Panel Seemingly Unrelated Regression with Dummy Variables For Economic Modeling Of Developed And Developing Country

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Abstract: - This study aims to identify the relationship between population density, inflation, and unemployment on the human development index, GNP, export-import, and urbanization in the category of developed and developing countries using the Panel Seemingly Unrelated Regression (Panel SUR) with a dummy variable as a slope component. This study uses economic data from 145 countries in the world obtained through the official websites of the World Bank and the International Monetary Fund. The results showed that the Fix Effect SUR model (AIC=84915.74) was better than the pooled SUR model (AIC=114936) and Random Effect SUR (AIC=1148415). The results of the analysis using the Fix Effect SUR model show that population density has a significant positive relationship with GNP, imports, and exports. There is a significant negative relationship between the unemployment rate and GNP. In addition, the results obtained show that the effect of population density on GNP in developed countries is positive and greater. The effect of the unemployment rate on GNP in developed countries is negative and greater than that of developing countries. The results of the analysis using the pooled SUR model and the Random Effect SUR are the same conclusion where population density has a significant positive relationship to GNP, imports, and exports. There is a significant negative relationship between inflation and GNP. The effect of population density and inflation on GNP, imports, and exports in developed countries is positive and greater than that of developing countries. The effect of the unemployment rate on GNP in developed countries is negative and greater than that of developing countries.

Key-Words: - Panel SUR, Pooled SUR, Fix Effect SUR, Random Effect SUR, World Economics, Developed, and Developing Country.

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1 Introduction

Seemingly Unrelated Regression (SUR) is a form of multivariate regression that can accommodate the residual correlation between equations (variety structure between equations) so that each equation in the model seems to stand alone or there is no relationship between equations, but actually between equations has linkage, [1], [2]. However, there is a problem related to multivariate regression analysis on economic variables, namely the dynamic nature of the data so that the data at one point in time is not sufficient to model the relationship between each predictor variable to each response variable. Therefore, data on the variables related to several time periods for each research object (panel data) are needed so that the regression model obtained is better at describing the relationship between the predictor variables and the response variables so that a method that can accommodate these problems is developed, namely multivariate regression analysis on panel data.

Therefore, the Seemingly Unrelated Regression Panel model was developed which is able to model the relationship of one or more predictor variables to more than one observed response variable from a research object during a certain period of time on panel data, [3]. The Seemingly Unrelated Regression Panel Model is one of the developments of multivariate regression analysis and panel data regression analysis that allows statistical users to analyze cause-and-effect relationships in data resulting from combining cross-section and time series data not only on one response variable.

Economic success is something that all countries in the world want to achieve, especially developing countries. A country is said to be developed if it has a high and evenly distributed economic level, a high standard of living, and sophisticated technology. Developing countries have a middle level of social welfare and there is no economic equality, [4]. The benchmark for economic success is economic development. Every country must strive for economic development from year to year so that the country does not have problems in the economic field and has a prosperous society. This is still a problem in developing countries where developing countries continue to strive for a better country's economy through various economic actions and policies.

Economic success is a process because it is a stage that must be lived by every country so it requires hard work and cooperation between the community, government, and other elements involved in the long term so that economic success can be achieved. Economic success can generally be seen through 2 aspects, namely the economic aspect which includes international trade and GNP, and the social aspect which includes the human development index and urbanization, [5].

Therefore, this study aims to form a mathematical model of the seemingly unrelated regression model and develop it on panel data with dummy variables and apply it in economic cases, namely the relationship between HDI, GNP, Imports, Exports, and Urbanization such as Population Density, Inflation and Unemployment Rates in developed and developing countries using the Pooled SUR, Fix Effect SUR and Random Effect SUR. Therefore, this study aims to show the differences in the effect of Population Density, Inflation, and Unemployment Rate on the Human Development Index, GNP, Imports, Exports, and Urbanization in developed and developing countries so that this research can be beneficial for developing countries in maximizing economic activities in their countries as well as aspects what needs to be improved to have a high economic level like in developed countries, apart from that this research can be useful for developed countries to maintain the economic level in their country.

2 Literature Review

2.1 Seemingly Unrelated Regression (SUR) Models

The SUR model uses m response variables as a function of p predictor variables which can be seen in equation (1), [6].

$$Y_{1i} = \beta_{10} + \beta_{11}X_{1i} + \beta_{12}X_{2i} + \cdots + \beta_{1p}X_{pi} + \beta_{20}(0) + \beta_{21}(0) + \cdots + \beta_{21}(0) + \cdots + \beta_{mp}(0) + \varepsilon_{1i}$$

$$Y_{2i} = \beta_{10}(0) + \beta_{11}(0) + \cdots + \beta_{1p}(0) + \beta_{20} + \beta_{21}X_{1i} + \beta_{22}X_{2i} + \cdots + \beta_{2p}X_{pi} + \beta_{m0}(0) + \beta_{m1}(0) + \ldots + \beta_{mp}(0) + \varepsilon_{2i}$$

$$\vdots$$

$$Y_{mi} = \beta_{10}(0) + \beta_{11}(0) + \cdots + \beta_{1p}(0) + \cdots + \beta_{(m-1)p}(0) + \beta_{m0} + \beta_{m1}X_{1i} + \beta_{m2}X_{2i} + \cdots + \beta_{mp}X_{pi} + \varepsilon_{mi} \tag{1}$$

Equation (1) can be simplified into equation (2).

$$Y_{li} = \beta_{l0} + \sum_{j=1}^{p} \beta_{lj}X_{ji} + \varepsilon_{li} \tag{2}$$

$$i = 1, 2, \ldots, n \,;\, l = 1,2, \ldots, \mathrm{m} \,;\, \mathrm{j} = 1,2, \ldots, p$$

The form of the SUR equation in the matrix can be seen in equation (3), [7].

$$\boldsymbol{Y}_{mn\times 1} = \boldsymbol{X}_{mn\times m(p+1)}\boldsymbol{\beta}_{m(p+1)\times 1} + \boldsymbol{\varepsilon}_{mn\times 1} \tag{3}$$

2.2 Panel Seemingly Unrelated Regression Models (Panel SUR)

The Panel Seemingly Unrelated Regression model is an extension of the Seemingly Unrelated Regression model that can be used in panel data. The Panel Seemingly Unrelated Regression model was developed to accommodate the dynamic nature of data (data patterns always change over time), [3]. The Panel Seemingly Unrelated Regression model is the same as the Seemingly Unrelated Regression model (equation (1)) except that there is an element of time presented in equation (4).

$$Y_{1it} = \beta_{10} + \beta_{11}X_{1it} + \beta_{12}X_{2it} + \cdots + \beta_{1p}X_{pit} + \varepsilon_{1it}$$

$$Y_{2it} = \beta_{20} + \beta_{21}X_{1it} + \beta_{22}X_{2it} + \cdots + \beta_{2p}X_{pit} + \varepsilon_{2it}$$

$$\vdots$$

$$Y_{mit} = \beta_{m0} + \beta_{m1}X_{1it} + \beta_{m2}X_{2it} + \cdots + \beta_{mp}X_{pit} + \varepsilon_{mit} \tag{4}$$

According to [3], there are 3 Panel Seemingly Unrelated Regression models as follows.

