Optimizing Failure Modes and Effects Analysis with Fuzzy Multiattribute Grey Theory and DEA

SAFIYE TURGAY
Department of Industrial Engineering
Sakarya University
54187, Esentepe Campus Serdivan-Sakarya,
TURKEY

Abstract: - The Failure Modes and Effects Analysis (FMEA) is one of the major approaches utilized for the risk analysis and risk management in many fields of human activity. The usual FMEA tools are not effective in dealing with complex systems institutional concentration of uncertainty over, and do not deliver the optimal solutions. To avoid this obstacle, the current study will fuse the successful managerial coupling of Fuzzy Multiattribute Grey Theory(FMGT) and Data Envelopment Analysis(DEA) to optimize the sequencing of FMEA process. The main strength of FMGT lies in its ability to develop/ construct an imprecise information and continual attributes which are related to failure modes and their influence on the system, while cost analysis done in DEA offers the idea of efficiency solutions that are optimal. By blending both control strategies of FMEGT and DEA within an integrated framework, FMEA analysis is able to reach greater effectiveness. Serving as a case study we do so in a series of specific tests and simulations, the approach proposed successfully analyzes critical failure modes, risk factors, and resource allocation. The results indicate that the suggested integrated way acts as a facilitator of decision-making by minimizing risk and making system wise reliability in complex industrial plants.

Key-Words: - Failure Modes and Effects Analysis (FMEA); Fuzzy Logic; Multi-attribute Evaluation; Grey Theory; Data Envelopment Analysis (DEA); Risk Assessment; Uncertainty; Optimization; Decision Making; Reliability Analysis

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1 Introduction

Failure Modes and Effects Analysis (FMEA) is a systematic and standard way of approach which is broadly used across all industries to identify, classify, and mitigating the potential failures within complex systems or process. This technique brings out the causes and outcomes of failure in a logical manner that leads to premonitory risk management and assists organizations to achieve the highest degree of reliability and safety in respect of their products or operations. In other words, conventional FMEA methods are often confronted with problems due to the fact that they don't perform adequately when immediate uncertainties and multiple attributes concerning failure modes and their impacts are available. With many systems growing to be more complex and interrelated, it is necessary to come up with more advanced ways to deal with the same issues and find the most efficient ways of solving these complexities.

In confrontation with the difficulties hauled with this possibility, the study gives the response coming from an innovative style of idea, which is Fuzzy Multiattribute Grey Theory (FMGT) in consistency with Data Envelopment Analysis (DEA) in a view to improve the FMEA process. It has attributed that FMGT is a potent tool for expressing and managing incomplete and complex information that characterized of failure modes and the consequences they can have. Through application of fuzzy logic and grey theory principles, FMGT becomes more thorough and flexible analyzing, which is the key when it is about situations where data are neither accurate nor unambiguous in general.

DEA, in conjunction to FMGT, is a great tool to maximize the efficiency by finding and perfecting the best solution when multiple goals, requirements and restrictions are always present. The decision-making process of FMEA improved by determining which failure modes are more/less efficient depending on how many risks they have [1], [2], [3].

DEA prioritizes the risk control and allocation of resources toward activities and practices that will diminish risks and costs in a well-considered manner.

Having constructed the new hybrid framework by collating the different components of FMEA and DEA, the proposed study aims to overcome the challenges, which were obvious in case of conventional FMEA techniques to effectively address complex industrial environments [4], [5], [6].
By making use of the connections between the fuzzy logic, multi-attribute assessment, grey theory, and efficiency analysis, the proposed methodology provides a universal solution to increasing the reliability, safety and efficacy of systems and processes that are subject to different failure modes and perils. Next sections of this paper will focus on theoretical basis, methodology, examples and application potential of coherent approach, which can increase the quality of decision-making and risk management procedures of various business industries {7], [8], [9].
The present research develops a gray-related fuzzy set model, where data envelopment analysis used to rank alternatives in a more objective way. Section 2 discusses the DEA, which is relevant to this work. In part 3, the gray-related technique is described. According to section 4, the ranking of failure modes by the severity assessment in a hypothetical FMEA analysis is demonstrated through multiple and conflicting criteria. Section 5 statistically analyzes the obtained results and the conclusions are given in Section 6.

2 Literature Survey

The Failure Modes and Effects Analysis (FMEA) method is an effective tool for risk assessment and management that commonly used in diverse industries, such as manufacturing and engineering, healthcare and aerospace. Several research works have been carried out over the years with the objective of optimizing and streamlining FMEA procedures so as to address the challenges presented by more and more complex systems and processes. In addition to that, the adoption of advanced methods like Fuzzy Multiattribute Grey Theory (FMGT) and Data Envelopment Analysis (DEA) drew substantial attention from research community as they are supposed to deal with the disadvantages of traditional FMEA and produce a better-optimized and robust solution for risk analysis [10], [11], [12], [13], [14], [15].

Fuzzy logic has been widely used in FMEA for uncertainties towards system behavior, failure modes, and their effects. Through the capability of the fuzzy logic system to incorporate imprecise or vague information, a seemingly more real and flexible assessment of risk factors can be achieved thus contributing to the accuracy of decision-making during the risk management processes [16], [17], [18]. Multi criteria rankings use FMEA as a methodology to include various attributes associated with failure modes and failure consequences. With elements such as severity, frequency, and visibility, holistic evaluation with multi attributes renders a useful framework for prioritizing risks and allocating resources in an efficient manner [19], [20], [21].

According to Grey Theory, which distinguishes itself by its capacity to treat limited, uncertain, or incomplete information, the analysis of complex systems and processes is enriched with very useful ideas. Accounting for uncertainties and variations of data, which are the base Grey Theory, is a way to improve the robustness and reliability of FMEA approaches whose reliability is especially critical where exact information may be unavailable or difficult to obtain. Combining the Grey Theory with the FMEA enables the approach to analyse risk factors within a broader context and regard their interrelationships as a whole [22], [23], [24].

