An experimental validation of model based control techniques for interacting nonlinear systems

KANTHALAKSHMI.S[1], WINCY PON ANNAL.A.S[2]
[1]Department of Electrical and Electronics Engineering,
PSG College of Technology, Coimbatore, Tamilnadu, INDIA
[2]Department of Electronics and Instrumentation Engineering,
Government College of Technology, Coimbatore, Tamilnadu, INDIA

Abstract: - Model based controllers are those controllers that has gained significant attention in the arena of nonlinear process control. Conical tank is a nonlinear process whose nonlinearity increases when it interacts with another conical tank. Maintaining the level of an interacting nonlinear process operating with constraints is the control objective of this paper. Model Predictive Control (MPC) has the capability of handling constraints and exerts a control action with optimization. MPC is employed for this process and the experimental results obtained are subjected to time domain analysis and the performances are compared with the performance of Proportional-Integral-Derivative (PID) and Internal Model controller based PID (IMC-PID) controller.

Key-Words: - Non linear system, proportional integral derivative control, internal model control, model predictive control

Received: July 17, 2022. Revised: October 29, 2023. Accepted: November 27, 2023. Published: December 31, 2023.

1 Introduction

The chemical process control is one area where automation has a significant impact. In fact, almost all of the processes are nonlinear. Nonlinear systems are chaotic, uncertain, or counterintuitive. Nonlinear systems are usually approximated by linear equations. This is effective with some precision and range for the input values, but some interesting phenomena like chaos and singularities are obscured by linearization.

To achieve perfection in operation of such a system, the choice of an appropriate control technique that will guarantee smooth running of the process at the desired level of performance is required. The factors for the selection of such a controller are that, it should be able to work with constraints, should be reliable and should have a robust design. When the process is nonlinear, robust control is a challenging task [1].

In industries, the PID controller is the most used controller for decades because of its feasibility and easy implementation. Its performance is not based on a process model and so it cannot compensate for process dynamics such as dead time and nonlinearity. The limitations of PID controller were reported in [2,3]..Paper [4] stated various tuning methods and modifications to be incorporated to design PID which may work with robustness and easily adopted in industries. Even if a good PID design is made, the PID controller cannot be robust compared to robust controllers when the system encounters multiple challenges from the operating environment of the system.

The development of robust control systems, like model-based control techniques, including Process Model Based Control, (PMBC) [5], Non Linear Model Predictive Control (NLMPC) [6] has resulted from the advancement of technology in control.

Among the control techniques based on model, Internal Model Control (IMC) has a considerable importance to be adopted for a process due to its effective design philosophy. [7]. The ideology behind IMC is the Internal Model Principle according to which, the only approach to acquire better control is to understand the process model. IMC is able to forecast output constraint violations and take remedial action using this model. Another advantage of IMC is that, only the controller and the nominal plant are responsible for its stability.

The IMC controller possesses the ability to cancel out any variations in the process variable from the requirement, hence, attaining prefect control. The following characters are known to be present in IMC: dual stability, perfect control, zero offset. Paper [8] made analysis of the various tuning techniques for PID controller. IMC based PID gives better performance in case of delays and results in good and robust settings

The major drawback of IMC is its constraint handling capacity. This problem can be overcome by MPC. It reduces the operating cost while satisfying the constraints. It can be used for processes whose manipulated and controlled variables are large in number. It enables the application of constraints to manipulated variables as well as control variables. It has the capability to operate nearer to constraints. It allows time delays, inverse response and inherent nonlinearities[9]. The paper [10] states that, MPC provides robust feasibility with trackable real time computation, with optimal closed loop dynamics. Paper [11] reported results on stability and computations of NMPC.

Level control of two conical tanks which are interacting to each other is the process considered here. The process dynamics is highly nonlinear with a significant dead time because of the interaction. Both the control algorithms are implemented and the results obtained are experimentally verified.