a. Pooled Seemingly Unrelated Regression Models

This model has the same structure as the general model of Seemingly Unrelated Regression with the addition of an element of time. The Pooled Seemingly Unrelated Regression model is presented in equation (5).

$$Y_{1it} = \beta_{10} + \beta_{11}X_{1it} + \beta_{12}X_{2it} + \cdots + \beta_{1p}X_{pit} + \varepsilon_{1it}$$

$$Y_{2it} = \beta_{20} + \beta_{21}X_{1it} + \beta_{22}X_{2it} + \cdots + \beta_{2p}X_{pit} + \varepsilon_{2it}$$

$$\vdots$$

$$Y_{mit} = \beta_{m0} + \beta_{m1}X_{1it} + \beta_{m2}X_{2it} + \cdots + \beta_{mp}X_{pit} + \varepsilon_{mit} \tag{5}$$

b. Fix Effect Seemingly Unrelated Regression Models

In the fixed effects model, it is assumed that there is a relationship between the characteristics of the object (α_{mi}) and the predictor variables for each response variable. The Fix Effect Seemingly Unrelated Regression model is presented in equation (6).

$$
\begin{aligned}
Y_{1it} &= \beta_{01i} + \beta_{11}X_{1it} + \beta_{12}X_{2it} + \cdots + \beta_{1p}X_{pit} + \varepsilon_{1it} \\
Y_{2it} &= \beta_{02i} + \beta_{21}X_{1it} + \beta_{22}X_{2it} + \cdots + \beta_{2p}X_{pit} + \varepsilon_{2it} \\
&\vdots \\
Y_{mit} &= \beta_{0mi} + \beta_{m1}X_{1it} + \beta_{m2}X_{2it} + \cdots + \beta_{mp}X_{pit} + \varepsilon_{mit}
\end{aligned}
\tag{6}
$$

c. Random Effect Seemingly Unrelated Regression Models

In the random effects model, it is assumed that there is no relationship between the object characteristics (α_{mi}) and the predictor variables for each response variable. The object characteristics are assumed to be random variables combined with random error terms. The Random Effect Seemingly Unrelated Regression model is presented in equation (7).

$$
\begin{aligned}
Y_{1it} &= \beta_{11}X_{1it} + \beta_{12}X_{2it} + \cdots + \beta_{1p}X_{pit} + v_{1it} \\
Y_{2it} &= \beta_{21}X_{1it} + \beta_{22}X_{2it} + \cdots + \beta_{2p}X_{pit} + v_{2it} \\
&\vdots \\
Y_{mit} &= \beta_{m1}X_{1it} + \beta_{m2}X_{2it} + \cdots + \beta_{mp}X_{pit} + v_{mit}
\end{aligned}
$$

with,

$$
\begin{aligned}
v_{1it} &= \beta_{01i} + \varepsilon_{1it} \\
v_{2it} &= \beta_{02i} + \varepsilon_{2it} \\
&\vdots \\
v_{mit} &= \beta_{0mi} + \varepsilon_{mit}
\end{aligned}
\tag{7}
$$

2.3 Slope Dummy Seemingly Unrelated Regression (SUR) Models

The Slope Dummy Panel Seemingly Unrelated Regression model is a development of the Panel Seemingly Unrelated Regression model which aims to determine the relationship of one or more predictor variables to more than one response variable and can accommodate differences between categories of dummy variables as follows.

a. Slope Dummy Pooled Seemingly Unrelated Regression Models

This model has the same structure as the general Seemingly Unrelated Regression model with the addition of time elements and dummy variables as slope components. The Slope Dummy Pooled Seemingly Unrelated Regression model is presented in equation (8).

$$
\begin{aligned}
Y_{1it} = {} & \alpha_{10} + \alpha_{112}(D_{2i} \times X_{1it}) + \alpha_{113}(D_{3i} \times X_{1it}) + \cdots + \alpha_{11k}(D_{ki} \times X_{1it}) + \alpha_{122}(D_{2i} \times X_{2it}) + \alpha_{123}(D_{3i} \times X_{2it}) + \cdots + \\
& \alpha_{12k}(D_{ki} \times X_{2it}) + \cdots + \alpha_{1p2}(D_{2i} \times X_{pit}) + \alpha_{1p3}(D_{3i} \times X_{pit}) + \cdots + \alpha_{1pk}(D_{ki} \times X_{pit}) + \beta_{11}X_{1it} + \beta_{12}X_{2it} + \cdots + \beta_{1p}X_{pit} + \varepsilon_{1it} \\
Y_{2it} = {} & \alpha_{20} + \alpha_{212}(D_{2i} \times X_{1it}) + \alpha_{213}(D_{3i} \times X_{1it}) + \cdots + \alpha_{21k}(D_{ki} \times X_{1it}) + \alpha_{222}(D_{2i} \times X_{2it}) + \alpha_{223}(D_{3i} \times X_{2it}) + \cdots + \\
& \alpha_{22k}(D_{ki} \times X_{2it}) + \cdots + \alpha_{2p2}(D_{2i} \times X_{pit}) + \alpha_{2p3}(D_{3i} \times X_{pit}) + \cdots + \alpha_{2pk}(D_{ki} \times X_{pit}) + \beta_{21}X_{1it} + \beta_{22}X_{2it} + \cdots + \beta_{2p}X_{pit} + \varepsilon_{2it} \\
& \vdots \\
Y_{mit} = {} & \alpha_{m0} + \alpha_{m12}(D_{2i} \times X_{1it}) + \alpha_{m13}(D_{3i} \times X_{1it}) + \cdots + \alpha_{m1k}(D_{ki} \times X_{1it}) + \alpha_{m22}(D_{2i} \times X_{2it}) + \alpha_{m23}(D_{3i} \times X_{2it}) + \cdots + \\
& \alpha_{m2k}(D_{ki} \times X_{2it}) + \cdots + \alpha_{mp2}(D_{2i} \times X_{pit}) + \alpha_{mp3}(D_{3i} \times X_{pit}) + \cdots + \alpha_{mpk}(D_{ki} \times X_{pit}) + \beta_{m1}X_{1it} + \beta_{m2}X_{2it} + \cdots + \beta_{mp}X_{pit} + \varepsilon_{mit}
\end{aligned}
\tag{8}
$$

b. Slope Dummy Fix Effect Seemingly Unrelated Regression Models

In the fixed effects model, it is assumed that there is a relationship between the characteristics of the object (α_{mi}) and the predictor variables for each response variable. The model of Slope Dummy Fix Effect Seemingly Unrelated Regression is presented in equation (9).