Classical propositional extraction is powered by mean link coefficients. Then it becomes undecipherable which ones are important and we just simply set every single link equal with each other. Here, we can use the method of each link objectively receive a weighed grade. In this regard, data envelopment analysis (DEA) is advocating an approach that is based on data as an alternative solution to this problem [25] and [26]. Being that done, it seems that recent literature hit a high spot in integrating FMGT, Grey Theory and DEA with FMEA to end with solving the problem of complicatedness and vagueness. The studies have even taken it a step further to analyze the theoretical underpinnings, pioneer new methods, conduct case studies and even apply the classifications in different industries [27], [28], [29]. By utilizing the affinity between fuzzy logic and multi-attribute decision making, two-tiered thinking, and efficiency analysis, researchers are intent on producing more robust and holistic frameworks for risk management and reliability and performance improvement.

3 Methodology

The actual scenario happens only when the mix of quantitative information, such as parameters analysis in FMEA evaluated. Trade-offs among multi-dimensions are involved and parameter estimates usually need to modify using some degree of information. Although there have been some uncertainties identified which calls for the use of Gray analysis tool in FMEA, interval fuzzy data representation seems the best tool on which to use. Not only do parameters of available choices being pulled from may contain uncertainty but also the attributes importance. The weights made after using DEA with MCDM approach thought to be reliable alternative in order to finding criteria of achieving goals of each index. This article offers a specific approach of gray-related fuzzy sets system using data envelopment analysis to provide a better comparative result by ranking alternatives more accurately. The objective is to carry out a parameter investigation, which will help to achieve the FMEA analysis with the proposed method. The emphasis made on selecting the alternative that most likely yields the desired outcome, if the decision maker expresses one, from the available choices. This research aimed to forecast the types of errors and their impact, error risks, applying possible means to avoid error-prone situations and the hybridization of these results in order to produce quality products. At the stage of estimating the incidence situations of failure events in the situation of uncertainty needs to be taken into account. Fuzzy grey multi-attribute-theory was used as an analytical tool by analyzing such an uncertain event. The process of performing an suggested model steps summarized in Figure 1.

3.1 Mathematical Model

The assumption is that F is the group of failure modes that addressed by the Failure Modes and Effects Analysis (FMEA). Therefore, E would correspond to the collection of effects involved. Each failure mode f∈F characterized by a few attributes, with them being severity(S_j), occurrence (O_j) and detectability(D_j), being the most relevant ones. On the contrary, intensity of every emotion type e∈E is shown as (S_e), while it's occurrence as (O_e) [30], [31], [32].

3.1.1. Fuzzy Multi-attribute Grey Theory (FMGT)

Model the fuzziness of failure modes attributes through fuzzy logic. Identify the fuzzy sets S_j^*, O_j^*, D_j^* and S_e^*, O_e^* that encompass the linguistic variables of severity, occurrence, and detectability. Use the method of fuzzy logic to find the membership grades of failure modes and effects for each attribute [33], [34].

$$\mu S_j^*(x), \mu O_j^*(x), \mu D_j^*(x), \mu S_e^*(x) \mu O_e^*(x) \tag{1}$$

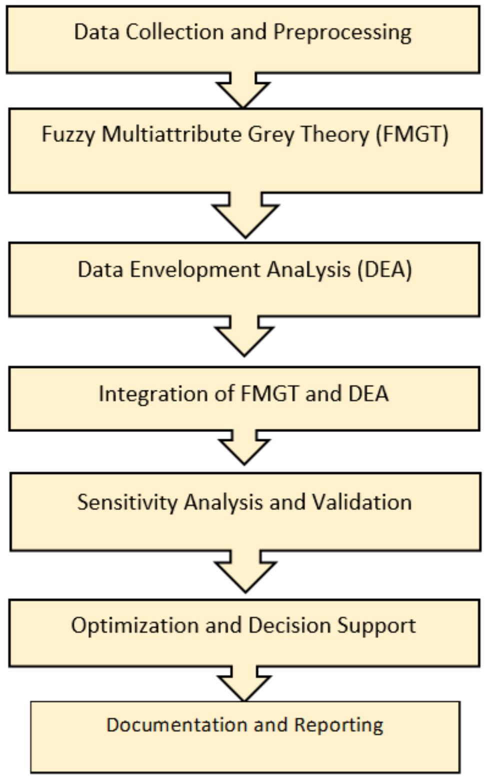

Figure 1 The proposed model steps

3.1.2. Data Envelopment Analysis (DEA)

Define input and output variables as depending on the identified attributes in the FMEA process. For X to stand for all the input variables such as severity (S_j^*), occurrence (O_j^*), and detectability (D_j^*), and Y as the set of output variables which consist of severity (S_e^*) and occurrence (O_e^*). Formulate a DEA model to assess the efficiency (θ_j) of every failure mode f∈F in converting inputs to outputs

$$\max \theta_j \tag{2}$$

Subject to:

$$\textstyle\sum_{x\in X} \lambda_{fx} S_{fx}^* \leq S_e^* \tag{3}$$

$$\textstyle\sum_{x\in X} \lambda_{fx} O_{fx}^* \leq O_e^* \tag{4}$$

$$\textstyle\sum_{x\in X} \lambda_{fx} D_{fx}^* \leq D_e^* \tag{5}$$

$$\textstyle\sum_{x\in X} \lambda_{fx} = 1 \tag{6}$$

$$\lambda_{fx} \geq 0,\ \forall z \in X \tag{7}$$

In which λ_{fx} means the weight assigned to each input variable used for determining the efficiency of failure mode f, $S^*_{fx}, O^*_{fx}, D^*_{fx}$ stands for the membership degree of severity, occurrence, and detectability of failure mode f corresponding to assigned attributes.

3.1.3 Integration of FMGT and DEA

Besides FMGTs use, integrate DEA results which will lead to the establishment of priorities in treating risk factors and their optimal solution. Simulate fuzzy infiltration systems or grey relational analysis to perform joint fuzzy multi attribute assessment of failure modes and the efficiency estimation delivered by DEA. Develop clear, implementable solutions and recommendations for the decision-makers, who will need to consider the overall picture of the risk factors including their interconnections.

$$\text{Integrated Scoref} = \alpha.\text{FMGT. Scorej} + (1-\alpha).\text{DEA Efficiency}_j \quad (8)$$

Which α represents the rating coefficient established in the assigned assessment (a mixture of FMGT and DEA evaluation ratios), Scoref_{Integrated} demonstrates the overall score for failure mode f. Modify α for an adaptable multiattribute fuzzy evaluation and standard analysis, or efficiency, in the FMEA optimization procedure..