2 Plant Modeling

Two conical tanks interacting with each other, and whose level has to be maintained constant is the plant chosen. These types of processes are most commonly used in pharmaceutical industries where proper drainage of the fermented products is very crucial. The importance of conical tank in fermentation process is reported in [12]. A single conical tank is nonlinear in nature and when it undergoes interaction with another conical tank it exhibits high nonlinearity in its character and it becomes difficult to control such a process.

The plant (Fig. 1) consists of three conical tanks of which, the two tanks, tank 1 and tank 2 which are interacting to each other are taken to implement the control algorithms. The inlets to both the tanks are from the sump along with the inlet from the other tank through the pipeline which causes interaction. The inlet flows of liquid from the sump to the tanks are controlled by the pneumatic control valves attached to their pipelines.

Design of effective controllers depends on how well the process dynamics is known for which modelling the process becomes essential. Modelling a process requires the knowledge of all the basic principles of operations. The significance of order of the system during modelling is analysed in detail in [13].

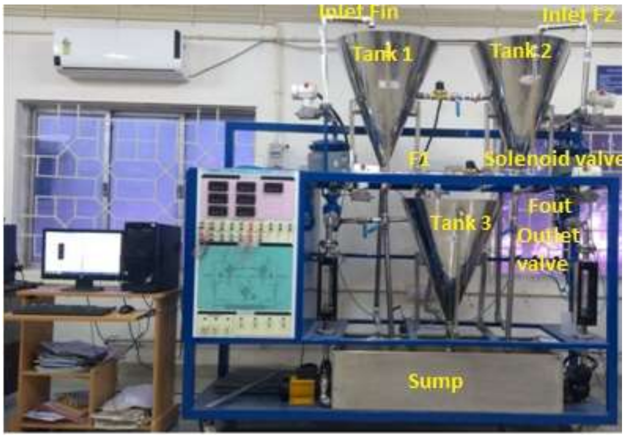

Fig. 1 Experimental setup of two interacting conical tanks

2.1 Experimental approach

Experimental modeling gives a better knowledge on how the process reacts to various set point changes and to disturbances. It also gives information regarding the dead time of the process, which is a crucial factor for the design of controller.

From the open loop response obtained using experimental modeling, a transfer function is obtained which is of the form,

$$H(s) = \frac{K}{\tau s + 1} e^{-t_d s} \tag{1}$$

which will have information on the dead time (td) of the process. Here, K is the gain of the process around the operating point, τ is the time constant of the process.

Approximation of dead time is done using first order Pade’s approximation

$$e^{-t_d s} = \frac{1 - \frac{t_d s}{2}}{1 + \frac{t_d s}{2}} \tag{2}$$

Therefore,

$$H(s) = \frac{K}{(\tau s + 1)} \frac{\left(1 - \frac{t_d s}{2}\right)}{\left(1 + \frac{t_d s}{2}\right)} \tag{3}$$

For the two interacting conical tanks used, the transfer function around the operating region of (16-30 cms) was obtained as,

$$H(s) = \frac{2.688}{(115s + 1)} e^{-40s} \tag{4}$$

whose dead time when approximated becomes,

$$H(s) = \frac{2.688}{(115s+1)} \frac{\left(1 - \frac{40s}{2}\right)}{\left(1 + \frac{40s}{2}\right)} \tag{5}$$

3. Controller Design

3.1 IMC controller

The method used to account for model uncertainty and disturbance is IMC. The major advantages of IMC controller in contrast with traditional feedback controllers is that, it is easy to tune and it shows clearly how time delay and right hand plane zeros influence the process's built in controllability. It is a compensation between closed loop behavior and robustness to model miscalculations using a single tuning parameter, λ. The restriction of this method is that, the system must be stable.

The process model was experimentally developed and its transfer function was found to be as in equation 4. This process model was factorized as,

$$H(s)_+ = e^{-40s} \tag{6}$$

which was the non-invertible portion and

$$H(s) = \frac{2.688}{(115s+1)} \tag{7}$$

which became the invertible component by the usage of all pass factorization.