$$
\begin{aligned}
Y_{1it} = {} & \alpha_{10i} + \alpha_{112}(D_{2i} \times X_{1it}) + \alpha_{113}(D_{3i} \times X_{1it}) + \cdots + \alpha_{11k}(D_{ki} \times X_{1it}) + \alpha_{122}(D_{2i} \times X_{2it}) + \alpha_{123}(D_{3i} \times X_{2it}) + \cdots + \\
& \alpha_{12k}(D_{ki} \times X_{2it}) + \cdots + \alpha_{1p2}(D_{2i} \times X_{pit}) + \alpha_{1p3}(D_{3i} \times X_{pit}) + \cdots + \alpha_{1pk}(D_{ki} \times X_{pit}) + \beta_{11}X_{1it} + \beta_{12}X_{2it} + \cdots + \beta_{1p}X_{pit} + \varepsilon_{1it} \\
Y_{2it} = {} & \alpha_{20i} + \alpha_{212}(D_{2i} \times X_{1it}) + \alpha_{213}(D_{3i} \times X_{1it}) + \cdots + \alpha_{21k}(D_{ki} \times X_{1it}) + \alpha_{222}(D_{2i} \times X_{2it}) + \alpha_{223}(D_{3i} \times X_{2it}) + \cdots + \\
& \alpha_{22k}(D_{ki} \times X_{2it}) + \cdots + \alpha_{2p2}(D_{2i} \times X_{pit}) + \alpha_{2p3}(D_{3i} \times X_{pit}) + \cdots + \alpha_{2pk}(D_{ki} \times X_{pit}) + \beta_{21}X_{1it} + \beta_{22}X_{2it} + \cdots + \beta_{2p}X_{pit} + \varepsilon_{2it} \\
& \vdots \\
Y_{mit} = {} & \alpha_{m0i} + \alpha_{m12}(D_{2i} \times X_{1it}) + \alpha_{m13}(D_{3i} \times X_{1it}) + \cdots + \alpha_{m1k}(D_{ki} \times X_{1it}) + \alpha_{m22}(D_{2i} \times X_{2it}) + \alpha_{m23}(D_{3i} \times X_{2it}) + \cdots + \\
& \alpha_{m2k}(D_{ki} \times X_{2it}) + \cdots + \alpha_{mp2}(D_{2i} \times X_{pit}) + \alpha_{mp3}(D_{3i} \times X_{pit}) + \cdots + \alpha_{mpk}(D_{ki} \times X_{pit}) + \beta_{m1}X_{1it} + \beta_{m2}X_{2it} + \cdots + \beta_{mp}X_{pit} + \varepsilon_{mit}
\end{aligned}
$$

(9)

c. Slope Dummy Random Effect Seemingly Unrelated Regression Models

In the random effects model, it is assumed that there is no relationship between the object characteristics (α_{mi}) and the predictor variables for each response variable. The object characteristics are assumed to be random variables combined with random error terms. The Random Effect Seemingly Unrelated Regression model is presented in equation (10).

$$Y_{1it} = \alpha_{112}(D_{2i} \times X_{1it}) + \alpha_{113}(D_{3i} \times X_{1it}) + \cdots + \alpha_{11k}(D_{ki} \times X_{1it}) + \alpha_{122}(D_{2i} \times X_{2it}) + \alpha_{123}(D_{3i} \times X_{2it}) + \cdots + \alpha_{12k}(D_{ki} \times X_{2it}) + \cdots + \alpha_{1p2}(D_{2i} \times X_{pit}) + \alpha_{1p3}(D_{3i} \times X_{pit}) + \cdots + \alpha_{1pk}(D_{ki} \times X_{pit}) + \beta_{11}X_{1it} + \beta_{12}X_{2it} + \cdots + \beta_{1p}X_{pit} + v_{1it}$$

$$Y_{2it} = \alpha_{212}(D_{2i} \times X_{1it}) + \alpha_{213}(D_{3i} \times X_{1it}) + \cdots + \alpha_{21k}(D_{ki} \times X_{1it}) + \alpha_{222}(D_{2i} \times X_{2it}) + \alpha_{223}(D_{3i} \times X_{2it}) + \cdots + \alpha_{22k}(D_{ki} \times X_{2it}) + \cdots + \alpha_{2p2}(D_{2i} \times X_{pit}) + \alpha_{2p3}(D_{3i} \times X_{pit}) + \cdots + \alpha_{2pk}(D_{ki} \times X_{pit}) + \beta_{21}X_{1it} + \beta_{22}X_{2it} + \cdots + \beta_{2p}X_{pit} + v_{2it}$$

□

$$Y_{mit} = \alpha_{m12}(D_{2i} \times X_{1it}) + \alpha_{m13}(D_{3i} \times X_{1it}) + \cdots + \alpha_{m1k}(D_{ki} \times X_{1it}) + \alpha_{m22}(D_{2i} \times X_{2it}) + \alpha_{m23}(D_{3i} \times X_{2it}) + \cdots + \alpha_{m2k}(D_{ki} \times X_{2it}) + \cdots + \alpha_{mp2}(D_{2i} \times X_{pit}) + \alpha_{mp3}(D_{3i} \times X_{pit}) + \cdots + \alpha_{mpk}(D_{ki} \times X_{pit}) + \beta_{m1}X_{1it} + \beta_{m2}X_{2it} + \cdots + \beta_{mp}X_{pit} + v_{mit}$$

with,

$$\begin{gathered} v_{1it} = \alpha_{10i} + \varepsilon_{1it} \\ v_{2it} = \alpha_{20i} + \varepsilon_{2it} \\ \vdots \\ v_{mit} = \alpha_{m0i} + \varepsilon_{mit} \end{gathered} \quad (10)$$

2.4 Parameter Estimation of Seemingly Unrelated Regression Model with Ordinary Least Squares Method

The ordinary least squares method (OLS) is a method for estimating the parameters of the Seemingly Unrelated Regression model in equation (1) by minimizing the sum of the squared errors as in equation (11).

$$\begin{gathered} \hat{\boldsymbol{\varepsilon}}'\hat{\boldsymbol{\varepsilon}} = (\boldsymbol{Y} - \boldsymbol{X}\hat{\boldsymbol{\beta}})'(\boldsymbol{Y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}) \\ \hat{\boldsymbol{\varepsilon}}'\hat{\boldsymbol{\varepsilon}} = (\boldsymbol{Y}' - \boldsymbol{X}'\hat{\boldsymbol{\beta}}')(\boldsymbol{Y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}) \\ \hat{\boldsymbol{\varepsilon}}'\hat{\boldsymbol{\varepsilon}} = \boldsymbol{Y}'\boldsymbol{Y} - \boldsymbol{Y}'\boldsymbol{X}\hat{\boldsymbol{\beta}} - \boldsymbol{Y}\boldsymbol{X}'\hat{\boldsymbol{\beta}}' + \boldsymbol{X}\hat{\boldsymbol{\beta}}\boldsymbol{X}'\hat{\boldsymbol{\beta}}' \\ \hat{\boldsymbol{\varepsilon}}'\hat{\boldsymbol{\varepsilon}} = \boldsymbol{Y}'\boldsymbol{Y} - 2\boldsymbol{X}'\hat{\boldsymbol{\beta}}'\boldsymbol{Y} + \boldsymbol{X}\hat{\boldsymbol{\beta}}\boldsymbol{X}'\hat{\boldsymbol{\beta}}' \end{gathered} \quad (11)$$

Then the sum of the squared errors is derived from the parameter estimator and equated to zero as in equation (12) to produce an estimator that is close to the parameter value.

$$\begin{gathered} \frac{\partial(\hat{\boldsymbol{\varepsilon}}'\hat{\boldsymbol{\varepsilon}})}{\partial(\hat{\boldsymbol{\beta}})} = \frac{\partial\left(\boldsymbol{Y}'\boldsymbol{Y} - 2\boldsymbol{X}'\hat{\boldsymbol{\beta}}'\boldsymbol{Y} + \boldsymbol{X}\hat{\boldsymbol{\beta}}\boldsymbol{X}'\hat{\boldsymbol{\beta}}'\right)}{\partial(\hat{\boldsymbol{\beta}})} = 0 \\ \frac{\partial(\hat{\boldsymbol{\varepsilon}}'\hat{\boldsymbol{\varepsilon}})}{\partial(\hat{\boldsymbol{\beta}})} = -2\boldsymbol{X}'\boldsymbol{Y} + 2\boldsymbol{X}'\boldsymbol{X}\hat{\boldsymbol{\beta}} = 0 \\ -\boldsymbol{X}'\boldsymbol{Y} + \boldsymbol{X}'\boldsymbol{X}\hat{\boldsymbol{\beta}} = 0 \\ \boldsymbol{X}'\boldsymbol{X}\hat{\boldsymbol{\beta}} = \boldsymbol{X}'\boldsymbol{Y} \end{gathered} \quad (12)$$

So that the regression parameter estimator is obtained using the ordinary least squares method as follows, [8].