The proposed mathematical model combines Fuzzy Multicriteria Grey theory and Data Envelopment Analysis to optimize Failure Modes an Effects Analysis, which provides a holistic approach of risk assessment as well as management for complex systems and processes.

FMEA is a technique perform risk analysis and error avoidance in planning and development procedure of products and manufacturing. In other words, the mission of these stages is to doubt and, if needed, upgrade the quality of the paintings. These stages form the groundwork for the subject's realism and accurate depiction. And by no means is it an easy process. It demands for each possible type of error in the system to be inspected, to determine its consequences, as well as effects on the system. We then need to classify the errors according to their contribution into the error rate. This research was aimed to forecast the types of errors and their impact, error risks, applying possible means to avoid error-prone situations and the hybridization of these results in order to produce quality products. At the stage of estimating the incidence situations of failure events in the situation of uncertainty needs to be taken into account. Fuzzy grey multi-attribute-theory was used as an analytical tool by analyzing such an uncertain event.

$$value_j = \sum_i w_i \times u(x_{ij}) \quad (9)$$

where for each alternative j, the relative weights of each attribute w_i denoted by the value $u(x_{ij})$ are measured over the given attributes i. These relative weights are capable of expressing the relative importance and, if the scores are not standardized, also the relative scale.

In fuzzy domains both w_i and $u(x_{ij})$ may be uncertain. Multi-attribute decision-making problem with interval numbers has m feasible options: $X_1,X_2,...,X_m$ and n indices: $G_1,G_2,...,G_n$ and the index value G_j of the j-th index of option X_i is an interval number, then i=1,2,...,m and j=1,2,...,n. Weights can also be expressed in terms of the number of steps which interval w_i is in. The problem that involves multiple attributes with interval numbers referred to as interval-valued indexed multi-attribute decision-making problem.

In terms of the methodology, Step 1 and 2 work on the data preparation and Step 3 deals with the scale differences. Step 4 is excellent as the vector. Step 5 computes the link coefficients using the discriminant coefficient selected by the decision maker. The optimization model in Step 6 delivers the objective weights that further used in Step 7 to sort the alternatives.

Step 1: Build interval numbers' index number matrix B.

$$X = \begin{bmatrix} (x^L_{11}, x^U_{11}) & (x^L_{12}, x^U_{12}) & \cdots & (x^L_{1n}, x^U_{1n}) \\ (x^L_{21}, x^U_{21}) & (x^L_{22}, x^U_{22}) & \cdots & (x^L_{2n}, x^U_{2n}) \\ . & . & & . \\ . & . & \cdots & . \\ . & . & & . \\ (x^L_{m1}, x^U_{m1}) & (x^L_{m2}, x^U_{m2}) & \cdots & (x^L_{mn}, x^U_{mn}) \end{bmatrix} \quad (10)$$

Step 2: Transform all "opposing indexes" into positive indexes.

A larger value of an index is more preferable such an index called a positive index. When a smaller value is better, the index referred to as inverse index. If -th index Gj is inverse, we can convert the opposite indexes into positive numbers.

$$\left[a^L_{ij}, a^U_{ij}\right] = \left[-x^L_{ij}, -x^U_{ij}\right], \quad i = 1, 2, ..., m$$

(11)

Moreover, we consider only positive indices.

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Step 3: Standardize the decision matrix - Standardize the interval numbers by the index number to get the decision matrix

$$R=\left[r_{ij}^{L},r_{ij}^{U}\right] \quad (12).$$

The column vectors of the decision matrix A with interval-valued indices A1,A2,...,An considered the element of the standardization decision matrix is defined $R=\left[r_{ij}^{L},r_{ij}^{U}\right]$ as follows:

$$\left[r_{ij}^{L},r_{ij}^{U}\right]=\frac{\left[x_{ij}^{L},x_{ij}^{U}\right]}{\left\|X_{j}\right\|},\quad i=1,2,...,m,\quad j=1,2,..,n \quad (13)$$

Here $\|X_j\|$ is the arithmetic mean of each column of the decision matrix $\|A_j\|$. But without losing generality, we could select the biggest number for every column in A if 0 is the minimum possible score of a_{ij}.

Step 4: Figure out the reference number sequence. The element reference number sequence consists of the effective weighted interval number index of each plan.4.

z={1,2,...,n} is called the reference number sequence.

$$U_{0}=\left(\left[u_{0}^{L}(1),u_{i0j}^{U}(1)\right],\left[u_{0}^{L}(2),u_{i0j}^{U}(2)\right],...,\left[u_{0}^{L}(n),u_{i0j}^{U}(n)\right]\right) \quad (14)$$

is called a reference number sequence if

$u_{0}^{L}(j)=\max_{1\le i\le m}r_{ij}^{L},u_{0}^{U}(j)=\max_{1\le i\le m}r_{ij}^{U}$, j=1,2,…,n (15).

Step 5: Make the calculation of the connection between the interval number standardizing sequence and the reference number sequence. Then, do the following: find the coupling coefficient $\xi_i(k)$ between the array of interval number standardizing index values of each plan and the reference number array. $\xi_i(k)$ formula:

$$U_{i}=\left(\left[c_{i1}^{L},c_{i1}^{U}\right],\left[c_{i2}^{L},c_{i2}^{U}\right],...,\left[c_{in}^{L},c_{in}^{U}\right]\right) \quad (16)$$

and reference number sequence

$$U_{0}=\left(\left[u_{0}^{L}(1),u_{0}^{U}(1)\right],\left[u_{0}^{L}(2),u_{0}^{U}(2)\right],...,\left[u_{0}^{L}(n),u_{0}^{U}(n)\right]\right) \quad (17).$$

The formula of $\xi_i(k)$ is:

$$\xi_{i}(k)=\frac{\min\limits_{i}\min\limits_{k}\left\|\left[u_{0}^{L}(k),u_{0}^{U}(k)\right]-\left[c_{ik}^{L},c_{ik}^{U}\right]\right\|_{k}+\rho\max\limits_{i}\max\limits_{k}\left\|\left[u_{0}^{L}(k),u_{0}^{U}(k)\right]-\left[c_{ik}^{L},c_{ik}^{U}\right]\right\|}{\left\|\left[u_{0}^{L}(k),u_{0}^{U}(k)\right]-\left[c_{ik}^{L},c_{ik}^{U}\right]\right\|_{k}+\rho\max\limits_{i}\max\limits_{k}\left\|\left[u_{0}^{L}(k),u_{0}^{U}(k)\right]-\left[c_{ik}^{L},c_{ik}^{U}\right]\right\|}$$

$$\left(\left[u_{0}^{L}(1),u_{0}^{U}(1)\right],\left[u_{0}^{L}(2),u_{0}^{U}(2)\right],...,\left[u_{0}^{L}(n),u_{0}^{U}(n)\right]\right) \quad (18)$$

Here ρ∈[0,1] may be called the discrimination coefficient. The lower ρ, the better the distinguishing traits. In fact, the value of ρ (ρ) can flexibility adapt to the practical situation.