The process model's invertible section was inverted and cascaded with a first order filter with filter coefficient λ that makes the controller proper.

$$q(s) = \frac{1}{2.688} \frac{(115s+1)}{(\lambda s+1)} \tag{8}$$

λ was adjusted to vary the speed of response and robustness.

3.2 IMC based PID controller

It makes advantage of dead time approximation for analysis. The major difference between IMC and IMC based PID is that, IMC based PID allows the controller to be improper so as to locate a controller that is comparable to a PID controller. Also it permits good set-point tracing for the process with a small delay time to time constant ratio. Fruehauf in 1990 reported improvement in IMC based PID controller performance.

The Pade's approximation of dead time was considered and the process transfer function was found to be as in equation 5 and the process model was factorized as,

$$H(s)_+ = \left(1 - \frac{40s}{2}\right) \tag{9}$$

which was the non invertible portions and,

$$H(s)_- = \frac{2.688}{(115s+1)} \frac{1}{\left(1 + \frac{40s}{2}\right)} \tag{10}$$

which was the invertible portion.

The process model's invertible section was inverted and cascaded with a filter with filter coefficient λ and the controller was designed.

$$q(s) = \frac{1}{2.688} \frac{(115s+1)\left(1 + \frac{40s}{2}\right)}{(\lambda s+1)} \tag{11}$$

On expansion,

$$q(s) = \frac{1}{2.688} \frac{\left(3300s^2 + 135s + 1\right)}{(\lambda s+1)} \tag{12}$$

The filter coefficient λ cannot be made small randomly. If so, then there will be a restriction on the performance of the IMC based PID strategy. Rivera et al. in 1986 recommended that λ>0.8td due to the model uncertainty caused by Pade's approximation. Morari and Zafiriou in 1989 recommended it to be that λ>0.25td for the PID plus lag formulation.

The PID parameter values were calculated comparing q(s) with the standard PID equation and the values obtained were Kc= 0.9658, τI=2.26 mins, τD= 0.2839 mins. The value of λ was tuned on line as a tradeoff between performance and robustness.

3.3 Model Predictive Controller (MPC)

MPC controller is analogous to IMC controller as the model performs laterally with the process and the outcomes serve as feedback. Yet the collocation of the control and target computation is an exclusive character of MPC. Additionally MPC has had larger brunt on industrial applications than IMC as it is apt for constraint MIMO problems. The formulation of multivariable systems with time delays is also made easy by this method.

It is a procedure that constructs controllers that can alter the control action in advance to the actual change in output target. This anticipating capacity, when mixed with conventional feedback operation, facilitates a controller to make variations that are mild and nearer to the optimal control action. The targets are computed from an optimization depending on the steady state model of the process. The targets are computed each time the control computations are performed.

The common optimizations comprises of profit function, reducing the cost function to a minimum, and increasing the production rate to a maximum. Here the optimization problem dealt with is, minimizing the cost function obtained by varying the M control moves, taking into consideration the modelling equations and the limitations on inputs and outputs. The three major steps involved in the working of a standard MPC are, estimating the system states, calculating the effective input which would minimize the required cost function over the prediction horizon, and implementing the first part of the optimal input until the next sampling instant. The general schematic of a process when implanting MPC is shown in Fig. 2

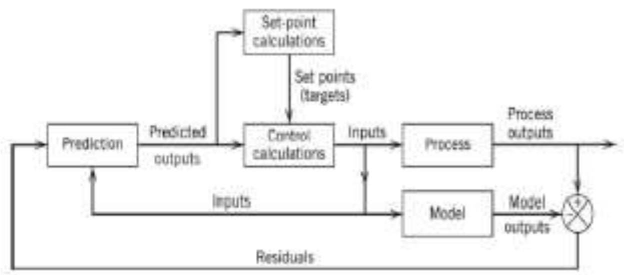

Fig. 2 General schematic of Model predictive controller

3.3.1 Choice of MPC parameters

State space model is more appropriate to be used as it requires less number of model parameters when compared to step response model to describe process behavior. The continuous state space model obtained for this process is,