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y} \quad (13)$$

2.5 Parameter Estimation of Seemingly Unrelated Regression Model with Generalized Least Squares Method

The generalized least squares method (GLS) is one of the methods for estimating the parameters of the Seemingly Unrelated Regression model in equation (1) with the assumption that matrix V (matrix of variance-covariance residual OLS) is known to be presented in equation (14), [9].

$$\hat{\boldsymbol{\beta}} = \left(\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{X}\right)^{-1}\boldsymbol{X}'\boldsymbol{V}^{-1}\boldsymbol{Y} \quad (14)$$

with the matrix **V** as follows.

$$\boldsymbol{V}^{-1} = \begin{bmatrix} \sigma_{11}\boldsymbol{I}_n & \sigma_{21}\boldsymbol{I}_n & \cdots & \sigma_{m1}\boldsymbol{I}_n \\ \sigma_{12}\boldsymbol{I}_n & \sigma_{22}\boldsymbol{I}_n & \cdots & \sigma_{m2}\boldsymbol{I}_n \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1m}\boldsymbol{I}_n & \sigma_{2m}\boldsymbol{I}_n & \cdots & \sigma_{mm}\boldsymbol{I}_n \end{bmatrix}^{-1}$$

$$\boldsymbol{V}^{-1} = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \cdots & \sigma_{m1} \\ \sigma_{12} & \sigma_{22} & \cdots & \sigma_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1m} & \sigma_{2m} & \cdots & \sigma_{mm} \end{bmatrix}^{-1} \otimes \boldsymbol{I}_n$$

$$\boldsymbol{V}^{-1} = \boldsymbol{\Sigma}^{-1} \otimes \boldsymbol{I}_n \quad (15)$$

In general, the matrix **V** is not known, so it is necessary to estimate the matrix **V** using a two-stage aiken, such as equation (16).

$$\hat{\boldsymbol{V}} = \begin{bmatrix} \hat{\sigma}_{11}\boldsymbol{I}_n & \hat{\sigma}_{21}\boldsymbol{I}_n & \cdots & \hat{\sigma}_{m1}\boldsymbol{I}_n \\ \hat{\sigma}_{12}\boldsymbol{I}_n & \hat{\sigma}_{22}\boldsymbol{I}_n & \cdots & \hat{\sigma}_{m2}\boldsymbol{I}_n \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\sigma}_{1m}\boldsymbol{I}_n & \hat{\sigma}_{2m}\boldsymbol{I}_n & \cdots & \hat{\sigma}_{mm}\boldsymbol{I}_n \end{bmatrix}$$

$$\hat{\boldsymbol{V}} = \begin{bmatrix} \hat{\sigma}_{11} & \hat{\sigma}_{21} & \cdots & \hat{\sigma}_{m1} \\ \hat{\sigma}_{12} & \hat{\sigma}_{22} & \cdots & \hat{\sigma}_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\sigma}_{1m} & \hat{\sigma}_{2m} & \cdots & \hat{\sigma}_{mm} \end{bmatrix} \otimes \boldsymbol{I}_n$$

$$\hat{\boldsymbol{V}} = \hat{\boldsymbol{\Sigma}} \otimes \boldsymbol{I}_n \quad (16)$$

with,

$$\hat{\sigma}_{rs} = \frac{1}{n-m}\sum_{i=1}^{n}(\hat{\varepsilon}_{ri}\hat{\varepsilon}_{si})$$
$$\hat{\sigma}_{rs} = \frac{1}{n-m}(\hat{\varepsilon}_r{}'\hat{\varepsilon}_s)$$
$$\hat{\sigma}_{rs} = \frac{1}{n-m}((\boldsymbol{Y_r} - \boldsymbol{X_r}\boldsymbol{\hat{\beta}_r})'(\boldsymbol{Y_s} - \boldsymbol{X_s}\boldsymbol{\hat{\beta}_s}))$$
$$r = 1,2,\dots,m; s = 1,2,\dots,m \tag{17}$$

$\boldsymbol{\hat{\beta}_r}$ and $\boldsymbol{\hat{\beta}_s}$ estimated using the ordinary least squares method for each equation so that the matrix **V** estimator is formed through a single equation with OLS, [10]. Then a matrix **V** is formed as in equation (18)

$$\boldsymbol{\hat{V}} = \begin{bmatrix} \hat{\sigma}_{11}\boldsymbol{I_n} & \hat{\sigma}_{21}\boldsymbol{I_n} & \cdots & \hat{\sigma}_{m1}\boldsymbol{I_n} \\ \hat{\sigma}_{12}\boldsymbol{I_n} & \hat{\sigma}_{22}\boldsymbol{I_n} & \cdots & \hat{\sigma}_{m2}\boldsymbol{I_n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\sigma}_{1m}\boldsymbol{I_n} & \hat{\sigma}_{2m}\boldsymbol{I_n} & \cdots & \hat{\sigma}_{mm}\boldsymbol{I_n} \end{bmatrix}$$
$$\boldsymbol{\hat{V}} = \begin{bmatrix} \hat{\sigma}_{11} & \hat{\sigma}_{21} & \cdots & \hat{\sigma}_{m1} \\ \hat{\sigma}_{12} & \hat{\sigma}_{22} & \cdots & \hat{\sigma}_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\sigma}_{1m} & \hat{\sigma}_{2m} & \cdots & \hat{\sigma}_{mm} \end{bmatrix} \otimes \boldsymbol{I_n}$$
$$\boldsymbol{\hat{V}} = \boldsymbol{\hat{\Sigma}} \otimes \boldsymbol{I_n} \tag{18}$$

The estimator of the SUR model using GLS is obtained as follows, [1].

$$\boldsymbol{\hat{\beta}} = (\boldsymbol{X'V^{-1}X})^{-1}\boldsymbol{X'V^{-1}Y} \tag{19}$$

3 Methodology

This study aims to model seemingly unrelated regression on panel data with dummy variables and its application in modeling economic success in developed and developing counties. This study uses the Seemingly Unrelated Regression Panel (SUR Panel) model because the five aspects of economic success (IPM, GNP, Import, Export, and Urbanization) are related and panel data is used so that the information obtained is better where it is not only at one time, rather it can cover multiple time periods at once. This study uses data on aspects of economic success (HDI, GNP, Import, Export, and Urbanization) and the factors that influence it (population density, inflation, and unemployment rate) in the world. This study uses balanced panel data so that it only uses samples that have complete data where there are 35 developing countries and 110 developed countries obtained from the official websites of the World Bank and IMF. The data source properties are presented in Table 1.

