumption, microbial growth, mediator evolution, and current production, while remaining sufficiently compact for equilibrium computation and nonlinear control analysis.

The state vector is defined by the substrate concentration S , the anodophilic biomass X_a , the methanogenic biomass X_m , the oxidized mediator concentration M_{Ox} , and the current I_{MEC} . Including both anodophilic and methanogenic populations makes it possible to represent the competition for substrate between an electroactive microbial route and a competing biological route that does not directly contribute to the desired electrochemical response, [10]. In addition, the mediator and current states provide an explicit description of the internal electron-transfer behavior of the cell, which is especially relevant when the interest is not limited to substrate removal but also includes electrochemical performance, [11].

Under these assumptions, the dynamic model is written as:

$$\begin{aligned} \frac{dS}{dt} &= D(S_{in} - S) - k_a \mu_a X_a - k_m \mu_m X_m \\ \frac{dX_a}{dt} &= (\mu_a - k_{d,a} - \alpha_a D) X_a \\ \frac{dX_m}{dt} &= (\mu_m - k_{d,m} - \alpha_m D) X_m \\ \frac{dM_{Ox}}{dt} &= \frac{\gamma}{V_r X_a} \left(\frac{I_{MEC}}{m F_1} \right) - Y_M q_a \\ \frac{dI_{MEC}}{dt} &= \frac{R_{Z1} R_{Z2} - R_{Z3} R_{Z4}}{I_{MEC} (R_{Z5})^2}. \end{aligned} \quad (1)$$

The first equation corresponds to the substrate balance. The term $D(S_{in} - S)$ represents the inlet-outlet contribution associated with the continuous-flow operation of the reactor, where D is the dilution rate and S_{in} is the influent substrate concentration. The remaining terms describe substrate consumption by the anodophilic and methanogenic populations. Therefore, the substrate dynamics result from the combined effect of hydraulic transport and biological conversion.

The second equation represents the anodophilic biomass balance. The term μ_a accounts for microbial growth, $k_{d,a}$ denotes endogenous decay. The parameters α_a and α_m denote the biofilm retention coefficients associated with the anodophilic and methanogenic populations, respectively. Although both coefficients are assigned the same nominal numerical value in the present study, separate notation is retained because they correspond to different microbial populations. Since the anodophilic biomass is directly associated with electron transfer to the anode, its dynamics play a key role in the electrochemical behavior of the MEC.

The third equation describes the methanogenic biomass balance. Its structure is analogous to that of the anodophilic biomass, but it represents

a competing microbial population. This state is relevant because methanogenic growth may consume part of the available substrate and, consequently, reduce the fraction that can be used for electroactive conversion.

The fourth equation corresponds to the oxidized mediator balance. The first term relates the mediator dynamics to the current through Faraday's law, scaled by reactor volume and anodophilic biomass concentration. The second term accounts for mediator consumption associated with the anodophilic metabolic activity. In this way, the mediator state acts as an intermediate variable linking microbial substrate conversion with electron-transfer processes.

The fifth equation describes the current dynamics. In the present work, this equation is written in compact form by using the auxiliary terms R_{Z1} – R_{Z5} :

$$\begin{aligned} R_{Z1} &= R_{Min} + (R_{Max} - R_{Min}) e^{-k_R X_a} \\ R_{Z2} &= -\frac{RT}{mF} \left(\frac{\frac{dM_{Ox}}{dt}}{M_{Total} - M_{Ox}} \right) \\ R_{Z3} &= E_{RCEF} + E_{applied} - \frac{RT}{mF} \ln \left(\frac{M_{Total}}{M_{Total} - M_{Ox}} \right) \quad (2) \\ R_{Z4} &= -k_R (R_{Max} - R_{Min}) e^{-k_R X_a} \frac{dX_a}{dt} \\ R_{Z5} &= R_{Min} + (R_{Max} - R_{Min}) e^{-k_R X_a}. \end{aligned}$$

These auxiliary expressions are introduced to organize the electrochemical contribution of the model and to rewrite the I_{MEC} equation in a more compact form. The term R_{Z1} represents the internal resistance, modeled as a biomass-dependent quantity. This dependence reflects the fact that the presence of electroactive biomass influences the effective charge-transfer conditions inside the reactor. The term R_{Z2} is associated with the variation of the oxidized mediator concentration and introduces the effect of mediator dynamics into the electrochemical potential. The term R_{Z3} contains the voltage contribution, including the applied voltage and a logarithmic term consistent with a Nernst-type dependence on mediator concentration. The term R_{Z4} accounts for the effect of variations in anodophilic biomass on the resistance dynamics, and R_{Z5} plays the role of an effective resistance term in the denominator of the current equation.

It is important to note that the second equation block is not only a matter of notation. Its introduction makes it possible to express the current dynamics in a more structured way, which is useful for the subsequent control-oriented reformulation. In particular, these terms collect in compact form the combined effects of biomass-dependent resistance, mediator availability, and electrochemical driving

force. This is especially convenient when the model is later expressed in a form suitable for Port-Hamiltonian analysis.

Overall, the model retains the main nonlinear interactions of the microbial electrolysis cell while preserving an explicit dependence on the dilution rate. This is essential in the present study because the dilution rate is chosen as the manipulated input, and the control objective is formulated in terms of substrate regulation together with a coherent internal evolution of the biological and electrochemical states, [5]. In that sense, the adopted representation provides an appropriate basis for the equilibrium-based and energy-oriented reformulation developed in the following sections, [12].