$$\dot{x} = \begin{bmatrix} -0.0587 & -0.0278 \\ 0.0156 & 0 \end{bmatrix} x + \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} u \quad (13)$$

$$y = \begin{bmatrix} -0.0935 & 0.2992 \end{bmatrix} x \quad (14)$$

Control interval k is chosen initially, and then is held constant to tune other controller parameters. As the value of k reduces, the anonymous disturbance rejection increases greatly. But as k becomes very small, the complexity in computation increases greatly. As a result, a compromise between performance and effort in computation is the best option.

When optimizing the manipulated variables at the given control interval k, the prediction horizon P is the number of future control intervals the MPC must calculate through prediction. The value of P is chosen so that the controller maintains the internal stability, predicts constraint violations quickly to permit remedial measures. To make the control system less responsive to model errors, the prediction horizon is chosen to be larger than control horizon. P is increased until any further increases have a negligible effect on performance. But at the same time, as P increases, the need for controller memory and the time to solve the quadratic problem increases.

The number of manipulated variable moves to be adjusted, so as to give the best result possible at a selected control interval k is the control horizon M. It must be generally greater than 1 and lesser than P. As M decreases, only fewer variables are to be computed in the quadratic problem solved at each time interval, which ultimately leads to a faster computation. As there is delay in a plant, M is chosen much lesser than P. When M is smaller it provides more chance for internal stability of the controller.

Least square formulation objective function is used because it penalizes large errors compared to smaller errors. It is of the form,

$$\Phi = \sum_{i=1}^{p} \left(r_{k+i} - \hat{y}_{k+i} \right)^2 + w \sum_{i=0}^{M-1} \Delta u_{k+i}^2 \quad (15)$$

where, P Prediction horizon, M – Control Horizon, r - Set point, Δu – change in manipulated input, w – Weights for changes in manipulated input, $\hat{y}$ - Predicted model output, k – control interval

The necessary condition for minimum Φ is obtained from $\frac{\partial \Phi}{\partial \Delta u}=0$, from which the optimal solution for the control parameter Δu could be obtained.

$$\Delta u(k_i)=\left[\Delta u(k_i) \quad \Delta u(k_i+1) \quad .... \quad \Delta u(k_i+M-1)\right]^T \tag{16}$$

where ki is the sampling instant,

Only the first element of Δu is used for implementing the control. The future state variables $x(k_i+1|k_i)$ to $x(k_i+P|k_i)$ are calculated sequentially using the set of future control parameters starting from

$$x(k_i+1|k_i)=Ax(k_i)+B\Delta u(k_i) \tag{17}$$

to

$$x(k_i+P|k_i)=A^P x(k_i)+A^{P-1}B\Delta u(k_i)+A^{P-2}B$$

$$\Delta u(k_i+1)+......+A^{P-M}B\Delta u(k_i+M-1) \tag{18}$$

From the predicted state variables the predicted output variables $y(k_i+1|k_i)$ to $y(k_i+P|k_i)$ are calculated.

$$y(k_i+P|k_i)=CAx(k_i)+CB\Delta u(k_i) \tag{19}$$

It continues till

$$y(k_i+P|k_i)=CA^P x(k_i)+CA^{P-1}B\Delta u(k_i)+CA^{P-2}B$$
$$\Delta u(k_i+1)+......+CA^{P-M}B\Delta u(k_i+M-1) \tag{20}$$

The predicted output is

$$Y=\left[y(k_i+1|k_i) \quad y(k_i+2|k_i) \quad ... \quad y(k_i+M-1|k_i)\right]^T \tag{21}$$

Based on the predicted output, the current output, the control action is taken and the process is repeated until the system meets the requirement.