Table 1. Data Source

Variables		Period	Data source
Predictor	Population Density (X_1)	2010-2019	World Bank (https://data.worldbank.org/) → Population, total
	Inflation (X_2)	2010-2019	World Bank (https://data.worldbank.org/) → Inflation, consumer prices (annual %)
	Open Unemployment Rate (X_3)	2010-2019	World Bank (https://data.worldbank.org/) → Unemployment, total (% of the total labor force) (modeled ILO estimate)
Response	HDI (Y_1)	2010-2019	Kaggle (https://www.kaggle.com/datasets/tjysdsg/human-development-index)
	Import (Y_3)	2010-2019	World Bank (https://data.worldbank.org/) → Imports of goods and services (current US$)
	Eksport (Y_4)	2010-2019	World Bank (https://data.worldbank.org/) → Exports of goods and services (current US$)
	Urbanization (Y_5)	2010-2019	World Bank (https://data.worldbank.org/) → Urban Population (% of total population)

Dummy	Country Category (D_2)	2010-2019	International Monetary Fund (https://www.imf.org/external/pubs/ft/weo/2022/01/weodata/groups.htm)

The procedure of this research is as follows.

1. Checking the relationship between predictor variables (assuming non-multicollinearity) using VIF statistics
2. Testing pooled SUR model parameters using the ordinary least squares (OLS) method, fix effects SUR, and random effects SUR model parameters using the general least squares (GLS) method
3. Testing the assumption of error correlation between equations in pooled SUR, fix effects SUR, and random effects SUR model using the Lagrange Multiplier test statistic
4. Testing the relationship of the predictor variables to the response variables of pooled SUR, fix effects SUR, and random effects SUR model simultaneously and individually
5. Selecting the best panel SUR model using the smallest AIC value and the largest adjusted R2
6. Interpretation of the best panel SUR model

4 Result and Discussion

This study uses the pooled SUR model which is estimated using OLS, Fix Effect SUR which is estimated using GLS, and Random Effect SUR which is estimated using GLS as follows.

4.1 Fix Effect SUR

The results of testing the Fix Effect SUR model with GLS give parameter estimators as shown in Table 2.

Table 2. Parameter Estimator for Fix Effect SUR

Beta	Estimate	t value	Pr(>|t|)
X1.1	0.003	0.0165	0.9868
X2.1	0	0.00017	0.9999
X3.1	-0.002	-0.00072	0.9994
X1D.1	0.003	0.00018	0.9999
X2D.1	-0.003	-0.00045	0.9996
X3D.1	-0.001	-0.00019	0.9998
X1.2	4.945	30.59201	<0.0001
X2.2	-0.659	-1.39751	0.1623
X3.2	-11.476	-4.06096	<0.0001
X1D.2	35.213	2.02937	0.0425
X2D.2	-10.281	-1.66973	0.095
X3D.2	-17.207	-3.99049	0.0001
X1.3	0.93	5.75196	<0.0001
X2.3	-0.06	-0.12767	0.8984
X3.3	-2.29	-0.81038	0.4178
X1D.3	13.618	0.78483	0.4326
X2D.3	3.732	0.60613	0.5445
X3D.3	-4.575	-1.06105	0.2887
X1.4	0.922	5.70326	<0.0001
X2.4	-0.042	-0.08848	0.9295
X3.4	-2.1	-0.74324	0.4574
X1D.4	21.095	1.21572	0.2241
X2D.4	0.434	0.07042	0.9439
X3D.4	-4.804	-1.1142	0.2652
X1.5	0.203	1.25671	0.2089
X2.5	-0.007	-0.01464	0.9883
X3.5	-0.067	-0.02369	0.9811
X1D.5	0.075	0.0043	0.9966
X2D.5	-0.079	-0.01286	0.9897
X3D.5	-0.01	-0.00225	0.9982

: Significant at α = 5%

Through the table, it can be seen that *Fix Effect Seemingly Unrelated Regression* (SUR) models are as in equation (20).

$$HDI_{it} = \alpha_{1i} + 0.003(D_{2i} \times Population\ Density_{it}) - 0.003(D_{2i} \times Inflation_{it}) - 0.001(D_{2i} \times Unemployment\ Rate_{it}) + 0.003 Population\ Density_{it} + 0.000080172 Inflation_{it} - 0.002 Unemployment\ Rate_{it}$$

$$GNP_{it} = \alpha_{2i} + 35.213(D_{2i} \times Population\ Density_{it}) - 10.281(D_{2i} \times Inflation_{it}) + -17.207(D_{2i} \times Unemployment\ Rate_{it}) + 4.945 Population\ Density_{it} -$$

$$0.659Inflation_{it} - 11.476Unemployment\ Rate_{it}$$

$$Import_{it} = \alpha_{3i} + 13.618(D_{2i} \times Population\ Density_{it}) + 3.732(D_{2i} \times Inflation_{it}) + -4.575(D_{2i} \times Unemployment\ Rate_{it}) + 0.93Population\ Density_{it} - 0.06Inflation_{it} - 2.29Unemployment\ Rate_{it}$$

$$Export_{it} = \alpha_{4i} + 21.095(D_{2i} \times Population\ Density_{it}) + 0.434(D_{2i} \times Inflation_{it}) - 4.804(D_{2i} \times Unemployment\ Rate_{it}) + 0.922Population\ Density_{it} - 0.042Inflation_{it} - 2.1Unemployment\ Rate_{it}$$

$$Urbanization_{it} = \alpha_{5i} + 0.075(D_{2i} \times Population\ Density_{it}) - 0.079(D_{2i} \times Inflation_{it}) - 0.01(D_{2i} \times Unemployment\ Rate_{it}) + 0.203Population\ Density_{it} - 0.007Inflation_{it} - 0.067Unemployment\ Rate_{it}$$

$$D_{2i} = \begin{cases} 1, if\ i-th\ Country\ is\ classified\ as\ a\ developed\ country \\ 0, if\ i-th\ Country\ is\ classified\ as\ a\ growth\ country \end{cases} \quad (20)$$

The intercept for each equation (α_{1i}, α_{2i}, α_{3i}, α_{3i} and α_{5i}) is different for each country because it uses the Fix Effect SUR model which assumes that there are characteristics for each country. The intercept for each country in each equation is presented in Table 3.

Table 3. The Intercept for Each Country

Country	α_{1i}	α_{2i}	α_{3i}	α_{4i}	α_{5i}
Albania	0.808	80.418	29.392	25.406	72.283
Algeria	0.712	-838.82	-35.711	-29.187	21.175
Angola	0.544	-614.763	-33.577	-14.171	86.023
Armenia	0.798	113.39	36.13	31.945	-86.489
Australia	0.823	569.63	-23.988	-195.421	75.376
□	□	□	□	□	□
Zambia	0.565	-314.262	-13.743	-10.973	40.904
Zimbabwe	0.534	-321.892	-22.386	-22.08	31.758

Through this model, it can be seen that in developing countries, every additional population will increase the human development index, GNP, imports, exports, and urbanization. Any increase in inflation will increase the human development index, but reduce GNP, imports, exports, and urbanization. Any increase in the unemployment rate will decrease the human development index, GNP, imports, exports, and urbanization.