The classical gray-related parameter rho (ρ) calculated as a height might be formally considered as an emphasis ratio. The sensitivity of the trend after which the results were first compiled is discussed in the current study [35], [36]. It highlights the fact that different ervice scores can give rise to different ranks [37], [38]. However, all shortcomings have a minimal impact only on global rank aggregation (see Figure 6 in [39], [40]. Our work has the leading position and creates some classical gray parameters based on the 0.5.

Once we have got the link coefficient $\xi_i(k)$ between the standardizing amount of all plans and the reference group numbers, we are going to workout the corresponding weight for the link coefficient $\xi_i(k)$.

Step 6. DEA-based gray-related analysis.

The DEA can be a tool for evaluating the performance of alternatives in terms of varied weights that ultimately weigh every element in each alternative. An example: the magnitude: w_j is unclear and not necessarily specific. DEA is proposed to get the set of weight that is optimal by maximizing the coupling coefficient $\xi_i(k)$ between the sequence of the index value that is standardized and made the number range of each plan close to the reference number sequence.

$$\theta_{0}=Max\quad\sum_{k=1}^{n}w_{k}\xi_{0}(k) \quad (19)$$

$$\sum_{k=1}^{n}w_{k}\xi_{0}(k)\le 1\quad i=1,2,...,m \quad (20)$$

{weight normalization constraint}

$$w_{k}\ge 0 \quad (21)$$

Just like in most of the classical evaluator studies, for example, [35] and [37], where the weights optimized in order to help in the calculating of efficiency values for the alternative measures, we can also use objective weights. It is well-known that weights calculated following the same definition as shown in equations (19-20) were proved to be correct by a controlled study.

In this sense, our version of the DEA differs from the standard DEA, where the criteria are put into inputs and outputs, as the "inverse index" is first transformed into a positive index in Step 2. Optional. Note that when there is no weight normalization constraint, the DEA model will yield more efficient than when there is a weight normalization constraint. The alternative score is finally obtained as an objective function dependent on the value of θ (0∈[1,2,...,m]). $\xi_i(k)$ being the utility data, came from Step 2, where we transformed the "contrarian index" into a positive index. Consequently, all alternatives considered and chosen from the ranking DEA score and the well-established option apparently selected with the largest score.

There are the different types of these weight normalization constraints. To illustrate this, assume that there an inequality constraint that makes the weight vector to belong to the unit simplex. This results in the following in equations (22-25),

$$\theta_0 = Max \quad \sum_{k=1}^{n} w_k \xi_0(k) \tag{22}$$

$$\sum_{k=1}^{n} w_k \xi_0(k) \leq 1 \quad i = 1, 2, ..., m \tag{23}$$

$$\sum_{k=1}^{n} w_k = 1 \quad i = 1, 2, ..., m \tag{24}$$

$$w_k \geq 0 \tag{25}$$

The normalisation musts always result into too many efficient (non-dominated) which are usually difficult to interpret because it does not have much flexibility of weights assignment. This solution is a plain measure of reducing the number of weight limits. Furthermore, model (7) omits all weights with a zero value. To solve this problem, it is necessary to particularize an assurance region scheme with the leader's decision [28]. Adopting the assurance region scheme like in DEA studies, the following model (8) can efficiently be developed so that the applicable as cone constraints.

$$\theta_0 = Max \quad \sum_{k=1}^{n} w_k \xi_0(k) \tag{26}$$

$$\sum_{k=1}^{n} w_k \xi_0(k) \leq 1 \quad i = 1, 2, ..., m \tag{27}$$

$$w = [w_k] \in P, \quad k = 1, 2, ..., n \tag{28}$$

where $P \subset E_{+}^{m}$ is a closed convex cone, and $IntP \neq \varnothing$ where is a closed convex cone.

The cone constraint in (20), defined by $w=\{w_k\}\in P$, can in fact be reduced to (21) when P is a polyhedral cone given by its "intersection-form",

$$a_k w_1 \leq w_k \leq \beta_k w_1, \quad k = 2, 3, ..., n \tag{29}$$

Step 7: Write down the needs from the most varied to the least varied and choose the plan of action that fits the most variable. The plan that yields rt for a given rp is called the optimal plan provided $r_t \geq \max_{1 \leq i \leq m} r_i$.

The Failure Mode and Effects Analysis technique gives us an ability to assess the parameters of probability, severity and detectability for each fault that is analyzed and facts are prioritized. FMEA can, on the other hand, be referred to as a description of the work of an engineer who would have discovered each of the recurrent problems using past experience and events such as designing systems. The Main Mission of this FMEA Technique is to evaluate the types of errors which may arise in the product and in its process, the effect of these errors on customers, and their risk situations. The objective of both process and product control in the manufacturing area is to avoid the occurrence of many process errors before they happen and to prevent their future occurrence. Assess the strategic products' design elements while considering the manufacturing and assembly processes that will bring the product characteristics to the accurate level of customer expectations.
All risk categories ranked to ensure failure type prioritized according to the risk of each one of them. Action plan encompassing the removal of some error types that result due to system design strategy, is a very useful measure.

4. Probabilistic and Design FMEA

Then, proper FMEA of Design done to avoid identifying any of the product functional performance-related flaws at the production phase. It provides design FMEAs to be completed regarded the types of defects likely to occur during the service and manufacturing phases of the product because design errors are included. Thus, handling of design issues is carried out. It guarantees that particularly when new products design or when amends to existing products design are needed the process is properly accomplished. It reveals cases of design flaws, what effects the errors have and what should be done to minimize them.