As the prediction horizon is substantially greater than the control horizon, control weighting is set to zero. The prediction horizon and control horizon are chosen as 30 and 7 respectively. The constraint considered is the inlet valve stem lift which has to vary only between 25% and 75 %.

4. Results

4.1.Open loop response

Keeping the manual outlet valve of tank 1 and tank 2, to drain tank, to open by 50 %, and allowing the solenoid valve fixed in the pipeline of interacting between the two tanks, tank 1 and tank 2, to open, letting 100% interaction between the tanks, the level of liquid in the tank is maintained by varying the pneumatic actuator's stem lift.

The open loop response of the system was obtained by initially maintaining the actuator stem lift at 30%, allowing the system to attain steady state, after which, additional pneumatic signal was given, which made the stem lift to 60%. The level of the tank whose output has to be controlled was noted until steady state had occurred.

From the response curve, it is noted that the initial steady state occurred at 16 cms of level in the tank 1, and the final steady state occurred at 30 cms. From the input fed to the tank 1, the output level obtained and the graph (Fig 3) which shows the dynamics of the plant, the system is represented by the transfer function given in equation4.

Even if the system transfer function seems to be first order, in practical case, the measuring device may at least be of first order, and the control valve which implements the controllers action will again be at least of first order, which ultimately says that a system in which a controller is implemented will be of at least third order, which can be seen from the time constant of the open loop response. Using Pade's approximation, it becomes as in equation5.

From the open loop response curve (Fig.3) and the transfer function obtained, it is evident that the system has a delay, and a larger time constant.

4.2 PID controller

The PID controller parameters were found by Cohen Coon's method of tuning, and the controller gains calculated are, proportional gain K_P is 1.15909, integral gain K_I is 0.0115, derivative gain K_D is 6.8401.

From the graph (Fig.4), it is noted that, on implementing PID controller, the system response speed has increased. It is due to the fact that the time constant of the system has reduced by a factor of ($1+K_P$ K). So as the proportional gain is increased, the speed of response increases. The rise time is about 500 seconds. Also the offset of the system decreases on increasing the proportional gain and is found to be 6.5%. Because integral controller takes action as long as an error persist, the system takes a longer time of around 800 seconds to settle. The controller response is stabilized by derivative action, which permits the usage of higher gains and lesser integral time constants, but it creates a noisy environment.

4.3. Internal Model Control based PID

From the graph (Fig. 5), it is seen that, compared to PID controller, the IMC based PID controller has a better performance. Irrespective of time delay approximation, the controller provides good set point tracking. The offset has reduced to 4 % from 6.5%. The ratio of time delay to time constant influences the performance of the controller. It provides flexibility in controller design, to attain the required performance. As the filter coefficient decreases, the performance improves. The speed of response has improved and the rise time has decreased to 400 seconds which is smaller compared to PID controller. The system has also settled quickly at almost 550 seconds which is a much better performance compared to PID controller. The performance indices based on error has also greatly improved (Table 1), which says that IMC-PID is much better controller than a PID controller.

4.4. Model Predictive Control

Without constraints control problems would not exist. So while designing a controller always the constraints are to be kept in mind. They lead to additional control objective. The main drawback of PID and IMC PID are that, they do not have the capability to handle constraints, which can be very well handled by MPC. The constraint considered is the inlet valve stem lift which has to vary only between 25% and 75 %.

Even if a good initial model for a controlled process is known, it may not be sufficient for effective control during long operations because of process non linearity which may cause the change in process characteristics based on the operating point. So the controller is designed such that it is less sensitive to model errors, for which, the prediction horizon is made much greater than control horizon. So prediction horizon is chosen as 30. Increasing prediction horizon has also lead to decrease in mean square error.

If control horizon is small, the peak time increases. Also it provides internal stability of the controller. If control horizon increases, the control moves get a tendency to become more violent. Larger weights are required to reduce the aggressiveness of control moves. So a control horizon of 7 which is much less when compared to prediction horizon is chosen. From the graph (Fig. 6), it is clear that the system has a rise time of only 200 seconds, which says that the system response speed has improved.