For the Microbial Electrolysis Cell, the volumetric hydrogen production rate at the operating temperature and pressure can be obtained by combining Faraday's law with the ideal gas equation:

$$Q_{H_2} = Y_{H_2} \left(\frac{I_{MEC}}{mF} \right) \left(\frac{RT}{P} \right). \quad (3)$$

Here, Q_{H_2} denotes the volumetric hydrogen production rate, Y_{H_2} is the dimensionless cathodic hydrogen recovery efficiency, I_{MEC} is the electric current generated in the MEC, m is the number of electrons transferred per mole of hydrogen ($m = 2$), and F is Faraday's constant. Furthermore, R is the ideal gas constant, T is the operating temperature, and P is the gas pressure. The term $I_{MEC}/(mF)$ represents the molar hydrogen production rate, whereas RT/P represents the molar gas volume. Consequently, Q_{H_2} has units of volume per unit time. Consistent units must be used for R and P , [13].

2.2 Control problem formulation

The control problem considered in this work is the regulation of the substrate concentration around desired operating conditions by using the dilution rate as the manipulated input. This choice is physically meaningful because the dilution rate appears directly in the substrate mass balance and therefore has an immediate effect on reactor loading and substrate renewal, [5]. Since this same input also influences microbial growth and, indirectly, the electrochemical variables, it provides a practical way of shaping the overall behavior of the microbial electrolysis cell, [11].

Rather than forcing the reactor to follow an arbitrary output signal, the proposed approach seeks convergence toward equilibrium conditions associated with the desired substrate level. This interpretation is more suitable for a nonlinear process, since the target is not just a numerical reference but an

admissible operating regime of the system, [14]. In this way, the control objective remains connected to the steady-state behavior that the reactor can actually sustain.

A satisfactory closed-loop response should therefore satisfy more than one requirement. Besides accurate substrate regulation, the controller should preserve bounded input behavior and a dynamically coherent evolution of the biological and electrochemical states, since these variables remain strongly coupled and evolve on different time scales in microbial electrolysis cells, [6]. This is especially relevant in practice, because a numerically acceptable response is not enough if the internal states cease to be biologically plausible, [10].

This viewpoint is also relevant from the application perspective, because continuous MEC operation must balance hydraulic manipulation, microbial activity, and electrochemical performance at the same time, [11]. For this reason, the control problem is better understood as the regulation of an interconnected bioelectrochemical process rather than as a purely input-output task, [12].

2.3 Equilibrium computation

A central element of the proposed methodology is the computation of equilibrium conditions associated with each desired operating regime. For a given substrate target and inlet condition, the equilibrium point is obtained by setting the time derivatives of the nonlinear model equal to zero and solving the resulting algebraic equations, [14].

Using equilibrium references gives the control objective a clearer physical meaning. Instead of regulating the reactor with respect to an arbitrary signal, the controller is designed to move the system toward steady-state conditions that are compatible with the process dynamics and the selected operating point, [15]. This is especially useful in nonlinear bioelectrochemical systems, where the relation between the manipulated input and the corresponding steady-state behavior is usually far from trivial.

This equilibrium-based viewpoint is particularly convenient for nonlinear bioelectrochemical processes, since the equilibrium depends simultaneously on biological kinetics, inlet conditions, and electrochemical interactions, [16]. For this reason, equilibrium computation not only provides a nominal operating point but also a physically interpretable reference for control analysis.

In addition, equilibrium values are used as reference quantities for the shifted-coordinate formulation of the Hamiltonian and for the comparison between actual and nominal closed-loop

trajectories. This makes it possible to evaluate the controller not only in terms of regulation error, but also in terms of convergence toward meaningful operating conditions, [17].

3 Port-Hamiltonian systems and the IDA-PBC control methodology

3.1 Port-Hamiltonian

To formulate the control problem in an energy-based setting, the nonlinear microbial electrolysis cell model is next interpreted within the Port-Hamiltonian framework. This reformulation is useful because it makes explicit the relation between stored energy, dissipative effects, and externally supplied power, which are key ingredients in passivity-based analysis, [18].

In the present work, the Port-Hamiltonian viewpoint is not introduced merely as a formal mathematical rewriting. Instead, it is used as a structured way of organizing the reactor dynamics so that the equilibrium-based control objective can be linked to an energy function and to an explicit damping injection mechanism, [12].

The Port-Hamiltonian (PH) model can be described as a system with the following general form, [18]

$$\begin{cases} \dot{x} = [J(x) - R(x)] \nabla H + g(x)u \\ y = g^T(x) \nabla H \end{cases} \quad (4)$$

The term $u^T y$ represents power, while $J = -J^T$ denotes the interconnection matrix that characterizes energy-conserving interactions within the system. The matrix $R = R^T \geq 0$ includes the dissipative effects, and H stands for the stored energy. As a result, Port-Hamiltonian systems are passive and satisfy the following energy balance:

$$\underbrace{\dot{H}}_{\text{stored}} \leq \underbrace{-\nabla H^T R \nabla H}_{\text{dissipated}} + \underbrace{u^T y}_{\text{supplied}} \quad (5)$$

The storage and dissipation phenomena are represented in aggregated form. The port variables characterize the power exchange between the system and its environment, while $J(x)$ describes energy-conserving interconnections and $R(x)$ represents dissipative effects, [18].