The kind of the Design FMEA method will be taught in this seminar along with its applications to the employees of the organizations, which would be carried out for design activities of the teams using systematic approaches.
- An ordered list of failure types with respect to the risk priority number,
- Classification of prioritized risks with/or set of crucial or possible blunders.
- To avoid input of the sort of error, the following anti-error precautions should be carried out:
- Parameter list possible for the list of items in check and detection of failures.

- A provision through which we compile actions against defect types that are critical and risky. makes possible errors of the risky type sneaking in examination.
- Unveils hypothetical system defects and interdependencies with subsystems. Use our automated writing assistant to find out more!

4.1. Process FMEA

Predict analyze manufacturing as well as assembly process This category targets the types of defect or assembly problems caused by either process or assembly inadequacies. The outcome of the process FMEA is also useful as the method of process improvement also the due to of the made improvements in the process. A list of major failure modes descriptions with their values displayed as ranking. It is recommended to come up with a list of critical error features about it.

Here are several of possible measures suggested in the table below for vital characteristics:

- An inventory of possible actions, to get rid of perfective faults, to reduce their frequency and to enhance their rate of detection.

- The enabling function is identified in the creation of preventative regulatory measures for the process inadequacies.
- Enables to create and control plans through identification the most important activities.
- Rank action for maintenance optimization.
- Assists installing the procedures of the manufacturing or assembly processes.
- It facilitates making a document for the goal of amendments to which they are made.

5 Case Study

In this study, based on the information received from experts, the types of faults encountered in cable manufacturing were listed. During this listing, the following criteria and their importance levels were taken into consideration: customer; fault type, fault cause, fault effect, available controllers. Failure Mode and Effects Analysis is an analysis technique that aims to predict failure risks and prevent failure before it occurs.

The methods in the methodology were applied in the FMEA analysis process. Multiple attribute data are given in Table 1.

There are four criteria.

C1 Cost - billions of dollars of cost must be minimized.

C2 Error Severity - must be minimized.

C3 Risk-The risk of producing a defective product must be minimized.

C4 Emergency - Emergency warnings from the areas where the product is used, factors affecting human life should be identified and such warning situations should be minimized.

Table 1. Data for FMEA fault types and effects in Cablo manufacturing company

	Cost	Importance Degree	Risk	Emergency Situation
S1. Connector error	60	40	Low	High
S2. Pinning Error	60	40	Medium	Medium
S3. Terminal Error	70	80	Low	Very high
S4. Common Errors	70	80	High	Medium
S5. Retouching Error	60	70	High	High
S6. Panel Error	50	30	Medium	Medium
S7. Socket Error	90	130	Very high	Very low
S8.Cable Construction Error	80	120	Very low	Very low
S9. Adapter Error	80	70	Medium	Low]
S10. Marking Error	90	100	Very high	Very low

Improvement- Improvement should be maximized in order to achieve the best quality, taking into account the cost factor during the production of the product.

Table 2. Table 2. Fuzzy Data for FMEA fault types and effects data.

	Cost	Error Type-Importance Degree	Risk	Emergency S
S1. Connector error	[0.50–0.85]	[0.40–0.60]	[0.10–0.30]	[0.70–0.90]
S2. Pinning Error	[0.50–0.85]	[0.20–0.40]	[0.40–0.60]	[0.40–0.60]
S3. Terminal Error	[0.50–0.70]	[0.70–0.95]	[0.10–0.30]	[0.90–1.00]
S4. Common Errors	[0.50–0.70]	[0.70–0.95]	[0.70–0.90]	[0.40–0.60]
S5. Retouching Error	[0.50–0.70]	[0.50–0.70]	[0.70–0.90]	[0.70–0.90]
S6. Panel Error	[0.40–0.60]	[0.10–0.40]	[0.40–0.60]	[0.40–0.60]
S7. Socket Error	[0.85–0.95]	[0.90–1.00]	[0.90–1.00]	[0.00–0.10]
S8. Cable Construction Error	[0.70–0.90]	[0.70–0.90]	[0.00–0.10]	[0.00–0.10]
S9. Adapter Error	[0.70–0.90]	[0.50–0.85]	[0.40–0.60]	[0.10–0.30]
S10. Marking Error	[0.85–1.00]	[0.70–0.90]	[0.90–1.00]	[0.00–0.10]

The raw data is expressed in fuzzy intervals as shown in Table 2. These data are standardized (fulfilling Step 3 of the previous section), with attractive values higher than low. Each criterion is now on a common 0-1 scale, where 0 represents the worst imaginable gain in a criterion and 1.00 represents the best possible gain.

The total value for each alternative site will be the sum product of the time performance of the weights. DEA used to identify non-dominated (efficient) alternatives by making these weight variables and optimizing the total value for each alternative site. Conversely, fuzzy weights assigned by the decision maker or decision-making group reflecting the relative importance of the criteria. Then DEA to identify used in a set of efficient sites.

The next step of the gray-related method (Step 4) is to obtain reference number sequences based on the optimal weighted interval number value for each alternative. Since we assumed in the initial analysis that all weights are in the range [0, 1], the reference number vector will be the maximum left range value over all alternatives for each criterion and the maximum right range value for each criterion. Table 3 gives this vector reflecting the range of value probabilities.

Table 3 Reference number vector

	Cost	Importance Level	Risk	Emergency Situation
Max(Min)	0.85	0.90	0.90	0.90
Max(Max)	1.00	1.00	1.00	1.00

Distances defined as the maximum value between each interval value and the extreme values generated in the reference number vector. Table 4 shows the distances calculated according to the alternatives. The maximum distance to the ideal for each alternative defined as the largest distance calculation in each cell of Table 4. These maxima shown in Table 5.