The process delay which is not compensated by PID, is well compensated by MPC. Also the weight on control is made zero as the prediction horizon is much larger than the control horizon.

From the graph (Fig 6), it is evident that, the MPC controller shows a much better performance compared to PID controller and IMC based PID controller because of its predicting capability. It has almost no offset and system has settled very quickly.

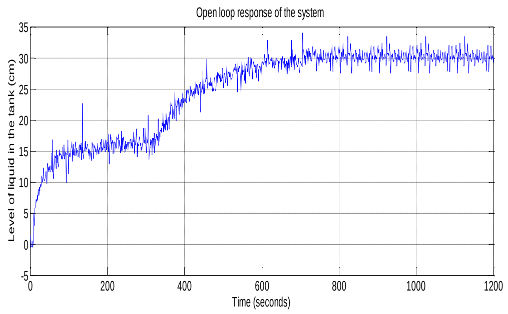

Fig. 3. Open loop response of the system

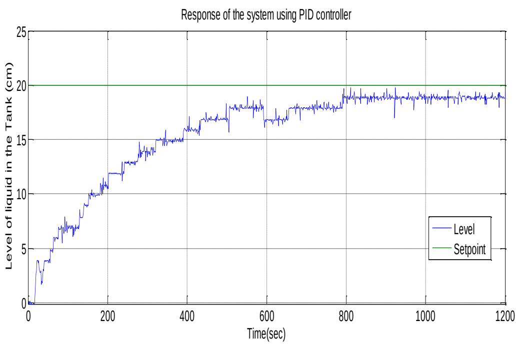

Fig. 4. Response of the system using PID controller

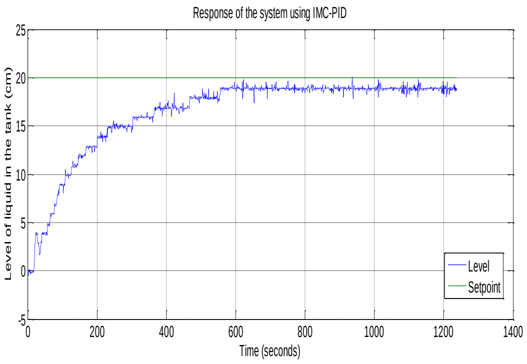

Fig. 5. Response of the system using IMC-PID controller

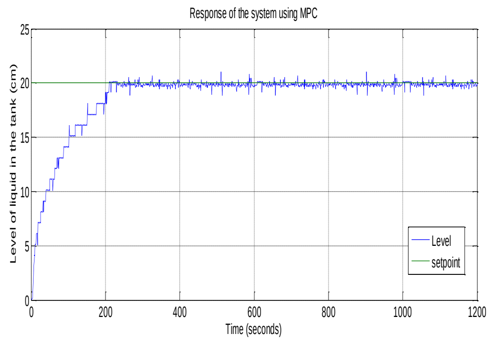

Fig.6. Response of the system using MPC controller

4.5. Comparison of results

Table 1shows the comparison of results of all the three controllers. From the table, it is clear that, the offset has reduced much in MPC compared to IMC-PID and PID. This is because the target is computed each time the control calculation is done. This has lead to effective control and ultimately better tracking of set point. The rise time has drastically reduced when using MPC because of the use of a larger control horizon. But the aggressiveness of the control action is minimized by the choice of a still large prediction horizon, which has also supported to reduce errors caused by model mismatches. When using PID and IMC-PID controllers the ISE produced is large which paved way to use it as the optimization function for MPC, as a result of which the large errors were penalized and a very small ISE compared to PID and IMC-PID was achieved. The reduction of the performance parameters IAE and ITAE by MPC controller also shows that any small errors caused and the errors that persist for a long time were also suppressed by MPC. Ultimately, from the table it is clear that MPC outperforms PID and IMC based PID controllers