In addition, it can be seen that in developed countries, each additional population will increase the human development index, GNP, imports, exports, and urbanization. Any increase in inflation will reduce the human development index, GNP, and urbanization, but increase imports and exports. Any increase in the unemployment rate will decrease the human development index, GNP, imports, exports, and urbanization.

Table 2 shows that population density has a significant positive relationship with GNP, imports, and exports. There is a significant negative relationship between inflation and GNP. The effect of population density and inflation on GNP, imports, and exports in developed countries is positive and greater than in developing countries. The effect of the unemployment rate on GNP in developed countries is negative and greater than in developing countries.

4.2 Random Effect SUR

The results of testing the Random Effect SUR model with GLS give parameter estimators as shown in Table 4.

Table 4. Parameter Estimator for Random Effect SUR

Beta	Estimate	t value	Pr(>|t|)
X1.1	0.00050	0.00464	0.9963
X2.1	0.01001	0.00496	0.9960
X3.1	0.04678	0.02141	0.9829
X1D.1	0.00433	0.00356	0.9972
X2D.1	0.15974	0.0072	0.9943
X3D.1	0.00804	0.00143	0.9989
X1.2	6.18306	57.41236	<0.0001
X2.2	-5.77721	-2.86327	0.0042
X3.2	-0.59539	-0.27251	0.7852
X1D.2	27.48431	22.58424	<0.0001
X2D.2	244.05	11.00268	<0.0001
X3D.2	15.39835	2.73385	0.0063
X1.3	1.19832	11.12692	<0.0001
X2.3	-0.85747	-0.42497	0.6709
X3.3	0.73194	0.33501	0.7376
X1D.3	7.79384	6.40431	<0.0001
X2D.3	83.81546	3.77871	0.0002
X3D.3	3.77439	0.67011	0.5028
X1.4	1.20177	11.15896	<0.0001
X2.4	-0.88664	-0.43943	0.6604
X3.4	0.746	0.34144	0.7328
X1D.4	8.51871	6.99994	<0.0001
X2D.4	78.88334	3.55635	0.0004
X3D.4	3.33352	0.59184	0.5540
X1.5	0.03325	0.3087	0.7576
X2.5	0.71171	0.35273	0.7243
X3.5	3.98798	1.82529	0.068
X1D.5	0.44179	0.36302	0.7166
X2D.5	15.70975	0.70825	0.4788
X3D.5	0.37756	0.06703	0.9466

: Significant at α = 5%

Through the table, it can be seen that *Random Effect Seemingly Unrelated Regression* (SUR) models as in equation (21).

$$HDI_{it} = 0.00433(D_{2i} \times Population\ Density_{it}) + 0.15974(D_{2i} \times Inflation_{it}) + 0.00804(D_{2i} \times Unemployment\ Rate_{it}) + 0.0005Population\ Density_{it} + 0.01001Inflation_{it} + 0.04678Unemployment\ Rate_{it}$$

$$GNP_{it} = 27.48431(D_{2i} \times Population\ Density_{it}) + 244.05(D_{2i} \times Inflation_{it}) + 15.39835(D_{2i} \times Unemployment\ Rate_{it}) + 6.18306Population\ Density_{it} - 5.77721Inflation_{it} - 0.59539Unemployment\ Rate_{it}$$

$$Import_{it} = 7.79384(D_{2i} \times Population\ Density_{it}) + 83.81546(D_{2i} \times Inflation_{it}) + 3.77439(D_{2i} \times Unemployment\ Rate_{it}) + 1.19832Population\ Density_{it} - 0.85747Inflation_{it} + 0.73194Unemployment\ Rate_{it}$$

$$Export_{it} = 8.51871(D_{2i} \times Population\ Density_{it}) + 78.88334(D_{2i} \times Inflation_{it}) + 3.33352(D_{2i} \times Unemployment\ Rate_{it}) + 1.20177Population\ Density_{it} - 0.8866Inflation_{it} + 0.746Unemployment\ Rate_{it}$$

$$Urbanization_{it} = 0.44179(D_{2i} \times Population\ Density_{it}) + 15.70975(D_{2i} \times Inflation_{it}) + 0.37756(D_{2i} \times Unemployment\ Rate_{it}) + 0.03325Population\ Density_{it} + 0.71171Inflation_{it} + 3.98798Unemployment\ Rate_{it}$$

$$D_{2i} = \begin{cases} 1, if\ i-th\ Country\ is\ classified\ as\ a\ developed\ country \\ 0, if\ i-th\ Country\ is\ classified\ as\ a\ growth\ country \end{cases} \quad (21)$$

Through this model, it can be seen that in developing countries, each additional population will increase the human development index, GNP, imports, and exports, but reduce urbanization. Any increase in inflation will reduce the human development index, GNP, imports, exports, and urbanization. Any increase in the unemployment rate will reduce GNP, imports, and exports, but increase the human development index and urbanization.

In addition, it can be seen that in developed countries, each additional population will increase the human development index, GNP, imports, exports, and urbanization. Any increase in inflation will increase the human development index, GNP, imports, exports, and urbanization. Any increase in the unemployment rate will increase the human development index, GNP, imports, exports, and urbanization.

Table 4 shows that population density has a significant positive relationship with GNP, imports, and exports. There is a significant negative relationship between unemployment and GNP. In addition, the results show that the effect of population density on GNP in developed countries is positive and greater than in developing countries and the effect of the unemployment rate on GNP in developed countries is negative and greater than in developing countries.

4.3 Pooled SUR

The results of testing the pooled SUR model with OLS give parameter estimators as shown in Table 5.

Table 5. Parameter Estimator for Pooled SUR

Beta	Estimate	t value	Pr(>|t|)
X0.1	0.6794	0.021	0.983161
X1.1	-1.7E-06	0	0.999987
X2.1	-0.002	-0.001	0.999237
X3.1	0.000734	0	0.99981
X1D.1	0.001328	0.001	0.999134
X2D.1	0.04745	0.002	0.99834
X3D.1	0.01169	0.002	0.998344
X0.2	67.34	2.092	0.036471
X1.2	6.133	55.645	<0.0001
X2.2	-6.968	-3.325	0.000887
X3.2	-5.159	-1.671	0.094678
X1D.2	27.19	22.201	<0.0001
X2D.2	232.9	10.217	<0.0001
X3D.2	15.76	2.798	0.005152
X0.3	51.14	1.589	0.112155
X1.3	1.161	10.53	<0.0001
X2.3	-1.762	-0.841	0.400458
X3.3	-2.734	-0.886	0.37579
X1D.3	7.568	6.18	<0.0001
X2D.3	75.36	3.306	0.000952
X3D.3	4.049	0.719	0.472242
X0.4	58.67	1.823	0.068397
X1.4	1.158	10.511	<0.0001
X2.4	-1.924	-0.918	0.358485
X3.4	-3.23	-1.046	0.295377
X1D.4	8.259	6.745	<0.0001
X2D.4	69.19	3.035	0.002416
X3D.4	3.648	0.648	0.517159
X0.5	54.21	1.684	0.092155
X1.5	-0.00674	-0.061	0.951218
X2.5	-0.2471	-0.118	0.906112
X3.5	0.3136	0.102	0.919063
X1D.5	0.2021	0.165	0.868902
X2D.5	6.749	0.296	0.767208
X3D.5	0.6684	0.119	0.905539