Let $H(x) : R^n \rightarrow R^+$ be an energy storage function, and let its time derivative be computed as follows

$$\begin{aligned} \dot{H} &= \nabla H^T \dot{x} \\ &= \nabla H^T [J(x) - R(x)] \nabla H + g(x)u \\ &= \nabla H^T J(x) \nabla H - \nabla H^T R(x) \nabla H + \nabla H^T g(x)u. \end{aligned} \quad (6)$$

Since $\nabla H^T g(x)u$ is a scalar quantity, it can be equivalently written as

$$\nabla H^T g(x)u = u^T g^T(x) \nabla H = u^T y.$$

Therefore, the derivative of the Hamiltonian becomes

$$\dot{H} = \nabla H^T J(x) \nabla H - \nabla H^T R(x) \nabla H + u^T y. \quad (7)$$

In addition, since the interconnection matrix is skew-symmetric, that is, $J^T(x) = -J(x)$, this property can be used to examine the term $\nabla H^T J(x) \nabla H$. In particular, this can be shown by taking the transpose of the expression as follows

$$\begin{aligned} (\nabla H^T J(x) \nabla H)^T &= \nabla H^T J^T(x) \nabla H \\ &= \nabla H^T (-J(x)) \nabla H \quad (8) \\ &= -\nabla H^T J(x) \nabla H. \end{aligned}$$

Therefore, the identity $\nabla H^T J(x) \nabla H = -\nabla H^T J(x) \nabla H$ can only hold when $\nabla H^T J(x) \nabla H = 0$, and consequently

$$\dot{H} = -\nabla H^T R(x) \nabla H + u^T y \leq u^T y. \quad (9)$$

Thus, the interconnection structure redistributes energy without creating or dissipating it, while $R(x)$ accounts for the dissipative effects of the system, [18].

3.2 IDA-PBC design

First, within the IDA-PBC framework, the desired equilibrium point must be selected. A practical way to do this is to take the system model and set all time derivatives equal to zero, [14]. This transforms the dynamic model into a set of algebraic equations. By solving them, the equilibrium pair (x_d, u) is obtained, where x_d denotes the target state and u is the input required to maintain the system at that point, typically in terms of the dilution rate. Although the specific details may vary from one case to another, the main idea is to identify an operating point that remains invariant. This equilibrium is then used as the reference point for the stabilization stage in the controller design, [19].

For a desired operating condition, let x_d denote the corresponding equilibrium state and let D^* denote the nominal dilution rate. The equilibrium pair (x_d, D^*) satisfies

$$0 = f(x_d) + G(x_d)D^* \quad (10)$$

Thus, D^* is the dilution rate required to maintain the MEC at the selected operating point in the absence of disturbances, [14], [19].

To distinguish the open-loop factorization from the shifted energy function used for regulation, a control-oriented quadratic Hamiltonian is first introduced as

$$H_c(x) = \frac{1}{2} x^T x \quad (11)$$

Its gradient is

$$\nabla H_c(x) = x \quad (12)$$

With this choice, the nonlinear MEC dynamics can be expressed in the control-oriented form

$$\dot{x} = Q(x) \nabla H_c(x) + G(x)D \quad (13)$$

where $Q(x)$ collects the biological and electrochemical drift terms, whereas $G(x)D$ contains the material-exchange and biomass-removal terms associated with the dilution rate.

Within this formulation, growth terms such as microbial kinetics $\mu_i x_j$ become part of the energy flow described by $Q(x) \nabla H_c(x)$. Material flow terms of the type $(x_{in} - x_i)D$ are integrated into the interconnection structure given by $G(x)$, [12]. Accordingly, the model is written in a control-oriented representation inspired by the Port-Hamiltonian framework, with

$$\nabla H_c(x) = x = \begin{bmatrix} S \\ X_a \\ X_m \\ M_{Ox} \\ I_{MEC} \end{bmatrix} \quad (14)$$

$$Q_c(x) = \begin{bmatrix} 0 & Q_{1,2} & Q_{1,3} & 0 & 0 \\ 0 & Q_{2,2} & 0 & 0 & 0 \\ 0 & 0 & Q_{3,3} & 0 & 0 \\ 0 & Q_{4,2} & 0 & Q_{4,4} & 0 \\ 0 & 0 & 0 & 0 & Q_{5,5} \end{bmatrix} \quad (15)$$

$$\begin{aligned} Q_{1,2} &= -k_a \mu_a \\ Q_{2,2} &= \mu_a - k_{d,a} \\ Q_{4,2} &= \frac{\gamma}{V_r X_a^2} \left(\frac{I_{MEC}}{m F_1} \right) \\ Q_{1,3} &= -k_m \mu_m \\ Q_{3,3} &= \mu_m - k_{d,m} \\ Q_{4,4} &= -\frac{Y_M q_a}{M_{Ox}} \\ Q_{5,5} &= \frac{R_{Z1} R_{Z2} - R_{Z3} R_{Z4}}{I_{MEC}^2 (R_{Z5})^2} \end{aligned} \quad (16)$$

For the local damping-injection implementation, the input vector is evaluated at the current equilibrium condition:

$$G(x) = G(x_d) = \begin{bmatrix} S_{in} - S \\ -\alpha_a X_a \\ -\alpha_m X_m \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

$$u = [D] \quad (18)$$

It is important to clarify the role of the matrix $Q_c(x)$ in the previous representation. In a strict Port-Hamiltonian realization, the matrix multiplying the Hamiltonian gradient must admit the decomposition

$$Q_{PH}(x) = J(x) - R(x) \quad (19)$$

where $J(x) = -J^T(x)$ and $R(x) = R^T(x) \succeq 0$. These structural properties are not assumed for the matrix $Q_c(x)$ introduced in the present formulation.

Instead, $Q_c(x)$ is defined as a control-oriented factorization of the nonlinear drift dynamics satisfying

$$f(x) = Q_c(x) \nabla H_c(x) \quad (20)$$

Its purpose is to separate the biological and electrochemical drift terms from the dilution-dependent material-exchange terms contained in $G(x)D$. Therefore, the resulting representation is interpreted as a control-oriented formulation inspired by the Port-Hamiltonian and passivity-based frameworks, rather than as a strict Port-Hamiltonian realization, [18], [19].