Table 4. Distances from fault types to reference number vector

	Cost		Importance Level		Risk		Emergency Situation				
	d1+	d1-	d2+	d2-	d3+	d3-	d4+	d4-	d*+	d*-	C*+
S1	0,519577177	0,47195	0,516639902	0,469349823	0,490086185	0,446	0,50708875	0,460893148	0,508216	0,461897449	0,476127
S2	0,332912158	0,47297	0,331030138	0,470358738	0,314016197	0,447	0,324910364	0,461882938	0,325633	0,462889485	0,587034
S3	0,687583677	0,44558	0,683696627	0,443499314	0,648556703	0,425	0,671057087	0,436754849	0,672549	0,437583892	0,394172
S4	0,477412213	0,33913	0,474713305	0,337791153	0,450314487	0,326	0,465937252	0,333453805	0,466973	0,334001941	0,416994
S5	0,610945909	0,33785	0,607492108	0,336521845	0,576268864	0,325	0,596261366	0,332215117	0,597587	0,332759649	0,357673
S6	0,326506313	0,54981	0,324660506	0,547403096	0,307973945	0,526	0,318658488	0,539591874	0,319367	0,540562778	0,628613
S7	0,599008233	0,45982	0,595621917	0,457222971	0,565008765	0,434	0,58461062	0,448770263	0,585911	0,44976816	0,434274
S8	0,477366418	0,46597	0,474667768	0,463349799	0,450271291	0,44	0,465892557	0,454839603	0,466929	0,455845821	0,493995
S9	0,306955751	0,4193	0,305220468	0,416948863	0,289533066	0,396	0,299577839	0,40930269	0,300244	0,410207072	0,57739
S10	0,409814185	0,49733	0,407497422	0,494523385	0,386553296	0,469	0,399963993	0,485381102	0,400853	0,486460408	0,548239

The minimum of the MIN column is 0.00 and the maximum of the MAX column is 0.95. Then, in Step 5 of the gray-related method, the link distances are calculated (results in Table 6). The link distances depend on the parameter ρ, which here is 0.80.

Table 5 Maximum distances

	Cost	Importance Level	Risk	Emergency Situation	MIN	MAX
S1	0,519577177	0,5166399	0,490086185	0,50708875	0,490086185	0,5195772
S2	0,472966108	0,47035874	0,446805083	0,461882938	0,446805083	0,4729661
S3	0,687583677	0,68369663	0,648556703	0,671057087	0,648556703	0,6875837
S4	0,477412213	0,4747133	0,450314487	0,465937252	0,450314487	0,4774122
S5	0,610945909	0,60749211	0,576268864	0,596261366	0,576268864	0,6109459
S6	0,549811945	0,5474031	0,525772334	0,539591874	0,525772334	0,5498119
S7	0,599008233	0,59562192	0,565008765	0,58461062	0,565008765	0,5990082
S8	0,477366418	0,47466777	0,450271291	0,465892557	0,450271291	0,4773664
S9	0,419300528	0,41694886	0,395694281	0,40930269	0,395694281	0,4193005
S10	0,49733492	0,49452339	0,46910639	0,485381102	0,46910639	0,4973349

Table 6 Connection distances

	Cost	Importance Level	Risk	Emergency Situation
S1	0,519577177	0,5166399	0,490086185	0,50708875
S2	0,472966108	0,47035874	0,446805083	0,461882938
S3	0,687583677	0,68369663	0,648556703	0,671057087
S4	0,477412213	0,4747133	0,450314487	0,465937252
S5	0,610945909	0,60749211	0,576268864	0,596261366
S6	0,549811945	0,5474031	0,525772334	0,539591874
S7	0,599008233	0,59562192	0,565008765	0,58461062
S8	0,477366418	0,47466777	0,450271291	0,465892557
S9	0,419300528	0,41694886	0,395694281	0,40930269
S10	0,49733492	0,49452339	0,46910639	0,485381102
maks	0,687583677	0,68369663	0,648556703	0,671057087

According to the results of the analysis in Table 7, the fault types with the highest risk factor can be listed as S3- Terminal Fault, S5- Retouching Fault, S7- Socket Fault, S6- Panel Fault. The faults with the lowest risk level are S9- Adapter Fault, S2- Pinning Fault, S4- General Faults.

Table 7. DEA solutions for all weights.

	Cost	Importance Level	Risk	Emergency Situation	MIN	MAX	
S1	0,519577177	0,5166399	0,490086185	0,50708875	0,490086185	0,5195772	0,50834800
S2	0,472966108	0,47035874	0,446805083	0,461882938	0,446805083	0,4729661	0,46300321
S3	0,687583677	0,68369663	0,648556703	0,671057087	0,648556703	0,6875837	0,67272352
S4	0,477412213	0,4747133	0,450314487	0,465937252	0,450314487	0,4774122	0,46709431
S5	0,610945909	0,60749211	0,576268864	0,596261366	0,576268864	0,6109459	0,59774206
S6	0,549811945	0,5474031	0,525772334	0,539591874	0,525772334	0,5498119	0,54064481
S7	0,599008233	0,59562192	0,565008765	0,58461062	0,565008765	0,5990082	0,58606238
S8	0,477366418	0,47466777	0,450271291	0,465892557	0,450271291	0,4773664	0,46704950
S9	0,419300528	0,41694886	0,395694281	0,40930269	0,395694281	0,4193005	0,41031159
S10	0,49733492	0,49452339	0,46910639	0,485381102	0,46910639	0,4973349	0,48658644
maks	**0,687583677**	**0,68369663**	**0,648556703**	**0,671057087**			

These averages comprised the basis of the grey-related method, which was designed to analyze the performance of alternatives in accordance with the TOPSIS approach, which measures the distance of alternatives from an inferior solution and from an ideal one. The DEA score is determined through the Equation (10-28) above which considers non-dominated solutions. The solution of the DEA model is provided twelve times, the number of alternatives corresponding to the one that is evaluated in each case.

Here $\xi_i(k)$ is utility information explicitly indicate in step 2, we changed the contrarian index to the positive index. Identified efficient collectives can be ranked and the most effective ones can be chosen. On the other hand, the criteria to settle a preorder relation within the set of non-dominated alternatives are equally baseless. As far as regard to relative weights is concerned, one should give more attention to the preferred choices. Let us assume that the decision maker's attitude toward quantifying the criterion just boils down to the following cone ratio form: The calculated output weights and the resulting certainty region DEA scores are shown in Table 8. In the cost-benefit analysis, the optimal weights are now all positive showing that all four of the standards are employed in the evaluation process. In Table 9, the DEA scores of the identified non-dominated solutions had been used, the ranking order of the sites would have been: S3-S5-S6-S7; S1-Retouch Error, S2-Socket Error, S4-Panel Error. Table 10 presents average rates of the all columns. Hence, those values are located in a larger area of figures with a higher sensitivity. On top of this, the average approach cannot account for the reality of the scores which is what the model does rationalizing the scores values. The results of the scores present no cause for alarm as they stem from a different weighting formula compared to the averages.