Table 4.1 Comparison of performance of the system subjected to different controllers

	Set point Level (cm)	Steady state Level (cm)	Offset error Level (cm)	Rise time (Sec)	Settling time (sec)	IAE	ISE	ITAE
PID	20	18.7	1.3	500	805	4.562	42.26015	0.001922
IMC-PID	20	19.2	0.8	400	550	3.729	32.87	0.0014
MPC	20	19.9	0.1	200	220	1.261	11.213	0.0002

5. Conclusion

An experimental comparison of MPC controller with PID and IMC-PID controller was made for a highly non linear process. It was noted that MPC outperforms the other two controllers. However, the optimization of MPC parameters could be obtained by using soft computing techniques. Also the model mismatches which are taken into consideration in MPC design could be better handled if online estimation of parameters were done.

References:

[1] FuatAlarçin, HakanDemirel, Ertugrul Su, M., AhmetYurtseven, Conventional PID and Modified PID Controller Design for Roll Fin Electro- Hydraulic Actuator, *ActaPolytechnicaHungarica,* Vol.XI, No.III, 2014, pp. 233-248.

[2] Fawzan Salem, Mohamed A Awadallah, Ehab E Bayoumi, Model Predictive Control for Beadbeat Performance of Induction Motor drives, *WSEAS Transaction on Circuits and Systems*, Vol.XIV, 2015, pp. 303-311.

[3] He, Y.Q., Han, J.D., Nonlinear Model Predictive Control with Regulable Computational Cost, *Asian Journal of Control,*Vol.XIV, No.I, 2011, pp. 300-307.

[4] Krivosheev, V.P., Yu. Torgashov, A.,Inverse Process Model-Based Optimal Control of a Multicomponent Distillation Column, *Journal of Computer System Sciences International*, Vol.XL, No.I, 2001, pp. 78-84.

[5] NedjeljkoPeric Ivan Branica, Ivan Petrovic,Modification and Application of Auto Tuning PID Controller, *Proceedings of the 8th IEEE Mediterranean Conference on Control, Automation,* 2000.

[6] RohitKawathekar, James B. Riggs,Nonlinear Model Predictive Control of a Reactive Distillation Column,*Control Engineering Practices*, Vol.XV, 2007, pp. 231-239.

[7] Scott Hurowitz, Joe Anderson, Marshall Duvall, James B. Riggs, Distillation control configuration selection, *Journal of Process Control,*Vol.XIII, 2003, pp. 357-362.

[8] M. Shamsuzzoha, Moonyong Lee,Analytical design of Enhanced PID Filter Controller for Integrating and First Order Unstable Processes with Time Delay, *Chemical Engineering Science*, Vol.LXIII, 2008, pp. 2717-2731.

[9] SigurdSkogestad,Simple Analytic Rules for Model Reduction and PID Controller Tuning, *Modelling, Identification and Control*, Vol.XXV, No.II, 2004, pp. 85-120.

[10] Speers, R.A., Scott Stokes,Effects of Vessel Geometry, Fermenting Volume and Yeast Repitchingon Fermenting Beer, *Journal of the Institute of Brewing,*Vol.CXV, No.II, 2009, pp. 148-150.

[11] Xiang Li, Thomas E Marlin,Model Predictive Control with Robust Feasibility, *Journal of Process Control,*Vol.XXI, 2011, pp. 415-435.

[12] Ya-Gang Wang, Wen-JianCai,Advanced Proportional-Integral-Derivative Tuning for Integrating and Unstable Processes with Gain and Phase Margin Specifications, *Industrial and Engineering Chemistry Research,* Vol.XLI, 2002, pp. 2910-2914.

[13] Yan Zhang, Hoa Nguyen, D. Subbaram Naid, Yun Zou, Chenxiao Cai Nanjing, Time Scale Analysis and Synthesis for Model Predictive Control, *WSEAS Transactions on Systems and Control*, Vol. 9, 2014, pp. 130-139

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.