For the local stabilization of the selected operating point, the regulation error is defined as

$$e = x - x_d \quad (21)$$

The desired Hamiltonian is selected as

$$H_d(x) = \frac{1}{2}(x - x_d)^T(x - x_d) = \frac{1}{2}e^T e \quad (22)$$

Therefore, $H_d(x)$ is positive definite with respect to the regulation error, satisfies $H_d(x_d) = 0$, and has an isolated minimum at $x = x_d$. Its gradient is

$$\nabla H_d(x) = x - x_d = \begin{bmatrix} S - S_d \\ X_a - X_{a,d} \\ X_m - X_{m,d} \\ M_{Ox} - M_{Ox,d} \\ I_{MEC} - I_{MEC,d} \end{bmatrix} \quad (23)$$

The desired closed-loop dynamics are characterized by the Hamiltonian $H_d(x)$ and a target matrix $Q_d(x)$, [19].

$$\dot{x} = Q_d(x) \nabla H_d(x) \quad (24)$$

The core step involves solving the matching equation

$$f(x) + G(x)b(x) = Q_d(x) \nabla H_d(x), \quad (25)$$

The matching equation can be analyzed through two complementary operations. Premultiplication by a left annihilator of $G(x)$ yields the matching conditions that constrain $\nabla H_d(x)$, whereas premultiplication by a left inverse of $G(x)$, whenever it exists, provides the static control component $b(x)$. When the original passive structure is preserved and $H_d = H$, additional energy shaping is not required, and stabilization can be pursued through damping injection, [19].

In the present MEC model, only one manipulated input, namely the dilution rate D , is available for a nonlinear system with five state variables. Therefore, solving the complete matching equation globally over the full state space is restrictive.

For this reason, the present design uses a local damping-injection implementation based on the IDA-PBC methodology. The feedforward component is selected as the equilibrium dilution rate,

$$b(x) = D^*. \quad (26)$$

This choice satisfies the matching condition at the desired equilibrium. Indeed, because the equilibrium pair (x_d, D^*) satisfies

$$f(x_d) + G(x_d)D^* = 0, \quad (27)$$

it follows that

$$f(x_d) + G(x_d)b(x_d) = 0. \quad (28)$$

The complete IDA-PBC matching equation is therefore not claimed to be solved globally. Instead, the feedforward component ensures compatibility with the selected equilibrium, while the stabilizing action is introduced through damping injection.

Consistently with the numerical implementation, the input vector used by the local controller is evaluated at the corresponding equilibrium,

$$G_d = G(x_d). \quad (29)$$

The local passive-output error is defined as

$$z_d(x) = G_d^T \nabla H_d(x). \quad (30)$$

For the subsequent stability analysis, the passive-output variable associated with the current plant input vector is also defined as

$$z(x) = G^T(x) \nabla H_d(x). \quad (31)$$

The unsaturated damping-injection control law is proposed as

$$D = D^* - K_p(x)G_d^T \nabla H_d(x), \quad K_p(x) > 0, \quad (32)$$

or equivalently,

$$D = D^* - K_p(x)z_d(x). \quad (33)$$

The equilibrium input D^* maintains the selected operating condition, whereas the damping-injection term acts when the state moves away from the equilibrium. Since $\nabla H_d(x_d) = 0$, the corrective contribution vanishes at $x = x_d$, [19], [20].

Lemma 1. Let $K_p(x) > 0$ and consider the unsaturated damping-injection control law given by Eq. (32). Assume that, in an admissible neighborhood Ω of x_d , the nominal dynamics associated with the equilibrium input satisfy

$$\nabla H_d^T(x) [f(x) + G(x)D^*] \leq 0, \quad x \in \Omega, \quad (34)$$

and that the current and equilibrium-evaluated port variables satisfy the local alignment condition

$$z(x)z_d(x) \geq 0, \quad x \in \Omega. \quad (35)$$

Then, in the region where the dilution-rate saturation limits do not dominate the control action,

$$\dot{H}_d(x) \leq -K_p(x)z(x)z_d(x) \leq 0, \quad x \in \Omega. \quad (36)$$

Proof. Substitution of the unsaturated control law (32) into the control-affine MEC model gives

$$\dot{x} = f(x) + G(x)D^* - G(x)K_p(x)G_d^T \nabla H_d(x). \quad (37)$$

The derivative of the desired Hamiltonian along the closed-loop trajectories is

$$\begin{aligned} \dot{H}_d(x) &= \nabla H_d^T(x) \dot{x} \\ &= \nabla H_d^T(x) [f(x) + G(x)D^*] \\ &\quad - K_p(x) \nabla H_d^T(x) G(x) G_d^T \nabla H_d(x). \end{aligned} \quad (38)$$

Using the definitions

$$z(x) = G^T(x) \nabla H_d(x), \quad z_d(x) = G_d^T \nabla H_d(x), \quad (39)$$

the derivative becomes

$$\dot{H}_d(x) = \nabla H_d^T(x) [f(x) + G(x)D^*] - K_p(x)z(x)z_d(x). \quad (40)$$

Therefore, under the local dissipativity condition (34) and the local port-alignment condition (35),

$$\dot{H}_d(x) \leq -K_p(x)z(x)z_d(x) \leq 0, \quad x \in \Omega. \quad (41)$$

Hence, the desired Hamiltonian is non-increasing along the unsaturated closed-loop trajectories in the considered local neighborhood. *Q.E.D.*

Since the desired Hamiltonian is positive definite with respect to the regulation error and has an isolated minimum at $x = x_d$. Consequently, the inequality $\dot{H}_d(x) \leq 0$ establishes local stability in the sense of Lyapunov.