Table 8. Weight values of the criteria considered as a result of the analysis

	Cost	Importance Level	Risk	Emergency Situation
W_j	0,261032934	0,25808993	0,232241642	0,248635492

Table 9 Input and output values for error types

		Input Measures		Output Measures	
	Fault Types	Cost	Importance Level	Risk	Emergency Situation
S1	1. Connector failure	0,75	0,4	0,210526316	0,842105263
S2	2. Pinning Error	0,75	0,24	0,526315789	0,526315789
S3	3.Terminal Error	0,66666667	0,66	0,210526316	1
S4	4.General Errors	0,66666667	0,66	0,842105263	0,526315789
S5	5.Retouch Error	0,66666667	0,48	0,842105263	0,842105263
S6	6. Panel Error	0,55555556	0,2	0,526315789	0,526315789
S7	7.Socket Error	1	1	1	0,053157895
S8	8.Cable Construction Error	0,88888889	0,92	0,842105263	0,053157895
S9	9.Adapter Error	0,88888889	0,54	0,710526316	0,210526316
S10	10.Marking Error	0,91666667	0,64	0,842105263	0,053157895

Table 10. Result ranking values

INPUT SCORE	#		OUTPUT SCORE
0,268966	S1	1. Connector failure	0,239898681 (5)
0,244855	S2	2. Pinning Error	0,218607238 (9)
0,355937	S3	3.Terminal Error	0,317470483 (1)
0,247139	S4	4.General Errors	0,220430314 (7)
0,316265	S5	5.Retouch Error	0,282085365 (2)
0,284798	S6	6. Panel Error	0,256267921 (4)
0,310085	S7	7.Socket Error	0,276573512 (3)
0,247115	S8	8.Cable Construction Error	0,220409169 (8)
0,217062	S9	9.Adapter Error	0,193663865 (10)
0,257452	S10	10.Marking Error	0,229629007 (6)

Preference reflected through the criteria specified by the ratio range. With the average approach, the weights have no easily visible meaning.

6 Conclusion

The combination of Fuzzy Multi-attribute Grey Theory (FMGT) with FMEA via Data Envelopment Analysis (DEA) shows to be an effective method to better tackle risks in systems where these are too complex. By the application of sophisticated techniques and methods, this integrated approach delivers important information on pinpointing critical failures, identifying the risk mitigation strategies, and concerning resource allocation. Gray related analysis gives tool to add uncertainties to the analysis. Through data envelopment analysis, the ideal non–dominated solution objectively found. In this paper, we provide a way to base ranking procedure using mean functional scores provided by gray related analysis and DEA as a foundation. The objective of the suggested DEA is to be different from the classical DEA where the criteria are grouped into inputs and outputs. We conducted a "reverse indexing procedure" in the previous step which converted the "opposing index" into a positive index. In the regular functional approach precisely the leading sense of weights is unclear. In the DEA approach, the decision maker's sentiment towards the criteria can be included by using the preference cone ratio in the model, and hence generating realistic benchmarks to ensure that the realism in DEA scores is maintained in the selection process. Different approaches to DEA [31], like super-efficient DEA, can be applied to achieve that purpose. Conversely, in some cases, when some inputs are near zero, calculated by Thrall [32], the application of DEA can be misleading due to the practicality and sensitivity problems as implemented in the linkage coefficient model ξi(k) whose values ranged between 0 and . The formulas we use below show some of the characteristics of the method. According to the results given in Table 7, 10 alternatives out of the dozen under survey not be dominated.

Despite this, the dominated solutions can still be the highest in a number of metrics as shown in the Table 8, but cannot be the first. In this case, the solutions, that are worse than the other solutions, still can rank high.

Industrial case study, which is used in this type analyses, is a real practical example of integrated approach application in automotive production. Through applying Fault Mode and Effects Analysis (FMGT) and Failure Modes, Effects, and Criticality Analysis (FMECA) to the system, critical failure modes were detected, best risk mitigation methodologies were decided, and resources were allocated in an efficient manner which has resulted in the enhancement of the reliability and performance of the system.

Finally, at the end, this approach that takes into account FMEA, FMGT, and DEA advantage concern organizations of different sectors. It is through refinement of the decision-making mechanism, allocation of resources judiciously and in the long run minimization of risks that the approach has therein become a useful weapon in the struggling for operational superiority and sustained competitive advantage in the modern cutthroat business world. The technologies and methods will keep on growing. From this, we can predict much more exciting findings in the future related to extending risk management practices and equipment safety performances.

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References:

[1] Constantin Zopounidis, C., Galariotis, E., Doumpos, M., Sarri, S., Andriosopoulos, K., Multiple criteria decision aiding for finance: An updated bibliographic survey, *European Journal of Operational Research*, Volume 247, Issue 2, 2015, Pages 339-348.

[2] Zeng, Y., Guo, W., Wang, H., Zhang, F., A two-stage evaluation and optimization method for renewable energy development based on data envelopment analysis, *Applied Energy*, Volume 262, 2020, 114363

[3] Wu, D.D., Olson, D.L., Fuzzy multiattribute grey related analysis using DEA, *Computers & Mathematics with Applications*, Volume 60, Issue 1, 2010, Pages 166-174

[4] Chai, J., Liu, J.N.K., Ngai, E.W.T., Application of decision-making techniques in supplier selection: A systematic review of literature, *Expert Systems with Applications*, Volume 40, Issue 10, 2013, Pages 3872-3885.

[5] Chang, K.H., Generalized multi-attribute failure mode analysis, *Neurocomputing*, Volume 175, Part A,2016,Pages 90-100,

[6] Turgay, S., Dinçer, E., Kazan, S., Navigating Uncertainty: A Comprehensive Approach to Risk Management in R&D Projects with the Gravity Search Algorithm Based MCDM. Industrial Engineering and Innovation Management (2023) Vol. 6: 95-103.

[7] Zhou, F., Chen, T.Y., A hybrid group decision-making approach involving Pythagorean fuzzy uncertainty for green supplier selection,

International Journal of Production Economics, Volume 261, 2023, 108875,

[8] Geng, X., Qiu, H., Gong, X., An extended 2-tuple linguistic DEA for solving MAGDM problems considering the influence relationships among attributes, *Computers & Industrial Engineering* (2017), 112, 135-148.