: Significant at α = 5%

Through the table, it can be seen that *Pooled Seemingly Unrelated Regression* (SUR) models are as in equation (22).

$$HDI_{it} = 0.6794 + 0.001328(D_{2i} \times Population\ Density_{it}) + 0.04745(D_{2i} \times Inflation_{it}) + 0.01169(D_{2i} \times Unemployment\ Rate_{it}) - 1.729E-06 Population\ Density_{it} - 0.002004 Inflation_{it} + 0.000733 Unemployment\ Rate_{it}$$

$$GNP_{it} = 67.34 + 27.19(D_{2i} \times Population\ Density_{it}) + 232.9(D_{2i} \times Inflation_{it}) + 15.76(D_{2i} \times Unemployment\ Rate_{it}) + 6.133 Population\ Density_{it} - 6.968 Inflation_{it} - 5.159 Unemployment\ Rate_{it}$$

$$Import_{it} = 51.14 + 7.568(D_{2i} \times Population\ Density_{it}) + 75.36(D_{2i} \times Inflation_{it}) + 4.049(D_{2i} \times Unemployment\ Rate_{it}) + 1.161 Population\ Density_{it} - 1.762 Inflation_{it} - 2.734 Unemployment\ Rate_{it}$$

$$Export_{it} = 58.67 + 8.259(D_{2i} \times Population\ Density_{it}) + 69.19(D_{2i} \times Inflation_{it}) + 3.648(D_{2i} \times Unemployment\ Rate_{it}) + 1.158 Population\ Density_{it} - 1.924 Inflation_{it} - 3.23 Unemployment\ Rate_{it}$$

$$Urbanization_{it} = 54.21 + 0.2021\left(D_{2i} \times Population\ Density_{it_{it}}\right) + 6.749(D_{2i} \times Inflation_{it}) + 0.6684(D_{2i} \times Unemployment\ Rate_{it}) - 0.006743 Population\ Density_{it} - 0.2471 Inflation_{it} + 0.3136 Unemployment\ Rate_{it}$$

$$D_{2i} = \begin{cases} 1, if\ i - th\ Country\ is\ classified\ as\ a\ developed\ country \\ 0, if\ i - th\ Country\ is\ classified\ as\ a\ growth\ country \end{cases} \quad (22)$$

Through this model, it can be seen that in developing countries, any increase in population will reduce the human development index and urbanization, but increase GNP, imports, and exports. Any increase in inflation will reduce the human development index, GNP, imports, exports, and urbanization. Any increase in the unemployment rate will reduce GNP, imports, and exports, but increase the human development index and urbanization.

In addition, it can be seen that in developed countries, each additional population will increase the human development index, GNP, imports, exports, and urbanization. Any increase in inflation will increase the human development index, GNP, imports, exports, and urbanization. Any increase in the unemployment rate will increase the human development index, GNP, imports, exports, and urbanization.

Table 5 shows that population density has a significant positive relationship with GNP, imports, and exports. There is a significant negative relationship between unemployment and GNP. In addition, the results show that the effect of population density on GNP in developed countries is positive and greater than in developing countries and the effect of the unemployment rate on GNP in developed countries is negative and greater than in developing countries.

4.4 Compare Pooled, Fix Effect, and Random Effect SUR

The best model used to model the relationship between population density, inflation, and the unemployment rate to the human development index, GNP, imports, exports, and urbanization is the model that has the smallest AIC value among the Pooled SUR, Fix Effect SUR and Random Effect SUR models.

Table 6. Compare between Pooled SUR, Fix Effect SUR, and Random Effect SUR

	Pooled SUR	Fix Effect SUR	Random Effect SUR
AIC	114936	84915.74	114845.7

Table 6 shows the best model for modeling the relationship between population density, inflation and unemployment on the human development index, GNP, export-import, and urbanization in the category of developed and developing countries is the Fix Effect SUR model because Fix Effect SUR has the smallest AIC value compared to the Pooled SUR model and Random Effect SUR. In the Fix Effect SUR model, the parameter estimators are not as significant as in the Pooled SUR model and the Random Effect SUR model. However, the Fix Effect SUR model can accommodate individual characteristics (countries) as well as the average difference in the value of the response variable between countries that are not obtained through the Pooled SUR and Random Effect SUR models.

4.5 Interpretation of The Best Panel SUR Model

Table 6 shows that the best model for modeling the relationship between population density, inflation, and unemployment to the human development index, GNP, export-import, and urbanization in developed and developing countries is the Fix Effect SUR as in equation (23).

$$
\begin{aligned}
HDI_{it} = {} & \alpha_{1i} + 0.003(D_{2i} \times Population\ Density_{it}) - 0.003(D_{2i} \times Inflation_{it}) - 0.001(D_{2i} \times Unemployment\ Rate_{it}) + 0.003 Population\ Density_{it} + 0.000080172 Inflation_{it} - 0.002 Unemployment\ Rate_{it} \\
GNP_{it} = {} & \alpha_{2i} + 35.213(D_{2i} \times Population\ Density_{it}) - 10.281(D_{2i} \times Inflation_{it}) + -17.207(D_{2i} \times Unemployment\ Rate_{it}) + 4.945 Population\ Density_{it} - 0.659 Inflation_{it} - 11.476 Unemployment\ Rate_{it} \\
Import_{it} = {} & \alpha_{3i} + 13.618(D_{2i} \times Population\ Density_{it}) + 3.732(D_{2i} \times Inflation_{it}) + -4.575(D_{2i} \times Unemployment\ Rate_{it}) + 0.93 Population\ Density_{it} - 0.06 Inflation_{it} - 2.29 Unemployment\ Rate_{it} \\
Export_{it} = {} & \alpha_{4i} + 21.095(D_{2i} \times Population\ Density_{it}) + 0.434(D_{2i} \times Inflation_{it}) - 4.804(D_{2i} \times Unemployment\ Rate_{it}) + 0.922 Population\ Density_{it} - 0.042 Inflation_{it} - 2.1 Unemployment\ Rate_{it} \\
Urbanization_{it} = {} & \alpha_{5i} + 0.075(D_{2i} \times Population\ Density_{it}) - 0.079(D_{2i} \times Inflation_{it}) - 0.01(D_{2i} \times Unemployment\ Rate_{it}) + 0.203 Population\ Density_{it} - 0.007 Inflation_{it} - 0.067 Unemployment\ Rate_{it}
\end{aligned} \quad (23)
$$

Through this model, the SUR fix effect model for developing countries (D_{2i}=0) is obtained as in equation (24).