To characterize asymptotic convergence, define

$$\mathcal{E} = \{x \in \Omega : \dot{H}_d(x) = 0\}. \quad (42)$$

According to LaSalle's Invariance Principle, the trajectories approach the largest invariant subset $\mathcal{M}$ contained in $\mathcal{E}$. Therefore, the desired equilibrium is locally asymptotically stable provided that

$$\mathcal{M} = \{x_d\}. \quad (43)$$

This condition is equivalent to requiring local detectability of the port variables in the considered operating neighborhood: a trajectory along which the dissipation vanishes identically must converge to the desired equilibrium, [21].

A similar control-oriented argument is suggested by the IDA-PBC approach for process engineering presented, [22]. For continuous biochemical fermentation systems, damping injection based on the passive output can produce a damping effect that is closely linked to biomass-associated states. Therefore, when these states become small, for instance near low-biomass or washout scenarios, the resulting damping action may be weakened. This observation provides a rationale for introducing state-dependent gains that preserve damping effectiveness under such operating conditions.

Following this control-oriented reasoning, and in order to avoid the singular behavior produced by a pure inverse-square gain, the damping matrix is regularized as

$$K_p = \frac{k_1}{x_1^2 + \varepsilon_1} \quad (44)$$

This choice guarantees boundedness of the gains:

$$0 < K_p \leq \frac{k_1}{\varepsilon_1} \quad (45)$$

Because dilution rates represent physically realizable operating variables, the calculated control action must be kept within admissible operational

limits. Accordingly, the implemented input is defined as

$$D_{\text{sat}} = \text{sat}_{[D_{\min}, D_{\max}]} (D^* - K_p G_d^T(x) \nabla H_d(x)) \quad (46)$$

The preceding dissipation inequality applies locally in the neighborhood Ω where the nominal dissipativity and port-alignment conditions hold and where the saturation limits do not dominate the control action. Accordingly, the result establishes local passivity-based stabilization around each fixed admissible equilibrium condition. No global stability result or explicit region-of-attraction estimate is claimed for the constrained nonlinear MEC system.

From this perspective, the proposed law preserves the logic of passivity-based design: the equilibrium input provides the reference operating condition, and the additional damping term shapes the transient response around that point. Such a structure is well aligned with the energy-based formulation developed for nonlinear interconnected systems, [18].

3.3 Simulation setup

The closed-loop performance of the proposed controller was evaluated through numerical simulations under successive changes in the operating conditions. The dilution rate D was used as the manipulated input, while the substrate concentration S was selected as the main regulated variable. For each desired operating regime, the corresponding equilibrium point was computed and used as the reference in the control law.

To assess the robustness of the strategy, variations were introduced in the inlet substrate concentration S_{in} during the simulation horizon. This enabled analysis of not only the controller's tracking capability with respect to the desired substrate level but also its behavior under variations in feeding conditions. The closed-loop response was evaluated by monitoring the evolution of S , X_a , I_{MEC} , and the applied dilution rate D , together with their corresponding equilibrium trajectories when applicable.

The variables monitored in the simulation were selected to provide information at different levels of the process. The substrate concentration was used as the main regulated variable, the anodophilic biomass was included to assess the reactor's biological adaptation, and the electrical current was used as an indicator of the reactor's electrochemical response.

The dilution rate was also tracked because it represents the control effort required to achieve regulation. Its evolution is relevant not only from the numerical point of view, but also from the operational perspective, since a useful control law

should preserve bounded input behavior while driving the system toward the desired operating region, [11].

Taken together, these monitored variables provide a compact but informative description of the closed-loop response. They make it possible to assess whether the controller achieves substrate regulation while maintaining consistency in the internal biological and electrochemical dynamics of the microbial electrolysis cell, [5].

4 Results and discussion

In order to evaluate the closed-loop performance of the proposed control strategy under realistic operating changes, variations were first introduced in the inlet substrate concentration. This is important because, in practice, the influent of a wastewater-based system is rarely constant, and any control strategy intended for this type of process should be able to minimize the effect of variable feeding conditions. Fig. 1 shows the profile of S_{in} used during the simulation. As can be seen, the inlet concentration was modified at different time instants, generating changes in the operating conditions of the reactor and providing a suitable scenario to assess the robustness of the controller.

Even in the presence of this exogenous perturbation, the substrate concentration remained well regulated. This can be observed in Fig. 2, where the evolution of S is compared with its corresponding equilibrium value S_{eq} . The response shows that the controlled variable moves quickly toward the new operating condition after each change and stays very close to the desired equilibrium trajectory. From a practical point of view, this is one of the most relevant results of the study, since it indicates that the controller is able to preserve the regulation objective even when the system is subjected to variations at the input. Another positive aspect is that the transitions do not exhibit oscillatory or erratic behavior, which suggests that the damping introduced by the control law is sufficient to maintain a smooth response.

An additional point worth emphasizing is that the substrate response is not simply following an arbitrary reference signal, but rather moving toward equilibrium conditions that are recomputed for each operating regime. This gives a clearer physical meaning to the tracking task, since the controller is not forcing the system toward unattainable values, but instead steering it toward admissible steady-state conditions associated with the updated inlet and operating requirements. From this perspective, the close agreement between S and S_{eq} provides evidence that the proposed strategy is able to connect the control objective with the intrinsic nonlinear behavior of the reactor.