[9] Liu, Y., Yang, Z., He, J., Li, G., Zhong, Y., A new approach to failure mode and effect analysis under linguistic Z-number: A case study of CNC tool holders, Engineering Failure Analysis, Volume 154, 2023, 107688

[10] Mandal, P., Samanta, S., Pal, M.,Failure mode and effects analysis in consensus-based GDM for surface-guided deep inspiration breath-hold breast radiotherapy for breast cancer under the framework of linguistic Z-number, Information Sciences, Volume 658, 2024, 120016

[11] Hashemi, S.H., Karimi, A., Tavana, M., An integrated green supplier selection approach with analytic network process and improved Grey relational analysis, *International Journal of Production Economics*, Volume 159, 2015, Pages 178-191,

[12] Zadeh, L.A., Fuzzy sets, *Information and Control*, 8 (1965), pp. 338–356

[13] Wu, D.D., Olson, D.L., Fuzzy multi attribute grey related analysis using DEA, Computers Mathematics with Applications, Volume 60, Issue 1, July 2010, Pages 166–174

[14] Wong, C. C. Lai, H. R., A grey-based clustering algorithm and its application on fuzzy system design, *International Journal of Systems Science*, 34 (4) (2003), pp. 269–281

[15] Turgay, S. ,Ayma, S.B., Determined by Tolerances with Rough Set Based MCDM, Industrial Engineering and Innovation Management (2021) 4: 34-47 Clausius Scientific Press,, DOI: 10.23977/ieim.2021.040105 ISSN 2522-6924.

[16] Aktürk, B., Turgay, S., Integrated Risk Assessment Analysis with Fuzzy Logic. Manufacturing and Service Operations Management (2022) Vol. 3: 8-18. DOI: http://dx.doi.org/10.23977/msom.2022.030402.

[17] Yu, J.R., Tzeng, Y.C., Tzeng, G.H., Yu, T.Y., Sheu, H. J., A fuzzy multiple objective programming to DEA with imprecise data, *International Journal of Uncertainty, Fuzziness & Knowledge-Based Systems*, 12 (5) (2004), pp. 591–600

[18] Sotoudeh-Anvari, A., The applications of MCDM methods in COVID-19 pandemic: A state of the art review, Applied Soft Computing, Volume 126, 2022, 109238

[19] Wu, X., Feng, Z., Yang, S., Qin, Y., Chen, H., Liu, Y., Safety risk perception and control of water inrush during tunnel excavation in karst areas: An improved uncertain information fusion method, Automation in Construction, Volume 163, 2024, 105421

[20] Suryawanshi, P., Dutta, P., Optimization models for supply chains under risk, uncertainty, and resilience: A state-of-the-art review and future research directions, Transportation Research Part E: Logistics and Transportation Review, Volume 157, 2022, 102553,

[21] Kaya, S.K., Kundu, P., Görçün, Ö.F., Evaluation of container port sustainability using WASPAS technique using on type-2 neutrosophic fuzzy numbers, Marine Pollution Bulletin, Volume 190, 2023, 114849,

[22] Akhtar, M. J., Naseem, A., Ahsan, F., A novel hybrid approach to explore the interaction among faults in production process with extended FMEA model using DEMATEL and cloud model theory, Engineering Failure Analysis, Volume 157, 2024, 107876.

[23] Kan, Z., Wei, Y., Zhao, T., Cao, J., Risk evaluation of submarine pipelines in FMEA by combination of gray relation projection and VIKOR method, Ocean Engineering, Volume 302, 2024, 117695.

[24] Bai, C., Sarkis, J., Evaluating supplier development programs with a grey based rough set methodology, *Expert Systems with Applications*, 38 (2011) 13505-13517.

[25] Deng, J.L. Control problems of grey systems, Systems and Controls Letters, 5 (1982), pp. 288–294

[26] Zhang, J. , Wu, , Olson, D.L., The method of grey related analysis to multiple attribute decision-making problems with interval numbers, *Mathematical and Computer Modelling*, 42 (9–10) (2005), pp. 991–998

[27] Zhou, F., Chen, T.Y., A hybrid group decision-making approach involving Pythagorean fuzzy uncertainty for green supplier selection,International *Journal of Production Economics*, Volume 261, 2023, 108875.

[28] Turgay, S., Aydın, A., Risk Mitigation for SMEs: A Step-by-Step Guide to Implementing an Effective Framework, Financial Engineering and Risk Management (2023), Vol. 4 Num. 1, DOI: 10.23977/ferm.2023.060808, ISSN 2616-3349

[29] Olabanji, O.M., Mpofu, K., Appraisal of conceptual designs: Coalescing fuzzy analytic hierarchy process (F-AHP) and fuzzy grey relational analysis (F-GRA), *Results in Engineering*, Volume 9, 2021,100194

[30] Toloo, M., Nalchigar, S., A new DEA method for supplier selection in presence of both cardinal and ordinal data, Expert Systems with Applications 38 (2011) 14726-14731.

[31] Zhang, Z., Chu, X., Risk prioritization in failure mode and effects analysis under uncertainity, Expert Systems with Applications, 38(2011) 206-214.

[32] Zolfani, S.H., Görçün, Ö.F., Kundu, P., Küçükönder, H., Container vessel selection for maritime shipping companies by using an extended version of the Grey Relation Analysis (GRA) with the help of Type-2 neutrosophic fuzzy sets (T2NFN), Computers & Industrial Engineering, Volume 171, 2022, 108376,

[33] Zavadskas, E. K., Evolving a linear programming technique for MAGDM problems with interval valued intuitionistic fuzzy information, *Expert Systems with Applications*, Volume 42, Issue 23, 2015, Pages 9318-9325,

[34] Haase O., Henrich, A., A hybrid representation of vague collections for distributed object management systems, *IEEE Transactions on Knowledge and Data Engineering*, 12 (2000), pp. 448–467

[35] Atanassov, K., Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986), pp. 87–96.

[36] Göcük, İ., Durmaz, E.D., Şahin, R., Interval type-2 fuzzy development of FUCOM and activity relationship charts along with MARCOS for facilities layout evaluation, Applied Soft Computing, 128(2022) 109414.

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