$$
HDI_{it} = \alpha_{1i} + 0.001 Population\ Density_{it} + 0.000 Inflation_{it} - 0.002 Unemployment\ Rate_{it}
$$

$$GNP_{it} = \alpha_{2i} + 28.680 Population\ Density_{it} - 0.055 Inflation_{it} - 10.358 Unemployment\ Rate_{it}$$
$$Import_{it} = \alpha_{3i} + 2.952 Population\ Density_{it} - 0.009 Inflation_{it} - 2.195 Unemployment\ Rate_{it}$$
$$Export_{it} = \alpha_{4i} + 2.657 Population\ Density_{it} + 0.002 Inflation_{it} - 2.019 Unemployment\ Rate_{it}$$
$$Urbanization_{it} = \alpha_{5i} + 0.103 Population\ Density_{it} - 0.009 Inflation_{it} - 0.072 Unemployment\ Rate_{it} \quad (24)$$

Meanwhile, the SUR fixed effect model for developed countries (D_{2i}=1) is as in equation (25).

$$HDI_{it} = \alpha_{1i} + (0.001 + 0.005) Population\ Density_{it} + (0.000 - 0.003) Inflation_{it} + (-0.002 - 0.001) Unemployment\ Rate_{it} = \alpha_{1i} + 0.006 Population\ Density_{it} - 0.003 Inflation_{it} - 0.003 Unemployment\ Rate_{it}$$
$$GNP_{it} = \alpha_{2i} + (28.680 + 12.931) Population\ Density_{it} + (-0.055 - 9.768) Inflation_{it} + (-10.358 - 16.019) Unemployment\ Rate_{it} = \alpha_{2i} + 41.611 Population\ Density_{it} - 9.823 Inflation_{it} - 26.377 Unemployment\ Rate_{it}$$
$$Import_{it} = \alpha_{3i} + (2.952 + 11.711) Population\ Density_{it} + (-0.009 + 3.776) Inflation_{it} + (-2.195 - 4.474) Unemployment\ Rate_{it} = \alpha_{3i} + 14.663 Population\ Density_{it} + 3.767 Inflation_{it} - 6.669 Unemployment\ Rate_{it}$$
$$Export_{it} = \alpha_{4i} + (2.657 + 19.459) Population\ Density_{it} + (0.002 + 0.471) Inflation_{it} + (-2.019 - 4.718) Unemployment\ Rate_{it} = \alpha_{4i} + 22.116 Population\ Density_{it} + 0.473 Inflation_{it} - 6.737 Unemployment\ Rate_{it}$$
$$Urbanization_{it} = \alpha_{5i} + (0.103 + 0.169) Population\ Density_{it} + (-0.009 - 0.081) Inflation_{it} + (-0.072 - 0.015) Unemployment\ Rate_{it} = \alpha_{5i} + 0.272 Population\ Density_{it} - 0.900 Inflation_{it} - 0.087 Unemployment\ Rate_{it} \quad (25)$$

4.6 Discussion

Through subbab 4.4, the best model for modeling the effect of population density, inflation and the unemployment rate on the human development index, GNP, imports, exports, and urbanization is the fixed effect SUR model because it has the smallest AIC value compared to the pooled SUR and random effect SUR models. The AIC values in the three models are very large due to the large number of estimators, sample sizes, and time periods used. In the fixed effect SUR model, more parameter estimators are used compared to the pooled SUR and random effect SUR models because the SUR fix effect model pays attention to differences in the average response variables for each country (country characteristics). In addition, the fixed effect SUR model has the smallest AIC value when compared to the pooled SUR and random effect SUR models. This can happen because the SUR fix effect model is able to accommodate country characteristics (differences in the average response variables for each country) and the parameter estimators are significant as shown in Table 4, Table 5, and Table 6, which means that there is a real difference between the average response variables of one country and the other. In the SUR fix effect model, a corrected determination coefficient value of 0.9846 is obtained, which means that 98.46% of the diversity of response variable values (human development index, GNP, imports, exports, and urbanization) can be explained by predictor variables (population density, imports, and exports) and 1.54% the rest is explained by other variables not included in the model. The SUR fix effect model obtained is presented in equation (22).

Through the SUR fix effect model in developed and developing countries, it can be seen that:

- For every increase of 1 million population, the human development index in developing countries increases by 0.001, while the human development index in developed countries increases by 0.006. For every 1 million population increases, GNP in developing countries increases by US$ 26.68 billion, while GNP in developed countries increases by US$ 41,611 billion. For every 1 million increase in population, imports to developing countries increased by US$ 2,952 billion, while imports to developed countries increased by US$ 14,633 billion. For every 1 million increase in population, exports to developing countries increased by US$ 2,657 billion, while exports to developed countries increased by US$ 12,116 billion. For every increase of 1 million population, urbanization in developing countries increases by 0.103%, while urbanization in developed countries increases by 0.272%.

- With every 1% increase in the inflation rate, the human development index in developing countries does not change at all, while the human development index in developed countries decreases by 0.003. For every 1% increase in the inflation rate, GNP in developing countries decreased by US$ 0.055 billion, while GNP in developed countries decreased by US$ 9.823 billion. For every 1% increase in the inflation rate, imports to developing countries decreased by US$ 0.009 billion, but imports to developed countries increased by US$ 3.767 billion. For every 1% increase in the inflation rate, exports to developing countries increased by US$ 0.002 billion, while exports to developed countries increased by US$ 0.473 billion. For every 1% increase in the inflation rate, urbanization in developing countries decreases by 0.009%, while urbanization in developed countries decreases by 0.09%.
- If the unemployment rate increases by 1%, the human development index in developing countries will decrease by 0.002, while the human development index in developed countries will decrease by 0.003. If the unemployment rate increases by 1%, GNP in developing countries will decrease by US$ 10,358 billion, while GNP in developed countries will decrease by US$ 26,377 billion. If the unemployment rate increases by 1%, imports in developing countries will decrease by US$ 2,195 billion, while imports in developed countries will decrease by US$ 6,669 billion. If the unemployment rate increases by 1%, exports to developing countries will decrease by US$ 2,019 billion, while exports to developed countries will decrease by US$ 6,737 billion. If the unemployment rate increases by 1%, urbanization in developing countries will decrease by 0.072%, while urbanization in developed countries will decrease by 0.087%.

5 Conclusion

The Fix Effect SUR model is the best model to model the relationship between population density, inflation, and the number of unemployed on the human development index, GNP, export-import, and urbanization in the category of developed and developing countries (AIC=84915.74). Through the Fix Effect SUR model, it is found that population density has a significant positive relationship with GNP, imports, and exports. There is a significant negative relationship between unemployment and GNP. In addition, the results show that the effect of population density on GNP in developed countries is positive and greater and the effect of the unemployment rate on GNP in developed countries is negative and greater than in developing countries. Therefore, it is expected that all countries pay attention to population density and the unemployment rate because it can affect the country's GNP, imports, and exports. It is hoped that developing countries will be able to carry out population efficiency so that with an increasing population, the influence on GNP, imports, and exports will be greater like developed countries where most of the population of developed countries has good quality human resources. On the other hand, it is hoped that developing countries will be able to add decent jobs to the population to reduce the unemployment rate so that the influence on GNP, imports, and exports will be greater like in developed countries where most of the population of developed countries are already working and have decent jobs and salaries.

This study is limited to using balanced panel data and dummy variables as slope components, so it is hoped that future research can use unbalanced panel data so that all countries can be sampled and use dummy variables as intercept components to distinguish the average value of the response variable in the category developed and developing countries.

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The authors have no conflict of interest to declare.