The evolution of the anodophilic biomass is presented in Fig. 3. In comparison with the substrate, the transient of X_a is naturally slower, which is consistent with the biological nature of this state. Biomass cannot adjust instantaneously in the same way as the substrate concentration, since its dynamics depend on microbial growth and adaptation. Even so, the trajectory of X_a follows the expected trend and approaches the equilibrium values associated with each operating regime. This is important because it shows that the controller is not only effective in the regulation of the main output but also preserves a biologically coherent internal response.

The response of the anodophilic biomass also suggests that the proposed controller remains compatible with the biological nature of the process. Instead of producing abrupt or unrealistic internal variations, the closed-loop behavior allows the biomass to evolve according to its own characteristic time scale while still moving toward the equilibrium region associated with each operating condition, [10].

This is a relevant qualitative result because, in bioelectrochemical systems, the usefulness of a control strategy depends not only on the behavior of the regulated output but also on whether the internal biological states remain dynamically plausible. In this sense, the observed evolution of X_a supports the consistency of the proposed formulation.

Fig. 4 shows the evolution of the current I_{MEC} during closed-loop operation. Its behavior remains smooth throughout the simulation, even when the operating conditions are modified. A clear increase in current can be observed when the reactor moves toward regimes associated with higher electrochemical activity, while a gradual decrease appears when the demand is reduced. This trend is consistent with the expected response of the microbial electrolysis cell, since the current reflects the internal electrochemical conversion linked to substrate consumption and microbial activity. Although I_{MEC} is not directly imposed as a controlled output, its trajectory remains physically coherent and supports the idea that the proposed control strategy preserves the internal electrochemical behavior of the system while achieving the regulation objective.

The current trajectory is particularly informative because it reflects the way in which the internal electrochemical activity adjusts during changes in the operating regime. Even though the control objective is formulated in terms of substrate regulation, the evolution of I_{MEC} shows that the electrochemical subsystem remains coupled to the rest of the reactor dynamics in a consistent manner, [5].

From the viewpoint of interconnected systems, this behavior is desirable because it indicates that the

controller is not regulating one variable in isolation. Instead, it is shaping the global closed-loop response of the microbial electrolysis cell in a way that remains compatible with the coupled nature of the process, [12].

The control effort is shown in Fig. 5, where the applied dilution rate D is compared with the equilibrium dilution rate D_{eq} . As expected, the manipulated input reacts mainly at the moments when the operating conditions change. These variations are necessary to steer the reactor toward the new equilibrium associated with the updated substrate reference and inlet condition. After each transient, the applied dilution rate remains close to its equilibrium value, which indicates that the corrective action becomes smaller once the system approaches the desired regime. This is a desirable feature from the operational point of view, since it avoids unnecessary control effort once the reactor has settled.

It is also important to note that the good tracking performance is achieved without an excessive deviation of the manipulated input from its equilibrium profile. This suggests that the additional damping term acts mainly as a transient correction mechanism, helping the system converge after each operating change, while the equilibrium input remains the dominant contribution once the reactor approaches the desired regime. Such a behavior is consistent with the control design philosophy adopted in this work, where the stabilization objective is pursued through an energy-based correction around the equilibrium rather than through aggressive input action.

Taken together, the results show that the proposed local Port-Hamiltonian-based control strategy provides a stable and effective way to regulate the substrate concentration in the microbial electrolysis cell. The different variables do not evolve on the same time scale, which is consistent with the multiphysical nature of the process: the substrate reacts quickly to the updated operating conditions, the anodophilic biomass exhibits a slower adaptation due to the underlying biological kinetics, and the current reflects the electrochemical adjustment associated with each new regime. At the same time, the manipulated input remains bounded and close to its equilibrium value after the transient periods. This overall behavior supports the suitability of the proposed formulation as a control-oriented framework for this type of bioelectrochemical system.

To complement the qualitative interpretation of the closed-loop trajectories, quantitative performance indicators were evaluated after the initial transient, considering the interval $t \geq 2$ d. Substrate regulation was assessed through the integral absolute

error IAE_S and the root-mean-square error $RMSE_S$. The corrective control action was quantified through IAE_D , calculated from the deviation between the applied dilution rate and its corresponding equilibrium value. The minimum and maximum applied dilution rates were also recorded. Finally, the average and accumulated hydrogen production and the maximum and final values of the local Hamiltonian were included to quantify the process and energy-based responses.

As reported in Table 1, the substrate response yielded $IAE_S = 0.290988$ and $RMSE_S = 0.332524$ over the selected evaluation interval. The accumulated deviation between the applied and equilibrium dilution rates was $IAE_D = 0.011708$. Moreover, the applied dilution rate remained between 1.117844 and 1.283960 d^{-1} , within the prescribed physical limits of 0.2587 and 1.7378 d^{-1} .

The average hydrogen production rate was 0.282144 L/d , resulting in an accumulated production of 5.078585 L during the evaluated interval.

To assess the influence of simultaneous parametric and operating variations on the closed-loop response, a global sensitivity analysis was carried out using the Latin Hypercube Sampling (LHS) method. A total of 500 parameter sets were generated by varying the selected kinetic, electrochemical, and operating quantities within $\pm 20\%$ of their nominal values. For each sample, the complete closed-loop simulation was performed and the overall performance index J was evaluated.

The monotonic dependence between each sampled quantity and J was quantified using the Spearman rank correlation coefficient, ρ_S . Figure 6 presents the corresponding absolute correlation coefficients, $|\rho_S|$. Therefore, larger values indicate a greater monotonic influence on the selected performance index, regardless of whether the relationship is positive or negative.

For each LHS realization, the closed-loop performance was quantified through the fraction of the evaluation interval during which the applied dilution rate reached either of its prescribed physical limits. The performance index was defined as

$$J = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} I[D_{\text{real}}(t) \in \{D_{\min}, D_{\max}\}] dt, \quad (47)$$

where $I[\cdot]$ denotes the indicator function. Accordingly, $J = 0$ indicates that the applied dilution rate remains strictly inside its admissible interval throughout the evaluation period, whereas larger values of J indicate that the manipulated input remains saturated for a greater fraction of time.

As observed in Fig. 6, the biomass-retention

coefficient α exhibited the strongest influence on the overall performance index, with an absolute Spearman coefficient of approximately 0.165 . This parameter directly affects the biomass-loss contribution associated with the dilution rate and, consequently, modifies the equilibrium and transient response of the biological populations.

The maximum anodophilic growth rate $\mu_{a,\max}$ and the anodophilic half-saturation constant K_{Sa} presented the next largest coefficients, with $|\rho_S|$ values of approximately 0.148 and 0.144 , respectively. These parameters determine the magnitude and substrate dependence of the anodophilic kinetics. Their influence on J indicates that variations in the anodophilic conversion capacity modify the corrective dilution action required to maintain the substrate concentration close to its reference value.

A lower influence was observed for the methanogenic half-saturation constant K_{Sm} , with $|\rho_S| \approx 0.070$, while $R_{\max}$ presented an absolute coefficient close to 0.050 . The remaining quantities yielded correlation coefficients below approximately 0.05 , indicating a comparatively smaller individual effect on the selected closed-loop index within the investigated parameter region.

To complement the rank-based sensitivity analysis, empirical admissible ranges were determined for the six parameters with the largest absolute Spearman coefficients. For each realization, the change in the performance index relative to the nominal closed-loop case was calculated as

$$\Delta J_s = J_s - J_0, \quad (48)$$

where J_0 and J_s denote the nominal and sampled performance indices, respectively. The realizations that completed the closed-loop simulation without nonfinite state trajectories and while respecting the imposed physical operating conditions were retained. The minimum and maximum values of the accepted subset were then used to construct the empirical parameter ranges reported in Table 2.

The accepted interval for α extended from 0.43318 to 0.64919 , corresponding to deviations of -19.930% and 19.998% from its nominal value. Similarly, g_D varied from 0.80060 to 1.19950 , retaining practically the complete uncertainty interval considered in the sampling design.

The anodophilic saturation constant K_{Sa} presented an accepted range from 16.007 to 23.989 , whereas the corresponding range for K_{Sm} extended from 64.001 to 95.973 . The maximum anodophilic growth rate $\mu_{a,\max}$ varied between 1.5772 and 2.3635 , and $R_{\max}$ remained within the interval from 160.08 to 239.99 .

Thus, all six quantities retained ranges close to the original $\pm 20\%$ intervals, and no appreciable contraction of the investigated parameter domain was observed. Together with the relatively small absolute Spearman coefficients, this result indicates a comparatively lower sensitivity of the selected saturation-based performance index within the explored region.

It should be emphasized that the intervals reported in Table 2 represent empirical closed-loop admissible ranges obtained from the accepted numerical realizations. Therefore, they should not be interpreted as a formal proof of robustness with respect to structural modeling errors, unmodeled dynamics, measurement noise, or operating conditions outside the region considered in the present analysis.

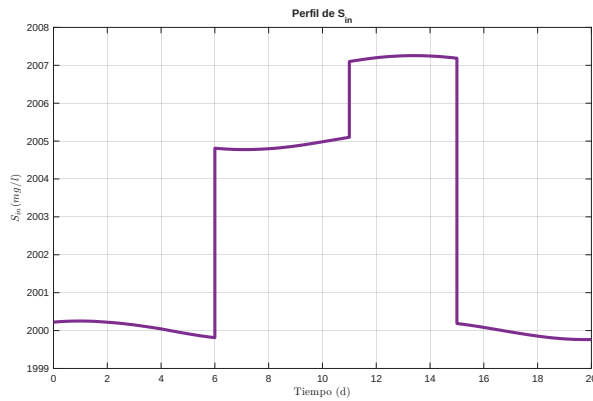


Fig. 1: Profile of the inlet substrate concentration S_{in} used to introduce perturbations in the reactor feeding conditions. (Source: created by the authors).

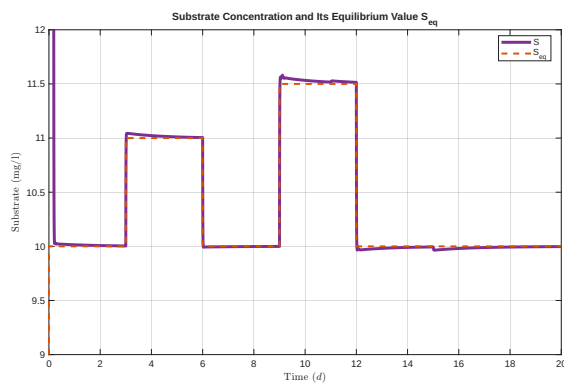


Fig. 2: Closed-loop response of the substrate concentration S and its equilibrium value S_{eq} under changes in the operating conditions. (Source: created by the authors).

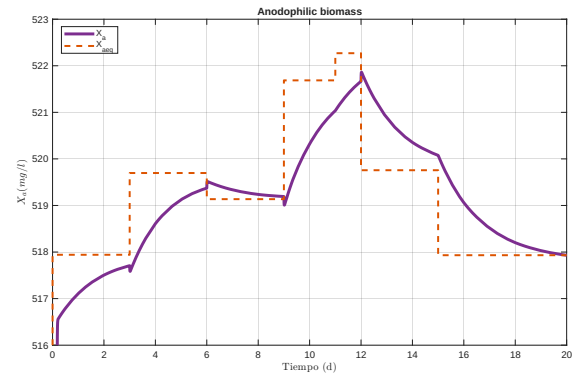


Fig. 3: Evolution of the anodophilic biomass X_a and its equilibrium trajectory during closed-loop operation. (Source: created by the authors).

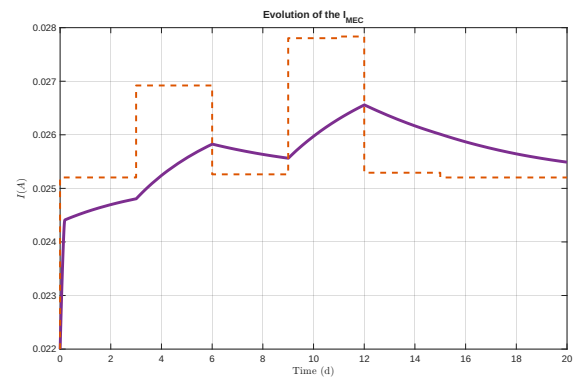


Fig. 4: Closed-loop evolution of the current I_{MEC} during the simulation. (Source: created by the authors).

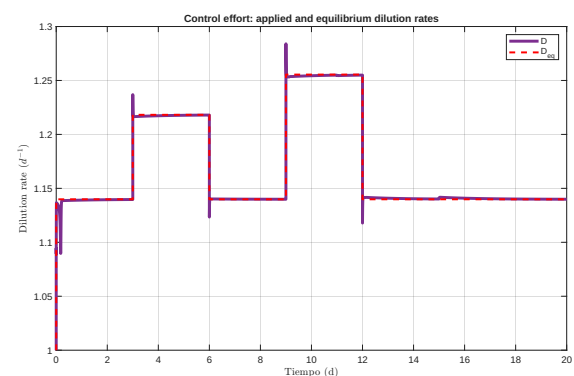


Fig. 5: Applied dilution rate D and equilibrium dilution rate D_{eq} during closed-loop operation. (Source: created by the authors).

Table 1. Quantitative closed-loop performance indicators evaluated for $t \geq 2$ d. (Source: created by the authors).

Performance indicator	Value
IAE_S	0.290988
$RMSE_S$	0.332524
IAE_D	0.011708
$D_{app,min}$ (d ⁻¹)	1.117844
$D_{app,max}$ (d ⁻¹)	1.283960
$\bar{Q}_{H_2}$ (L/d)	0.282144
$H_{2,total}$ (L)	5.078585

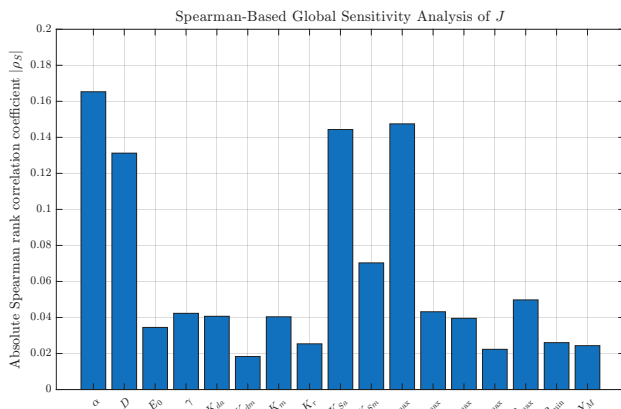


Fig. 6: Global sensitivity analysis of the overall performance index J using absolute Spearman rank correlation coefficients. The results were obtained from 500 Latin Hypercube samples with simultaneous variations of $\pm 20\%$ around the nominal parameter values. (Source: created by the authors).

Table 2. Empirical admissible ranges of the most influential parameters obtained from the accepted LHS realizations. (Source: created by the authors)

Parameter	Min.	Min. (%)	Max.	Max. (%)
α	0.43318	-19.930	0.64919	19.998
D	0.80060	-19.940	1.19950	19.952
K_{Sa}	16.007	-19.964	23.989	19.945
K_{Sm}	64.001	-19.999	95.973	19.966
$\mu_{a,max}$	1.5772	-19.939	2.3635	19.976
R_{max}	160.08	-19.959	239.99	19.993

5 Conclusion

A Port-Hamiltonian-based control strategy was developed for a microbial electrolysis cell model using the dilution rate as the manipulated input. The proposed formulation combines an equilibrium-based representation with an additional damping injection term inspired by the IDA-PBC methodology. In this

way, the control objective was expressed in terms of substrate regulation while preserving a physically coherent response in the internal states of the system.

The simulation results showed that the controller was able to maintain accurate tracking of the substrate concentration under successive changes in the operating conditions and under variations in the inlet substrate concentration. In addition, the anodophilic biomass and the current evolved consistently with the new equilibrium regimes, and the control effort remained bounded throughout the simulation horizon. These results indicate that the proposed strategy is effective for stabilizing the reactor around admissible operating points while retaining a meaningful interpretation from the energy-based control perspective.

Future work will consider a systematic comparison with conventional and advanced control strategies under identical operating conditions, common performance criteria, independent controller tuning, and equivalent treatment of the physical input constraints.

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Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

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Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

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René Flores-Estrella: Supervision, Validation, Data curation, Writing review & editing.
Ixbalank Torres: Supervision, Validation, Writing review & editing.
Victor Alcaraz-Gonzalez: Supervision, Conceptualization, Methodology, Resources, Project administration, Writing review & editing.

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Conflicts of Interest

The authors declare no conflicts of interest.

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